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The Role of Catastrophe Bonds in Mitigating Flood Risk

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Introduction

The thesis focuses on the description of an innovative model to price Catastrophe bonds triggered by multiple events with an extreme dependence structure.

The first chapter introduces one of the most serious risks faced by people, government and companies: the catastrophic risk. A catastrophic event is a tragic event induced by natural causes which cause devastation and suffering.

The emergency related to catastrophic risk has become increasingly serious in recent years due to both climate change and the growth in economic exposure. In particular, the most frequent and severe catastrophic event in Europe and in Italy is flood, which every year causes numerous victims, devastating damage and heavy economic losses. The European Commission defines flood risk as “the most common and most costly natural disasters in Europe”.

The actual situation is alarming because it has highlighted the inadequacy of the government and insurance industry to mitigate this type of risk.

The second chapter presents the techniques used by the insurance and reinsurance companies to mitigate catastrophic risk. In particular, presenting securitization, which is the process of transferring the risk associated with the assets, liabilities or cash flows from the balance sheet of an insurance company to the capital market, through the creation of tradable securities.

The third chapter describes the most important financial instrument for securitization, which is the Catastrophe bond. This security became popular in the mid-1990s, with the increasing frequency and severity of catastrophic events. This type of security represents a real Act of God for the insurance industry, because given the low contingency of the bond's cash flows and high return of the Catastrophe bond, this type of security offers an efficient way to transfer insurance related risk to the capital market, providing insurers more capacity and allowing investors to benefit from diversification and additional returns of the Catastrophe bond.

The fourth chapter explores the models used to evaluate and price the Catastrophe bond. Catastrophe Bond pricing requires different models than traditional bonds, as catastrophe risk is characterized by sudden events and incomplete markets. Catastrophe bond pricing is based on aggregate loss modelling, combining the frequency through the Poisson process, and the severity through extreme value theory, of catastrophe events. The Extreme Value Theory allows for the estimation of the severity, and analyze the dependencies between variables with the Copula function.

Despite various approaches, there is still no universal model for determining the price of catastrophe bonds.

Finally, the fifth chapter conducts an empirical analysis, focusing on a simulation of the price of a multi-trigger Catastrophe bond designed to cover flood risk in the Italian regions. The method adopted to price the Multi trigger Catastrophe bond is the CIR-Copula-POT Model, which uses the Peak over threshold (POT) approach to describe the probability that an extreme event exceeds a determined threshold, the copula function theory to better describe the dependency between the multiple trigger indicators and the Cox-Ingersoll-Ross (CIR) model to compute a stochastic estimate of the interest rate of the Catastrophe bond.

Chapter 1: Catastrophic risk in the insurance industry

Risk is defined as “the probability that an event could occur and cause a loss” (A. Coviello, R. Somma, 2021, page 27). It represents an integral and inevitable aspect of business. Companies and Governments operating in complex economic environments must face risks.

The insurance business is vital for today’s economies; insurance companies play a crucial role in mitigating and transferring many risks, providing financial protection against potential losses. However, one of the most unpredictable forms of risk is associated with catastrophic events.

1.1 Extreme Catastrophic Events

A catastrophic event is defined as “an unpredictable and uncontrolled event that causes suffering, destruction, and difficulties to a large part of humanity”¹. Catastrophic events include natural disasters like floods, heatwaves, earthquakes, tsunamis, extreme storms, volcanic eruptions, and outer space-related events, as well as the possibility of nuclear and chemical accidents and pandemics. Such events usually occur infrequently but cause huge losses over a short period.

Catastrophic events are typically characterized by four features. The most important is the unpredictability of future events because the risk exists when there is the possibility of manifestation of a future event. On the contrary, risk doesn’t exist when there is the certainty that a determined event will not happen or is impossible.

The second feature is the objectivity of uncertainty. The uncertainty of a determined natural or social event constitutes a risk when it does not depend on the subject's sensibility or the cultural level.

The third feature is the versatility of the economic effects of uncertain events. Because there is a risk, it is necessary that an uncertain fact can give results different from those expected and have a concrete economic manifestation.

The final feature is the difference between peril and hazard. Peril is the cause of possible economic losses, representing unpredictable events out of human control, such as fire,

¹ Source: <https://library.fiveable.me/key-terms/ap-hug/catastrophic-event>

lightning, diseases, or hurricanes. A hazard is a condition that may create or increase the chance of a loss caused by a peril.

Floods, droughts, heatwaves, cyclones, earthquakes, tsunamis, and volcanic eruptions have always been an integral part of the earth system, however, the severity and frequency of catastrophic events have reached alarming levels in recent years, with notable economic and social consequences. According to the 2024 Global Risk Report, extreme weather events are considered the most relevant global sources of risk faced in 2024. The main reason is that such events have direct consequences in all aspects of society: health, economics, safety and geopolitics.

Observing Figure 1.1, it is intuitive that catastrophic events are increasing. One contributing factor is the growth in economic exposure; as society's wealth and property continue to expand, so do the losses associated with natural disasters. Moreover, the real estate growth is concentrated in high-risk areas, such as floodplains or coastal areas. The second factor is deforestation and land use, which significantly alters the environmental and climatic balance and increases greenhouse gas emissions. Forests, especially rainforests like the Amazon, absorb large amounts of carbon dioxide, one of the major greenhouse gases responsible for global warming.

The most important factor is climate change, which intensifies the frequency and severity of natural catastrophes. Higher temperatures are the main problem of climate change; they increase the energy available for extreme natural phenomena, making them more destructive. Prolonged periods of high temperature contribute to the melting of polar ice and, as a consequence, the rise of sea levels, the increase of heatwaves and wildfire, making them harder to control and increasing economic and environmental damage. Climate change also transforms precipitation; some areas of the world experience more intense rainfall in short periods and prolonged droughts.

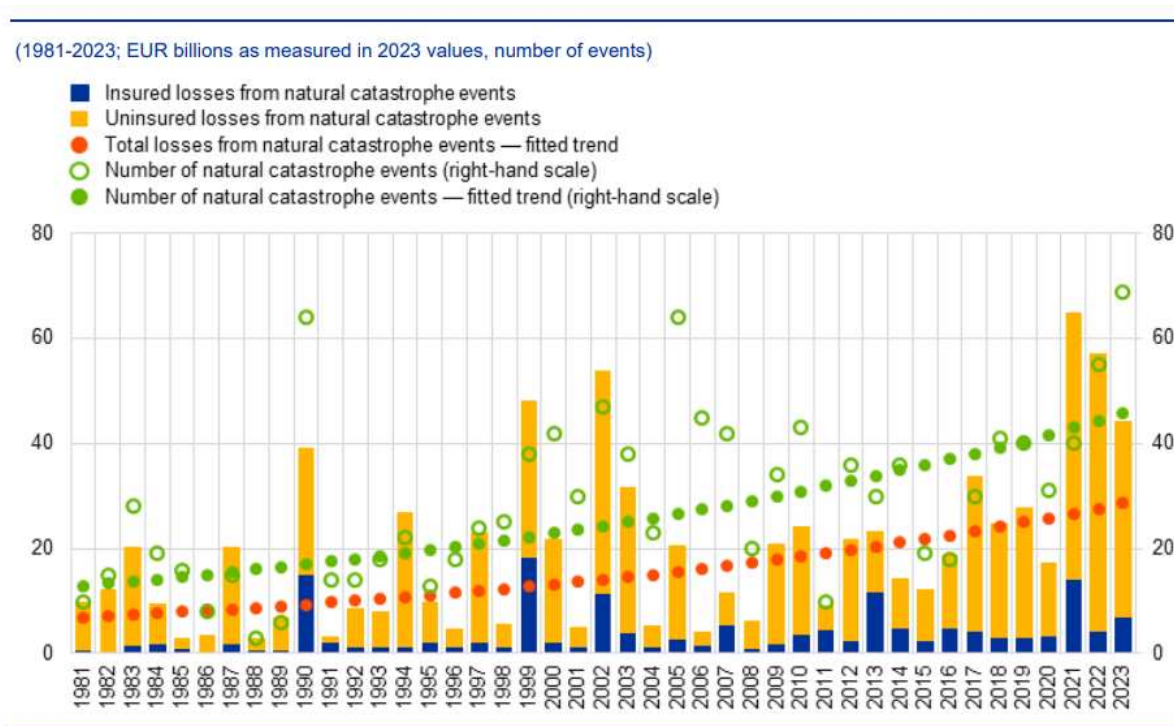
For instance, between 1981 and 2023, Europe reported €900 billion in direct economic losses related to natural catastrophe extreme events, with more than a fifth of the losses occurring in the last three years². In 2021, weather and climate disasters accounted for \$105 billion in insured losses (Swiss Re, 2021), making it the fourth most costliest year on record for insurers. These events create the most uncertainty for insurers, so it is

² Data reported by the European Central Bank and represented in Figure 1.1.

more important than ever for them to manage and understand catastrophic risk, especially in the face of climate change.

To make better business decisions about policies, pricing, and portfolio management, insurers need an adapted approach to risk that matches their specific needs and helps them prepare for the future.

Figure 1.1: Economic losses of natural catastrophes in the EU



Source: Towards a European system for natural catastrophe risk management, European Central Bank, 2024, Eiopa.

Some catastrophic events result in more frequent and more extensive losses. Flooding, in particular, represents one of the most powerful global risks. It is the catastrophe most dependent on and most affected by climate change. Floods represent a significant threat to global economies and societies.

1.2 Flood risk: causes, consequences, and climate change impacts

Flooding is a natural hydrologic phenomenon that forms an integral part of the annual cycle and plays a crucial role in providing essential ecosystem services. These services

offer both direct and indirect benefits derived from natural resources that support human well-being and livelihoods, including enhancing soil fertility, recharging groundwater, and providing diverse habitats for wildlife.

However, when communities and infrastructure are inadequately prepared to manage the risk of inundation, the consequences can be catastrophic. Floods represent a major peril, causing huge economic losses around the world, affecting more people than any other natural catastrophes, and are a leading cause of natural disaster fatalities across the world.

According to the Swiss Re Institute (2024), around 29% of the world's population is exposed to flood risk. Globally, floods represent roughly one-third of all reported events and economic losses arising from natural catastrophes since 2011, and floods are becoming the most frequent natural catastrophe.

Recent flood events highlight the increasing severity of this risk. One of the most devastating occurred in the Spanish city of Valencia in 2024. During autumn, a low-pressure system typically forms at high altitudes near the Mediterranean coast, causing cold air masses to meet warm, humid air in the lower atmosphere, leading to heavy rainfall. In Valencia, rainfall reached 500 mm in a single day, which is equivalent to the annual average of rainfall in the region. Locally, over 600 millimeters fell in 24 hours, some areas recording up to 180 millimeters in just one hour. The total damage amounted roughly to \$ 11 billion, of which \$ 4.2 billion was insured.

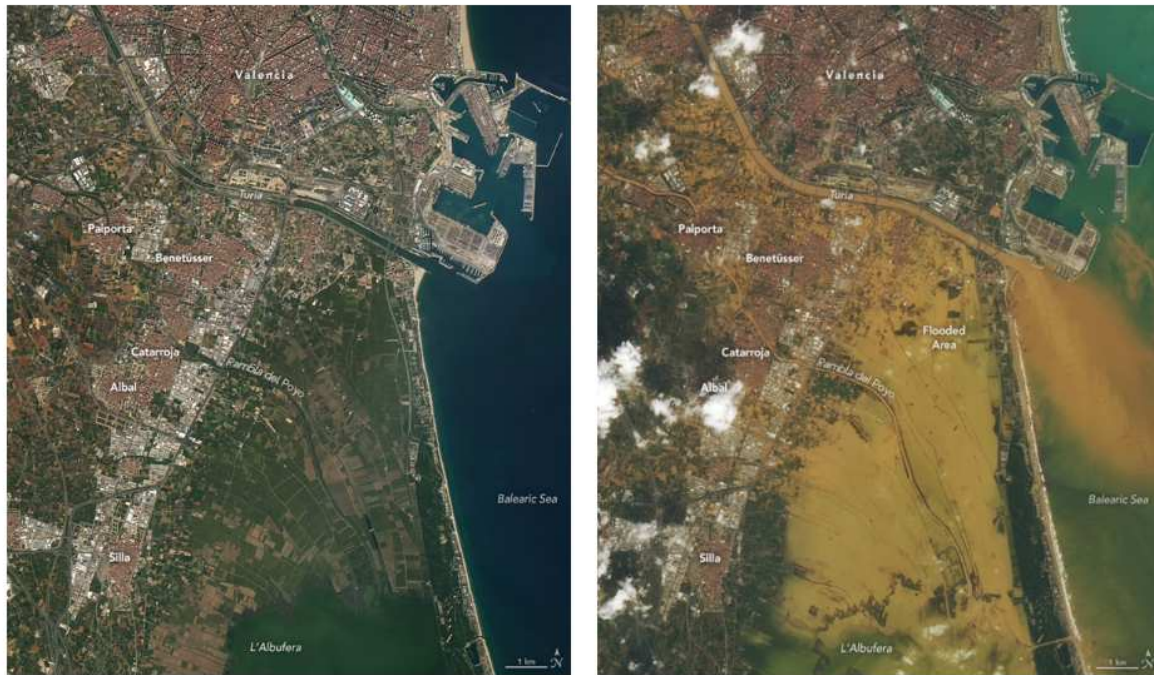
As shown in Figure 1.2, this event was particularly severe. Satellite images from NASA illustrate the transformation of the region of Valencia caused by the flood catastrophes in 2024. The before-and-after comparison highlights the extension of the inundation, with entire neighbourhoods submerged and infrastructure severely damaged.

Additionally, flood catastrophes in Europe during 2024 occurred in June in Germany and neighbouring countries and in September in Central and Eastern Europe, causing damage totalling more than \$9 billion, around \$4 billion of which was covered by insurance.

Drivers of flood risk are weather, geography, environmental and physical land features and human actions. Weather factors include heavy or prolonged rain, snowmelt, and

storm surges from tropical cyclones. Droughts can also increase flood risk by making the soil too dry to absorb rain or by enhancing wildfires.

Figure 1.2: Satellite imagery showing before and after of flood event in Valencia.



Source: Satellite images from NASA

Geographical factors involve proximity to coasts, rivers, and lakes, especially in floodplains. Building infrastructures near water can increase the exposition to flood risk and also damage the environment.

Physical and environmental factors include soil type, vegetation, drainage, earthquakes, and deforestation, all of which can have some impact on water flow and the risk of flash floods.

Human activities contribute significantly to increasing flood risk. Poor urbanization planning, construction along riverbanks and excessive extraction of water from riverbeds disrupt the natural flow of water. Dams and levees can encourage construction in flood-prone areas and may fail during extreme events. Human ground-covering infrastructure, such as impervious surfaces and inadequate drainage reduce the ground's ability to absorb and drain water, while groundwater overuse leads to land subsidence.

Climate change is increasing the probability of extreme precipitation in many regions of the world, and more frequent flash flooding and flood disasters are the consequences. Rising temperatures in the atmosphere and warming sea surfaces are also influencing precipitation patterns. Since warmer air can absorb more moisture, this increases the potential for heavy rainfall. Numerous scientific studies analysed by the Intergovernmental Panel on Climate Change (IPCC)³ indicate a likely rise in flash floods and river floods in many regions of the world.

One of the most critical effects of climate change is sea level rise. There is a gradual increase in sea levels globally, raising the risk and frequency of coastal flooding, primarily as a result of land and sea ice melting due to increased global temperatures. Additionally, the thermal expansion of warming oceans plays a significant role. NASA currently estimates that seas are rising at a rate of 3.3 millimetres per year (Nasa, 2025), caused primarily by climate change induced by increasing greenhouse gases. In certain locations, subsiding land erosion and tidal changes intensify the risk.

The second effect is storm surge, which occurs when strong storms cause sea levels to rise temporarily, resulting in coastal flooding. The actual mechanism of storm surges attacking the coast is the onshore flow of water due to the low air pressure that allows the water to rise near the coast. They can also be aggravated by heavy rains combined with storms and hurricanes, as well as by rising sea levels.

The Centre for Research on the Epidemiology of Disasters classified floods into three categories: coastal or surge floods, riverine or fluvial floods, and flash or pluvial floods. This classification is clarified in Figure 1.3.

Coastal or surge floods occur in areas that lie on the coast of a sea, ocean, or other large body of open water as a consequence of extreme tidal conditions caused by severe weather and tsunamis, which can result from offshore earthquakes. High winds from tropical cyclones or other storms push water onshore to create a storm surge, which is

³ “The Intergovernmental Panel on Climate Change (IPCC) is the United Nations body for assessing the science related to climate change. It was created to provide policymakers with regular scientific assessments on climate change, its implications and potential future risks, as well as to put forward adaptation and mitigation options”. Source: <https://www.ipcc.ch/>

the leading cause of coastal flooding and represents the greatest threat and damage associated with such storms, which can overwhelm low-lying land.

Riverine floods occur when extreme rainfall over prolonged periods results in a river exceeding its capacity. These floods can also be the consequence of heavy snowmelt or earthquake-induced landslides that block rivers. Damage from a riverine flood can be extensive, as overflow can impact rivers downstream, often leading to the failure of dams and dykes, which inundate surrounding areas. There are two types of riverine floods: overbank floods and fluvial floods. The first occurs when water rises and overflows the edges of a river. This is the most common type of riverine flood and can occur in any size channel, including small streams and large rivers. The latter is characterized by the high intensity and velocity of water that occurs in an existing river channel. These floods can be extremely dangerous and destructive because of the force of water, the fact that they can occur with little to no notice, and the speed and power at which debris can be swept up in the flow.

Flash floods occur when the ground can no longer absorb water during a heavy rainfall, creating a flood event independent of the overflow of a body of water. These floods often happened in combination with coastal and fluvial flooding. Although in some cases resulting water only reaches a few centimetres deep, flash floods can cause significant property damage. Flash floods commonly occur in two situations: when intense rainfall overwhelms an urban drainage system, often called a microburst or rain bomb, which causes water to overflow into streets and nearby structures, or when heavy rain falls on hillsides that cannot absorb the water, resulting in rapid and dangerous flooding. One aspect that is often underestimated is that flash floods can occur almost anywhere, even a long distance from any watercourses. Consequently, they are impossible to predict, making short-term loss reduction measures largely ineffective.

As climate change intensifies and natural landscapes continue to be altered by human activities, the risk of flooding is expected to increase. Effective risk management, urban planning, and adaptation strategies are considered essential to mitigate future disasters.

Insurance plays a crucial role in mitigating flood risk. The catastrophic nature of flood risk means that flood events are typically low-frequency and high-severity events, it is

impossible for insurers to predict the exact year in which a catastrophic event will occur. As a result, the insurer is unable to collect the precise amount of premium needed in any given year.

Figure 1.3: Summary of key characteristics of major flood types based on data from the International Actuarial Association.

Type	Cause	Threatened Areas	Loss Factors	Damage	Loss Prevention	Effects of Climate Change
Coastal floods	• High level of water due to wind set-up, high waves • Tsunamis	• Relatively narrow strip of coast	• Salt water • Wave forces	• Low frequency (good protection measures, sea defences) • Very high losses	• Land-use rules • Early warning • Dykes • Evacuation	• Sea level rise • Glacial melt
Riverine floods	• Prolonged, copious large-scale precipitation (possibly snow melt)	• Recurrently the same areas near rivers	• Prolonged impact of water • Water contamination (e.g., by oil)	• Low frequency • High loss potential	• Land-use rules • Early warning • Structural flood control • Temporary protection of property • Placement of hard-to-move objects somewhere else • Evacuation	• Glacial melt • Increased rainfall
Flash floods	• Mostly local torrential rain (thunderstorms)	• Practically anywhere, including places far from watercourses	• Mechanical impact of fast-flowing water • Possibly large amounts of sediment	• High frequency (but not at the same place) • Relatively minor losses (in individual events) • Erosion	• Adequate drainage • Effective building construction and maintenance • Quick evacuation	• More intense rainfalls • More intense and stalled cyclones

Source: International Actuarial Association, *Flood Risk: Discussion Paper*, Resource and Environment Working Group, 2019.

To protect themselves from financial volatility and possible catastrophic losses, insurance companies purchase reinsurance. Reinsurance allows an insurer to reduce its exposure to catastrophic risk, ensure its obligations to policyholders, and remain financially sustainable.

However, the rise of natural catastrophes is expected to increase insurance and reinsurance demand to hedge catastrophic risk, exacerbating the natural catastrophe insurance protection gap.

The management of catastrophic risk is a serious challenge for insurance and reinsurance companies. These companies are not able to hedge the number of losses caused by catastrophic events. For this reason, insurance companies have adopted different approaches to mitigate catastrophic risks and reduce losses, which will be discussed in the following chapter.

Chapter 2: Methods of Catastrophic Risk Transfer

The insurance sector is a very diversified industry, it can be divided into primary and secondary insurance. Primary insurance is the first form of insurance coverage purchased by an individual or a company to protect itself from specific risks. It is split into two lines, one regarding life insurance and the other regarding non-life insurance. The first line consists of contracts paid at the death of the insured or earlier, when the insurer is still alive, together with the health insurance, which covers the losses caused by illness or injury of individuals. The second line is also called property and casualty insurance. It covers damages to or losses of the policyholders' property and takes legal liability for damages caused to other people or their property.

On the other hand, the secondary insurer represents reinsurance companies. Their role is to reimburse primary insurers for claims paid, in this way, insurers can increase their capital and business. Also, reinsurers can enter into a reinsurance agreement with other reinsurers, called *Retrocession*, or alternatively, they can use another method to transfer risk, which is called *Securitization*.

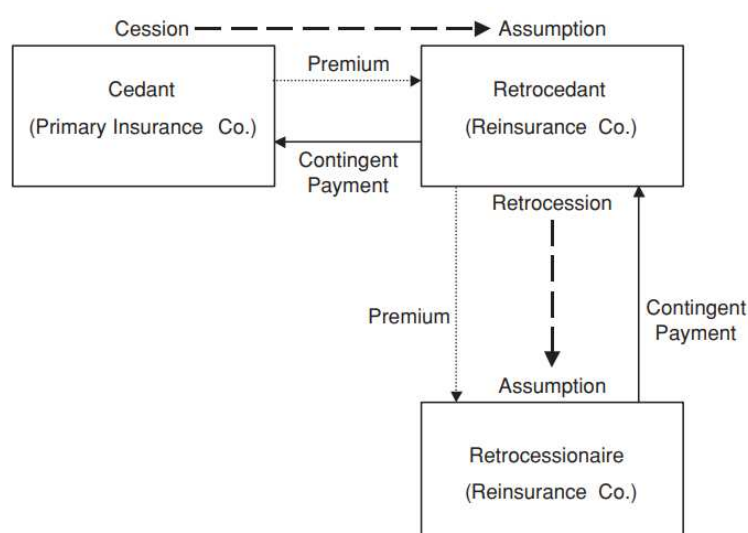
2.1 Reinsurance

The traditional way for primary insurers to transfer underwritten risk is Reinsurance. Reinsurance plays a crucial role in transferring the financial risks associated with catastrophic events, which due to their rarity and unpredictability can provoke significant losses. Traditional reinsurance is a mechanism used to help insurers manage their risks by absorbing, diversifying and redistributing some of the underwritten risks to reduce the likelihood of paying a large obligation resulting from an insurance claim. This process allows primary insurance companies to continue their growth and innovation and issue more policies with higher limits than would otherwise be possible.

The global reinsurance industry is dominated by large reinsurance companies, most of which are concentrated in one of the well-known "reinsurance centres" located in Germany, Switzerland, the US, Bermuda, the United Kingdom, France and Japan. Europe is home to the five largest reinsurers of the world with a market share of around 54% of the global market (ESRB, 2015, page 4), covering risks around the globe.

The insurance and reinsurance process starts from the need of individuals and companies to insure themselves to limit their exposure to risks and, of course, to losses. They enter into an agreement with an insurance company, transferring part of the risk in payment of a premium to the insurer. The primary insurer is responsible for covering potential losses in return for regular payments. In the case of a catastrophic event, the primary insurer must pay the losses suffered by the insured party as specified in the policy. However, certain situations such as large-scale catastrophic or extraordinary events can lead to a serious risk to the financial stability of the primary insurer. To manage this risk, the primary insurer enters into an agreement with a reinsurance company. The coverage offered by the reinsurance company allows the primary insurer to limit its exposure to significant losses and maintain sufficient capital reserves. The way reinsurance works involves the primary insurer paying a premium to the reinsurer for the coverage, and in the case a claim arises, the primary insurer receives contingent payments from the reinsurance company (see Figure 2.1).

Figure 2.1: Reinsurance and Retrocession.



Source: Christopher Weber, Insurance Linked Securities, The role of Banks, Gabler Research, 2011.

Reinsurance offers some benefits to the primary insurers. Reinsurance allows primary insurers to manage risk efficiently, reducing volatility and severity of claims, particularly caused by large-scale catastrophic events. It stabilizes premium rates by preventing a

sudden increase after the events so that insurers can reduce their reserve for risk retention.

Reinsurers are not able to assume excessive risk, even though they try to diversify risk both geographically and by type. Consequently, reinsurers to optimize their risk management use two approaches: by ceding it to another reinsurer or by accessing the capital markets. The first approach is called Retrocession (see Figure 2.1); the second approach is known as Securitization. These approaches allow reinsurers to effectively diversify their portfolios and enhance their capacity to manage potential losses.

2.1.1 Types of Reinsurance Agreements

Reinsurance is an agreement between an insurer, which represents the ceding part and another insurer, which represents the reinsurance company. From a technical point of view, the main aim of the reinsurance transfer is to find protection against portfolio ruin. The basic idea underlying any reinsurance agreement form is to split the portfolio random payment, $X[P]$ into two parties:

$$X^{[P]} = X^{[ret]} + X^{[ced]} \quad (2.1)$$

The random amount $X^{[ced]}$ is the ceded part of the total payment, so the part paid by the reinsurer to the ceding part. The random amount $X^{[ret]}$ is the retained part of the total payment, the net payment of the cedant (Olivieri and Pitacco, 2015).

Generally, reinsurance agreements fall into two main categories: *Facultative reinsurance* and *Treaty reinsurance*. Facultative Reinsurance⁴ agreement is purchased to cover single or multiple risks held in the business's portfolio of the primary insurer. Facultative reinsurance is a one-time agreement, which covers a single transaction with the insurance company. It is usually used to cover part of specified large, hazardous or unusual risk exposures, to limit their potential impact upon the cedent. However, since the risk has to be considered individually, facultative reinsurance is time-consuming and expensive for both parties.

⁴ Facultative reinsurance has this name because the insurer decides facultatively, on a case by case basis, to cede a risk to the reinsurer.

Unlike Facultative reinsurance, Treaty reinsurance is a permanent agreement where the insurer is obliged to cede a portion of a specified risk and the reinsurer is obliged to take all the cessions offered by the insurer specified in the treaty during a determined period. The treaty is a complex document with many clauses, which describe in detail the kind of risks included, the business line, the region, the period covered, and further terms and conditions. The sum of all risks insured is called the reinsured portfolio. Within the reinsured portfolio, the amount payable by the reinsurer can be limited by the agreement of a maximum underwriting limit.

Treaty reinsurance can be classified into different reinsurance arrangements according to several criteria. In particular, the classification into global reinsurance arrangements, which are on a portfolio basis, and individual arrangements defined on a policy basis. When a reinsurance arrangement is defined on a policy basis, the significant parameters concern the particular risks.

There is another type of reinsurance agreement called semi-obligatory reinsurance, which is a combination between treaty and facultative reinsurance but is very rare. While one party is under contractual obligation, the second party is free to choose the risks it wants to cede or accept. A semi-obligatory agreement is split into two forms. The first form is facultative-obligatory reinsurance, which gives the primary insurer the freedom to decide which risk to cede, and the reinsurer has the obligation to take all risks offered. The second form is called obligatory facultative reinsurance. It obliges the primary insurer to cede all risks within the terms of the treaty but gives the reinsurer the right to decide if to accept the cession in part or in full or to decline it.

According to another criterion, reinsurance agreements can be classified into *proportional* and *non-proportional* arrangements. In a Proportional reinsurance arrangement, the reinsurer and insurer share the original premiums and claims based on a pre-agreed ratio. The obligation to pay is divided between the primary insurer and reinsurer on the basis of the sum insured, defined in the primary contract. This category includes two different types: the Quota-share treaty and the Surplus-share treaty. A quota-share treaty involves reinsuring a fixed percentage of each subject policy. In this agreement, as shown in the left part of Figure 2.2, the insurer cedes a fixed percentage of

the policy to the reinsurer, which takes a proportional share of the premiums and assumes a proportional share of the risks and claims.

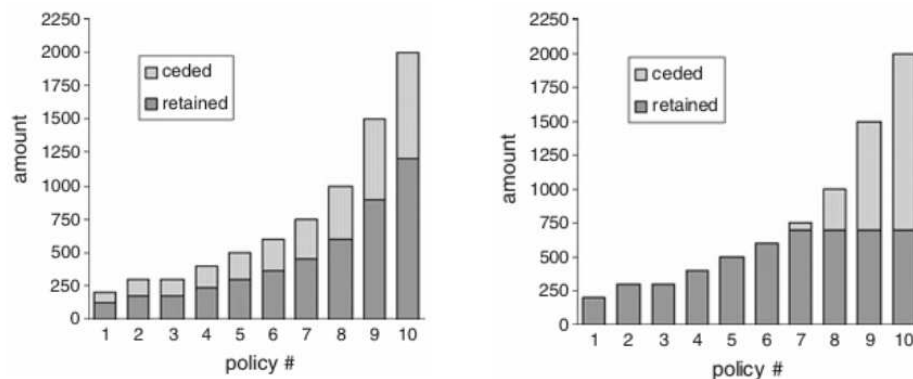
In contrast, the Surplus-share treaty reinsures only the part of an underwritten risk which exceeds the $x^{[re]}$ proportion of the sum insured, which indicates the treaty's specified net line retention.

For a generic risk with a sum insured $x^{(j)}$, the distribution of liability is determined as follows:

- the amount $\min\{x^{(j)}, x^{[ret]}\}$ is retained;
- the amount $\max\{0, x^{(j)} - x^{[ret]}\}$, the surplus is ceded.

The effect on the sum insured is illustrated by Figure 2.2, which shows on the right side, that the sum insured is reduced when it exceeds a determined threshold and the surplus is transferred to the reinsurer.

Figure 2.2: Quota-share and Surplus-share reinsurance.



Source: Olivieri and Pitacco (2015)

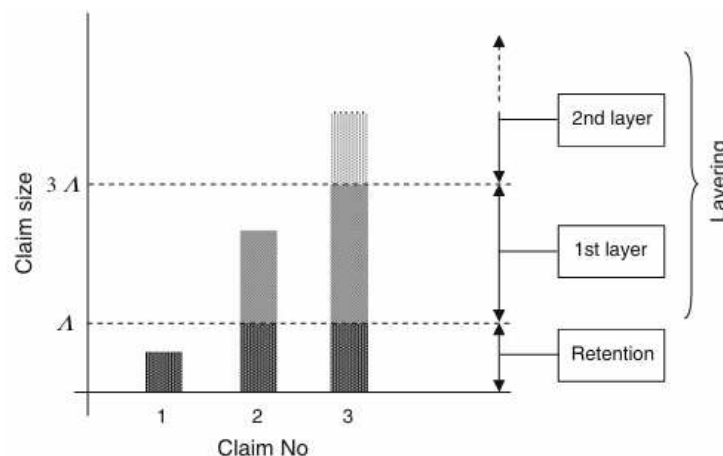
In a Non-Proportional reinsurance arrangement, the reinsurer pays out only when claims exceed a predetermined amount. This means that the insurance company assumes the risk up to a certain amount known as the retention limit, while the reinsurance covers only the losses that exceed that limit. The retention limit is determined depending on the severity of each claim, the number of claims in the portfolio, or the total portfolio payment.

Non-proportional reinsurance is generally used for coverage of natural and man-made catastrophes but also in insurance liability or professional indemnity.

The forms and types of non-proportional agreements differ concerning the definition of reinsured loss. The Excess of Loss or XL reinsurance is the most common form of non-proportional reinsurance on a policy basis, in which the reinsurer indemnifies or compensates the ceding party for losses that exceed the pre-defined deductible, also called excess point. The example in Figure 2.4 shows that for a generic claim with a random size $x_k^{(j)}$ and an upper limit, the reinsurance cover is set to $h\Lambda$. The possible situations are as follows:

- if $x_k^{(j)} \leq \Lambda$, then the insurer retains the total claim amount;
- if $\Lambda < x_k^{(j)} \leq h\Lambda$, then the XL cover exhausts the cession;
- if $x_k^{(j)} > h\Lambda$, then the insurer has still to cede $x_k^{(j)} - h\Lambda$, through a second XL cover (or possibly more XL covers), with another reinsurer.

Figure 2.3: Excess of loss reinsurance.



Source: Olivieri and Pitacco (2015)

Another type specifically designed for catastrophe events is Catastrophe reinsurance. This reinsurance agreement is aimed at protecting the reinsured against the cost of the aggregated losses from an event, covering losses above the deductible up to a maximum limit.

A less frequent form of non-proportional cover is Stop-loss reinsurance, which provides direct protection against the portfolio's total payment default. Under this treaty, the reinsurer covers any part of the total annual loss which exceeds a specified amount, known as the stop-loss retention or priority. This allows the primary insurer to stabilize its financial performance by mitigating the impact of large claim fluctuations from year to year. Typically, this form of reinsurance is triggered only when the primary insurer incurs a technical loss, that is, when claims and administrative costs exceed premiums. This type of treaty offers primary insurers the most extensive level of coverage.

Figure 2.3 provides a summarized representation of the concepts just discussed. Specifically, it illustrates how reinsurance agreements are categorized.

Figure 2.4: Reinsurance arrangements.

		PROPORTIONAL	NON-PROPORTIONAL
Level	POLICY	Quota-share Surplus	Excess-of-loss (XL)
	PORTFOLIO		Stop-loss Catastrophe

Source: Olivieri and Pitacco (2015).

2.1.2 Catastrophe Reinsurance

Catastrophe excess of loss reinsurance, often referred to as Cat-XL, is a particular type of non-proportional reinsurance that operates on a portfolio basis. It's specifically designed to protect the portfolio and the insurance company against the financial risks associated with a single large-scale catastrophic event, which causes a huge number of claims in the portfolio itself. The particularity of this arrangement is that the reinsurer intervenes only when the event exceeds a certain level of severity. Since an event is usually considered a catastrophe if a given minimum number of claims occur in a time interval of a given maximum duration, let consider c as the number of claims that occurred within a specified time interval and let K denote the random number of claims,

the reinsurer will intervene only if $K \geq c$. Furthermore, let's denote $X^{[P]}$ the consequent total payment before the reinsurer's intervention and λ the deductible in terms of the number of claims. The ceding company retains the cost of the first λ claims, and the reinsurer covers the excess number of claims beyond λ .

Then, the cedant's retention and the reinsurer's intervention are respectively given by (Olivieri and Pitacco, 2015):

$$X^{[ret]} = \min\{X^{[P]}, \frac{\lambda}{K} X^{[P]}\} \quad (2.2)$$

$$X^{[ced]} = \max\{0, \frac{K-\lambda}{K} X^{[P]}\} \quad (2.3)$$

Note that, according to this definition, if $X[P]$ is large, then $X[ret]$ is also large. Thus, the reinsurance arrangement is effective if each individual claim has approximately the same amount, and if the total payment $X[P]$ mainly depends on the number of claims.

The essential difference compared to excess loss per risk is that Cat-XL does not consider the loss as the individual loss per policy, but the aggregate loss caused by one event within the insurance portfolio. Therefore, an excess of loss per event represents an effective risk mitigation against large catastrophe losses composed of the sum of potentially hundreds or thousands of relatively small losses caused by the same event.

2.2 Insurance-linked securities: Transferring risks to capital markets

Securitization is the process of transferring the risk associated with the assets, liabilities or cash flows from the balance sheet of an insurance company to a third party, generally a Special-purpose vehicle or SPV, through the creation of tradable securities.

The process started in the banking sector in the late 1980s and early 1990s. The need for savings and loan associations to turn their mortgage loans into cash led Wall Street banks to pool mortgage loans so these institutions had the possibility to finance local housing demands. To achieve this objective and facilitate securitization, the Government National Mortgage Association created SPVs structured as trusts, which held mortgage loan pools and issued trust certificates, which were sold as securities to investors.

The earliest developments started with pools of residential mortgages called Mortgage Backed Securities (MBS) and Collateralized Mortgage Obligations (CMO), followed by the inclusion of commercial real estate. The next step was Asset Backed Securities (ABS),

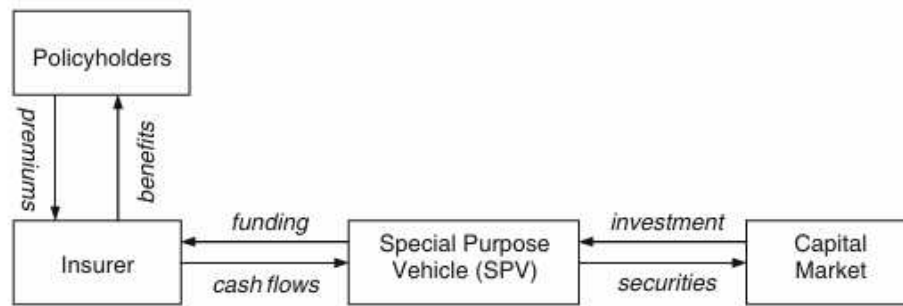
which started to include pools of receivables, leases and any kind of assets which can be securitised.

In the early 1990s, global investment banks extended the process to credit markets by pooling loans into Collateralized Loan Obligations (CLO) and bonds into Collateralized Bond Obligations, together forming Collateralized Debt Obligations (CDOs).

In 1992, Hurricane Andrew signed a turning point for the securitization process, which from that time started a significant rise. This tragic event highlighted the inadequacy of traditional reinsurance to hedge extreme catastrophic risks. Consequently, it primed the creation of new financial instruments for risk transfer and capital increase, called the *Insurance-Linked Securities* or ILSs. The main goal of ILSs is to offer insurers and reinsurers an alternative form of transfer risk while providing a source of capital. With these financial instruments, companies can better manage their risk exposure and maintain adequate capital levels.

The organizational aspects of an ILS transaction are quite complex. It involves three parties: the insurer or cedent, the special purpose vehicle and the investor. The insurer uses the security to transfer the risk from its balance sheet. The Special Purpose Vehicle is a legal entity created with the purpose of managing the cash flows and administering the payables and receivables. It acts as an intermediary, ensuring the efficient allocation of funds between policyholders and investors. The investor is an individual who invests in the security with the intention to receive interest payments. Figure 2.5 illustrates a simple design, focusing on the main agents involved. The transaction starts in the insurance market where underwritten policies generate cash flows. The ceding party sells the right to some cash flows to an SPV. The SPV issues securities to the capital market backed by the cash flows, raising funds. The capital raised from investors is held in a collateral account, and it is used to compensate the insurer for transferring the risk in the occurrence of a trigger event.

Figure 2.5: The securitization process.



Source: Olivieri and Pitacco (2015).

Once the securities are issued, the principal remains invested and investors are paid periodic returns until maturity, unless a trigger event occurs, in which case investors could lose some or all of their principal.

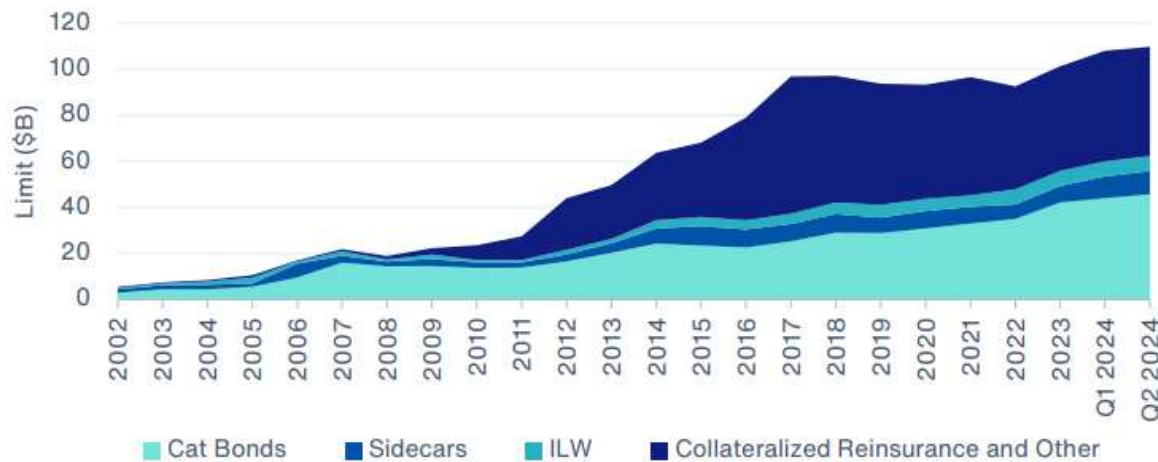
The ILSs market has developed very rapidly and in large numbers. In Figure 2.6 we can observe that over the past two decades, the increasing demand for alternative risk transfer⁵ solutions and the attractive investment opportunities offered by ILS have led the ILS market to grow significantly. In particular in 2024, exceeding 100 billion dollars for the first time, detecting a 7% increase from 2023(A. Mukhopadhyay, 2024). A significant portion of this growth occurred in relation to the catastrophe bond market, which saw a remarkable increase in 2023 equal to 7 billion dollars, 21% more respect the previous year(A. Mukhopadhyay, 2024).

There are several reasons why they developed very rapidly. ILSs, and in general securitization, offer financial protection to insurance and reinsurance companies, corporations, governments, and other organisations from insurance risk, in particular from severe natural catastrophes. ILSs allow an increase in capital and liquidity, permitting companies to offer more financial services. In some cases, securitization allows loans to be removed from balance sheets, improving financial ratios. From an investor perspective, Insurance-Linked Securities offer attractive potential for

⁵ Alternative risk transfer (ART), includes all the risk transfer methods, as reinsurance, securitization and insurance-linked securities.

diversification due to their attractive risk-adjusted returns and low correlation with other asset classes.

Figure 2.6: The rise of insured linked securities from 2002 to 2024 (Limit in \$B)



Source: Akankshita Mukhopadhyay, *Alternative capital surpasses \$100bn in 2023, cat bond market hits all-time high*, Aon, 2024.

The continued growth is also due to market trends, which include the emergence of new types of ILS, expansion into new regions, and the application of technology to improve risk modelling and simplify the issuance process. Another justification for this sudden growth is inflation. In inflationary environments, the insured values of insurance companies increase along with rising property values and construction costs.

ILSs include a wide range of financial instruments, each one with its main characteristics, objectives and structures.

Hurricane Andrew in 1992 questioned the capacity of the insurance and reinsurance sectors to deal with large catastrophic events. As a result, the reinsurance industry began to explore alternative measures for mitigating catastrophic risk. The first effort was computed in 1992 when the CBOT introduced catastrophe futures and options. They were redesigned in 1995 but eventually, their volume was reduced and they were deleted in 2000.

Catastrophe futures and options are standardized exchange-traded contracts to pay or receive payments at a specified time. Their value typically depends on a loss index caused by catastrophic events. For example the CBOT used the Property Claim Services or PCS index, which computes the insured losses in the property and casualty branches. The PCS calculates the index values daily, based on continuously updated estimates of losses, furthermore, the losses are classified according to the geographical area of origin. In a catastrophe futures contract the buyer assumes a long position and the seller a short position according to the underlying loss index. First, an initial margin payment is required by both parties (to cover possible future losses) and then the contract is aligned to market value, with additional margin calls issued according to the direction of value fluctuations.

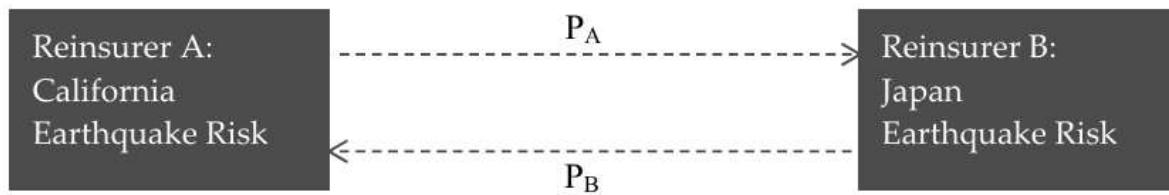
The reasons that led to the failure of the CBOT contracts are excessive basis risk, lack of insurer expertise in options trading, low liquidity, counterparty credit risk, and uncertainty about regulatory accounting treatment.

Also, the Bermuda Commodities Exchange tried to launch catastrophe options, but it failed. In 2007, the New York Mercantile Exchange (NYMEX), the Chicago Mercantile Exchange (CME), and the Insurance Futures Exchange (IFEX) introduced futures and options contracts on the Chicago Climate Exchange (CCX). Deutsche Bank and Swiss Re have joined the IFEX effort as market makers. These efforts have generated some trading volume, even if still small compared to other ILS⁶, but the market has potential for future growth.

Catastrophe swaps are another type of ILS. There are two types of catastrophe swaps: the pure risk swaps, between reinsurance companies, and the event-linked swaps that allow non-reinsurers to transfer catastrophic risk. In a pure risk swap transaction, the two reinsurers exchange uncorrelated catastrophe risk exposures to improve portfolio risk diversification and reduce regulatory capital requirements. An example is given in Figure 2.7.

⁶ Catastrophe futures and options didn't succeed as much as ILWs and CAT bonds due to their high basis risk and limited flexibility.

Figure 2.7: Example of a “pure risk” CAT swap.



Source: S. B. Ammar Semir, A. Braun, M. Eling, *Alternative Risk Transfer and Insurance-Linked Securities: Trends, Challenges and New Market Opportunities*, 2015, page 34.

In both catastrophe swap cases, the cedent agrees to pay a series of fixed premium payments to the counterparty in exchange for floating payments triggered by the occurrence of a specified insured event. They can be negotiated directly with the counterparty or placed through another financial intermediary.

Concerning other ILS, Swaps are simpler and cheaper and can cover multiple risks. On the other hand, they are illiquid assets and not fully collateralized, therefore they expose the buyer to counterparty credit risk.

The type of ILS that has gained much attraction in recent years and which holds the larger market share (see Figure 2.6) is Collateralized reinsurance or ColRe. Collateralized reinsurance agreements are privately structured deals which reinsure a portfolio of specific insurance policies against losses caused by a predefined peril. This type of ILS allows reinsurance companies to access the capital markets by issuing securities supported by collateral that is provided up-front. The collateral covers the full potential claim obligation and enables unrated entities, such as hedge funds or dedicated cat bond funds, to take on catastrophe risk exposure⁷. Similar to traditional reinsurance, ColRe contracts are typically closed with one-year maturity, but they provide a high flexibility both in terms of structural characteristics and underlying insurance risks. From an investor's perspective, the biggest disadvantage is that ColRe is a very complex product, which can be managed only by expert investors, but it offers high protection for the buyer and seller, and high spreads.

⁷ The collateral can be cash or liquid securities, in case of loss, it is used to cover the payments.

An innovative financing vehicle with direct access to capital markets similar to conventional reinsurance is Sidecar. Sidecars appeared in 2002 but became more famous in 2005, following the hurricane season⁸. Sidecars are special purpose vehicles sponsored by reinsurers to provide additional capacity, usually for property catastrophes and marine risks. The owner of sidecars is usually a holding company, which raises capital by issuing equity and debt.

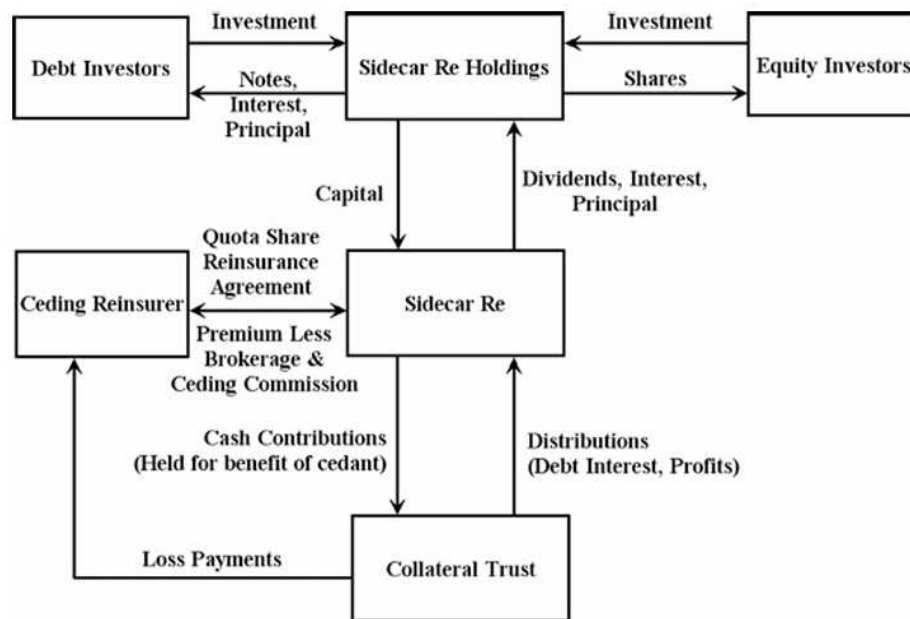
The sidecar transaction, shown in Figure 2.8, is a reinsurance contract, usually a quota share agreement, between a ceding reinsurer and the sidecar. The capital raised by the sidecar is held in a collateral trust for the benefit of the ceding reinsurer. The sidecar receives premiums for the reinsurance by the cedent and in return it must pay the losses consistent with the terms of the reinsurance contracts. An advantage for the ceding reinsurer is that through the transaction with the sidecar, it can earn profits from the ceding commissions. Thus, the ceding part can replace underwriting profit income based on risk with fee income, simply transferring the risk to the sidecar.

In comparison with traditional financing through debt or equity securities, sidecars do not affect the capital structure of the reinsurance, because the sidecar is a separate entity, and its liabilities and assets do not appear on the reinsurance company's balance sheet.

An alternative important instrument that covers risk arising from natural catastrophes is the Industry loss in warranty or ILW. Industry Loss Warranties are contracts that allow investors to take on the risk associated with an industry-wide loss in the event of a catastrophe. The key aspect of an ILW is its dual-trigger mechanism, meaning that a joint event must occur.

⁸ During the 2005 Atlantic hurricane season, 28 tropical and non-tropical storms followed one to another.

Figure 2.8: The Structure of a Typical Sidecar.



Source: J. David Cummins, Mary A. Weiss. *Convergence of Insurance and Financial Markets: Hybrid and Securitized Risk-Transfer solutions*. The Journal of Risk and Insurance, 2009.

Hence an ILW is triggered if a specified industry loss exceeds a certain threshold and at the same time, the losses suffered by the issuing insurer equal or exceed a specified amount. The former is called the index trigger or warranty, and the latter is the indemnity trigger. When both conditions are fulfilled, the investor helps cover the losses. It is important to notice, that generally, the threshold for the index trigger of ILWs is usually much higher than the level of the indemnity trigger, which leads to a high basis risk.

Compared to other ILWs characterized by a coverage ranging from 1 million to 250 million dollars in losses, ILWs offer higher coverage and have a short duration, in fact rarely exceeding one year of coverage.

The most common Insurance-Linked Securities instrument used against catastrophic events is the Catastrophe bond. Catastrophe bonds are considered the more efficient

financial instrument that allows investors and reinsurers to diversify their portfolios and reduce their exposure to risk.

Chapter 3: Catastrophe Bonds

In the mid 1990s, the increasing frequency and severity of catastrophic events created significant pressure inside the traditional reinsurance system. This scenario highlighted the inadequacy of the reinsurance industry to manage catastrophic risk and provide capital to cover the consequent losses. Consequently, it has become necessary to research alternative forms to mitigate catastrophic risk.

The most important development is signed by the introduction by the Chicago Board of Trade (CBOT) of the “Acts of God” often referred to as Catastrophe bonds or CAT bonds (BSIC, 2024). These financial instruments represent a significant innovation in managing catastrophic risk. They offer an efficient way to transfer insurance related risk to the capital market, providing insurers and reinsurers more capacity. They are also an optimal portfolio diversification instrument, allowing investors to benefit from additional returns.

3.1 Definition and purpose of catastrophe bond

Catastrophe bond is one of the principal financial instruments for the securitisation of insurance risk. It is a high-yield bond whose interest payments and principal repayments are conditioned on the occurrence of a predefined and specified event. The purpose of a catastrophe bond is to transfer the risk derived from large-scale natural disasters to the capital market.

CAT bonds offer a range of advantages and disadvantages for both investors and insurers.

The main reason why investors are attracted to CAT bonds is their historically relatively high returns, which have two characteristics: low volatility and low correlation with traditional asset classes. The correlation of CAT bonds with other financial assets is near zero, therefore they fall into the category of zero-beta assets, which includes financial activities with zero correlation with the financial market returns. This is due to the fact that the occurrence of disasters, especially those due to natural events, has no significant connection with phenomena that influence the performance of traditional financial instruments, such as interest rates, exchange rates or other macroeconomic variables.

Another advantage for investors is the low probability of Trigger⁹. The probability of a CAT bond being triggered is relatively low due to the low frequency of catastrophic events. Additionally, CAT bonds often offer attractive risk premiums, making them a potentially lucrative investment. They compensate investors for event risk with higher expected returns.

Hence, the insertion of catastrophic risk into a financial portfolio leads to relatively small increases in risk, providing investors with considerable positive portfolio diversification effects.

From the point of view of the insurer or the reinsurer, CAT bonds are attractive for several reasons. First of all, they are financial instruments that offer optimal risk diversification. Insurance or reinsurance companies benefit from CAT bonds by diversifying their risk exposure in space and in time. By transferring risk to the capital markets, they reduce the concentration of risk on their balance sheets.

Secondly, while traditional reinsurance is generally renewed every year, CAT bonds can be issued anytime and provide multi-year coverage at a fixed price, therefore they offer flexibility and price stability. This advantage represents a great attraction for insurers and reinsurers because in general, one-year coverage requires annual renegotiation, which is time consuming and risk carriers for the cedents. Therefore, multi-year contracts reduce this administrative burden and offer better strategic risk management planning.

Thirdly, for insurers CAT bonds actually act as collateralized reinsurance contracts. In case of a loss due to a catastrophic event, the funds raised from the bond issuance can be used to reimburse policyholders, thus providing a source of capital for claims payments. Hence, full collateralisation is a key benefit of CAT bonds for the cedent, as it notably minimizes both interest rate and credit risk in the structures and reduces the uncertainty pertaining to the availability of funds in the case of a trigger event.

Fourthly, CAT bonds are highly transparent and can ensure rapid payments in case of a trigger event, without delay compared to conventional reinsurance or government action.

⁹ The probability of trigger refers to the probability of the occurrence of a catastrophic event.

Finally, due to the recent spread tightening, CAT bond coverage is offered at highly competitive prices relative to traditional reinsurance and retrocession deals.

CAT Bonds also have some possible drawbacks. The CAT bond market is considered more complex than the classic bond market, always linked to the probability of a catastrophic event occurring. The severity of the event is not predictable, above all, the losses cannot be quantified precisely in monetary terms. This problem could lead investors to lose entirely or partially the principal and the insurers may not receive payouts equal to actual losses. CAT bonds typically have lower liquidity compared to more traditional debt instruments, such as corporate or government bonds. The CAT bond market is much smaller than traditional bond markets such as government bonds or corporate bonds. This means that both investors and sponsors have fewer options available than in larger, more established markets.

Finally, they have a complex structure with specified trigger mechanisms and only a limited number of investors are willing or able to understand and evaluate them.

3.2 Historical developments of catastrophe bonds

Catastrophe bonds are the best-known type of Insurance Linked Security for transferring catastrophic risk to capital market investors and the second largest in terms of volume.

Over the past 25 years, the CAT bond market has grown starting as a small part of the insurance industry and becoming a fundamental instrument for managing insured natural catastrophe losses. The increase in catastrophe dimensions, their frequency and their great costs of compensation have stressed the incapacity of the reinsurance industry to effectively aggregate and diversify risk, supplying the necessary capital to cover damages. It is, therefore, clear that the traditional reinsurance institute is not sufficient to guarantee the insurability of damages by catastrophe.

The first attempts at catastrophe bond market development took place at the beginning of the 1990s in the USA, probably because of the unfavourable geo-climatic and hydrologic conditions that affected the country. In particular, after Hurricane Andrew in 1992, the capital market was directly considered as a source of financing and participation in the risks following possible catastrophic future events (A. S. Ben, A.

Braun, M. Eling, 2015, pages 17-36). Hurricane Andrew was a very powerful tropical cyclone that devastated the Bahamas, Florida, and Louisiana during August 1992, causing nearly 49.4 billion dollars¹⁰ in insured loss and leading to the failure of 63 insurance companies.

The first successful CAT Bond was issued in 1997, for 85 million dollars, with the participation of American International Group (AIG), Hannover Re, St. Paul Re and the US government. Initially, the CAT bonds concerned only one type of disaster in a single region, the Atlantic Basin hurricane, with a horizon of one year.

In 1999, Oriental Land Co. was the first company not involved in financial activities to issue CAT Bonds against the earthquakes at Tokyo Disneyland.

The CAT Bond Kamp Re 2005 Ltd. was the first failed CAT Bond. The main reason is that in late August 2005, Hurricane Katrina devastated the city of New Orleans. Hurricane Katrina was one of the most powerful, devastating tropical cyclones in history. It caused 1,392 fatalities and the total damages were estimated at 125 billion dollars concentrated in the city of New Orleans and its surrounding area. The investors of Kamp Re reported losses of 190 million dollars.

But Catastrophe bonds gained popularity for their role in diversifying risk. After the reported losses from Hurricane Katrina, between 2006 and 2007 reinsurance prices jumped (see Figure 3.1) and attracted significant amounts of capital to the Catastrophe bond market. This flow of capital accounted for 4.7 billion dollars in 2006 and 7.1 billion dollars in 2007.

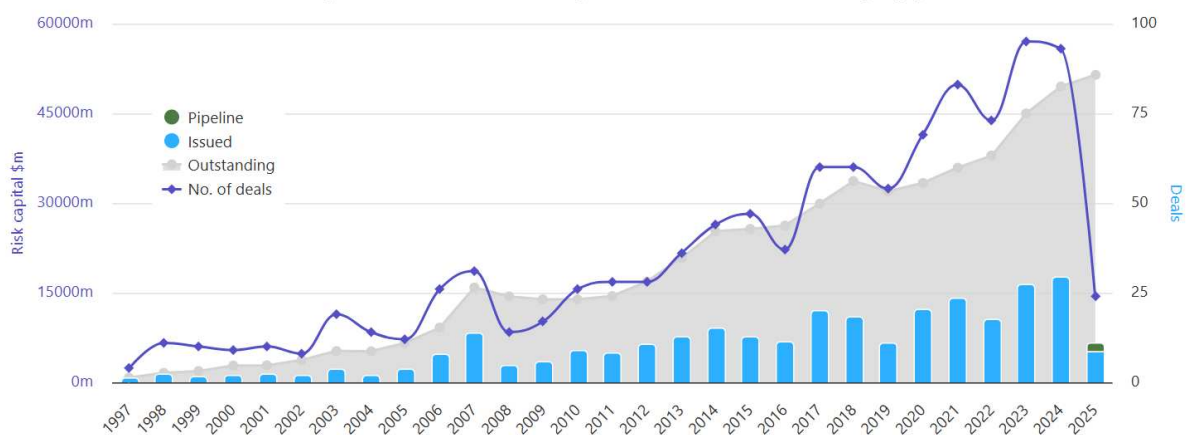
However, the issuance of CAT bonds fell in the financial crisis of 2008, following the collapse of Lehman Brothers in September, which had been an active counterparty in some major global deals. During this period investors determined that the CAT bond structure exposed them to too much risk. Between September 2008 and January 2009, the CAT bond issuance suffered a complete block (see Figure 3.1) until more secure counterparty structures were developed. After the collapse of Lehman Brothers, the

¹⁰In 1992, Hurricane Andrew caused insurance losses worth 15.5 billion dollars, equivalent to a value of 49.4 billion dollars in 2018.

structure of CAT bonds was changed, the Special purpose vehicle was introduced as intermediary. So, at the end of 2009, investors returned to the CAT bond market.

The CAT bond market has continued to grow strongly in the post-crisis years. For instance, between 2010 and 2017, the amount of outstanding CAT bonds has more than doubled. Given the low yields on long-term Treasury bonds and tight corporate bond spreads, many institutional investors have been attracted by the high yield combined with uncorrelated risk that catastrophe bonds offer. In addition, innovations in catastrophe bond modelling have enabled both CAT bond issuers to collateralize a wider range of risks and made it easier for institutional investors to understand the underlying risks.

Figure 3.1: Catastrophe bonds and ILS issued and outstanding between 1997 to early 2025.



Source: Artemis, Catastrophe bonds & ILS issued and outstanding by year, 2025.

Figure 3.1 illustrates a great rise in the catastrophe bond market from 2017 to 2025, but two major slowdowns can be noticed in conjunction with two important occurrences. In 2020, with the spread of the COVID-19 pandemic, volatility hit financial markets. In 2022, the prevailing conditions were inflation rising, rates increasing and high political and economic uncertainties. CAT bond investors, worried about the potential rise in claims and the risks, demanded higher spreads. As a result, the CAT bond spreads have reached their highest levels in over ten years. This increase in spreads has had a big impact and even bonds that were not affected by any disasters lost value.

In the first half of 2022, new CAT bond issues totalled \$8 billion, which is lower than the \$12.8 billion issued in 2021. However, the total CAT bond market recorded a growth of 6.8% compared to the end of 2021. The end of 2022 signaled an increase of uncertainty in terms of capital, losses and the possibility of new issuance also caused by the impact of Hurricane Ian (September 2022).

Some CAT bonds have recently been given additional features, such as links to environmental, social and governance (ESG) concerns. ESG CAT bonds are an extension of the basic CAT bond model that could attract investors interested in climate change mitigation or green projects.

An example of ESG CAT bond is the Lion III Re DAC, issued by the Italian insurance company Generali, in 2021, to cover multiple perils, in storms in Europe and earthquakes in Italy, it raised EUR 200 million. The funds reached with the green CAT bond issue are regulated by the framework ESG. It is responsible for the regulation of what types of projects can have green CAT bonds issued for them and how the allocation of the freed-up capital should be conducted. The framework has two main approaches. The first is to use the freed-up capital benefit for green assets and underwriting. The second is to invest the proceeds contained in the special-purpose vehicle in a portfolio of green investments for growth.

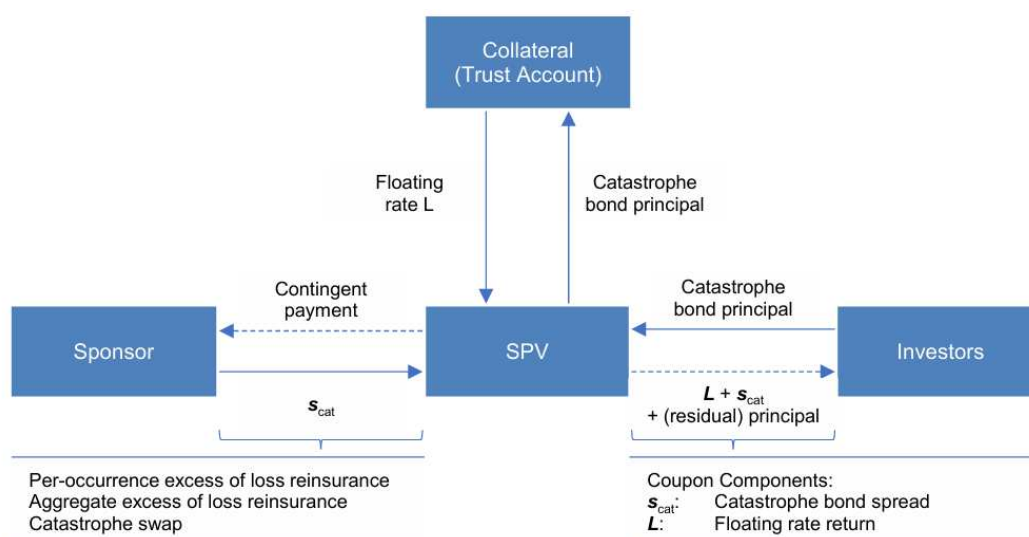
3.3 Structure of Catastrophe bond

Catastrophe bond uses the securitization process to transfer the risk derived from a catastrophic event through a financial instrument to the capital market. This process is depicted in Figure 3.2. The typical catastrophe bond transaction is based on a reinsurance agreement between the sponsor and the special purpose vehicle (SPV).

The sponsor is defined as the entity that needs to transfer the catastrophic risk from its balance sheet directly to the capital market.

Usually, the sponsor is an insurance or reinsurance company, but other entities can also issue CAT bonds, such as large corporations¹¹, sovereign nations¹² and supranational entities¹³.

Figure 3.2: Typical structure of CAT bond.



Source: OECD, *Fostering Catastrophe Bond Markets in Asia and the Pacific, The Development Dimension*, OECD Publishing, Paris, 2024.

¹¹ A famous example of a large private corporation issuing a CAT bond refers to the case of Quake Bonds issued by the Tokyo Disneyland's owner. In 1999 the Japanese owner of Tokyo Disneyland issued a CAT bond, called Quake bond, to hedge the earthquake risk in Japan. Source: L:A Times, *Owner of Tokyo Disneyland sells Quake bonds*, 1999.

¹² In 2023, the Government of Chile with the international Bank for Reconstruction and Developments (IBRID) issued a CAT bond and a swap transaction against the risk of earthquakes in Chile. Source: Swiss Re, *Cat bond provides Chile with financial protection against earthquakes*, 2023.

¹³ In 2017, the World Bank issued the first Pandemic Bond, with the aim to provide financial support to the Pandemic emergency. This was the first time that a CAT bond was used to finance efforts against infectious diseases, and the first time that pandemic risk was included in the securitization process. Source: World Bank Group, *World Bank Launches First-Ever Pandemic Bonds to Support \$500 Million Pandemic Emergency Financing Facility*, 2017.

The SPV is a legally independent entity. It operates as an intermediary, occasionally established to issue securities and assumes all related obligations. The SPV realizes three operations:

1. Issuing the CAT bond to raise capital from the capital market.
2. Creating a reinsurance agreement consistent with the CAT bond.
3. Investing the capital raised in high-quality securities.

The SPV issues the bond in which the payment of interest, the repayment of principal, or both are inversely linked to the future value of the variable representing a specific risk. The funds that the SPV raises by issuing the CAT bond are invested in order to pay for a specific risk. If a catastrophic event occurs, the funds that normally would be paid to investors are used to compensate the sponsor for the losses suffered.

At the same time as the bond is issued, the SPV enters into a reinsurance contract with the sponsor. It is important to note that the contract conditions are aligned with those of the bond. For this reason, the coverage amount must be equal to or represent a high percentage, near 80 or 90%, of the bond's nominal value, the duration of the contract must not exceed the duration of the bond, typically one or two years, the covered risk must be identical to the risk of the CAT bond, and losses must be calculated based on the same specified metric. Lastly, the type of coverage has to reflect how investors participate in the losses. If the investor tolerates only a specified portion of the loss, the reinsurance contract has the form of a quota share reinsurance. If the investors decide to cover all the losses exceeding a predetermined threshold, the contract follows the structure of a Cat-XL reinsurance agreement.

The SPV has another function in the securitization process, which is to invest the raised capital in safe financial assets such as government bonds or AAA corporate bonds held in a trust account. The capital raised operates as both a collateral for the bond and a reserve fund for covering possible sponsor's losses. The SPV's sources of revenue include the capital raised from the bond issuance, the reinsurance premiums paid by the sponsor and the returns generated from the invested collateral.

The SPV also has mutually exclusive expenses, the interest payments to investors, the repayment of principal at bond maturity and indemnity payments to the sponsor in the

event of a catastrophe. If the catastrophic event does not occur, the SPV does not need to compensate the sponsor and can use the reserve fund to pay the investor's interests. However, if the event happens, the investors lose a part or all expected payments, and the reserve fund is used to repay the sponsor's losses.

The coupon offered by the CAT bond is composed of a floating interest rate (indicated in Figure 3.2 with L), which generates interest from safe securities and a fixed spread (s_{cat}), which represents the risk premium determined at issuance. CAT bonds use a floating rate and have a high quality collateral that permits to carry minimal interest rate and credit risk. The solvency of the cedent does not represent the main risk for the investor, as the capital is guaranteed by adequate safe investments. On the contrary, the possibility of losing capital is affected by the occurrence of a certain event during the duration of the bond.

The payment mechanism can be binary or proportional to an underlying metric. In the binary case, the capital is completely transferred to the sponsor if a catastrophic event occurs, conversely, the latter is proportional to the losses suffered. To determine whether the CAT bond should make a payment, several trigger mechanisms are used. The different trigger mechanisms will be discussed in the following section.

The simple structure of the CAT bond transaction, illustrated in Figure 3.2, can present some variations. A reinsurance company may act as an intermediary between the sponsor and the SPV. Under this arrangement, the reinsurer absorbs the basis risk. If a catastrophe event occurs, the sponsor would receive the reinsurance recoverable amounts based on the insured losses, while the SPV would make the payments based on a particular trigger mechanism.

Most CAT bonds can be equipped with a special clause defined as *Extension Period clause*. This clause allows the sponsor to extend the maturity of the bond only if a covered event has occurred but the amount of losses has not yet been determined.

This extension of the maturity allows the sponsor to calculate the real impact that the catastrophic event had on its balance sheet, and it ensures that investors do not prematurely redeem their investment before the full extent of the bond's obligation

towards the covered losses is determined. Finally, after the “Extension Period”, or at the bond’s maturity, investors receive the remaining principal, net of payouts to the sponsor.

3.4 Trigger mechanisms

Traditional reinsurance pays claims on an indemnity basis, the sponsor of a CAT bond uses a variety of trigger mechanisms to determine whether a natural catastrophe qualifies for coverage. The selection of the trigger is computed by the Sponsor. Its selection is based on various factors, including basis risk, transparency, length of time between an event and the claim settlement, and accounting implications.

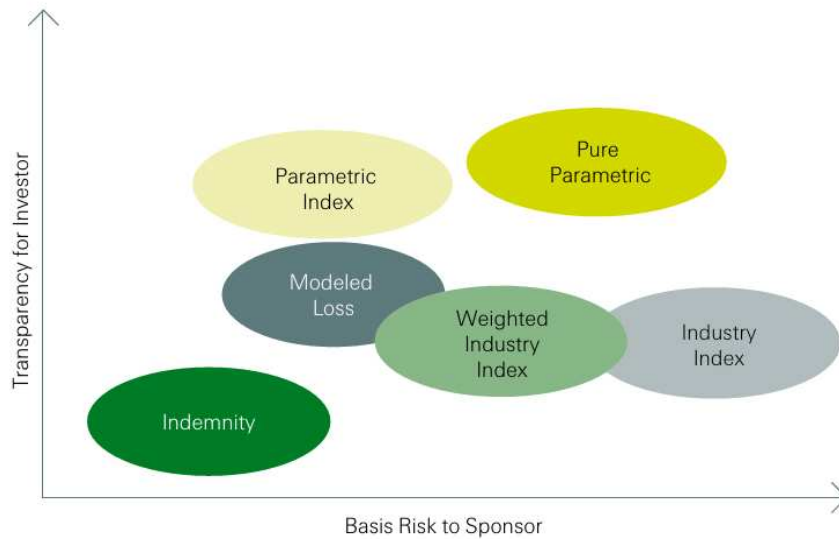
Trigger mechanisms are divided into two categories: the single triggers and the multiple trigger indexes. The single trigger indexes include five main flavours categorized into two main groups: indemnity index and non-indemnity trigger. Non-indemnity or index triggers include: industry index, pure parametric, parametric index and modelled loss trigger.

The first CAT bonds issued at the end of 1990 predominantly adopted the indemnity triggers. Over time, ceding companies began to adopt industry loss, parametric, and modelled loss triggers. These different trigger mechanisms coexist in the market, reflecting the various needs of ceding companies and investor preferences.

Choosing which trigger mechanism to use is always a trade-off, and depends on the characteristics of each trigger type. As shown in Figure 3.3, these five types of triggers differ in the combination of levels of transparency for investors and basis risk to the sponsor. The trade-off in selecting a trigger type lies between moral hazard risks and potential settlement speed on one side and basis risk on the other.

Moral hazard is defined as “the behaviour that occurs when the cost of loss-control efforts by the sponsor exceeds the benefits from debt forgiveness” (OECD, 2024).

Figure 3.3: Trigger comparison.



Source: Swiss Re Capital Markets.

Meanwhile, the basis risk affecting the CAT bond refers to the “gap between the sponsor's actual loss and the composite index of losses that prevents the sponsor from receiving full risk hedging” (OECD, 2024, page 10). However, it is defined as risk when the index reports losses lower than those actually suffered by the sponsor, preventing it from receiving full coverage of the risk and causing financial instability.

Sponsors often prefer indemnity triggers to minimize basis risk. However, CAT bonds with indemnity triggers should incur higher risk premiums because of moral hazard and information asymmetries.

Non-indemnity or index triggers tend to be favoured by investors, as they minimise moral hazard. CAT bonds with index triggers may also reduce sponsors' basis risk, although empirical evidence is ambiguous in this regard.

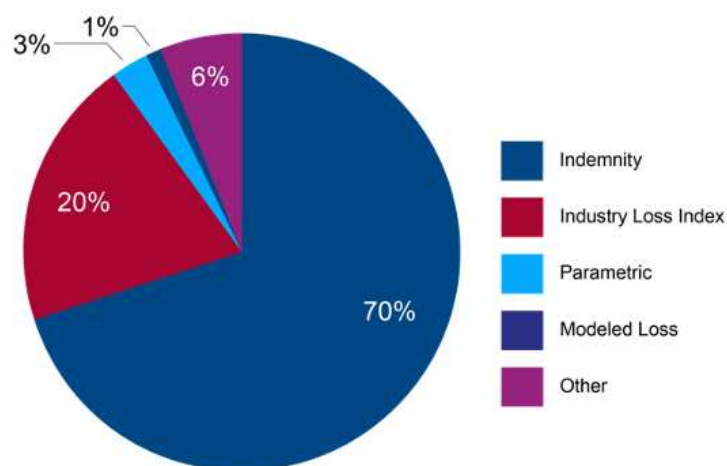
3.4.1 Indemnity trigger

As illustrated in Figure 3.4, the *indemnity trigger* is one of the most widely favoured mechanisms. In 2022 nearly two-thirds of the outstanding CAT bonds had an indemnity trigger. As said before this type of trigger is preferred from the sponsor's perspective because it is not subject to basis risk, as it is linked to the direct actual losses sustained

by the ceding party. The coverage provided by the CAT bond becomes effective only if the ceding party incurs a predetermined level of losses called attachment point. Once this threshold is exceeded, the actual losses suffered by the ceding party are reimbursed according to the contract policy.

While this structure has the advantage of providing full catastrophe coverage, which is highly beneficial to the sponsor, it is less favourable to investors. Investors are exposed not only to natural catastrophic risk but also to the effects of unexpected secondary losses. They are also subject to operational risk associated with the originator's underwriting and claims functions. Furthermore, rating agencies tend to require additional stress testing due to operational risk exposure, which may result in a lower rating.

Figure 3.4: Trigger types in the catastrophe bond market (March 2021).



Source: C. Kousky, A. Braun, *Catastrophe bonds*, EDF, 2022.

Another drawback for investors is the lack of direct visibility into the sponsor's insurance portfolio details. The less transparency between the sponsor and the investors introduces information asymmetry and moral hazard issues. Indemnity trigger CAT bonds require more documentation regarding the sponsor's exposures and underwriting standards, for this reason they are associated to have higher transaction

costs. Furthermore, they may take longer to settle following an event because the sponsor's losses must be evaluated.

Additionally, indemnity triggers make the process of payment very long. Verifying received claims, loss reporting, reimbursement approval and actual liquidation make the process lengthy and can delay the payment of claims to insurers and the notification to investors of how much capital they have lost.

To compensate for these risks, investors may demand higher spreads for indemnity trigger transactions.

3.4.2 Non-indemnity triggers

The highest level of transparency for investors and the highest level of basis risk for sponsors is associated with the pure parametric trigger. It references the physical characteristics of the actual event instead of the insured losses. This trigger mechanism is activated based on specific measurable characteristics of the catastrophic event, such as the water level for floods or the magnitude of an earthquake, once they exceed predefined thresholds at predetermined locations. Data for the determination of the parameters are reported by multiple stations in a given geographical area and entered into a predefined formula, for the definition of the parametric index trigger. Information about the event parameters is typically available shortly after the occurrence of an event, allowing transactions to be settled more rapidly than when an indemnity trigger is involved. Moreover, a parametric trigger can reduce information asymmetry due to the higher level of transparency associated with more objective official measurements.

However, the parametric trigger is not in favor of the sponsor as it introduces basis risk. This is because the parameters used to trigger the bond may not perfectly match the actual insurance losses incurred. As a result, there may be scenarios where significant losses incurred by the sponsor do not result in a payout from the CAT bond, or the payout does not fully cover the losses. Investors are therefore able to track the trigger parameters precisely and sponsors benefit from a quick liquidation after the event (C. Kousky, A. Braun, 2022).

Parametric index triggers are an adaptation of pure parametric triggers. The parametric index simply refines the pure parametric trigger by using a greater number of locations

and applying different weights to each location to reflect the ceding company's exposure to events in the area. This formula more accurately tracks losses to the ceding company's portfolio than the formula for a pure parametric trigger. In this way, the high level of transparency is maintained, but the level of basis risk is reduced.

The *industry loss index trigger* is based on the aggregate losses of the entire insurance industry following a catastrophic event of large size, and not directly on the losses of a single specific insurer. It is triggered when the aggregate losses of the entire insurance industry exceed a pre-specified financial threshold.

Aggregate losses are typically calculated by an independent third party, such as Property Claims Services (PCS) in the US or Perils in Europe, which collects and consolidates loss reports into an industry loss index. This method effectively mitigates the moral hazard problem, as payments are not directly linked to the individual insurer's claims handling or underwriting standards. However, despite these advantages, the industry-loss trigger exposes the sponsor to basis risk to the extent that its actual losses differ from that of the industry as a whole, meaning there may be scenarios where the CAT bond does not trigger despite the sponsor suffering significant losses. If the discrepancy is substantial, the sponsor may remain exposed to the industry risks it had sought to hedge, or, alternatively, may benefit from an unexpected recovery. In addition, data collection and aggregation after a catastrophe is a time-consuming process, which may delay the determination of whether the obligation's trigger condition has been satisfied.

A solution that can reduce the basis risk is the weighted industry index. It tries to replicate the geographical distribution of the sponsor's underwriting portfolio and thus reduces basis risk. Considering a covered area divided in various subregions, by applying weighted calculation factors to the subregions, the sponsor can obtain a coverage more in line with its portfolio of risks. This method enhances risk transfer efficiency and minimizes discrepancies between the sponsor's actual losses and the broader industry index.

A weighted PCS Index is the most frequently adapted solution in the market. Using an index-based approach for reinsurance protection, rather than a sponsor's specific book of business, offers two key advantages. First, it helps the ceding company protect

proprietary information from disclosure to competitors. Second, it increases transparency for investors, addressing concerns about adverse selection (the fear that the sponsor cedes only its riskiest exposures), moral hazard (the risk that offloading exposure alters the sponsor's incentives), and unsound underwriting practices.

Nevertheless, there is usually no perfect transparency for investors, as the index is based on self-reported data from the insurance industry and is often calculated according to proprietary rules. Industry loss triggers may be associated with substantial basis risk for the sponsor as many insurers cover different geographic areas that are more or less affected by a specific natural disaster. Thus, the sponsor's loss can significantly differ from that of the industry as a whole. To overcome this problem, it is possible to reference a weighted industry index which tries to replicate the geographical distribution of the sponsor's underwriting portfolio and thus reduces basis risk.

The last type of single trigger, similar to the parametric index, is the modelled loss trigger. This trigger is based on the estimation of the losses computed by a modelling firm¹⁴, comparing the physical parameters of events to entries in the firm's database of industry exposures. Alternatively, the CAT bond sponsor may opt for using a representative sample of its portfolio of exposures instead of the modelling firm's industry database when calculating the modelled losses attributable to a specific catastrophe event. The CAT bond is triggered when the modelled losses exceed a specified threshold.

Unlike indemnity triggers, which are estimated on actual reported losses, modelled loss triggers are based on the financial impact of an event determined by predefined models. Although this approach was designed to accelerate payouts, it may be subject to model risk, where the difference between modelled losses and actual losses is large.

Modelled loss triggers worked similarly to indemnity triggers but relied on third-party modelling firms to estimate expected losses at specific loss levels. Instead of determining payouts based on actual claims, after a catastrophe, the physical parameters of the event are included in the model, simulating expected losses for the

¹⁴ Modelling firm, or modelling company refers to a company that provides mathematical and statistical modeling services, used to assess and manage risk for insurance companies.

ceding company's portfolio. To allow transparency, both the loss model and the insurer's reference portfolio are predetermined and placed in escrow.

This predefined structure reduces basis risk compared to parametric triggers because the loss estimation process cannot be influenced by the sponsor.

Despite the advantage of rapid payment compared to traditional compensation mechanisms, this type has been progressively eliminated from the Catastrophe bond market, mainly due to its complexity and lack of transparency.

3.4.3 Multiple trigger indexes

In addition to a single-trigger index, the payment of a Catastrophe bond can be activated through a multiple-trigger index.

The first multiple-trigger CAT bond was issued by Swiss Re in 2002. This date marked an important moment in the reinsurance sector. The introduction of multiple-trigger indexes continued the development of CAT bonds attracting numerous investors.

A multiple-trigger catastrophe bond is triggered when multiple predefined events or conditions occur. Unlike traditional CAT Bonds, which are based on a single-trigger criterion, these instruments require the simultaneous or sequential occurrence of more than one parameter to activate the payout.

For example, the first multi-trigger CAT bond issued by Swiss Re used a double trigger based on the magnitude of an earthquake exceeding a certain value and the aggregate economic losses in the insurance sector exceeding a predefined threshold. The idea was to avoid the activation of the bond only based on the magnitude of the event, introducing a second criterion linked to the actual economic impact. This would reduce the risk of premature activation and make the bond more attractive to investors.

A subcategory of the multiple-trigger indexes is the hybrid trigger mechanisms. While multiple-trigger CAT bonds require two or more conditions to be met simultaneously or sequentially to be activated, belonging to the same category, for example, two distinct natural events or to different categories, hybrid trigger CAT bonds, on the other hand, combine different types of triggers. Hybrid triggers are divided into two main categories. In the first category, different types of triggers are used, each for a different peril in a

single operation. The second category is more complex because it applies sequentially different triggers for a single covered event. If all the triggers are met in the expected sequence, the CAT bond is activated and the funds are used to cover losses.

Multiple-trigger indexes are very attractive to investors for several reasons. First, they reduce the probability of losing the principal. The increasing trend of global catastrophe loss increases the probability that a claim of a single trigger CAT bond will occur and consequently increases the probability for the investor to lose the principal. Conversely, on a multiple-trigger CAT bond, the investor's principal is lost if two claim-triggering events occur sequentially or simultaneously, consequently, the probability for investors to lose the principal is lower.

Secondly, a multiple-trigger CAT bond provides more transparency than a single-trigger CAT bond and reduces moral hazard, because it better describes the catastrophe severity that occurs. It is clear because the catastrophe severity in a multiple-trigger CAT bond is measured by more than one trigger so it is more precise, whereas in a single-trigger CAT bond, it is only measured by one trigger that sometimes does not reflect the real losses suffered.

Thirdly, using a CAT bond with a multiple-trigger, basis risk can be reduced by incorporating an indemnity component in the trigger mechanism with another trigger.

Lastly, they offer a better hedge against financial market volatility and can improve portfolio diversification because, unlike single-trigger mechanisms, two conditions must be met.

Multiple Trigger Cat Bonds are an evolution in the market for catastrophe bonds, offering investors greater protection and better risk management for insurance companies. Their structure reduces the likelihood of capital loss and improves transparency. As global catastrophe events increase, their use is set to grow, attracting more and more investors.

Chapter 4: A Multiple-Trigger Catastrophe Bond Pricing Model

The correct pricing of Catastrophe bonds plays a crucial role in risk coverage, but it still remains an open question. Numerous academics and professionals have developed many different methods for the pricing of Catastrophe bonds. For a correct analysis, it is important to dwell on the considerations developed by Cox and Pedersen (2000). Cox and Pedersen demonstrated that the price of a CAT bond cannot be hedged by a portfolio of traditional bonds or common stocks, for two important reasons. Firstly, the Catastrophe bond market is considered an incomplete market mainly because CAT bonds are financial instruments whose primary risk is the occurrence of a catastrophic event, which is rare and unpredictable. Secondly, unlike corporate bonds, CAT bonds are uncorrelated with the underlying financial market variables¹⁵.

The value of CAT bonds relies on specified catastrophic loss indices, making pricing not as uniform as in the full market. Consequently, for CAT bond pricing, different methodologies are applied relying on stochastic modelling of catastrophic risks.

Modelling catastrophic risk of a specified peril means integrating the analysis of four elements: hazard, vulnerability, exposure and loss. With these elements, it is possible to construct a model to determine the potential losses caused by an extreme event.

Therefore, the pricing of the CAT bond requires specification of the catastrophic dynamics (i.e., the counting process), the aggregate loss function and the threshold.

The majority of pricing methods use methods based on the aggregate loss process, which combine the frequency and the severity of the occurrences of an extreme event. The Poisson process is used for the former; instead, the Extreme value theory is applied for the latter.

Baryshnikov et al. (2001), Burnecki and Kukla (2003), and Ma and Ma(2013) have modelled a double stochastic Poisson process, also known as a Cox process, to describe the price dynamics of the zero-coupon CAT bond. They tested various distributions for aggregate loss modelling function, including logarithmic normal distributions, Pareto's, Weibull's and Gamma's distributions, and parameters derived from them using the maximum likelihood criterion. The authors tend towards the choice of a logarithmic

¹⁵ E.g. interest rate levels or aggregate consumption

normal distribution due to its better adherence to real data. These models highlight the impact of the trigger level on the bond value, showing that a higher trigger level results in a higher bond price due to the decreased likelihood of losing the principal.

The Extreme Value Theory plays an important role in analysing statistical models of extreme events. To represent the tail characteristic of the catastrophic losses distribution, the Extreme Value Theory provides two methods: the Block Maximum method model and the Peaks Over Threshold approach. In particular, the Peaks Over Threshold model is more effective in describing the probability that an extreme event exceeds a determined threshold. It provides a more accurate CAT bond pricing.

A fundamental contribution that has had a significant impact on the understanding and pricing structure of these financial instruments is given by a linear relationship model proposed by Lane (2000). The author focuses on how to calculate a fair price for CAT bonds, taking into account catastrophic risk factors and how these risks can be quantified to calculate expected returns for investors. Based on the model of Lane (2000), Bodoff and Gan (2009) contributed by developing a linear approach to bond pricing. The model allows the calculation of the spread at the issue of a CAT bond as the sum of the expected loss value and the risk premium, which compensates investors for the uncertainty associated with the bond's trigger event and the stochastic nature of interest rates. This approach provides a more complete and efficient framework for the pricing of Catastrophe bonds, integrating both catastrophic risk and the time value of money in a unified way.

In recent years, the increasing frequency and severity of natural calamities of great magnitude have changed the approach to pricing CAT bonds. Investors' interest in single trigger CAT bonds is declining because of the increasing probability of the occurrence of the claim. An alternative structure of CAT bonds with the objective to attract more investors is the Multiple Trigger Catastrophe bond (MTCB). MTCB is a CAT bond activated if two trigger claims occur. MTCB is attractive both for investors and sponsors. For investors, it reduces the possibility of losing the principal, while for the sponsor, it reduces the moral hazard. Moreover, MTCB describes the actual catastrophe severity

better than a Single trigger CAT bond. However, the main obstacle of MTCB is the complexity of pricing, because quite a lot of factors have to be considered.

An efficient model for pricing a Multiple trigger Catastrophe bond is the CIR-Copula-POT Model. This model combines three components: the Cox-Ingersoll-Ross interest rate model, the Copula function theory and the Peaks Over Threshold model.

In the following section, a correct pricing of a Multi Trigger CAT bond will delve in detail into all the needed steps, starting from the modelling of catastrophe risk, determining the aggregate loss process and the Cox-Ingersoll-Ross model, and finally performing an in-depth analysis of the classic models to determine the CAT price.

4.1 Structure of Catastrophe Risk Modelling

The issuance of a CAT bond involves a complex quantitative analysis process, which starts with the analysis of the risks.

The risk analysis is conducted using a specific approach based on modelling an extreme natural phenomenon, through the construction of physical and statistical models of the event.

Catastrophe risk modelling is a recent multidisciplinary method, born from the necessity to assess, predict and manage the risk from natural disasters. It is based on the combination of four main components: hazard, exposure, vulnerability, and loss (S. Zanardo, J. L. Salinas, 2021).

The starting point of the catastrophe risk modelling is the construction of a *hazard* model. This model is different depending on the type of peril being analyzed because it describes the physical processes that cause the natural disaster and its consequences, measuring the potential damages. The hazard model focuses on two fundamental aspects of risk, which are the intensity and frequency of the peril. The frequency is a variable that uses historical data of the peril to determine the frequency forecasts through the creation of a probability distribution. The intensity reflects the physical consequences of the event, with particular attention to quantities such as the extension of the affected areas or the involved destructive force. It is a multivariate analysis which concentrates on the causal relationships and dependencies between physical variables.

The second component is the exposure. The Exposure is compared to a catalogue of assets at risk considered by the model. Each asset is described with its own relevant information, such as location, measures and characteristics about the susceptibility of the property to the peril considered. In this step, it is necessary to classify all the variables and their causal relationships which are exposed to the event in the determined area.

The third step is *vulnerability* analysis, which converts the relative damage produced by the event into actual economic losses. The relative damages are calculated by dividing absolute damages by the asset values.

The last element is the financial model of loss. It converts the expected physical damages into monetary losses. In the model, the damage information for each event and location in the exposure catalogue is aggregated to produce a loss for each of the simulated events. The loss is provided by the following formula, which combines the four components

$$Loss = \sum_i^N V_i(H_i)E_i \quad (4.1.1)$$

In the formula, i indicates any location in the hazard area, N is the total number of locations, E is the exposure at risk, and $V(H)$ represents the relative damage produced by a certain hazard level indicated with H , based on the appropriate vulnerability curve.

Among the four components of a catastrophe model, it is the hazard that models the physics of the peril, and therefore, it is the one that differs most markedly between catastrophe models for different perils.

The catastrophe risk modelling is fundamental to determining the trigger of a CAT bond. The data collected by the risk analysis is used to provide a quantitative method to determine when the catastrophic event is severe enough to exceed a threshold and activate the bond. In addition to determining the trigger of a CAT bond, the risk modelling influences the calibration, structure, pricing and revision over time. It is essential to ensure the reliability of the financial instrument because without an

accurate modelling, the trigger may be ineffective or contestable, making the CAT bond less reliable for investors and insurance companies.

4.1.1 Challenges in Flood Risk Modelling

The Flood risk modelling is particularly challenging. This section will discuss four of the more relevant challenging characteristics associated with flood risk modelling (S. Zanardo, J. L. Salinas, 2021).

The first challenge encountered by flood risk modelling is the high resolution and large scale of flood events. The impact of an extreme flood event is very different from other perils, like wind or earthquake, which usually cause more homogenous damage in a limited area. A single extreme flood event can cause much damage in different countries and at the same time, have extremely high local variability.

Different properties in the same city or also in the same neighbourhood can report different levels of flood damage during an event. This variability is due to factors such as topography and land use.

With respect to other perils models, the flood models are described with a higher resolution to consider each variation in the land with precision. While a wind model has a resolution of a few kilometres, the flood risk models use a resolution from 50-100 metres down to a few metres. This approach, of course, constitutes an important challenge for modelling flood risk, and involves a very elevated computational power and memory.

The second important challenge, that has to be considered in the flood risk modelling, is the spatial and temporal correlation of flood risk. When an extreme flood event occurs, different regions are impacted simultaneously or almost simultaneously. The insurance systems and governments need to understand how losses are distributed in time and space to correctly estimate the aggregate risk. To predict the spatial impact of an extreme flood event, it is appropriate to consider some factors. The precipitation and its patterns are one of the main natural factors influencing flood risk and increasing the risk of large-scale flooding. The other factor is the river networks, because the connections between rivers contribute to the propagation of flood risk. The floods tend to propagate

downstream along rivers, increasing the spatial correlation between regions located near the same river.

Also, the hydrological history of a region can affect the likelihood of future floods. The temporal correlation of flood events can be linked to some factors as the precipitation seasonality and the temporal dynamics of catchment processes.

The third challenge, which needs to be included, refers to the flood defenses. The presence and performance of flood defenses are essential to correctly estimate economic damages. However, the flood defenses are complex and vary from one country to another. The main reason is that some countries invest in costly, permanent dikes along main rivers, which provide long-term protection, while other countries rely on temporary measures of rapid implementation, such as sandbags. Another reason is that in many regions, there is no reliable data on the distribution and characteristics of defenses, including their protection levels. Even when the defense data can be identified correctly, it is still challenging to assess when and how the defense could fail. Since it is impossible to represent all defences accurately, catastrophe risk models use simplified approaches. For example, when the data on defences are not completely available, the protection levels can be estimated indirectly by looking at the urbanisation of areas prone to flood risk. In some cases, the pattern recognition algorithms on digital elevation models (DEMs) are used to identify possible defence structures.

The last challenge is the flood damage assessment. Determining the damages caused by a flood to buildings, infrastructure, and other assets is an essential part of loss modelling.

The flood damages are classified into two main categories: direct and indirect damage. Direct damage is caused by direct contact with water, such as damaged buildings, submerged vehicles or compromised machinery. On the contrary, indirect damage is the consequence of flood events, such as economic losses to a business or psychological impacts on the population. Also, in this case, the data reported is not totally reliable and creates uncertainty. High levels of uncertainty in flood damage models can influence investment decisions in flood protection.

4.2 The aggregate loss process

The aggregate loss is the total sum of losses incurred due to catastrophic events during a period of time that coincides with the duration of the CAT bond.

Modelling the aggregate losses consists of determining the frequency and severity of an extreme event. The frequency of the occurrence of natural disasters is described through the nonhomogeneous Poisson process, while the severity of the events is modelled through the Peaks Over Threshold model using the Generalized Pareto distribution.

The *aggregate loss process* is defined as:

$$L(t) = \sum_{i=1}^{N(t)} X_i \quad (4.2.1)$$

where $\{N(t): t \in [0, T]\}$ is a Poisson process with intensity rate $\lambda_1 > 0$, and $\{X_i: i = 1, 2, \dots, N(t)\}$ is a sequence of independent and identically distributed continuous random variables, representing the i -th loss. $\{N(t): t \in [0, T]\}$ and $\{X_i: i = 1, 2, \dots, N(t)\}$ are assumed to be independent.

4.2.1 Poisson process

The Poisson process is a mathematical model used to compute the number of occurrences of an event over a given interval of time or space. It is commonly used to describe the frequency of extreme catastrophic events.

To properly describe the Poisson process, it is necessary to present the following assumptions (A. F. Mcneil, R. Frey, P. Embrechts, 2015):

1. The probability p of the occurrence of a single event during a short interval, denoted by d , is proportional to its duration, so that $p = \alpha d$, where α is a positive constant representing the average occurrence rate per unit of time, known as the intensity rate.
2. The probability that more than one event will happen in an infinitely small time interval is almost zero, ensuring that the events occur one at a time and not simultaneously.

3. The number of occurrences in one interval is independent of the number of occurrences in other non-overlapping intervals.
4. The number of events occurring in two intervals with the same duration has the same distribution of probabilities.

Given these assumptions, we suppose a counting process $\{N(t), t \geq 0\}$, which is a stochastic process that records the number of catastrophic events that occur in a determined time period $(0, t]$. For all $t \geq 0$, the counting process $N(t)$ is a non-negative and non-decreasing function of time t . When $0 \leq s < t$, $N(t) - N(s)$ denotes the number of extreme events in the time interval $(s, t]$.

The Poisson process can be divided into three categories: the homogeneous Poisson process, the Non-homogeneous Poisson process and the Cox process.

The Homogeneous Poisson process (HPP) assumes that the events occur at a constant rate $\lambda = \alpha t$, which denotes the average number of disasters in one unit time interval. The HPP relies on three properties:

- The distribution of increments $N(t_j) - N(t_{j-1})$, when $0 \leq t_1 < t_2 < \dots < t_k$, depends on the length of the time interval $\Delta t_j = t_j - t_{j-1}$
- The increments $N(t_j) - N(t_{j-1})$ are non-overlapping increments and independent of each other.

The Homogeneous Poisson process is denoted as

$$\mathbb{P}\{N(t) = k\} = \frac{(\lambda)^k \exp(-\lambda)}{k!} \quad (4.2.3)$$

where $k = 0, 1, 2, \dots, \lambda > 0$ and $E[N_t] = \text{Var}[N_t] = \lambda$.

Conversely, the Non-homogeneous Poisson process or NHPP assumes $\lambda(t)$ a function of time t .

$$\mathbb{P}\{N(t) = k\} = \frac{(\lambda)^k \exp(-\lambda)}{k!(1 - \exp(-\lambda))} \quad (4.2.4)$$

where $k = 1, 2, \dots$ and $\lambda > 0$.

The Cox process is different from the other Poisson processes because it determines λ as a stochastic process, representing a rate that changes over time and also follows a random process. The Cox process is represented by the following equation:

$$\mathbb{P}\{N(t) = k\} = \frac{1}{k!} \left(\int_s^t \lambda(u) du \right)^k e^{-\int_s^t \lambda(u) du} \quad (4.2.5)$$

4.3 Extreme value theory

The Extreme Value Theory is a branch of probability theory used to describe the limiting laws for extreme values in large samples. The theory is applied to developing models for extreme behaviour of financial risk factors. It describes upper-order statistics, sample values exceeding high thresholds and the distribution of the maximum and the minimum of a relevant event. For this reason, it is an optimal statistical application to determine the severity of a natural calamity.

The Extreme Value Theory provides two main approaches for the practical extreme analysis. The traditional approach is the Block Maxima Method model (BMM model); instead, the modern method is the Peaks Over Threshold model (POT model). The BMM model is concentrated on the description of the behaviour of the sample, dividing the observation period into non-overlapping, continuous and equal intervals, considering only the maximum observation of each interval. The maxima of each interval can be fitted into the Generalised Extreme Value distribution. This method could not be precise because considering only the maximum observations it is possible to miss some data. The POT model offers a more accurate analysis of data, as it selects all the relevant observations exceeding a certain high threshold. To approximate the observations, the POT model uses the Generalized Pareto Distribution.

To obtain a more realistic estimate of the risk is appropriate to combine the Extreme Value Theory with the Copula function. The Copula function analyzes the dependencies between the variables considered by the Extreme Value Theory.

4.3.1 Block Maxima Method - Generalized Extreme Value distribution

The Block Maxima Method or BMM approach in Extreme Value Theory (Shengyu Zheng, 2022) consists of dividing a given sample of N observations, during a determined period,

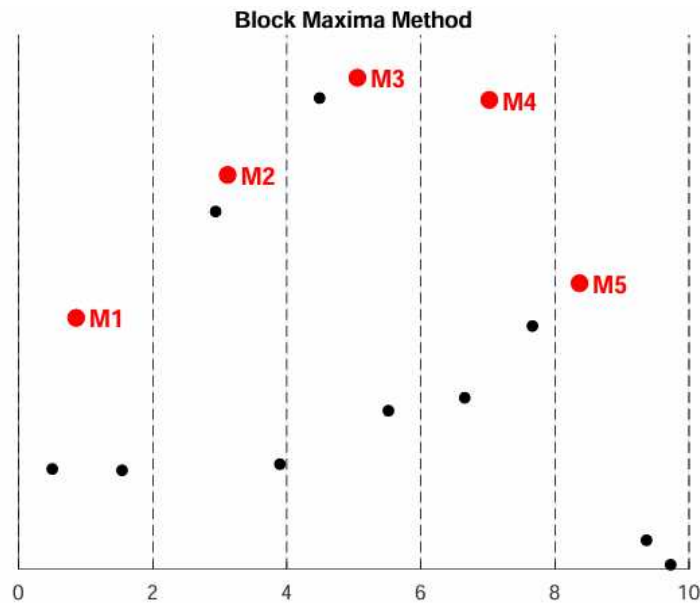
into M non-overlapping intervals with n observations. In each equal interval it is picked the maximum value:

$$M_n = \max(M_1, M_2, \dots, M_n) \quad (4.3.1)$$

An example of the BMM approach is depicted in Figure 4.1.

The distribution of the n maximum intervals M_n can be approximated by the three parameters of the Generalized Extreme Value distribution. Based on the extreme value theorem, the GEV distribution is the limit distribution of properly normalized maxima of a sequence of independent and identically distributed random variables.

Figure 4.1: Illustration of the Block Maxima Method model.



Source: Computation by the author.

Let us suppose that M_n is a sequence of independent and identically distributed random variables, which converge in distribution as $n \rightarrow \infty$ under an appropriate normalization, and let us consider a sequence of constants $\{a_n > 0\}$ and $\{b_n\}$, and a non-degenerate distribution function H such that

$$\lim_{n \rightarrow \infty} P\left(\frac{M_n - a_n}{b_n} < x\right) = \lim_{n \rightarrow \infty} F^n(a_n x + b_n) = H(x) \quad (4.3.2)$$

where F is the maximum domain of attraction of H ($F \in MDA(H)$).

The definition of the Generalized Extreme Value distribution or GEV is given by:

$$H_\xi(x) = \begin{cases} \exp(-(1+\xi x)^{-1/\xi}), & \text{if } \xi \neq 0 \\ \exp(-e^{-x}), & \text{if } \xi = 0 \end{cases} \quad (4.3.3)$$

where $1 + \xi x > 0$, ξ is the tail index of the GEV distribution and H_ξ defines the type of distribution.

The GEV distributions have three subtypes corresponding to different tails according to the value of ξ (see Figure 4.2):

- when $\xi < 0$, it is a Weibull distribution

$$G(x) = \exp\left\{-\left(-\frac{x-b}{a}\right)^\alpha\right\}, \quad x < b, a > 0, \alpha > 0, \quad -\infty < b < +\infty \quad (4.3.4)$$

- when $\xi = 0$, it is a Gumbel distribution;

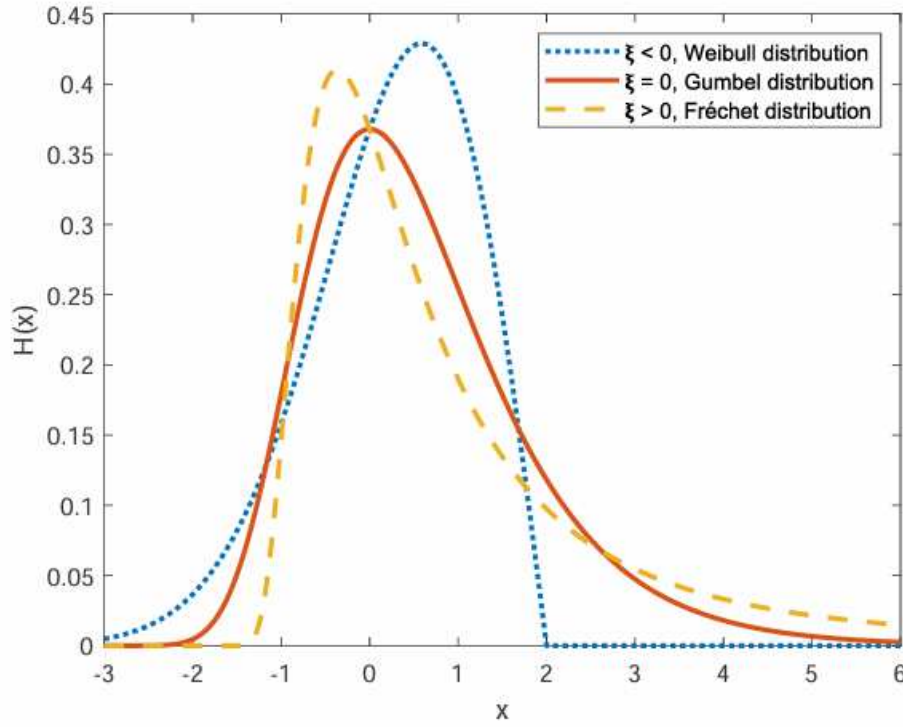
$$G(x) = \exp\{\exp[-(\frac{x-b}{a})]\}, \quad -\infty < x < +\infty, a > 0, -\infty < b < +\infty \quad (4.3.5)$$

- when $\xi > 0$, the distribution is referred to the Fréchet distribution;

$$G(x) = \exp\left\{-\left(\frac{x-b}{a}\right)^{-\alpha}\right\}, \quad x < b, a > 0, \alpha > 0, \quad -\infty < b < +\infty \quad (4.3.6)$$

where $\alpha = \frac{1}{\xi}$

Figure 4.2: Representation of Probability Density functions for Fréchet, weibull and Gumbel distributions.



Source: Computation by the author.

4.3.2 Peaks Over Threshold model- Generalized Pareto Distribution

The peaks-over-threshold approach selects only the observations that exceed a certain high threshold (see Figure 4.3). It is considered more efficient than the BMM model because it can make full use of the extreme value data.

As shown in Wen C. and Zou H. (2018), considering a random sample of $X_i = X_1, X_2, \dots, X_n$, which are independently and identically distributed with a common distribution function $F(x)$. Given a threshold u , the distribution $F_u(y)$ of the excess $Y_{[u]} = X - u$ conditionally on $X > u$, is thus given by

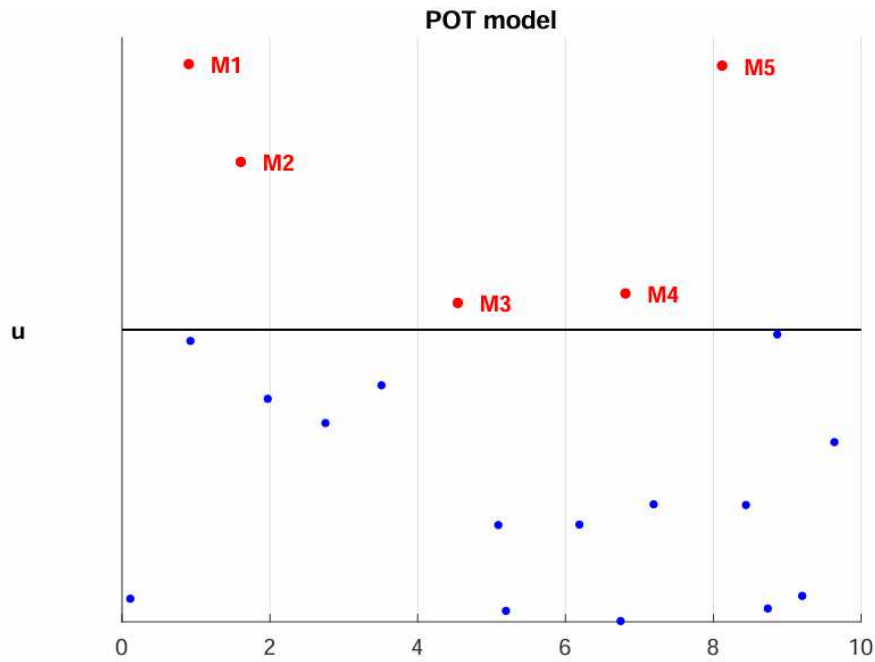
$$F_u(y) = \mathbb{P}\{X - u \leq y \mid X > u\} = \frac{\mathbb{P}\{u < X \leq y + u\}}{\mathbb{P}\{X > u\}} = \frac{F(y+u) - F(u)}{1 - F(u)} \quad (4.3.7)$$

The distribution of the threshold exceedances $Y[u]$ can be approximated by the Generalized Pareto distribution. Considering $y^* = \sup\{x \in \mathbb{R} : F(x) < 1\}$

$$G_{\xi, \sigma}(y) = \begin{cases} 1 - (1 + \frac{\xi y}{\sigma})^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1 - \exp(-\frac{y}{\sigma}), & \text{if } \xi = 0 \end{cases} \quad (4.3.8)$$

where ξ and σ are, respectively, the shape parameter and the scale parameter.

Figure 4.3: Illustration of the Peaks Over Threshold model



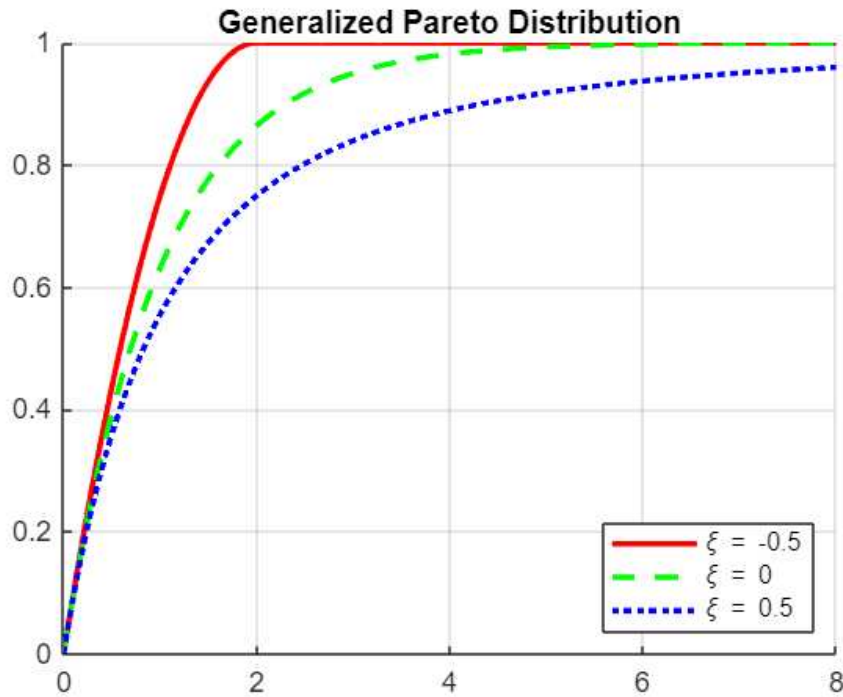
Source: Computation by the author.

Then substituting (4.3.7) into (4.3.8), we obtain

$$F(y) = \begin{cases} (1 - F(u))(1 - (1 + \frac{\xi y}{\sigma})^{-1/\xi}) + F(u), & \text{if } \xi \neq 0 \\ (1 - F(u))(1 - \exp(-\frac{y}{\sigma})) + F(u), & \text{if } \xi = 0 \end{cases} \quad (4.3.9)$$

Figure 4.4 represents the shapes of the Generalized Pareto distribution based on different values of the shape parameter and the scale parameter equal to one

Figure 4.4: Shapes of Generalized Pareto Distribution



Source: Computation by the author.

4.3.3 Copula Function Theory

A copula function is a mathematical tool which describes the dependence among the marginal variables. A d -dimensional copula is a distribution function on $[0, 1]^d$ with standard uniform marginal distributions (A. F. McNeil, 2015). The common notation for a copula distribution function is $C(u) = C(u_1, \dots, u_d)$.

A function is defined as a copula function if three properties are held:

1. $C(u_1, \dots, u_d) = 0$ if $u_i = 0$ any i .
2. $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$ for all $i \in \{1, \dots, d\}$, $u_i \in [0, 1]$.

3. For all $(a_1, \dots, a_n), (b_1, \dots, b_d) \in [0, 1]^d$ with $a_i \leq b_i$ we have

$$\sum_{i_1=1}^2 \dots \sum_{i_d=1}^2 (-1)^{i_1+\dots+i_d} C(u_{1i_1}, \dots, u_{di_d}) \geq 0, \quad (4.3.9)$$

where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in \{1, \dots, d\}$.

The second property indicates that the required marginal distributions are uniform.

For a joint distribution $G(x_1, \dots, x_n)$ with marginal distribution function G , for the i th component, the copula C is determined by

$$C(u_1, \dots, u_d) = G(G_1^{-1}(u_1), \dots, G_n^{-1}(u_n)), \quad (4.3.10)$$

where $(u_1, \dots, u_n) \in [0, 1]^n$ (Y. Tang, C. Ling, C. Wen, 2023).

Archimedean copulas are a special class of copula functions represented by a convex and decreasing generator $\varphi(t): [0, 1] \rightarrow (0, \infty)$ satisfying $\varphi(1) = 0$. The parameter θ controls its tail dependence. The copula function is given by

$$C(u_1, \dots, u_n) = \varphi(\varphi^{-1}(u_1), \dots, \varphi^{-1}(u_n)) \quad (4.3.11)$$

The three common types of Archimedean copulas are illustrated in Table 4.1.

Table 4.1: Archimedean copulas.

Copula	Generator	$C(u_1, \dots, u_m)$	Parameters
Gumbel	$(-\ln t)^\theta$	$\exp\left(-\left(\sum_{i=1}^m (-\ln u_i)^\theta\right)^{1/\theta}\right)$	$\theta \geq 1$
Clayton	$\frac{t^{-\theta}-1}{\theta}$	$\left(\sum_{i=1}^m u_i^{-\theta} - m + 1\right)^{-1/\theta}$	$\theta \geq 0$
Frank	$-\ln \frac{e^{-\theta t}-1}{e^{-\theta}-1}$	$-\frac{1}{\theta} \ln \left(1 + \prod_{i=1}^m \frac{(e^{-\theta u_i}-1)}{(e^{-\theta}-1)^{m-1}}\right)$	$\theta \neq 0$

Source: Y. Tang, C. Ling, C. Wen, *Pricing Multi-event Triggered Catastrophe Bonds Based on Copula-POT Model*, 2023.

4.4 Cox-Ingersoll-Ross Interest Rate Model

The Cox-Ingersoll-Ross model, or CIR model, is a mathematical model applied in finance to describe the behaviour of interest rates. It is an extension of the Vasicek model, introduced by John C. Cox, Jonathan E. Ingersoll and Stephen A. Ross(1985).

The Vasicek model was introduced in 1977 by Oldřich Vašíček (Niko Herrala, 2009, pages 9-11.). This model uses a mean-reverting stochastic process to model the evolution of short-term interest rates, considering a single source of market risk. The innovation brought by Vasicek concerns the mean reversion. This feature explains the tendency of the interest rate to return to the mean long-term equilibrium. For example, considering a market situation with high interest rates causing slowdowns in the economy, borrowers require less funding, bringing the rates to decline to the long-term equilibrium mean. In the opposite situation, with interest rates too low, an increase in borrower demand leads to interest rates rising towards the long-term mean.

The model employs the following stochastic differential equation:

$$dr(t) = a(b - r(t))dt + \sigma dW(t) \quad (4.4.1)$$

where $r(0)$, a , b and σ are positive constants. While the condition $2ab > \sigma^2$ guarantees that the process $r(t)$ is positive.

Being $r(t)$ a diffusion process ensures the efficiency of the market. A diffusion process is a continuous function of time following a Markovian process based on the property that the future developments of the short rate are independent of past movements.

The first part of the formula $a(b - r(t))$ is the drift term of the stochastic differential equation, which represents the expected change of interest rate at time t .

The formula b represents the mean long-term equilibrium value towards which the interest rate reverts. The parameter a is the speed of reversion, governing the speed of adjustment; it is responsible for guaranteeing the stability of the long-term value. If $r > b$ and the coefficient $a > 0$, the drift term is negative and the interest rate falls

towards b . If $r < b$ the drift term became positive for $a > 0$, it would generate a tendency for the interest rate to move toward equilibrium (Niko Herrala, 2009, pages 9-11).

The drift term is the deterministic part instead, the second term is defined as the stochastic part, composed of the parameter σ , which is the volatility and the $dW(t)$ which is a Wiener process, representing the random source of market risk. The stochastic part of the formula aims to capture the instantaneous volatility caused by an infinite number of unpredictable factors.

The CIR model is an extension of the Vasicek model. Although Vasicek's model was the first to capture mean reversion, it allows the interest rate to theoretically become negative. This shortcoming was fixed by Cox, Ingersoll and Ross.

The CIR model is described by the following Stochastic Differential Equation:

$$dr(t) = a(b - r(t))dt + \sigma\sqrt{r(t)}dW(t) \quad (4.4.2)$$

With respect to Vasicek's model, the CIR model adds the $\sqrt{r(t)}$ term, which avoids the possibility of negative interest rates for all positive values of the parameters a and parameter b .

It also avoids the possibility of a zero interest rate. When the rate $r(t)$ is close to zero, the term $\sigma\sqrt{r(t)}$ also became very small and its evolution is dominated by the drift factor, which pushes the rate towards equilibrium.

4.5 Pricing model for the CAT bonds

As highlighted by Cox and Pedersen (2000), Catastrophe bond pricing deviates from traditional methods applied to corporate bonds. Since catastrophic losses occur suddenly and discontinuously, the catastrophic loss process cannot be modelled with continuous distributions (e.g., Brownian motion in Black-Scholes models) like traditional financial processes. The catastrophic loss process is characterised by sudden

jumps, which implies that markets are incomplete and there is no single probability measure for pricing.

The pricing of Catastrophe bonds requires an incomplete market framework because no portfolio of primitive securities replicates the catastrophe risk bond. Fortunately, the uncorrelation of catastrophe risk with other underlying economic variables makes the incomplete market theory simpler than the case of significant correlation.

Despite the simplicity of the incomplete market theory, a model for pricing a CAT bond is still a point to be defined. Many authors have developed models for pricing CAT bonds, but the unpredictability of catastrophe risk has not yet led to the development of a general model such as for corporate bonds.

One of the earliest empirical studies was authored by Lane (2000), who developed a CAT bond pricing model following an actuarial approach. The author based its model on the general agreement that the price of an insurance-linked security is determined by two parts. The first part refers to the financing risk, represented by the risk-free rate (e.g. LIBOR). The second part is related to the insurance risk, represented by a spread adjusted to equivalent risk periods and day counts. This adjusted spread is split into two components: the expected losses (EL) and the expected excess return (EER).

The former compensates the investor for the losses, instead the latter compensates the investor for assuming the risk of the investment.

To represent the loss of a CAT bond with an attachment point α and an exhaustion point $\alpha + h$ (point of last loss), we define a non-negative random variable $X_{(a,a+h]}$ which represents the total loss as follows:

$$X_{(a,a+h]} = \begin{cases} 0, & \text{if } X \leq a, \\ X - a, & \text{if } a < X \leq a + h, \\ h, & \text{if } X > a + h. \end{cases} \quad (4.5.1)$$

From the previous formula, it is intuitive that if X is less than or equal to the attachment point, then no loss occurs. If the X lies between the attachment point and the exhaustion

point, then the loss is given by $X - a$. If X is greater than the attachment point, then the loss equals the exhaustion point less a .

The Expected Loss (EL) is expressed as the integral of the Survival function:

$$EL(X) = \int_0^{\infty} S_x(x) dx \quad (4.5.2)$$

where $S(x)$ is the Survival Function of the bond.

The simplest form of the Survival Function is given by:

$$S(x) = \alpha \exp(-\beta x^\gamma) \quad (4.5.3)$$

where α and β are the parameters to be determined.

From the Survival function, it is possible to determine the Probability of First Loss (PFL) and the Probability of Exhaustion (PE).

$$S(0) = \alpha = \text{Probability of First Loss (PFL)} \quad (4.5.4)$$

$$S(1) = \alpha * \exp(-\beta) = \text{Probability of Exhaustion (PE)} \quad (4.5.5)$$

The Expected loss can be rewritten as:

$$EL = \int S(x) dx = \int \alpha \exp(-\beta x) dx = \alpha/\beta (1 - \exp(-\beta)) \quad (4.5.6)$$

where β can be solved as PFL and $\beta = \ln(PFL/PE) = \ln(CEL)$.

Lane(2000) defines the Expected Excess Return(EER) as “the expected profit on the transaction over and above his financing return and expected losses”, and represents it

as a relation between the frequency and the severity of the extreme event. Let us express the EER as a Cobb-Douglas Production function:

$$ERR = \gamma(PFL)^\alpha \times (CEL)^\beta \quad (4.5.7)$$

In the formula, the frequency is expressed by the Probability of First Loss (PFL), instead, the severity is defined by the Conditional Expected Loss (CEL).

$$CEL = EL \div PFL \quad (4.5.8)$$

The simplest model reported by Lane assumes the spread to be a multiple of the EL and EER, hence, the price of the bond can be expressed as:

$$p(x) = LIBOR + Spread \quad (4.5.9)$$

with

$$Spread = EL + EER \quad (4.5.10)$$

$$Spread = PFL \times CEL + \gamma(PFL)^\alpha \times (CEL)^\beta \quad (4.5.11)$$

Obtaining

$$p(x) = LIBOR + PFL \times CEL + \gamma(PFL)^\alpha \times (CEL)^\beta \quad (4.5.12)$$

Further important references in the literature relating to actuarial models applied to Cat Bond pricing and catastrophe risk management, were conducted, firstly, by Bantwal & Kunreuther (2000) and subsequently by Wang (2004). Their studies focus on the investor's decisional behaviours under conditions of extreme risk and uncertainty. Based on the theoretical assumption that, from a risk perspective, CAT bond prices should reflect the expected loss associated with the trigger event, it is noted that the occurrence of a catastrophic event is inherently uncertain and that investors are not purely risk-neutral. Therefore, the price of CAT bonds is influenced not only by the expected loss but also by a number of factors.

Lane (2000) had previously explored the pricing of Catastrophe bonds, providing a robust model based on the frequency and severity of losses. His work suggests that market prices for CAT bonds reflect a risk premium above the expected loss, as investors demand greater compensation for bearing an unhedgeable catastrophic risk.

Building upon this foundation, Wang (1998, 2000, 2004) developed a method that has been shown to reconcile financial and insurance approaches for the pricing of CAT bonds. He introduced a distortion of the cumulative distribution function of the loss variable using a transformation known as the Wang Proportional Hazard (P-H) transform.

Wang transforms the Survival function with the distortion operator, as follows:

$$S_x^+(x) = (\Phi(\Phi^{-1}(S_x(x)) + \lambda) \quad (4.5.13)$$

where Φ^{-1} is the standard normal distribution function.

The Wang P-H transform for a loss variable X is given as follows:

$$\Pi_p(X) = E[\Pi_p(X)] = \int_0^\infty (S(x))^{1/p} dx \quad (4.5.14)$$

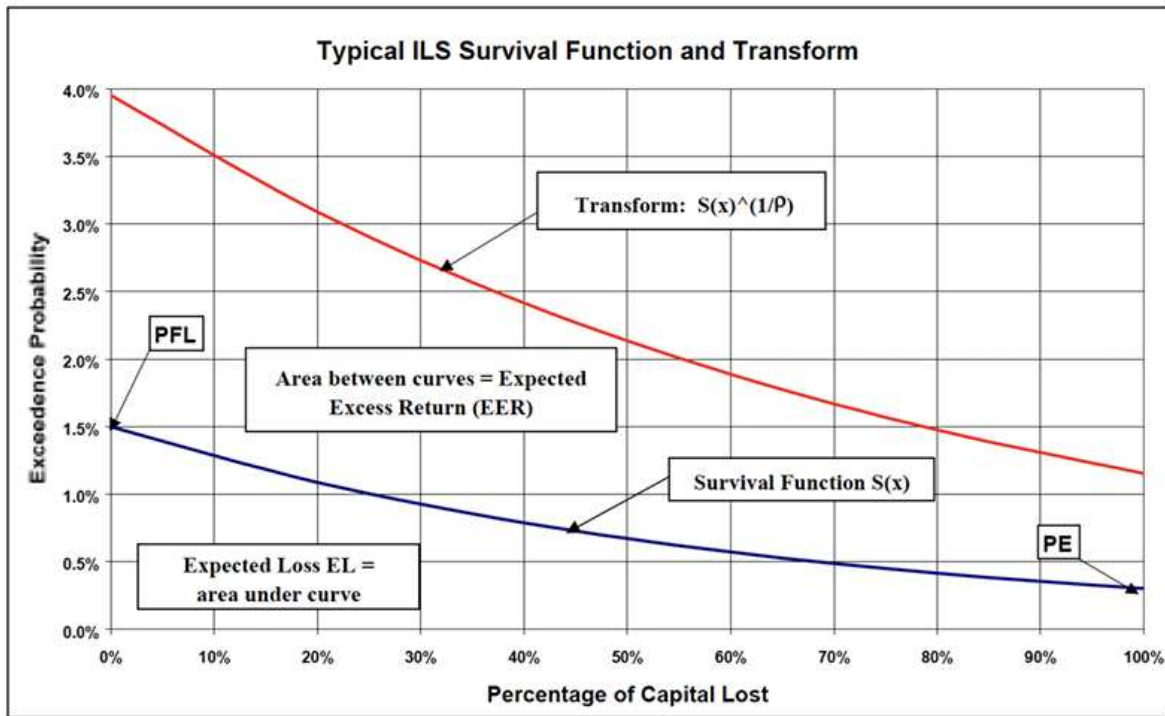
where $\Pi_p(X)$ is the risk-adjusted premium at a Risk aversion level (RAL) with $p \geq 1$, $S(x) = \mathbb{P}(X > x)$ is the Survival Function of X. From the formula is notable that the price increases as the risk aversion level (p) increases.

The use of the distortion function captures the asymmetry of risk and allows for obtaining the best-estimate distribution according to the levels of uncertainty, market competition, and portfolio diversification. In Figure 4.5, it can be seen how the P-H transform proposed by Wang changes and adjusts the Survival Function to a real value. The area between the two lines can be interpreted as the risk loading or Expected Excess Return (EER).

Another study describing market prices of CAT bonds was developed by Bodoff and Gan (2009). They provide a framework for measuring the price, modelling spreads as a

linear function of the expected loss of the bond, which varies by peril and geographical area. Starting with the model of Lane(2000), Bodoff and Gan argued that the compensation for the risk exposure is expressed as an additional rate of return.

Figure 4.5: ILS Survival Function and its P-H transform.



Source: Stavros Christofides, 2004.

In this model, the spread of a CAT bond depends on two factors: the uncertainty of the estimate of the expected losses and the required rate of return on downside risk capital in a portfolio context.

$$\begin{aligned} \text{Spread \%} &= \text{expected loss \%} \\ &+ \text{peril specific required rate of return on capital \%} \end{aligned} \quad (4.5.15)$$

$$\text{Spread \%} = \text{expected loss \%} + \text{peril specific margin \%} \quad (4.5.16)$$

The model demonstrates that if the expected loss increases, then the spread and the margin also tend to increase.

$$\begin{aligned} \textit{Peril specific margin \%} = & \textit{peril specific flat margin \%} \\ & + \textit{peril specific factor} \times \textit{expected loss \%} \quad (4.5.17) \end{aligned}$$

Combining equations (4.5.16) and (4.5.17), we obtain:

$$\begin{aligned} \textit{Spread \%} = & \textit{expected loss \%} + \textit{peril specific flat margin \%} \\ & + \textit{peril specific factor} \times \textit{expected loss \%} \quad (4.5.18) \end{aligned}$$

Catastrophe bond pricing is a complex process influenced by various theoretical models. Although the well established models of Lane and Wang, a universal approach has not yet been developed due to the unpredictable nature of catastrophe risk. The difficulty in modelling catastrophe risk leads the market to constantly evolve.

Chapter 5: Empirical analysis: Pricing flood multi-trigger catastrophe bond with risk simulation

The prominence of catastrophic risk is linked to the rise in fame of CAT bonds as financial tools to mitigate catastrophic risks.

Since the increased attention required by catastrophic events, because of their devastating consequences, the Italian government recently introduced a new regulation, the Budget Law 2024 (L. 213/2023), with the objective of protecting the national productive fabric from catastrophic events. This measure is reserved exclusively for commercial establishments, it obliges them to purchase insurance against natural disasters and catastrophic events.

Floods are one of the most common and dangerous catastrophes in Europe, particularly in Italy. For this reason, the conducted empirical analysis has the objective of determining the price of a flood multi-trigger CAT bond for each region of Italy.

Considering the nonlinear dependency and the tail behaviour of the triggered indexes of a multiple-trigger catastrophe bond, this analysis applies the Extreme Value Theory and the Copula function for the pricing of a multiple-trigger CAT bond. It applies the CIR-Copula-POT pricing model, incorporating a CIR stochastic interest rate and estimates the bond price using the Monte Carlo simulation method.

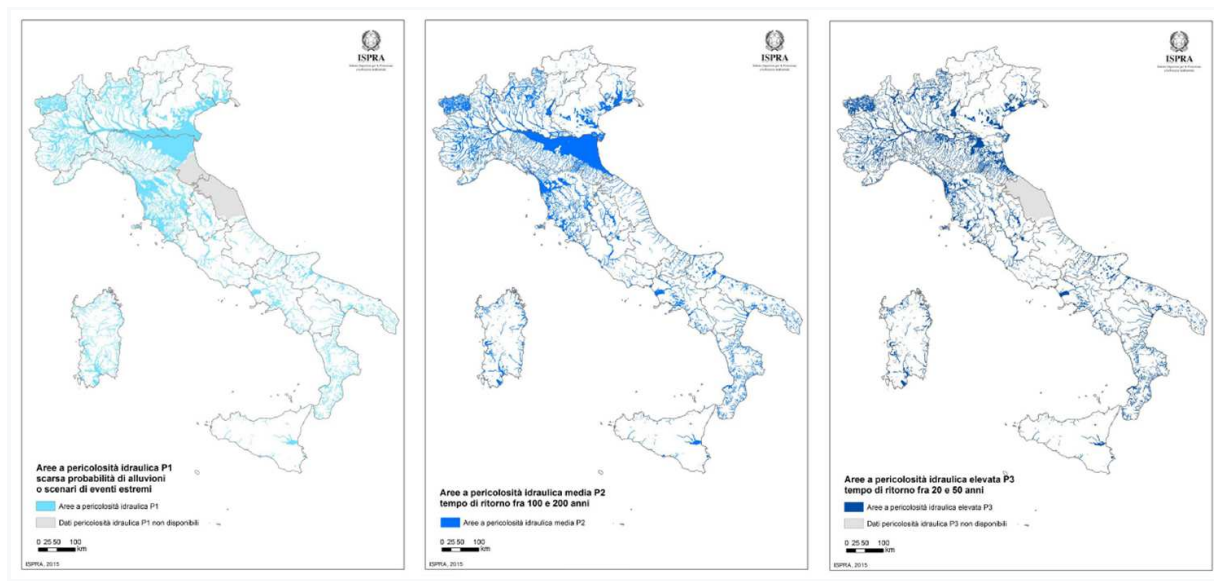
5.1 Flood risk in Italy

Italy is one of the European countries particularly exposed to hydrogeological risk and severely affected by floods. The geomorphological characteristics and intense urbanization, which characterize the country, are the main reasons. Even if the Italian government developed a flood risk management strategy, the flood risk continues to be underestimated.

Hydrological risk is more relevant in Italy than in other countries for the population, environment, cultural heritage, infrastructure and economic and productive impacts. Italy is characterized by an intense urbanization, and as documented in many reports, floods have a severe and dramatic impact on the country.

In Figure 5.1, all the regions exposed to hydrological risk are highlighted. The image on the left shows the areas at low risk, the middle image the areas at medium risk and the image on the right the areas exposed to high risk. As a result, the Italian environment has to hedge against a serious risk of flooding.

Figure 5.1: Hydrological risk exposure of Italian regions.



Source: Ivan Faiella, Filippo Natoli, Natural catastrophes and bank lending: the case of flood risk in Italy, *Questioni di Economia e Finanza*, 2018.

As shown in Figure 5.1, it is very intuitive to define Italy as a country very exposed to flood risk, both in the areas of Northern Italy that tend to be more affected by storms and river floods, and in the areas of Southern Italy surrounded by the sea. The main reason is the meteo-climatic, topographic, morphological and geological characteristics of the country, but also the increasing urbanization, which after the Second World War, increased the elements exposed to risk in areas subject to landslides and floods. The ISPRA's report (*Dissesto idrogeologico in Italia: pericolosità e indicatori di rischio*, 2021) stated that “artificial surfaces have gone from 2.7% in the 1950s to 7.11% in 2020”, which represents a big change for the country, also supported by the abandonment of rural mountain and hill areas.

Also, the climate changes in progress are increasing the catastrophic risk in Italy, caused by the increasing frequency of rainfall events.

Another important reason is the lack of regulation. Many buildings are located in high-risk areas where the occurrence of a catastrophic event can cause important damage.

5.2 Budget Law 2024 (L. 213/2023)

On 30 December 2023, the Italian government published the Budget Law 2024 (L. 213/2023) in the "*Gazzetta Ufficiale della Repubblica Italiana*". This law is a fundamental and innovative measure that defines the financial policy of the state for the current year (2024) and plans revenues and expenses for the following three years (2024-2026). It regulates many aspects, but in particular, this law addresses the new challenge against catastrophic events, introducing the obligation for all companies resident in Italy and listed on the "Registro delle imprese" to purchase insurance against catastrophic events. Catastrophic events include earthquakes, floods and landslides.

The reason for introducing this measure, as reported by Unipol, is that "Italy is one of the European countries with the highest seismic risk and hydrogeological disturbance, with almost 94% of municipalities at risk of landslides, floods or coastal erosion. There are 4.5 million active enterprises in these municipalities".

The Confindustria Research Centre has estimated that 30-40% of small and medium enterprises could suffer losses of 5-10% by 2040 due to business interruption for damages caused by catastrophic events.

5.3 The maximum likelihood estimation of the Cox-Ingersoll-Ross process

In the context of the valuation of financial instruments linked to catastrophic events, such as multi-trigger catastrophe bonds, it is essential to also consider the uncertainty linked to the financial markets. In particular, to describe stochastic interest rate developments, it is appropriate to use the Cox-Ingersoll-Ross term structure model (1985). It is considered a fundamental element for the current calculation of future cash flows associated with the bond.

This continuous time model, based on one or more stationary processes, permits modelling the interest rates' evolution and avoiding the possibility of negative values.

The model is represented by the stochastic differential equation:

$$dr(t) = a(b - r(t))dt + \sigma\sqrt{r(t)}dW(t) \quad (5.3.1)$$

If the a, b and σ parameters are positive constants, the condition $2ab > \sigma^2$ is guaranteed and the process follows a gamma distribution.

In this analysis, the Cox-Ingersoll-Ross (CIR) model (1985) has been adopted in the formulation proposed by Kamil Kladivko (2007) for the modelling of the evolution of interest rates.

The version proposed by Kamil Kladivko (2007) is distinguished for investigating the maximum likelihood estimation of the square root process (CIR process) for interest rate time series.

In order to estimate the parameters with maximum likelihood, transition densities with a closed-form expression are necessary. Given r_t at time t , the transition density of $r_{t+\tau}$ at time t is defined as:

$$p(r_{t+\tau}|r_t; \tau) = c \cdot e^{-u-v} \left(\frac{v}{u}\right)^{\frac{q}{2}} I_q(2\sqrt{uv}) \quad (5.3.2)$$

where

$$c = \frac{2k}{\sigma^2(1-e^{-k\tau})}$$

$$u = cr_t e^{-k\tau}$$

$$q = \frac{2k\theta}{\sigma^2} - 1$$

and $I_q(2\sqrt{uv})$ is a modified Bessel function of the first kind and of order q .

The maximum likelihood function for an interest rate time series with N observations $\{r_{t_i}, i = 1, \dots, N\}$, equally spaced with Δt time steps, is given by the formula:

$$L(\theta) = \prod_{i=1}^{N-1} p(r_{t_{i+1}}|r_{t_i}; \theta, \Delta t) \quad (5.3.3)$$

To simplify, it is computationally convenient to use the lognormal form of the formula:

$$\ln L(\theta) = \sum_{i=1}^{N-1} p(r_{t_{i+1}} | r_{t_i}; \theta, \Delta t) \quad (5.3.4)$$

We obtain the log-likelihood function of the CIR:

$$\ln L(\theta) = (N - 1) \ln c + \sum_{i=1}^{N-1} \left\{ -u_{t_i} - v_{t_{i+1}} + 0.5q \ln \left(\frac{v_{t_{i+1}}}{u_{t_i}} \right) + \ln \{ I_q(2\sqrt{u_{t_i} v_{t_{i+1}}}) \} \right\} \quad (5.3.5)$$

where $u_{t_i} = cr_{t_i} e^{-\alpha \Delta t}$ and $v_{t_{i+1}} = cr_{t_{i+1}}$

Following the algorithm proposed in Kladienko (2007), the Maximum Likelihood estimates $\hat{\theta}$ of the parameter vector θ by maximising the log-likelihood function (5.3.5) it is obtained by solving the following optimisation problem:

$$\hat{\theta} \equiv (\hat{\alpha}, \hat{\mu}, \hat{\sigma}) = \arg \max_{\theta} \ln L(\theta) \quad (5.3.6)$$

As reported by Kladienko (2007), since the logarithmic function is monotonically increasing, maximising the log-likelihood function also maximises the likelihood function.

In the process of calibration of the CIR model to the data, to guarantee the convergence of the global optimisation, it is important to have a good initial value of the parameters $\hat{\alpha}, \hat{\mu}, \hat{\sigma}$.

The parameters were first estimated with the Ordinary Least Squares (OLS) method, using the formula:

$$r_{t+\Delta t} - r_t = \alpha(\mu - r_t) \Delta t + \sigma \sqrt{r_t} \varepsilon_t \quad (5.3.7)$$

where ε_t is a white noise process, normally distributed, with zero mean and variance Δt .

Dividing both parts for $\sqrt{r_t}$ we obtain:

$$\frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} = \frac{\alpha\mu\Delta t}{\sqrt{r_t}} - \alpha\sqrt{r_t}\Delta t + \sigma\varepsilon_t \quad (5.3.8)$$

Minimising the OLS objective function, we obtain the drift initial estimates:

$$(\hat{\alpha}, \hat{\mu}) = \arg \min_{\alpha, \mu} \sum_{i=1}^{N-1} \left(\frac{r_{t_{i+1}} - r_{t_i}}{\sqrt{r_{t_i}}} - \frac{\alpha\mu\Delta t}{\sqrt{r_{t_i}}} + \alpha\sqrt{r_{t_i}}\Delta t \right)^2 \quad (5.3.9)$$

The initial estimate of the diffusion parameter $\hat{\sigma}$ is determined as the standard deviation of the residuals. The initial values of the parameters $\hat{\alpha}$, $\hat{\mu}$, $\hat{\sigma}$ are shown in Table 5.1.

Table 5.1: The initial value of parameters $\hat{\alpha}$, $\hat{\mu}$, $\hat{\sigma}$.

alpha ($\hat{\alpha}$)	mu ($\hat{\mu}$)	sigma ($\hat{\sigma}$)
+0.287969	+0.350287	+0.038694

The objective function (5.3.5) is then adjusted by evaluating the modified Bessel function $I_q(2\sqrt{uv})$ as $I_q(2\sqrt{uv})\exp(-2\sqrt{uv})$:

$$\ln L(\theta) =$$

$$(N - 1) \ln c + \sum_{i=1}^{N-1} \left\{ -u_{t_i} - v_{t_{i+1}} + 0.5q \ln \left(\frac{v_{t_{i+1}}}{u_{t_i}} \right) + \ln \{ I_q(2\sqrt{u_{t_i} v_{t_{i+1}}}) \} + 2\sqrt{u_{t_i} v_{t_{i+1}}} \right\} \quad (5.3.10)$$

Kladivko (2007) proposed a second method to implement the log-likelihood function in MATLAB. Kladivko substituted the Bessel function with a more numerically stable and preferred approach, which is the noncentral X^2 probability density function (PDF).

The results of estimating the CIR process for a time series of weekly observations of the Euribor from 2nd January 2023 to 21st March 2025 (Appendix II) are reported in Table 5.2.

Table 5.2: Estimated values of parameters $\hat{\alpha}$, $\hat{\mu}$, $\hat{\sigma}$.

	alpha ($\hat{\alpha}$)	mu ($\hat{\mu}$)	sigma ($\hat{\sigma}$)	Log-likelihood ($\ln L$)
ML besseli	+0.280344	+0.350786	+0.038714	+4.360375

Using these values, we computed the simulation of the future values of Euribor interest rates.

5.4 Data

In this empirical analysis, the data reported by ISPRA (Istituto Superiore per la Protezione e la Ricerca Ambientale) between 2000 and 2022 have been considered. ISPRA is a public research body operating in Italy for the protection and safeguarding of the environment, with the aim of researching, monitoring, and evaluating the environment, as well as managing and disseminating environmental information.

The dataset considered in this analysis refers to two indicators: the number of victims caused by floods and the annual amount of rainfall in millimetres recorded in each Italian region (data collected by the SINAnet Groupware¹⁶).

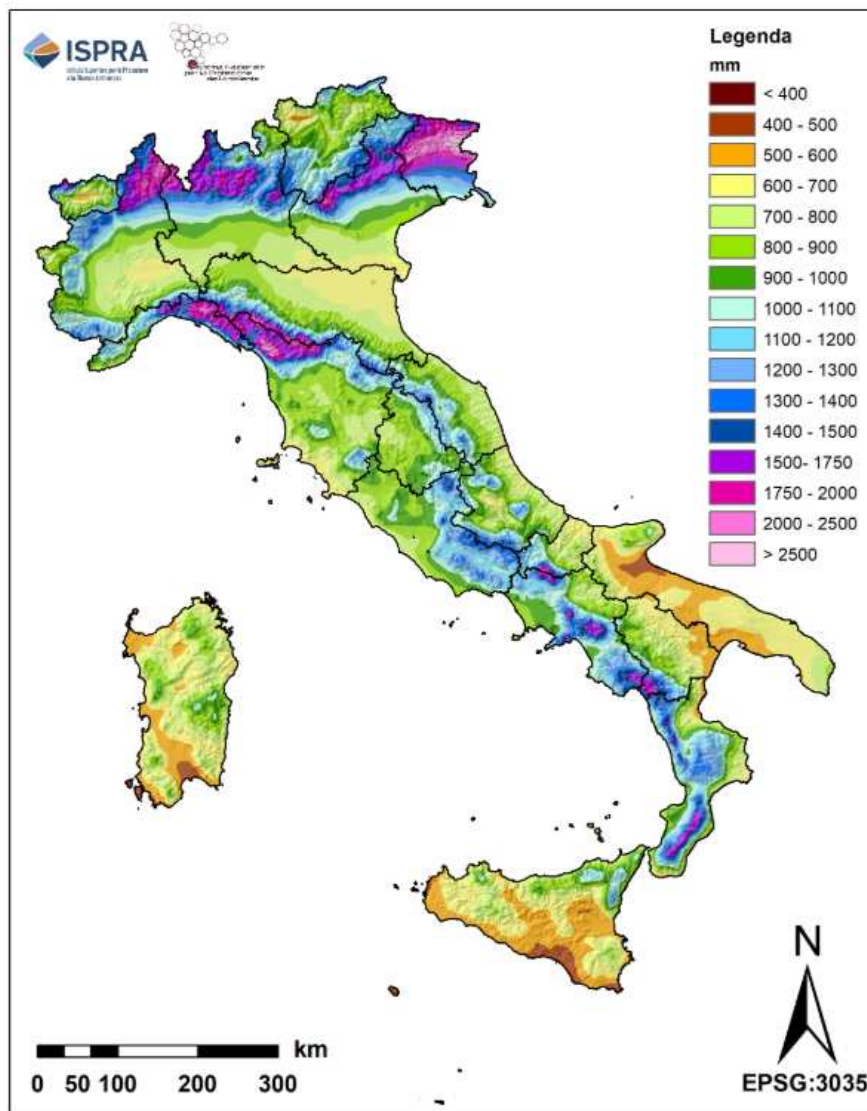
The choice to focus on the period between 2000 and 2022 stems from the availability of data with a higher resolution, due to improved collection of data on catastrophic events in recent years.

Given the significant climatic and meteorological heterogeneity between the different regions of the country, this analysis seeks to analyze each region separately with the aim of determining the price of a multi-trigger CAT bond for each individual region. The use of a multi-trigger structure for CAT bonds was driven by a combination of: historical impact of floods, recorded mortality and heterogeneity in precipitation exposure. As illustrated in Figure 5.3, which represents the annual average rainfall in millimetres between 1951 and 2023, Italy has regions more exposed to heavy rainfall than others,

¹⁶ Collaborative online system used within the National Environmental Information System (SINA), managed by ISPRA.

however, a lower level of rainfall doesn't mean less exposure to extreme events. In particular regions which present a lower annual rainfall, such as Sicily or Puglia (see Table 5.3), still have a high number of victims caused by floods.

Figure 5.3: Representation of the annual average rainfall millimetres by region between 1951–2023 by ISPRA.



Source: ISPRA, 2023,

<https://indicatoriambientali.isprambiente.it/it/risorse-idriche-e-bilancio/precipitazio>

Table 5.3: Representation of the average rainfall millimetres and number of victims by region in a year in Italy.

Regions	mm rained	Victims	Analysed regions
Abruzzo	20733.2	3	
Basilicata	18979.9	4	
Calabria	24152.1	36	✓
Campania	25349.8	10	
Emilia Romagna	19744.9	8	✓
Friuli Venezia Giulia	40637.9	5	✓
Lazio	22247.2	4	
Liguria	29821.0	30	✓
Lombardia	24813.5	9	✓
Marche	20783.0	18	✓
Molise	21470.8	0	
Piemonte	23596.4	44	✓
Puglia	15110.9	16	✓
Sardegna	15526.2	29	✓
Sicilia	16078.2	28	✓
Toscana	22858.1	29	✓
Trentino	24108.8	1	
Umbria	20260.1	2	
Valle d'Aosta	22535.1	0	
Veneto	25748.7	7	✓

Table 5.3 presents all the regions with the respective data of rainfall in millimetres and victims between 2000 and 2022. The regions shown in grey are those regions which have obtained reliable values to obtain a robust analysis, while those in yellow are these regions analysed in depth to which a presentation has been devoted in the following paragraph.

An attempt has been made to focus the analysis on those regions which tend to have many extreme events¹⁷. As shown in Table 5.3, it is not surprising that a region like Sicily has registered many extreme events like Liguria, despite their different regional climates.

5.5 Empirical Analysis

To illustrate the pricing mechanism applied, this section develops an accurate description of the empirical analysis carried out for the pricing of the multi-trigger CAT bond using two main risk indicators: rainfall and victims of floods in Italy. For a more accurate description of the analysis, we report the results of the two more significant regions, Lombardy and Sicily. Section 5.5.1 presents descriptive statistics for the two indicators. In section 5.5.2, the Extreme Value Theory is elaborated, given the non-normality of the data, the Peaks-Over-Threshold method and the Copula function are applied. Finally, in section 5.5.3, the formulas used for the pricing of the multi-trigger CAT bond are calculated.

5.5.1 Descriptive Analysis of Data.

We conducted the description analysis of the two risk indicators used. From Table 5.4, it is reported that for the Lombardy region, the distribution of precipitation shows a moderate symmetry with a slight tendency to the right and a Kurtosis lower than normal, suggesting a low frequency of extreme events. While the distribution of the number of victims shows a strong positive asymmetry and a high Kurtosis, typical characteristics of rare but potentially severe phenomena, with a clear prevalence of

¹⁷ With extreme events the author wants to indicate the triggers' activation for victims of floods and rainfall millimeters that exceed the threshold set at 80th percentile (it will be explained more precisely in the next paragraph).

events without victims. These results are graphically consistent with the exponential QQ plots in Figure 5.4.

On the contrary, from Table 5.5, it is reported that for the Sicily region, the distribution of precipitation is substantially symmetrical and characterised by a kurtosis lower than normal, suggesting a concentration around the average values and a low incidence of extreme events.

While the number of victims presents a strongly asymmetrical distribution and high kurtosis, with a predominance of low mortality events, but a significant probability of occurrence of serious events. These results are graphically consistent with the exponential QQ plots in Figure 5.5.

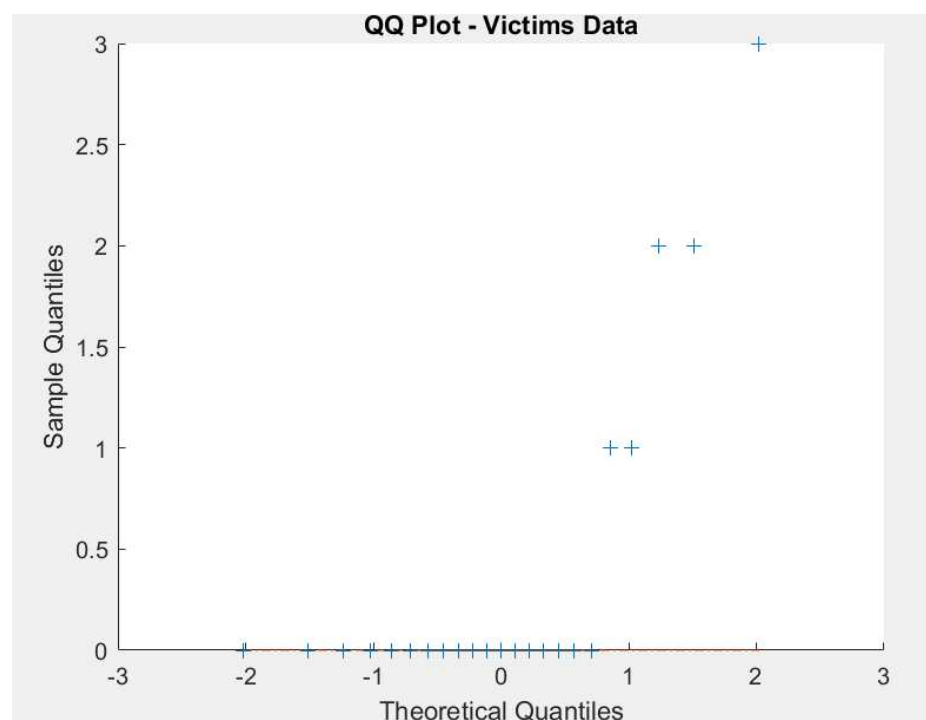
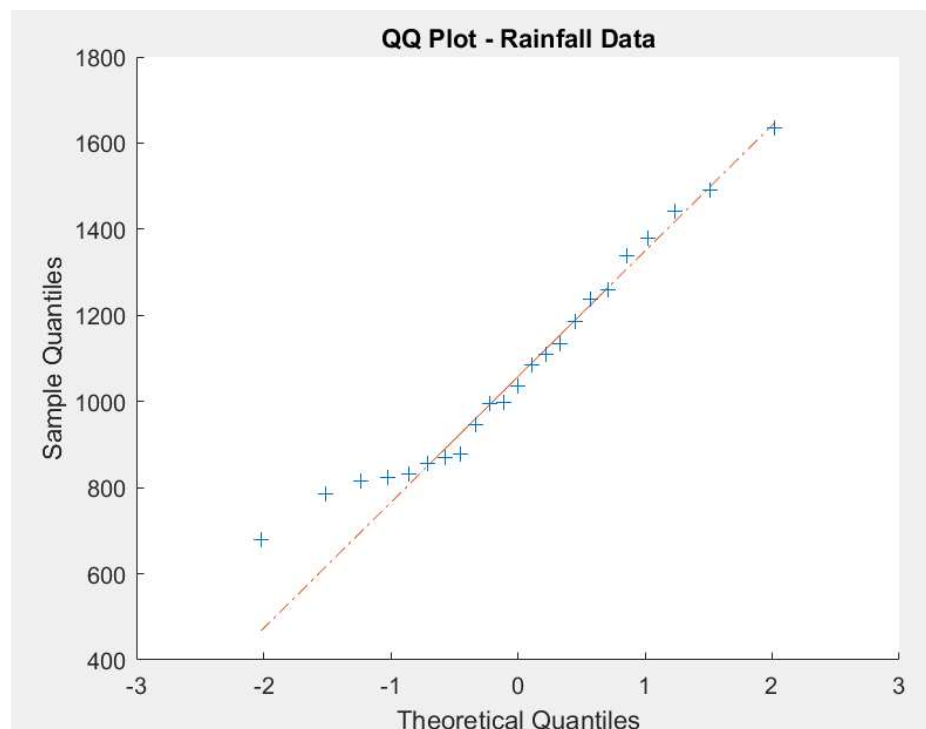
Table 5.4: Descriptive statistics of the data for the Lombardy region.

Statistics	Rain	Victims
{ 'Maximum' }	1634.25	3.00
{ 'Minimum' }	678.68	0.00
{ 'Mean' }	1078.85	0.39
{ 'Median' }	1036.39	0.00
{ 'Skewness' }	0.48	2.00
{ 'Kurtosis' }	2.28	5.80

Table 5.5: Descriptive statistics of the data for the Sicily region.

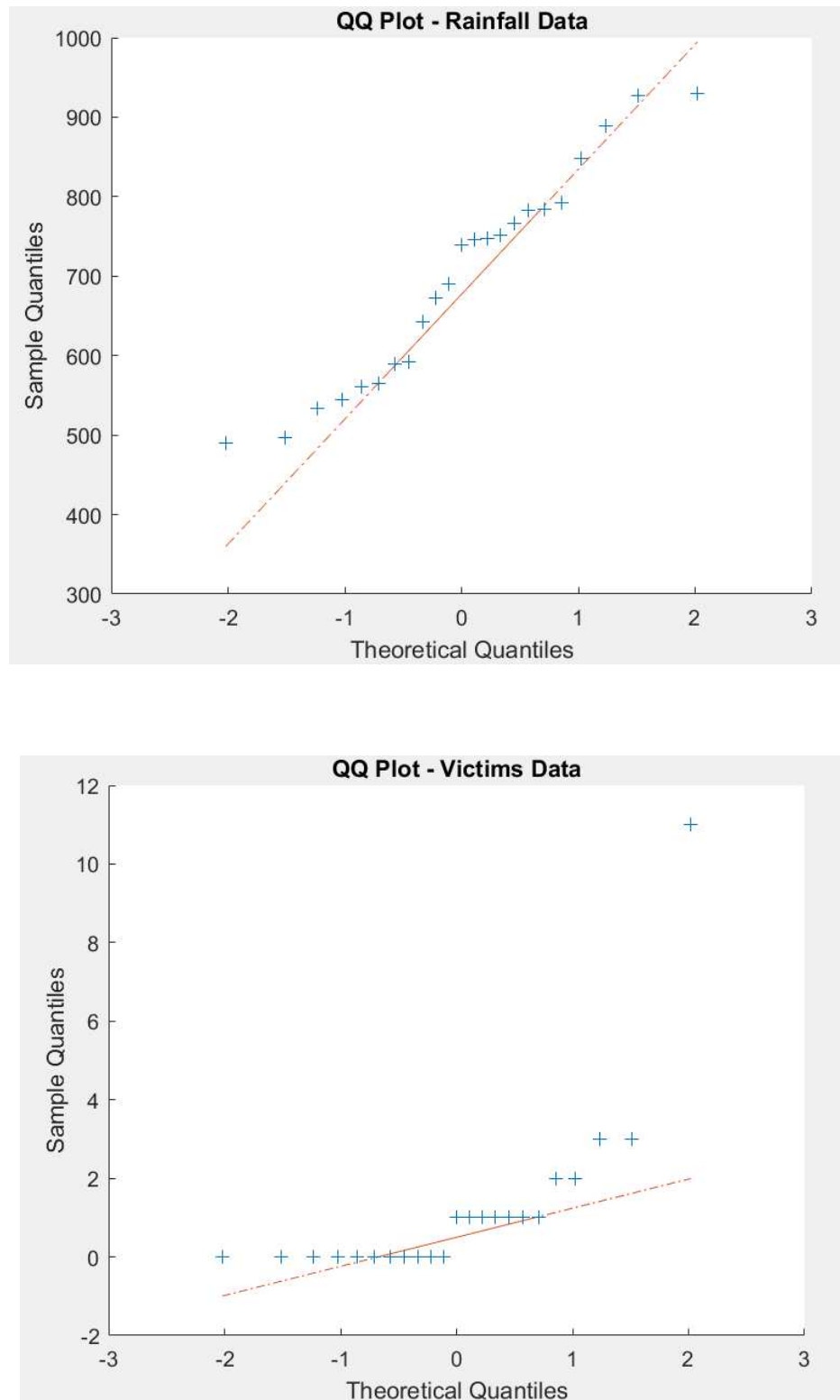
Statistics	Rain	Victims
{ 'Maximum' }	929.37	11.00
{ 'Minimum' }	489.74	0.00
{ 'Mean' }	699.05	1.22
{ 'Median' }	738.69	1.00
{ 'Skewness' }	0.07	3.39
{ 'Kurtosis' }	1.93	14.72

Figure 5.4: QQ plot for the Lombardy region.



Source: computed by the author.

Figure 5.5: QQ plot for the Sicily region.



Source: computed by the author.

5.5.3 Application of Extreme Value Theory

The thresholds for triggering the two triggers, relative to the millimetres of rain and the number of victims, respectively, have been set at the 80th percentile of the historical distribution of data. The analysis considered various thresholds between the 70th and 90th percentiles; based on the results obtained, the 80th percentile was selected as the most suitable threshold. In order to determine the two threshold levels, the mean residual life plots in Figure 5.6 for the region Lombardy and in Figure 5.7 for the Sicily region have been examined. Then, we estimate the stability of the scale and shape parameters (see Table 5.6) involved in the Generalized Pareto distribution of the threshold excesses.

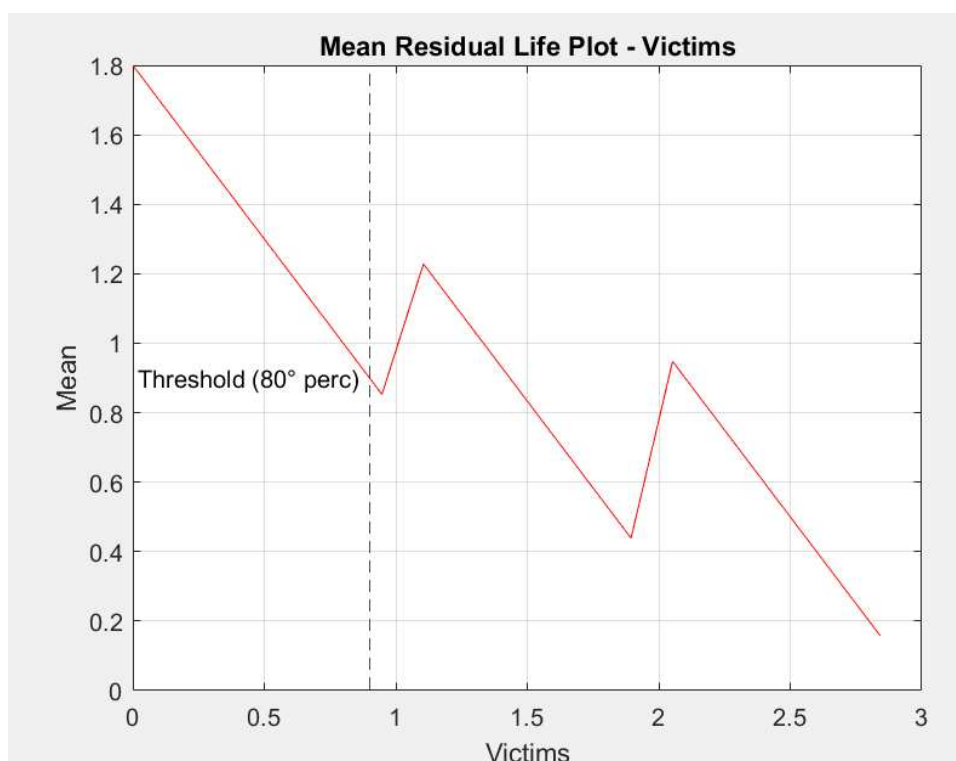
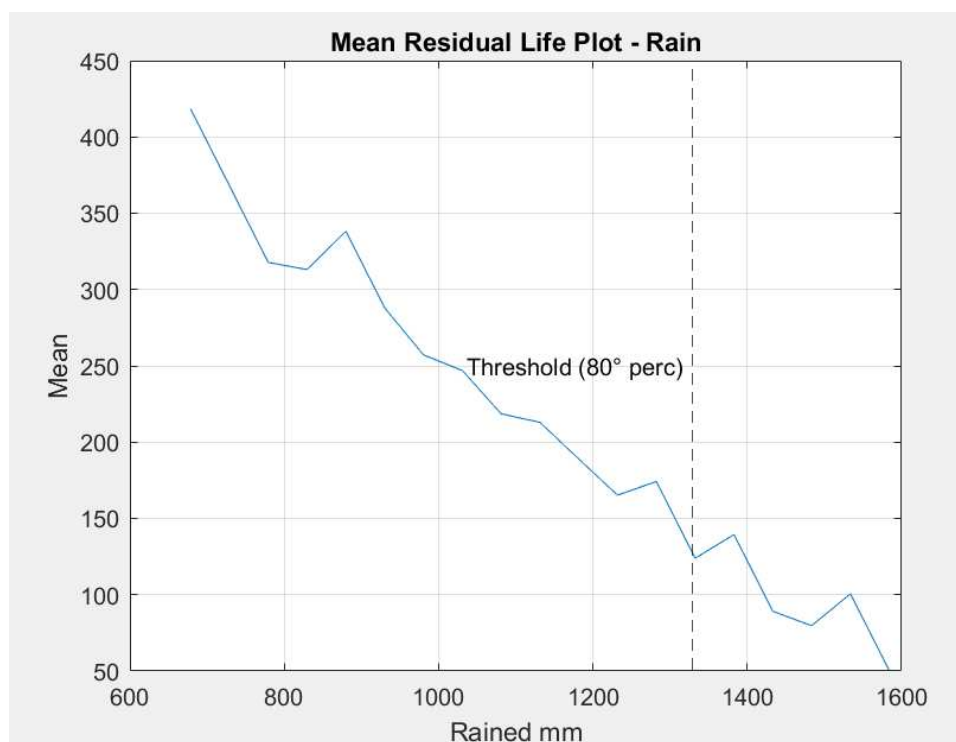
Table 5.6: Parameters of the Generalized Pareto distribution estimated from the residual data.

Parameters	Lombardy	Sicily
Parameter k (rain)	-1.1225	-1.4189
Parameter k (victims)	341.9238	195.6511
Parameter sigma (rain)	-1.0945	1.1149
Parameter sigma (victims)	2.2985	0.5727

Table 5.6 reports all the estimated values for the scale(k) and shape (sigma) parameters of the Generalized Pareto Distribution (GPD) for two regions: Lombardy and Sicily.

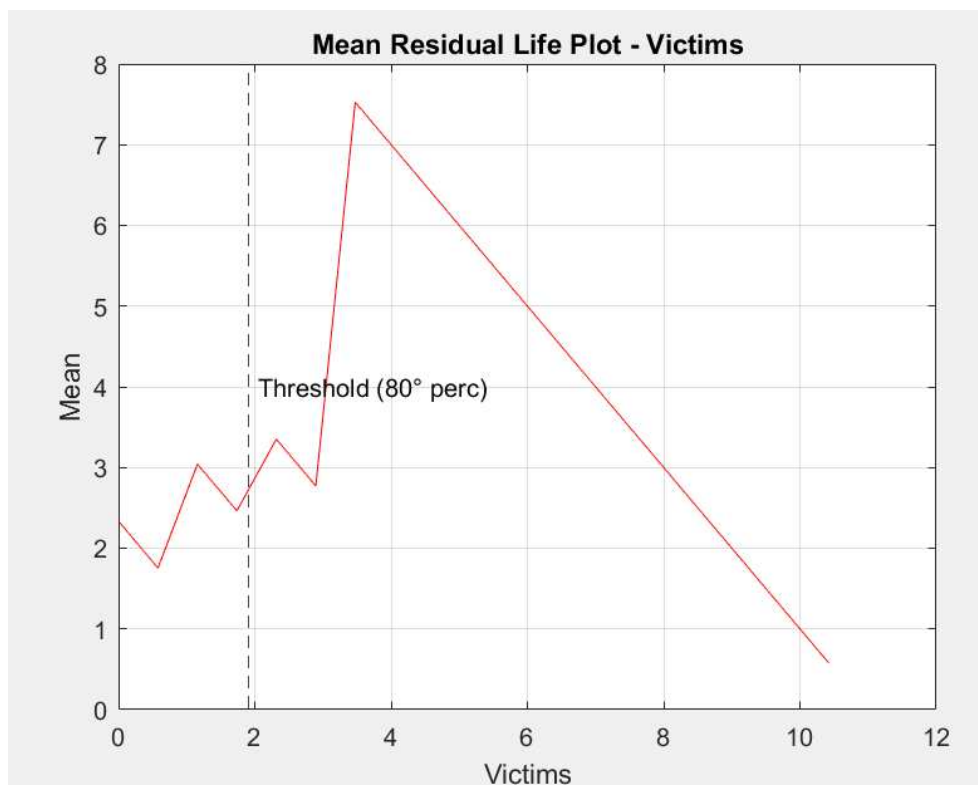
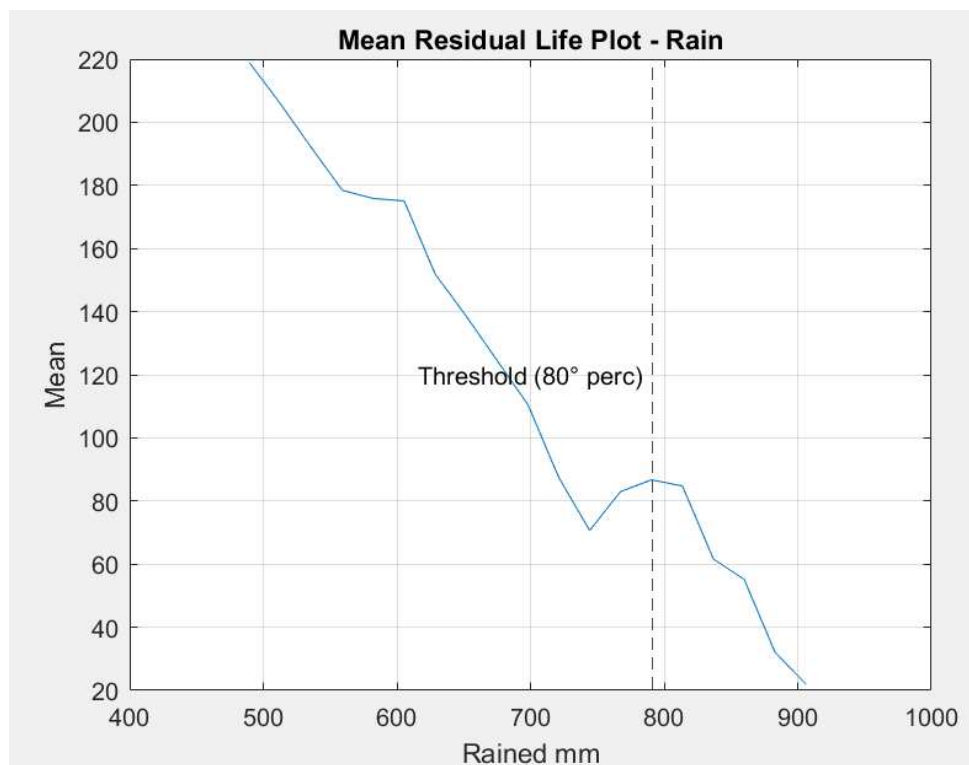
Given the distributions of each trigger indicator, for both regions analysed, Lombardy and Sicily, the parameters of the three Archimedean copula functions (Gumbel, Clayton and Frank) were estimated, and subsequently the goodness of fitting was evaluated through the Kolmogorov-Smirnov test (KS). The results obtained are summarised in Table 5.7.

Figure 5.6: Mean Residual Life Plot for Lombardy.



Source: computed by the author.

Figure 5.7: Mean Residual Life Plot for Sicily.



Source: computed by the author.

Table 5.7: Estimated parameters of the Archimedean copula functions.

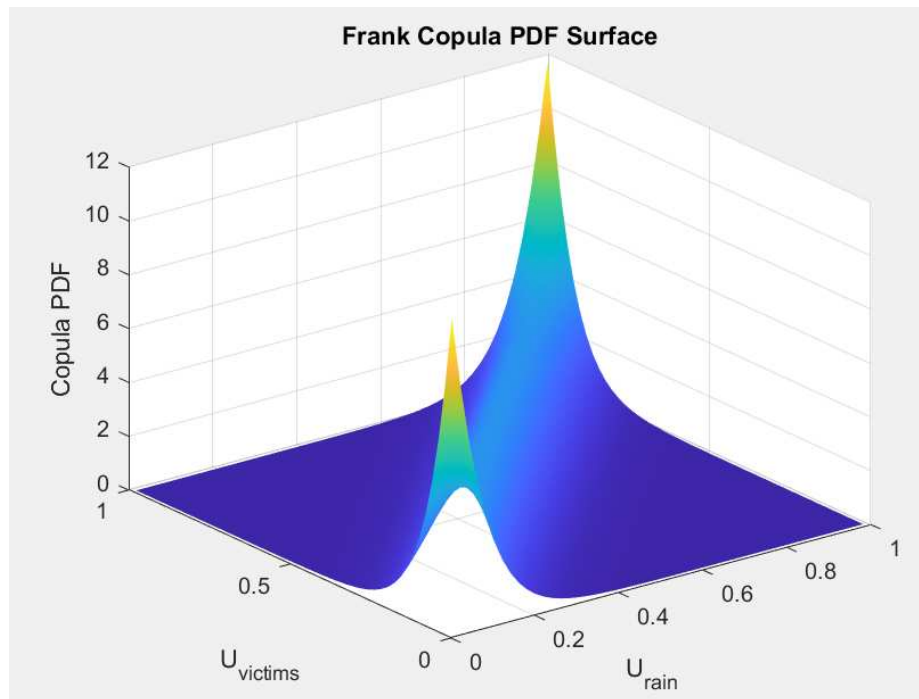
	Copula	Theta	KS	p-value
Lombardy	Gumbel	2.739	0.434	0.156
	Clayton	1.681	0.444	0.162
	Frank	6.262	0.440	0.159
Sicily	Gumbel	3.029	0.521	0.288
	Clayton	3.545	0.523	0.283
	Frank	8.615	0.523	0.286

Table 5.7 reports the estimated Theta, Kolmogorov-Smirnov parameters and the p-value of the Archimedean copula functions.

For Lombardy, the values of the Kolmogorov-Smirnov (KS) test are between 0.434 (Gumbel) and 0.444 (Clayton), while the corresponding p-values are all higher than the conventional threshold of statistical significance ($\alpha = 0.05$), ranging from 0.156 to 0.162. The Kolmogorov-Smirnov (KS) values are slightly higher for the Sicily region, while the p-values are significantly higher.

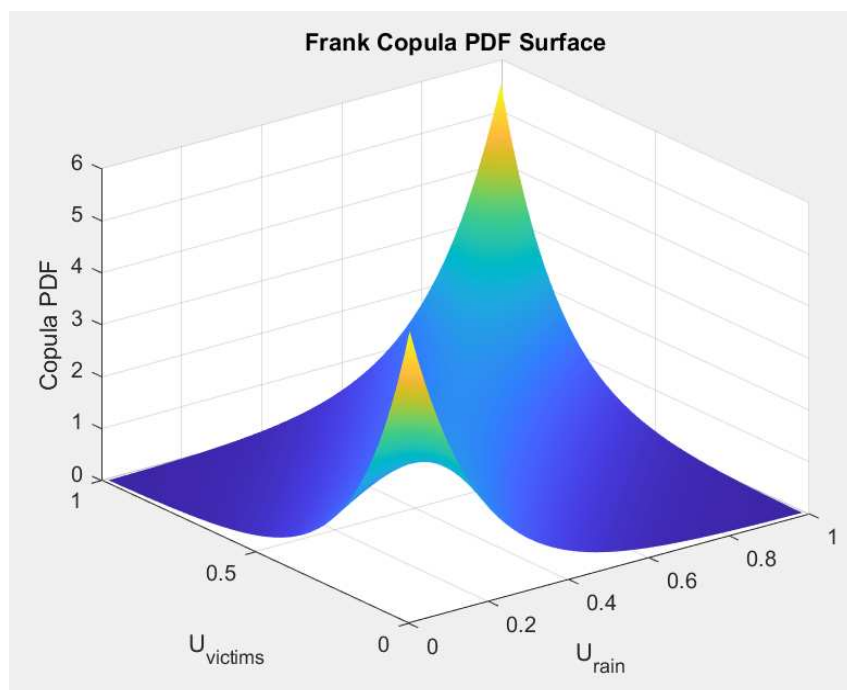
These results conclude that, for both regions, all the Archimedean copula functions fit as a model of dependence between the variables considered. The Frank copula function was applied, as the final model for both regions, to determine the dependence between the two triggers and represented with the probability density function (PDF) (see Figure 5.8 for Lombardy and 5.9 for Sicily). The reason for this choice is related to the ability of the Frank copula function to model a symmetric dependence characterizing the data.

Figure 5.8: PDF Frank Copula function for Lombardy.



Source: computed by the author.

Figure 5.9: PDF Frank Copula function for Sicily.



Source: computed by the author.

5.5.3 Mathematical Framework for Multi-trigger CAT Bond Pricing

Referring to the method applied by Chao and Zou (2018) and later by Y. Tang and C. Wen (2023), this analysis considers the determination of the price of a multi-trigger CAT bond due to a single disaster, where both the coupon and the principal are at risk, partially or completely, in the case of an extreme event.

Let us consider a bond's contract where investors receive at the end of each year a certain percentage of coupons and at the maturity date a certain percentage of the principal. The payment of the CAT bond depends on two indicators; in case these indicators exceed certain thresholds (in this analysis set at the 80th percentile), the investor receives only a part of the coupon or capital.

Unlike the original model (Y. Tang and C. Wen, 2023), two thresholds were proposed in this analysis, which allow the payment of the coupon and capital to be triggered gradually, depending on the severity of the event.

As in the model developed by Chao and Zou (2018), we consider three assumptions:

- I. $\{N(t), t \geq 0\}$ is a Poisson process with the intensity $\lambda > 0$;
- II. $\{X_i, i = 1, 2, \dots\}$ and $\{Y_i, i = 1, 2, \dots\}$, denote the average rainfall in millimetres and the number of victims of floods, respectively;
- III. $\{X_i, i = 1, 2, \dots\}$, and $\{Y_i, i = 1, 2, \dots\}$, are two sequences of independent and identically distributed random variables;
- IV. $\{N(t), t \geq 0\}$, $\{X_i, i = 1, 2, \dots\}$ and $\{Y_i, i = 1, 2, \dots\}$ are mutually independent.

When only one of the two indicators exceeds the threshold, the coupons are paid in proportion to the extreme events that occurred up to that moment. Similarly, the principal is paid according to the overall level of occurred events, but only when both indicators are activated simultaneously. The structure of the payoff is represented by the following equations:

The coupon paying framework:

$$C = \begin{cases} \prod_{i=1}^{N(t)} \left(1 - \frac{(X_i - I_X)_+}{X_i} \right) \left(1 - \frac{(Y_i - I_Y)_+}{\max\{Y_i, 1\}} \right) C, & \text{if } N(t) > 0; \\ C, & \text{if } N(t) = 0; \end{cases} \quad (5.5.1)$$

The principal paying framework:

$$D_T = \begin{cases} \prod_{i=1}^{\sum_{t=1}^T N(t)} q_i^D D, & \text{if } \sum_{t=1}^T N(t) > 0; \\ D, & \text{if } \sum_{t=1}^T N(t) = 0; \end{cases} \quad (5.5.2)$$

where

$$q_i^D = \begin{cases} \left(1 - \frac{(X_i - I_X)_+}{X_i} \right) \left(1 - \frac{(Y_i - I_Y)_+}{\max\{Y_i, 1\}} \right), & \text{if } X_i > I_X, Y_i > I_Y; \\ 1, & \text{others,} \end{cases} \quad (5.5.3)$$

where I_X and I_Y are the attachment points for the rainfall and for the victims, C is the coupon value, D is the principal value, T is the time to maturity and q_i^D is the percentage of the principal.

Finally, the price of the multi-trigger CAT bond is denoted as the present value of all future cashflows, indicated with P_t , and is given by:

$$P_t = \sum_{s=t+1}^T \mathbb{E}\{C_{t,s}\} \cdot p(t, s) + \mathbb{E}\{D_{t,s}\} \cdot p(t, T) \quad (5.5.4)$$

where

$$p(t, s) = \mathbb{E}\left\{ \exp\left(-\int_t^s r(u) du\right) \right\} \quad (5.5.5)$$

$\{r(t), t \geq 0\}$ is the interest rate process.

To describe the continuous time interest rate process $\{r(t), t \geq 0\}$, the Cox-Ingersoll-Ross model has been adopted.

$$p(t, T) = A(t, T)e^{-B(t, T)r(t)} \quad (5.5.6)$$

where

$$A(t, T) = \left[\frac{2\eta e^{(\kappa+\eta)(T-t)/2}}{(\kappa+\eta)(e^{\eta(T-t)} - 1) + 2\eta} \right]^{2\kappa\theta/\varepsilon^2} \quad (5.5.7)$$

$$B(t, T) = \frac{2(e^{\eta(T-t)} - 1)}{(\kappa+\eta)(e^{\eta(T-t)} - 1) + 2\eta}, \quad (5.5.8)$$

$$\eta = \sqrt{\kappa^2 + 2\varepsilon^2}. \quad (5.5.9)$$

5.6 Results

The price estimation was performed through a Monte Carlo simulation approach. The steps of the simulation were implemented as follows:

1. Generate $T = 3$ random numbers N_1, N_2, N_3 independently from a Poisson distribution with intensity measure $\lambda_1, \lambda_2, \lambda_3$.

λ is the expected frequency of joint extreme events. It is estimated both empirically and by copulation, and the maximum is taken between the two (Appendix I).

2. Suppose that the interest rate $\{r(t), t \geq 0\}$ follows the CIR process, we applied the discount factor $p(t, T)$ to simulate the interest risk (Equation 5.5.6).
3. Given a trigger level (u_1, u_2) , we calculate the price of the CAT bond based on the cash flows in Equation (5.5.1) and Equation (5.5.2) and the interest risk model in Equation (5.5.6).
4. Finally, we implement all the results in Equation 5.5.4.

The calculations were repeated for 10^4 simulations and then the sample mean of the price P_t of a 3 years multi-trigger CAT bond was computed.

Table 5.8 shows the results of the analysis, the prices and the yield to maturity for each Italian region. We considered a contract with the principal equal to 100 and the coupons equal to 5%. The results show that the bond prices vary significantly between regions, from a minimum of about 97.6606 (Puglia) to a maximum of about 104.9744 (Toscana).

Table 5.8: The price of the multi-trigger cat bond for each region analyzed.

Region	Price	Yield to Maturity (%)
Calabria	104.9705	3.23
Emilia Romagna	103.0453	3.90
Friuli Venezia Giulia	102.2425	4.19
Liguria	101.9161	4.31
Lombardia	104.9705	3.23
Marche	102.2159	4.20
Piemonte	99.5549	5.16
Puglia	97.6606	5.87
Sardegna	103.9040	3.60
Sicilia	102.3533	4.15
Toscana	104.9744	3.23
Veneto	100.7273	4.73

The Yield to maturity is inversely related to the price. Some regions with lower prices have higher yields (e.g. Puglia with price 97.6606 and YTM 5.87%), while regions with high prices have lower yields (e.g. Toscana with price 104.9744 and YTM 3.23%).

In conclusion, the analysis shows a significant regional variation in the prices of CAT bonds with multiple triggers, mainly due to differences in trigger thresholds and perceived risk levels. Regions with the lowest trigger percentiles tend to have lower prices and higher returns, reflecting a higher risk of triggering. Whereas regions with higher trigger thresholds show higher prices and lower returns, indicating a lower risk. These results highlight the importance of regional risk profiles in determining CAT bond prices and provide useful information for investors assessing risk-return trade-offs in different Italian regions.

Conclusion

The objective of this thesis is to present the role of Catastrophe bonds in Mitigating Flood Risk. Catastrophe bonds, especially Multi-trigger Catastrophe bonds, are innovative and efficient financial instruments in transferring the catastrophic risk from the insurance and reinsurance industry to the capital market. Since flood risk is one of the most dangerous and frequent risks in Europe and Italy, this thesis aims to conduct an empirical analysis of the pricing of flood multi-trigger catastrophe bonds.

The introduction of Catastrophe bonds has marked a significant change both for the insurance industry and the market. For investors, Catastrophe bonds are very attractive for two characteristics: the low volatility and low correlation with traditional asset classes, leading to high returns and great advantages regarding portfolio diversification. While for the insurance and reinsurance industry, Catastrophe bonds offer optimal risk diversification, flexibility, transparency and price stability and allows the expansion of their business.

Pricing of Catastrophe bonds plays a crucial role in risk coverage, but the catastrophe bond's value is based on specific catastrophe loss indices, making the pricing not as uniform as in the general market. Consequently, different methodologies based on stochastic catastrophe risk modelling are applied to the pricing of Catastrophe bonds.

However, the challenge of developing a universally applicable pricing Catastrophe bond model remains a central challenge, given the intrinsically random and non-replicable nature of catastrophic risk.

The thesis focuses on the empirical analysis of the pricing of a multi-trigger Catastrophe bond, using the CIR-Copula-POT pricing model and Monte Carlo simulation techniques. This analysis was applied to data gathered by ISPRA for the 20 Italian regions, based on rainfall and victims of floods that hit Italy between 2000 and 2022. The results highlight a marked territorial heterogeneity in the prices and yields of Catastrophe bonds, which reflects the different characteristics and risk profiles in the Italian regions. In particular, regions with a higher probability of bond activation show lower prices and higher

expected yields. Conversely, regions with a lower probability of bond activation show higher prices and lower yields, consistently with a lower perceived risk.

In conclusion, Catastrophe bonds represent a flexible and potentially effective solution to integrate a correct management of catastrophe risk in Italy. However, large-scale adoption requires a continuous refinement of pricing models, greater availability and updating of data and involvement of public and private stakeholders. In light of ongoing climate change, which increases the frequency and intensity of extreme events, the Catastrophe bonds market could play an increasingly relevant role in strengthening the financial stability of exposed territories, promoting a more efficient and sustainable management of catastrophe risk.

Appendix

A.1 MATLAB Code

The purpose of this appendix is to provide the reader with a useful reference of the Matlab code of the Multi-trigger CAT bond pricing.

```
%% PRICING MULTI-TRIGGER FLOOD CAT BOND

% Using the CIR-POT-Copula model

% This analysis applies Extreme Value Theory (EVT) to evaluate
% the marginal distributions of rainfall (in mm) and flood-related
% fatalities in Italy from 2000 to 2022. EVT is combined with
% Copula functions to model the joint distribution of rainfall and
% fatalities.

% To incorporate interest rate dynamics, the CIR stochastic model is used.
% Parameter estimation follows the methodology of Kamil Kladvko.

clc; clear; close all;

format bank

%% LOAD DATA

DATA_RAIN = readtable('DATA_RAIN.xlsx');
DATA_VICTIMS = readtable('DATA_VICTIMS.xlsx');

% Extract numerical data

rain_data = DATA_RAIN{2:end, 2:end};
victims_data = DATA_VICTIMS{2:end, 2:end};

% Year range and region names

year = 2000:2022;

colNames = string(year);

regions = [
    "abruzzo", "basilicata", "calabria", "campania", "emilia romagna", ...
    "friuli venezia giulia", "lazio", "liguria", "lombardia", "marche", ...
    "molise", "piemonte", "puglia", "sardegna", "sicilia", "toscana", ...
    "trentino", "umbria", "valle d aosta", "veneto"
];

%% SELECT REGION

% DATA FOR A SPECIFIED REGION
```

```

% Index
% ABRUZZO = 2;
% BASILICATA = 3;
% CALABRIA = 4;
% CAMPANIA = 5;
% EMILIA_ROMAGNA = 6;
% FRIULI_VENEZIA_GIULIA = 7;
% LAZIO = 8;
% LIGURIA = 9;
% LOMBARDIA = 10;
% MARCHE = 11;
% MOLISE = 12;
% PIEMONTE = 13;
% PUGLIA = 14;
% SARDEGNA = 15;
% SICILIA = 16;
% TOSCANA = 17;
% TRENTINO = 18;
% UMBRIA = 19;
% VALLE_D_AOSTA = 20;
% VENETO = 21;

% We select a region (for example 'Sicilia NUMBER 16') and we define the
price of the
% CAT bond for this specified region.
regionIndex = 14;
rain = DATA_RAIN{regionIndex, 2:24};
victims = DATA_VICTIMS{regionIndex, 2:24};
% Display data
dataMatrix = [rain; victims];
f3 = figure('Name','Region','NumberTitle','off');
T = array2table(dataMatrix, ...
    'RowNames', {'Rain', 'Victims'}, ...
    'VariableNames', cellstr(colNames));
uitable(f3, ...
    'Data', table2cell(T), ...

```

```

    'RowName', T.Properties.RowNames, ...
    'ColumnName', T.Properties.VariableNames, ...
    'Units', 'normalized', ...
    'Position', [0, 0, 1, 0.3]);

%% DESCRIPTIVE STATISTICS

rain_stats = [max(rain); min(rain); mean(rain); median(rain); skewness(rain);
kurtosis(rain)];

victim_stats = [max(victims); min(victims); mean(victims); median(victims);
skewness(victims); kurtosis(victims)];

Statistics = {'Maximum'; 'Minimum'; 'Mean'; 'Median'; 'Skewness';
'Kurtosis'};

statsTable = table(Statistics, rain_stats, victim_stats, 'VariableNames',
{'Statistics', 'Rain', 'Victims'});

disp(statsTable);

%% EVT THRESHOLDS AND POT MODEL

threshold_rain = prctile(rain, 80);
threshold_victims = prctile(victims, 80);

% Extract excesses
excess_rain = rain(rain > threshold_rain) - threshold_rain;
excess_victims = victims(victims > threshold_victims) - threshold_victims;

% Fit Generalized Pareto Distribution (GPD)
[paramEsts_rain] = gpfith(excess_rain);
[paramEsts_victims] = gpfith(excess_victims);

fprintf('GPD Parameters - Rain: k = %.4f, sigma = %.4f\n', paramEsts_rain(1),
paramEsts_rain(2));

fprintf('GPD Parameters - Victims: k = %.4f, sigma = %.4f\n',
paramEsts_victims(1), paramEsts_victims(2));

% Fit distributions
pd_rain = fitdist(excess_rain, 'GeneralizedPareto');
pd_victims = fitdist(excess_victims, 'GeneralizedPareto');

%% COPULA FITTING

idx_extreme = (rain > threshold_rain) & (victims > threshold_victims);
n_extreme = sum(idx_extreme);
u_rain = tiedrank(rain(idx_extreme)) / (n_extreme + 1);
u_victims = tiedrank(victims(idx_extreme)) / (n_extreme + 1);
U = [u_rain(:), u_victims(:)];

```

```

families = {'Gumbel', 'Clayton', 'Frank'};
results = struct('Family', {}, 'Theta', {}, 'KS', {}, 'pValue', {});
for i = 1:length(families)
    family = families{i};

    try

        theta = copulafit(family, U, 'Method', 'ML');
        cdf_vals = copulacdf(family, U, theta);
        [f_emp, x_emp] = ecdf(cdf_vals);
        ks_val = max(abs(f_emp - x_emp));
        [~, p_val] = kstest(cdf_vals);

        results(i) = struct('Family', family, 'Theta', theta, 'KS', ks_val,
'pValue', p_val);

    catch

        results(i) = struct('Family', family, 'Theta', NaN, 'KS', NaN,
'pValue', NaN);

        fprintf('Copula % It is not possible obtain a correct estimation.\n',
family);

    end
end

fprintf('%-10s %-10s %-10s %-10s\n', 'Copula', 'Theta', 'KS', 'p-value');
for i = 1:length(results)

    fprintf('%-10s %-10.3f %-10.3f %-10.3f\n', ...

            results(i).Family,    results(i).Theta,    results(i).KS,
results(i).pValue);

end

valid = ~isnan([results.Theta]);
if any(valid)

    [~, idx_best] = min([results(valid).KS]);
    idx_best = find(valid, idx_best, 'first');
    best_family = results(idx_best).Family;
    theta_best = results(idx_best).Theta;

    fprintf('Miglior copula: %s con theta = %.3f\n', best_family, theta_best);

else

```

```

        error('Nessuna copula stimata correttamente.');
```

end

```

%% PROBABILITY CALCULATIONS

p_rain = (length(excess_rain) / length(rain)) * (1 - cdf(pd_rain, max(rain) -
threshold_rain));

p_victims = (length(excess_victims) / length(victims)) * (1 - cdf(pd_victims,
max(victims) - threshold_victims));

u_r = cdf(pd_rain, max(rain) - threshold_rain);
u_v = cdf(pd_victims, max(victims) - threshold_victims);
joint_prob = 1 - copulacdf(best_family, [u_r, u_v], theta_best);
fprintf('\nJoint Trigger Probability: %.4f\n', joint_prob);

% Event intensity (lambda)
lambda_empirical = sum(idx_extreme) / length(rain);
baseline_prob = mean(copulacdf(best_family, U, theta_best));
lambda = lambda_empirical * joint_prob / baseline_prob;
fprintf('Lambda (event intensity): %.4f\n', lambda);

%% CIR MODEL ESTIMATION

T_r = readtable('DATA_RATE.csv');
data = T_r{:, 2};
data=data/10;

Model = struct('Data', data, 'TimeStep', 1/52, ...
               'Disp', 'y', 'MatlabDisp', 'iter', 'Method', 'besseli');

Results = CIRestimation(Model);

alpha = Results.Params(1);
mu = Results.Params(2);
sigma = Results.Params(3);
X0 = data(end);

%% MONTE CARLO SIMULATION

N_sim = 10000;

T = 3;
dt = 1/52;
steps = T / dt;

price_sim = zeros(1, N_sim);
coupon = 5;
principal = 100;

```

```

for i = 1:N_sim
    N_j = poissrnd(lambda, [1, T]);
    Ct_all = zeros(1, T);
    triggered_events = zeros(100, 2);
    event_count = 0;

    for t = 1:T
        Ct = coupon;
        for k = 1:N_j(t)
            u_sample = copularnd(best_family, theta_best, 1);
            X_i = threshold_rain + icdf(pd_rain, u_sample(1));
            Y_i = threshold_victims + icdf(pd_victims, u_sample(2));
            red_X = max((X_i - threshold_rain)/X_i, 0);
            red_Y = max((Y_i - threshold_victims)/max(Y_i, 1), 0);
            Ct = Ct * (1 - red_X) * (1 - red_Y);
            triggered_events = [triggered_events; X_i, Y_i];
        end
        Ct_all(t) = Ct;
    end

    DT = principal;
    for j = 1:size(triggered_events,1)
        X_i = triggered_events(j,1);
        Y_i = triggered_events(j,2);
        qD = (1 - max((X_i - threshold_rain)/X_i, 0)) * ...
            (1 - max((Y_i - threshold_victims)/max(Y_i,1), 0));
        DT = DT * qD;
    end

    r = zeros(1, steps+1); r(1) = X0;
    for j = 1:steps
        dW = sqrt(dt) * randn;
        r(j+1) = r(j) + alpha * (mu - r(j)) * dt + sigma * sqrt(max(r(j), 0))
* dW;
    end

    P_tT_sim = zeros(1, T);
    for s = 1:T

```

```

gamma = sqrt(alpha^2 + 2*sigma^2);

B = (2*(exp(gamma*s) - 1)) / ((gamma + alpha)*(exp(gamma*s) - 1) +
2*gamma);

A = ((2*gamma*exp((alpha + gamma)*s/2)) / ((gamma +
alpha)*(exp(gamma*s) - 1) + 2*gamma))^(2*alpha*mu/sigma^2);

P_tT_sim(s) = A * exp(-B * r(end));

end

% fprintf('Simulation %d: Final Coupon year %d = %.4f, principal = %.4f,
final rate = %.4f\n', i, t, Ct, DT, r(end));

price_sim(i) = sum(Ct_all .* P_tT_sim(1:T)) + DT * P_tT_sim(end);

end

estimated_price = mean(price_sim);

fprintf('Estimated CAT Bond Price: %.4f\n', estimated_price);

```

A.2 Data on Victims and Rainfall (mm)

Regions	abruzzo	basilicata	calabria	campania	emilia rom	fruli/venez	lazio	liguria	lombardia	marche	molise	piemonte	puglia	sardegna	sicilia	toscana	trentino	umbria	aosta	veneto
2000	757.5	583.0	914.2	799.4	806.8	1885.1	890.9	1570.3	1378.0	725.1	729.0	1389.5	424.6	548.0	544.5	1033.8	1285.3	850.2	1348.8	1154.1
2001	728.8	605.3	761.4	820.0	783.0	1411.4	743.9	1108.6	995.7	794.6	675.9	767.1	460.8	429.3	489.7	851.1	937.7	719.1	959.1	936.7
2002	935.2	813.5	991.3	1030.9	1004.3	1929.3	934.2	1581.2	1491.9	938.7	948.9	1571.1	759.6	641.1	565.1	1032.4	1310.4	900.5	1491.0	1437.6
2003	914.2	789.2	989.1	970.2	708.9	1278.5	763.3	1027.2	785.4	659.1	1044.6	776.2	711.2	773.7	848.6	778.4	804.0	676.0	753.0	821.5
2004	1105.2	837.9	1130.3	1173.8	943.6	1822.2	1138.8	939.6	998.8	955.5	1093.2	881.2	715.4	842.7	766.2	1020.3	883.6	1134.6	965.3	1198.9
2005	1043.2	809.4	963.3	1196.7	967.1	1435.0	1134.2	884.3	824.0	1086.3	1081.8	775.6	699.4	654.7	747.5	982.1	774.9	1029.9	798.8	1088.3
2006	674.9	763.4	901.2	966.0	677.1	1356.1	708.2	1082.9	856.6	745.5	769.8	826.7	656.9	644.3	689.6	775.5	795.0	686.5	896.7	865.8
2007	655.9	744.6	870.1	841.0	680.2	1451.7	588.6	794.6	832.9	686.2	636.0	758.0	614.3	580.2	752.1	716.2	860.2	597.2	979.6	891.2
2008	940.6	774.2	933.6	1107.0	903.2	2312.3	1183.2	1499.5	1337.3	876.4	908.6	1280.3	631.3	765.4	591.8	1095.8	1289.4	993.5	1269.5	1383.6
2009	1054.0	1024.9	1439.2	1353.7	936.5	2058.3	1050.8	1591.9	1184.5	850.6	1184.3	1128.2	897.0	723.1	929.4	1042.0	1064.2	888.7	950.3	1235.7
2010	1000.8	1151.3	1444.2	1610.0	1166.1	2332.5	1277.7	1742.9	1440.5	1131.6	1083.9	1274.0	764.8	866.8	783.0	1404.2	1214.7	1234.2	1044.8	1530.4
2011	741.4	736.0	1044.6	896.7	646.2	1533.6	764.0	1196.5	870.5	721.5	809.9	1015.9	602.6	693.5	783.6	709.7	913.6	601.1	972.1	908.6
2012	844.5	827.8	1081.6	1114.1	767.4	1926.8	958.2	1212.3	1036.4	926.9	841.1	961.6	674.7	594.5	672.4	995.1	1184.5	932.5	1001.0	1060.7
2013	1131.0	1068.8	1246.0	1436.1	1090.5	2016.1	1199.0	1735.9	1237.6	1189.5	1139.2	1175.3	738.7	855.5	738.7	1250.2	1196.4	1148.0	1063.7	1323.5
2014	957.3	816.2	1118.4	1201.2	1185.5	2771.0	1278.7	2142.3	1634.3	1143.0	943.9	1447.1	713.0	544.4	642.4	1369.5	1432.2	1104.6	1035.8	1681.8
2015	961.4	885.8	1250.5	1096.7	804.8	1321.0	882.5	940.9	814.6	1037.3	975.9	916.7	687.0	620.9	927.1	823.9	805.0	729.2	868.9	808.9
2016	912.3	884.8	1068.8	1079.5	884.5	1763.1	898.7	1285.5	1086.1	1011.1	950.1	1085.1	684.7	585.7	589.0	1139.6	1017.2	983.3	971.7	1142.0
2017	924.1	620.6	805.1	844.8	674.0	1764.8	733.9	840.2	879.2	944.9	900.5	655.2	538.7	444.3	560.9	748.1	997.1	716.1	760.3	918.7
2018	1125.8	951.4	1338.5	1255.9	906.5	1588.1	1224.1	1582.4	1134.2	953.4	1132.7	1349.6	759.4	1038.5	888.8	1089.5	1109.7	943.8	1255.4	1165.4
2019	886.5	795.3	1017.8	1170.4	1081.5	2022.0	1160.3	1766.1	1260.3	953.4	955.4	1257.6	649.3	730.7	745.8	1234.4	1277.0	952.7	999.9	1371.6
2020	843.8	824.6	900.6	983.1	809.6	1887.0	899.8	1352.0	1110.9	867.3	833.4	910.6	603.9	698.1	533.3	994.2	1231.0	903.9	753.4	1133.8
2021	881.5	832.7	1093.9	1282.6	648.8	1618.7	1044.5	1176.7	945.1	823.6	985.9	818.1	527.8	759.2	792.4	923.6	963.7	753.2	796.9	946.5
2022	713.4	839.7	848.3	1120.1	669.0	1153.2	789.9	767.2	678.7	761.3	846.7	585.8	576.0	511.5	496.5	848.6	782.0	801.4	599.0	743.3

Source: ISPRA, RS_ANNUAL_1951-2023, Sinanet, 2023.

Regions	abruzzo	basilicata	calabria	campania	emilia rom	fruli venez	lazio	liguria	lombardia	marche	molise	piemonte	puglia	sardegna	sicilia	toscana	trentino	umbria	aosta	veneto
2000	0.00	0.00	14.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00	34.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2001	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2002	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2003	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00	2.00	0.00	0.00	0.00	0.00
2004	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2005	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2006	0.00	0.00	4.00	4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2007	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2008	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.00	2.00	0.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
2009	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	1.00	0.00	0.00	0.00	2.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
2010	1.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	3.00	0.00	1.00	0.00	1.00
2011	1.00	2.00	0.00	1.00	2.00	0.00	2.00	15.00	1.00	1.00	0.00	0.00	0.00	0.00	3.00	3.00	0.00	0.00	0.00	0.00
2012	0.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	8.00	0.00	0.00	0.00	0.00
2013	1.00	1.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.00	18.00	1.00	2.00	0.00	0.00	0.00	1.00
2014	0.00	0.00	0.00	0.00	1.00	0.00	0.00	3.00	2.00	3.00	0.00	1.00	2.00	0.00	1.00	2.00	0.00	0.00	0.00	4.00
2015	0.00	0.00	1.00	1.00	3.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00
2016	0.00	0.00	0.00	0.00	1.00	1.00	0.00	1.00	0.00	0.00	0.00	1.00	1.00	0.00	2.00	8.00	0.00	0.00	0.00	0.00
2017	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
2018	0.00	0.00	13.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00	11.00	0.00	0.00	0.00	0.00	1.00
2019	0.00	0.00	0.00	0.00	0.00	1.00	0.00	2.00	0.00	0.00	0.00	3.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00
2020	0.00	1.00	0.00	1.00	0.00	0.00	0.00	1.00	3.00	0.00	0.00	1.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
2021	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	3.00	0.00	0.00	0.00	0.00	0.00
2022	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	12.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Source: ISPRA, *Elenco degli eventi di frana, inondazione e allagamento con vittime*,2021,2022 2023.

A.3 Data on interest rate

date	interest rate				
		13/02/2023	2.379	27/03/2023	2.879
02/01/2023	1.876	14/02/2023	2.342	28/03/2023	2.787
03/01/2023	1.834	15/02/2023	2.379	29/03/2023	2.9
04/01/2023	1.882	16/02/2023	2.387	30/03/2023	2.866
05/01/2023	1.878	17/02/2023	2.345	31/03/2023	2.898
06/01/2023	1.89	20/02/2023	2.393	03/04/2023	2.895
09/01/2023	1.882	21/02/2023	2.375	04/04/2023	2.876
10/01/2023	1.889	22/02/2023	2.386	05/04/2023	2.875
11/01/2023	1.879	23/02/2023	2.392	06/04/2023	2.886
12/01/2023	1.886	24/02/2023	2.379	11/04/2023	2.902
13/01/2023	1.895	27/02/2023	2.344	12/04/2023	2.862
16/01/2023	1.904	28/02/2023	2.339	13/04/2023	2.862
17/01/2023	1.895	01/03/2023	2.368	14/04/2023	2.894
18/01/2023	1.887	02/03/2023	2.365	17/04/2023	2.882
19/01/2023	1.905	03/03/2023	2.393	18/04/2023	2.9
20/01/2023	1.899	06/03/2023	2.381	19/04/2023	2.87
23/01/2023	1.893	07/03/2023	2.348	20/04/2023	2.878
24/01/2023	1.893	08/03/2023	2.382	21/04/2023	2.886
25/01/2023	1.894	09/03/2023	2.373	24/04/2023	2.882
26/01/2023	1.899	10/03/2023	2.378	25/04/2023	2.905
27/01/2023	1.884	13/03/2023	2.378	26/04/2023	2.88
30/01/2023	1.894	14/03/2023	2.317	27/04/2023	2.888
31/01/2023	1.895	15/03/2023	2.415	28/04/2023	2.872
01/02/2023	1.907	16/03/2023	2.424	02/05/2023	2.91
02/02/2023	1.97	17/03/2023	2.482	03/05/2023	2.91
03/02/2023	2.107	20/03/2023	2.565	04/05/2023	2.902
06/02/2023	2.148	21/03/2023	2.712	05/05/2023	2.95
07/02/2023	2.291	22/03/2023	2.827	08/05/2023	3.012
08/02/2023	2.342	23/03/2023	2.854	09/05/2023	3.078
09/02/2023	2.382	24/03/2023	2.878	10/05/2023	3.096
10/02/2023	2.359				

11/05/2023	3.126	26/06/2023	3.382	09/08/2023	3.647
12/05/2023	3.127	27/06/2023	3.394	10/08/2023	3.647
15/05/2023	3.145	28/06/2023	3.391	11/08/2023	3.628
16/05/2023	3.147	29/06/2023	3.415	14/08/2023	3.641
17/05/2023	3.112	30/06/2023	3.362	15/08/2023	3.608
18/05/2023	3.137	03/07/2023	3.388	16/08/2023	3.637
19/05/2023	3.129	04/07/2023	3.378	17/08/2023	3.64
22/05/2023	3.132	05/07/2023	3.388	18/08/2023	3.634
23/05/2023	3.152	06/07/2023	3.398	21/08/2023	3.631
24/05/2023	3.115	07/07/2023	3.393	22/08/2023	3.628
25/05/2023	3.152	10/07/2023	3.395	23/08/2023	3.64
26/05/2023	3.125	11/07/2023	3.386	24/08/2023	3.638
29/05/2023	3.146	12/07/2023	3.387	25/08/2023	3.648
30/05/2023	3.14	13/07/2023	3.391	28/08/2023	3.628
31/05/2023	3.122	14/07/2023	3.393	29/08/2023	3.638
01/06/2023	3.154	17/07/2023	3.381	30/08/2023	3.651
02/06/2023	3.121	18/07/2023	3.379	31/08/2023	3.645
05/06/2023	3.134	19/07/2023	3.395	01/09/2023	3.642
06/06/2023	3.153	20/07/2023	3.402	04/09/2023	3.636
07/06/2023	3.136	21/07/2023	3.394	05/09/2023	3.622
08/06/2023	3.153	24/07/2023	3.401	06/09/2023	3.628
09/06/2023	3.128	25/07/2023	3.392	07/09/2023	3.645
12/06/2023	3.136	26/07/2023	3.401	08/09/2023	3.659
13/06/2023	3.131	27/07/2023	3.422	11/09/2023	3.639
14/06/2023	3.159	28/07/2023	3.465	12/09/2023	3.632
15/06/2023	3.202	31/07/2023	3.455	13/09/2023	3.651
16/06/2023	3.234	01/08/2023	3.532	14/09/2023	3.674
19/06/2023	3.209	02/08/2023	3.61	15/09/2023	3.69
20/06/2023	3.312	03/08/2023	3.638	18/09/2023	3.715
21/06/2023	3.363	04/08/2023	3.643	19/09/2023	3.777
22/06/2023	3.401	07/08/2023	3.637	20/09/2023	3.878
23/06/2023	3.396	08/08/2023	3.64	21/09/2023	3.895

22/09/2023	3.875	07/11/2023	3.891	21/12/2023	3.852
25/09/2023	3.883	08/11/2023	3.878	22/12/2023	3.872
26/09/2023	3.882	09/11/2023	3.892	27/12/2023	3.857
27/09/2023	3.881	10/11/2023	3.896	28/12/2023	3.883
28/09/2023	3.892	13/11/2023	3.875	29/12/2023	3.9
29/09/2023	3.872	14/11/2023	3.846	02/01/2024	3.848
02/10/2023	3.858	15/11/2023	3.865	03/01/2024	3.887
03/10/2023	3.889	16/11/2023	3.875	04/01/2024	3.872
04/10/2023	3.885	17/11/2023	3.871	05/01/2024	3.892
05/10/2023	3.885	20/11/2023	3.841	08/01/2024	3.888
06/10/2023	3.898	21/11/2023	3.887	09/01/2024	3.869
09/10/2023	3.896	22/11/2023	3.887	10/01/2024	3.872
10/10/2023	3.878	23/11/2023	3.89	11/01/2024	3.871
11/10/2023	3.897	24/11/2023	3.895	12/01/2024	3.884
12/10/2023	3.882	27/11/2023	3.873	15/01/2024	3.88
13/10/2023	3.874	28/11/2023	3.852	16/01/2024	3.882
16/10/2023	3.862	29/11/2023	3.862	17/01/2024	3.886
17/10/2023	3.876	30/11/2023	3.838	18/01/2024	3.878
18/10/2023	3.891	01/12/2023	3.839	19/01/2024	3.858
19/10/2023	3.879	04/12/2023	3.887	22/01/2024	3.885
20/10/2023	3.89	05/12/2023	3.849	23/01/2024	3.882
23/10/2023	3.883	06/12/2023	3.84	24/01/2024	3.849
24/10/2023	3.88	07/12/2023	3.865	25/01/2024	3.851
25/10/2023	3.875	08/12/2023	3.843	26/01/2024	3.895
26/10/2023	3.881	11/12/2023	3.845	29/01/2024	3.878
27/10/2023	3.872	12/12/2023	3.858	30/01/2024	3.883
30/10/2023	3.883	13/12/2023	3.854	31/01/2024	3.832
31/10/2023	3.887	14/12/2023	3.835	01/02/2024	3.843
01/11/2023	3.868	15/12/2023	3.877	02/02/2024	3.877
02/11/2023	3.885	18/12/2023	3.835	05/02/2024	3.864
03/11/2023	3.887	19/12/2023	3.865	06/02/2024	3.876
06/11/2023	3.865	20/12/2023	3.848	07/02/2024	3.877

08/02/2024	3.888	25/03/2024	3.885	13/05/2024	3.895
09/02/2024	3.884	26/03/2024	3.875	14/05/2024	3.888
12/02/2024	3.876	27/03/2024	3.835	15/05/2024	3.872
13/02/2024	3.875	28/03/2024	3.853	16/05/2024	3.885
14/02/2024	3.875	02/04/2024	3.87	17/05/2024	3.875
15/02/2024	3.852	03/04/2024	3.863	20/05/2024	3.883
16/02/2024	3.882	04/04/2024	3.875	21/05/2024	3.894
19/02/2024	3.893	05/04/2024	3.89	22/05/2024	3.893
20/02/2024	3.886	08/04/2024	3.888	23/05/2024	3.875
21/02/2024	3.873	09/04/2024	3.875	24/05/2024	3.881
22/02/2024	3.86	10/04/2024	3.879	27/05/2024	3.863
23/02/2024	3.899	11/04/2024	3.876	28/05/2024	3.879
26/02/2024	3.907	12/04/2024	3.88	29/05/2024	3.869
27/02/2024	3.885	15/04/2024	3.898	30/05/2024	3.892
28/02/2024	3.878	16/04/2024	3.863	31/05/2024	3.889
29/02/2024	3.879	17/04/2024	3.855	03/06/2024	3.828
01/03/2024	3.872	18/04/2024	3.865	04/06/2024	3.83
04/03/2024	3.866	19/04/2024	3.891	05/06/2024	3.812
05/03/2024	3.882	22/04/2024	3.866	06/06/2024	3.848
06/03/2024	3.861	23/04/2024	3.84	07/06/2024	3.794
07/03/2024	3.87	24/04/2024	3.851	10/06/2024	3.716
08/03/2024	3.88	25/04/2024	3.868	11/06/2024	3.717
11/03/2024	3.893	26/04/2024	3.883	12/06/2024	3.65
12/03/2024	3.874	29/04/2024	3.852	13/06/2024	3.615
13/03/2024	3.884	30/04/2024	3.802	14/06/2024	3.626
14/03/2024	3.881	02/05/2024	3.858	17/06/2024	3.641
15/03/2024	3.89	03/05/2024	3.893	18/06/2024	3.612
18/03/2024	3.871	06/05/2024	3.886	19/06/2024	3.59
19/03/2024	3.875	07/05/2024	3.868	20/06/2024	3.648
20/03/2024	3.878	08/05/2024	3.882	21/06/2024	3.622
21/03/2024	3.868	09/05/2024	3.878	24/06/2024	3.619
22/03/2024	3.875	10/05/2024	3.892	25/06/2024	3.616

26/06/2024	3.607	09/08/2024	3.642	24/09/2024	3.382
27/06/2024	3.618	12/08/2024	3.63	25/09/2024	3.355
28/06/2024	3.612	13/08/2024	3.649	26/09/2024	3.367
01/07/2024	3.641	14/08/2024	3.609	27/09/2024	3.369
02/07/2024	3.63	15/08/2024	3.605	30/09/2024	3.383
03/07/2024	3.608	16/08/2024	3.621	01/10/2024	3.328
04/07/2024	3.608	19/08/2024	3.652	02/10/2024	3.348
05/07/2024	3.607	20/08/2024	3.628	03/10/2024	3.39
08/07/2024	3.605	21/08/2024	3.605	04/10/2024	3.395
09/07/2024	3.608	22/08/2024	3.625	07/10/2024	3.391
10/07/2024	3.608	23/08/2024	3.635	08/10/2024	3.392
11/07/2024	3.609	26/08/2024	3.631	09/10/2024	3.402
12/07/2024	3.61	27/08/2024	3.622	10/10/2024	3.382
15/07/2024	3.595	28/08/2024	3.609	11/10/2024	3.408
16/07/2024	3.589	29/08/2024	3.606	14/10/2024	3.4
17/07/2024	3.597	30/08/2024	3.623	15/10/2024	3.405
18/07/2024	3.626	02/09/2024	3.608	16/10/2024	3.34
19/07/2024	3.622	03/09/2024	3.619	17/10/2024	3.366
22/07/2024	3.612	04/09/2024	3.602	18/10/2024	3.339
23/07/2024	3.579	05/09/2024	3.624	21/10/2024	3.345
24/07/2024	3.579	06/09/2024	3.608	22/10/2024	3.39
25/07/2024	3.624	09/09/2024	3.628	23/10/2024	3.26
26/07/2024	3.642	10/09/2024	3.64	24/10/2024	3.165
29/07/2024	3.595	11/09/2024	3.587	25/10/2024	3.149
30/07/2024	3.643	12/09/2024	3.602	28/10/2024	3.171
31/07/2024	3.607	13/09/2024	3.518	29/10/2024	3.152
01/08/2024	3.642	16/09/2024	3.492	30/10/2024	3.144
02/08/2024	3.645	17/09/2024	3.442	31/10/2024	3.116
05/08/2024	3.62	18/09/2024	3.42	01/11/2024	3.131
06/08/2024	3.628	19/09/2024	3.363	04/11/2024	3.129
07/08/2024	3.626	20/09/2024	3.351	05/11/2024	3.142
08/08/2024	3.619	23/09/2024	3.362	06/11/2024	3.145

07/11/2024	3.095	23/12/2024	2.857	10/02/2025	2.666
08/11/2024	3.135	24/12/2024	2.878	11/02/2025	2.661
11/11/2024	3.148	27/12/2024	2.935	12/02/2025	2.658
12/11/2024	3.145	30/12/2024	2.912	13/02/2025	2.643
13/11/2024	3.147	31/12/2024	2.924	14/02/2025	2.613
14/11/2024	3.167	02/01/2025	2.911	17/02/2025	2.639
15/11/2024	3.172	03/01/2025	2.901	18/02/2025	2.668
18/11/2024	3.173	06/01/2025	2.898	19/02/2025	2.652
19/11/2024	3.142	07/01/2025	2.88	20/02/2025	2.634
20/11/2024	3.157	08/01/2025	2.875	21/02/2025	2.641
21/11/2024	3.141	09/01/2025	2.904	24/02/2025	2.63
22/11/2024	3.138	10/01/2025	2.925	25/02/2025	2.649
25/11/2024	3.166	13/01/2025	2.915	26/02/2025	2.641
26/11/2024	3.163	14/01/2025	2.926	27/02/2025	2.628
27/11/2024	3.163	15/01/2025	2.916	28/02/2025	2.607
28/11/2024	3.152	16/01/2025	2.916	03/03/2025	2.622
29/11/2024	3.128	17/01/2025	2.903	04/03/2025	2.649
02/12/2024	3.16	20/01/2025	2.921	05/03/2025	2.624
03/12/2024	3.116	21/01/2025	2.909	06/03/2025	2.624
04/12/2024	3.102	22/01/2025	2.921	07/03/2025	2.561
05/12/2024	3.136	23/01/2025	2.925	10/03/2025	2.562
06/12/2024	3.142	24/01/2025	2.913	11/03/2025	2.532
09/12/2024	3.159	27/01/2025	2.903	12/03/2025	2.418
10/12/2024	3.138	28/01/2025	2.882	13/03/2025	2.425
11/12/2024	3.09	29/01/2025	2.856	14/03/2025	2.424
12/12/2024	3.09	30/01/2025	2.849	17/03/2025	2.421
13/12/2024	3.076	31/01/2025	2.801	18/03/2025	2.413
16/12/2024	3.095	03/02/2025	2.787	19/03/2025	2.401
17/12/2024	3.04	04/02/2025	2.722	20/03/2025	2.406
18/12/2024	2.945	05/02/2025	2.735	21/03/2025	2.404
19/12/2024	2.895	06/02/2025	2.694	Source: https://www.rivaluta.it/#gsc.tab <u>=0</u>	
20/12/2024	2.862	07/02/2025	2.694		

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