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Passive, Active and Sentiment Based mean-variance allocation

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INTRODUCTION

This thesis wants to concentrate on behavioral finance and in particular on the method used by investors to face the problem of risk and return in the capital market and, especially, how the main macroeconomic events affect the performance of the securities and the investment decisions.

The traditional theory was based on the idea of being able to make coherent decisions in the financial market and was founded on two basic concepts. On one side, the efficiency of the markets that allows the agents to access to the same information, and on the other side the perfect rationality of individuals that are able to identify the best alternative which can lead them to maximize their utility.

In the current environment, however, individuals develop routines that cannot be defined as entirely rational, being moved by emotions and instincts that lead the behavior towards forms of irrationality. Here, the classical financial theory fails to give explanation to these phenomena and we are witnessing the birth of a new current of thought, the Behavioral Finance, developed by Kahneman and Tversky.

The studies of the two psychologists have demonstrated how the assessments and the decisions of individuals do not conform to the rules of logic and statistics, but are based on an intuitive reasoning which is founded on the individual experience and the emotional reactions to the information in the environment.

Their works are fundamental in the understanding of how the brain processes information and makes decisions, which form the basis of subsequent developments, leading cognitive psychologists and the most open-minded economists to understand the role of intuition in the decision making process. A beneficial role because it allows to reduce the efforts to make a decision, but also disadvantageous in areas such as the financial market that should be based on a strictly quantitative and analytical approach.

In behavioral finance and economy, there are three main topics:

- *Heuristics*, which states how often individuals make decisions based on rules of thumb, not following strictly rational analysis. An example is the cognitive biases and bounded rationality;

- *Framing*, which states that the way in which the agents deal with a problem or decision is influenced by how it is presented;
- *Inefficiencies of the Market*, stating that there are explanations for observed market outcomes which are contrary to rational rules and efficiency of the market, these include: the incorrect assessment of the price, not rational decision-making processes, and anomalies on the returns.

These three topics interact with each other, and in the development of thesis are used in order to highlight anomalous returns in the financial market due to irrational behaviors. The market-wide anomalies cannot be explained only by individuals suffering from cognitive biases, since an individual prejudice often does not produce a sufficient effect to change the prices and the market returns. Furthermore, the individual biases can potentially cancel each other. Cognitive biases have real anomalous effects only if there is a social contamination with a strong emotional content (collective greed or fear), leading to a more widespread phenomenon as the masses behavior and groupthink. Behavioral finance is, therefore, not rely on the individual psychology, but on the events that affect the society as a whole.

In the development of the elaborate I will try, so, to understand how the cognitive psychology is reflected in market prices and resource allocation. To assess this, I will build a portfolio of twenty stocks that will be tested during the period that goes from 1 January 2016 to 31 December 2016, in order to observe how the portfolio is sensitive to macro events occurred in the international market during the period considered. This portfolio will be allocated in three different ways: Passive, Active and Sentiment Based.

- The Passive allocation consists in a portfolio where the stocks enter with equal weight (1/N Portfolio).
- The Active allocation consists in a portfolio where the stocks enter with the weights of the mean-variance optimal allocation (Mean-Variance Portfolio).
- The Sentiment Based allocation consists in a portfolio where the stocks are reallocated after a significant macro event by implementing speculative techniques in order to get more gain exploiting the mass behavior of individuals.

Initially, my analysis will focus on the definition of the mean-variance model following the development of the portfolio theory of Harry Markowitz. In particular I will describe the concepts of return and risk, the latter expressed as a variance or a measure of variability of returns.

I will define, then, the frontier of efficient portfolios and determine the Capital Market Line (CML) including a risk free asset in the set of portfolios.

This first part will be useful to conduct my analysis to the Capital Asset Pricing Model (CAPM), a mathematical model of the portfolio theory and of the financial markets equilibrium, published for the first time by William Sharpe in 1964 (Nobel Prize for this contribution in 1990 together Markowitz and Miller), and then taken up by other scholars (Lintner and Mossin especially). A model that surely in recent years stood out for being the most used evaluation methods of the price of a share, for its use in the calculation of cash flows, the quantification of the cost of equity and the cost of capital (WACC) and in general all methods of quantitative evaluation of a share. I will specify, therefore, the model variables and the relationship that is the basis of the return of a security and its risk profile, expressed through the beta coefficient (β), a measure of systematic risk namely the risk that is not diversifiable or eliminable. I will explain then the Stock Market Line (SML) and its applicability and credibility in the field of investment and business valuation, noting the criticism and the actual limits.

Afterwards, I will select the stocks which may come into a possible security portfolio, to do this I have to consider some of the main stocks in the world scene, which could record an interesting performance in 2016 and that are each both geographically and sectorally diverse, in order to build a well-diversified investment portfolio. The geographical areas that will be considered are the European, the US and Global, which consider stocks of multinational companies, that act in a wider scenario not limited by political geographical boundaries.

From this selection process, I will identify thirty securities listed on several stock exchanges, and once selected I will proceed to skim the group through analysis of time series for the period 2010-2015, noting whether they meet the equilibrium condition defined by Capital Asset Pricing Model or not, using the Standard and Poor's 500 (S & P 500) as a market index and the three-month T-Bill as a risk-free rate, and evaluating then the individual performance of the securities through-out the Sharpe ratio. Thanks to these

two processes, I will determine the twenty securities that will be used to compose the portfolio, then, it will be allocated in the three different ways described above.

Initially, I will proceed to test the portfolio with passive allocation, in which the securities compose the portfolio all with the same weight, exactly equal to 1/20 (5%). It then will be tested the performance of this portfolio throughout the year 2016, calculating the return, the risk and the Sharpe Ratio.

After that, I will move to set what would be the frontier of efficient portfolios which is obtained from the optimization process of the twenty shares for the period 2010-2015. It will thus be possible to calculate the minimum variance portfolio and the tangency portfolio, essential to set the active allocation because the stocks will come in the new portfolio with the weights of the tangency. The active allocated portfolio will also be tested throughout the year 2016, and evaluating its performance in comparison with the passive allocation portfolio.

This analysis, however, will then surpassed by the setting the last portfolio, the portfolio with sentiment based allocation.

Consequently, I will explain the theory of behavioral finance starting from the classical theory, highlighting the limits and observing the theory of the efficient markets, after that, I will move on to describe the expected utility theory in which I will observe the preferences of the investors. This will introduce the behavioral finance, following the developments of Prospect Theory of Amos Tversky and Daniel Kahneman, and considering the latter as the overcoming of errors generated by the classical theory, showing the main heuristics and the main irrational behavior of individuals.

With these theories, I will move, then, to observe their applicability in the financial sector coming to evaluate the effects on the capital market of the mass behavior of investors and bringing the main empirical evidence.

This will pose the basis to proceed with the sentiment based allocation portfolio, in which, starting from the tangent weights, it will be reallocated during the whole 2016, changing the weights of the securities, whenever it meets a spread macro event that generates instability. Evaluating, consequently, the variations and the performances of this portfolio.

In the final part will be exposed the conclusions related to the study that I have computed through this thesis, which will bring me to evaluate the three portfolios and to bring to the light the fragility of the classical theory with respect to behavioral finance.

Chapter 1

THE PORTFOLIO SELECTION

Summary: 1.1 The Portfolio Theory – 1.2 The Return and Risk of a Security – 1.3 The Return and Risk of a Portfolio – 1.4 The Diversification Effect – 1.5 The Efficient Frontier – 1.6 Borrowing or Investing in Risk Free rate – 1.7 Alternative Optimizations

1.1 THE PORTFOLIO THEORY

The portfolio theory was developed by Harry Markowitz around the fifties and made public in an article published in the 1952 entitled "*Portfolio Selection*". His study is based on the analysis of the process that generates the demand and the supply of financial assets based on the risk/return ratio expressed by them[1].

The basic principle, governing the Markowitz theory, is that in order to build an efficient portfolio, a combination of securities should be identified as to minimize the risk and maximize the overall performance compensating the asynchronous performance of the individual stocks. So that it happens, the securities in the portfolio will have to be uncorrelated or, rather, not perfectly correlated. In this way the stocks, not having the same trend, reduce the standard deviation or the variance of the portfolio, otherwise the dispersion of possible performances (the risk of an investment).

The risk of any stock can be separated into two components: *Specific risk*, peculiar to each stock, and *Systematic risk*, which is derived from variations in the entire portfolio. With the diversification is possible to eliminate the specific risk, but not the systematic.

[1] Custom Publishing, *"Analisi e copertura dei fabbisogni finanziari",* Mc Graw-Hill Education, Milan 2013.

The fundamental assumptions of the portfolio theory according to Markowitz are as follow[2]:

- Investors intend to maximize the final wealth, and are risk averse.
- The investment period is unique.
- Taxes are void.
- The expected value and the standard deviation are the only parameters that guide the choice (The prices of the shares if taken with short time frequencies are distributed as normal variable).
- The capital market is perfectly competitive:
  i) All information is fully and readily available to all investors, so that all investors possess homogeneous information on asset returns.
  ii) Investors can purchase or short-sell unlimited amount of any assets.
  iii) Markets are frictionless, so that there are no transaction costs or any constraints to trading.
  iv) Assets are infinitely divisible.

1.2 THE RETURN AND RISK OF A SECURITY

The return of a financial asset is defined as the ratio between the initial capital and the earnings generated by the investment operations, or by a trade, of a specified period of time.
In case of securities, this ratio is computed with the price value considered at different instants of time. In particular, the expected return of a security can be expressed as:

$$R_t = \frac{P_t}{P_{t-1}} - 1$$

The risk of a security can be defined as the degree of uncertainty that the market expresses towards the effective realization of the expected returns, that consists in the volatility. To express the volatility of an investment are used the variance and standard deviation (the standard deviation is the square root of the variance) of the return of a stock. The variance is calculated as an average of the squared deviation of the return from the expected value.

[2] E. Agliardi, G. Chiesa, "*Economia dei Mercati Finanziari*", Carocci Editore 2003.

$$variance\ (R_t) = expected\ value\ of\ (R_t - R_m)^2$$

$$\text{with } Standard\ Deviation = \sqrt{var(R_t)}$$

where R_t is the effective return, whereas R_m is the average return[3].

These two moments of a random variable, the mean and standard deviation, determine uniquely a normal distribution. The prices are, therefore, considered as generated by a random process that expresses an expected average value of μ and a variance equal to σ^2.

1.3 THE RETURN AND RISK OF A PORTFOLIO

With the aim to calculate the risk and the return of a portfolio consisting of N assets is necessary to refer, in addition to the return and to the variance, also to a statistical measure of interdependence between the shares defined as the covariance.

In general, the expected return of a portfolio is a linear combination of the expected returns of the stocks that compose it and is calculated through the weighted average of the average returns of any individual assets, where the weights of average are the percentages invested in individual assets.

$$\mu = \sum_{i=1}^{n} \omega_{ip}\mu_i$$

$$\text{Subject to } \sum_{i=1}^{n} \omega_{ip} = 1$$

Where μ represents the vector z of every single stock return z_i.

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}$$

The value of the portfolio will initially be equal to th initial wealth W_0, of the investor who selects such portfolio. Given such initial wealth, W_0, the vector *w* will be important in determining the final value of the portfolio, *p*.

[3] Custom Publishing, *"Analisi e copertura dei fabbisogni finanziari",* Mc Graw-Hill Education, Milan 2013.

The return of the portfolio corresponds to the weighted average, with weights given by the proportion of wealth invested in the individual assets, of the assets' returns. And in this way we can then calculate the final wealth of an investor, equal to:

$$W_1 = \big(1 + \mu_p\big)W_0 = (1 + \omega' z)W_0$$

The risk of a portfolio return is measured by the variance, however, for the calculation we must consider the covariance between the various activities that compose the portfolio.

The covariance is a measure of the degree to which two variables move together, and it is equal to the product between the correlation coefficient ρ_{12} and the two standard deviations of the stocks:

$$covariance\ between\ stocks\ 1\ and\ 2 = \sigma_{12} = \rho_{12}\sigma_1\sigma_2$$
$$\text{with } -1 < \rho < 1$$

In general, the variance of the portfolio return is given by the sum of all the elements that compose the matrix formed by the covariance of all securities in the portfolio.

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \sigma_{1,3} & \sigma_{1,4} & \cdots & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_2^2 & \sigma_{2,3} & \sigma_{2,4} & \ldots & \sigma_{2,N} \\ \sigma_{3,1} & \sigma_{3,2} & \sigma_3^2 & \sigma_{3,4} & \ldots & \sigma_{3,N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{N,1} & \sigma_{N,2} & \sigma_{N,3} & \sigma_{N,4} & \ldots & \sigma_N^2 \end{bmatrix}$$

The variance-covariance matrix is identified with V_R. All the cells, along the main diagonal, contain the variances of the shares weighted by the square of the units invested in the same shares. The other cells contain the covariance between the couple of securities considered, weighted by the product of shares invested in such securities. In this way, the global variance of the portfolio is, therefore, equal to:

$$\sigma_p^2 = \omega_p^T V_R \omega_p$$

where the superscript T denotes the "transposed" of the vector ω_p of the portfolio weights. In formal terms, compute the sum of all the cells means perform the matrix calculation:

$$variance\ of\ the\ Portfolio = \sum_{i=1}^{n}\sum_{J=1}^{n} w_i w_j \sigma_{ij}$$

In the computation of the variance of a portfolio consisting of N assets, we must also include the weights of the securities that compose it. In particular we have, that the individual weights define a vector of size (N*1) equal to ω:

$$vector\ of\ weights = \{\omega\}^{N}_{i=1}$$

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \end{bmatrix}$$

Given that short-selling assets is permitted, the weights in the vector ω can be negative. Nevertheless, individuals cannot invest more than their wealth, so these weights must add up to 1, as mentioned above.

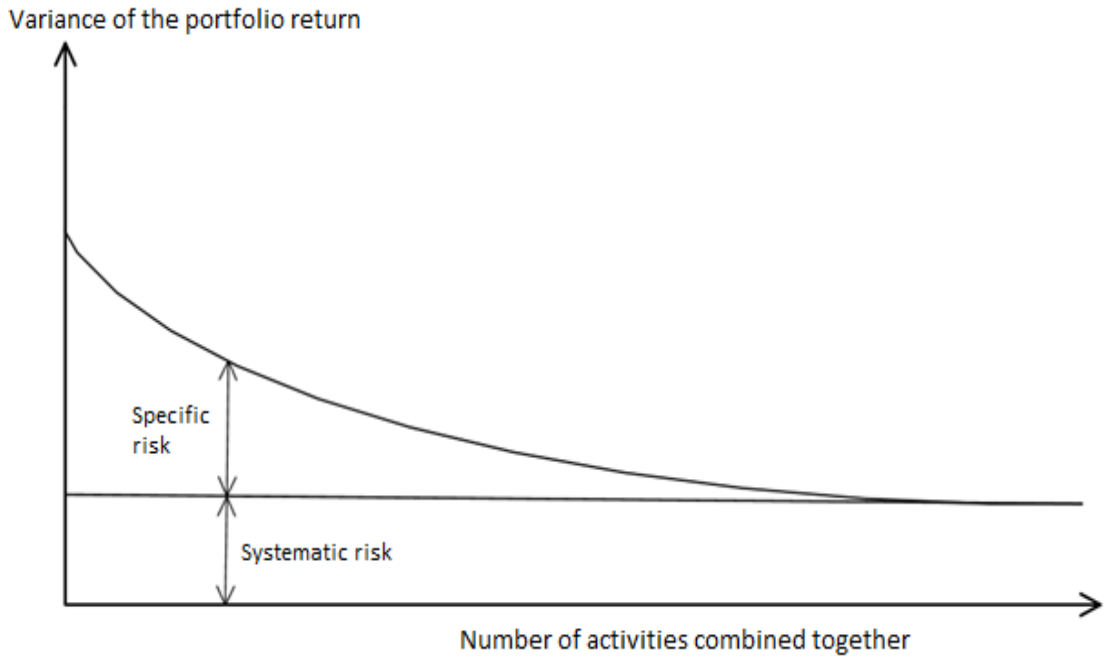
1.4 THE DIVERSIFICATION EFFECT

The variance of portfolio can be reduced by the effect of the diversification, namely composing the portfolio with shares that present a price movement not agree among each other. In this way, it is possible to have both the correlation coefficients among the securities and the covariance close to zero, therefore the variance of portfolio results diminished.

The portfolio diversification reduces the risk only when the correlation coefficient between the securities is less than one ($\rho < 1$). The best result that we can get is when two stocks are negatively correlated, although this almost never happens in the reality.

If $\rho_{12} = -1$, namely if the returns of the two activities have unitary negative correlation, it would be possible to build a portfolio with zero risk.

However, with the practice of diversification it is only possible to reduce the specific component of the risk (not set to zero), while the systematic component of the risk is inevitable.



***The Diversification**: the diversification eliminates the specific risk. but persist the systematic risk. This graph assumes that every stock has constant variance and covariance and the securities in the portfolio are taken with the same portion[4].*

With a portfolio of many activities, the effect of the diversification reduces the variability, and the variance is mainly reflected in the covariance of the securities.

In fact, if it is assumed to be handling a portfolio made up of equal size investments in each of the N shares, the proportion invested in each share is 1/N; and consequently, in each cell of the variances we have $(1/N)^2$ for the variance and in each cells of the covariance we have $(1/N)^2$ for the covariance. There are N cells with the variance and cells N^2-N with the covariance.

[4] Richard Brealy, Stewart Myers, Franklin Allen, Sandro Sandri, *"Principi di Finanza Aziendale"*, Sixth Edition, Mc Graw-Hill, Milan 2011.

Then:

$$variance\ of\ Portfolio =$$
$$= N\left(\frac{1}{N}\right)^2 * mean\ variance + (N^2 - N)\left(\frac{1}{N}\right)^2 * mean\ covariance$$
$$= \frac{1}{N} * mean\ covariance + \left(1 - \frac{1}{N}\right) * mean\ covariance =$$
$$= mean\ covariance + \frac{1}{N} * (mean\ variance - mean\ covariance)$$

When the number of activities grows (N increases), the portfolio variance approximates the mean covariance. If the mean covariance were zero it would be possible to eliminate all the risk by holding a sufficient number of securities. Unfortunately, the stocks in reality have agreed trends and they are not independent. So the majority of the shares that an investor can purchase is linked to a positive covariance that sets limits to the benefits of the diversification. The Systematic risk is the mean covariance of all securities. This is the "hard" risk that remains after that diversification has exerted its effects. Thus, the risk of a well-diversified portfolio depends on the systematic risk of the securities included in the portfolio.

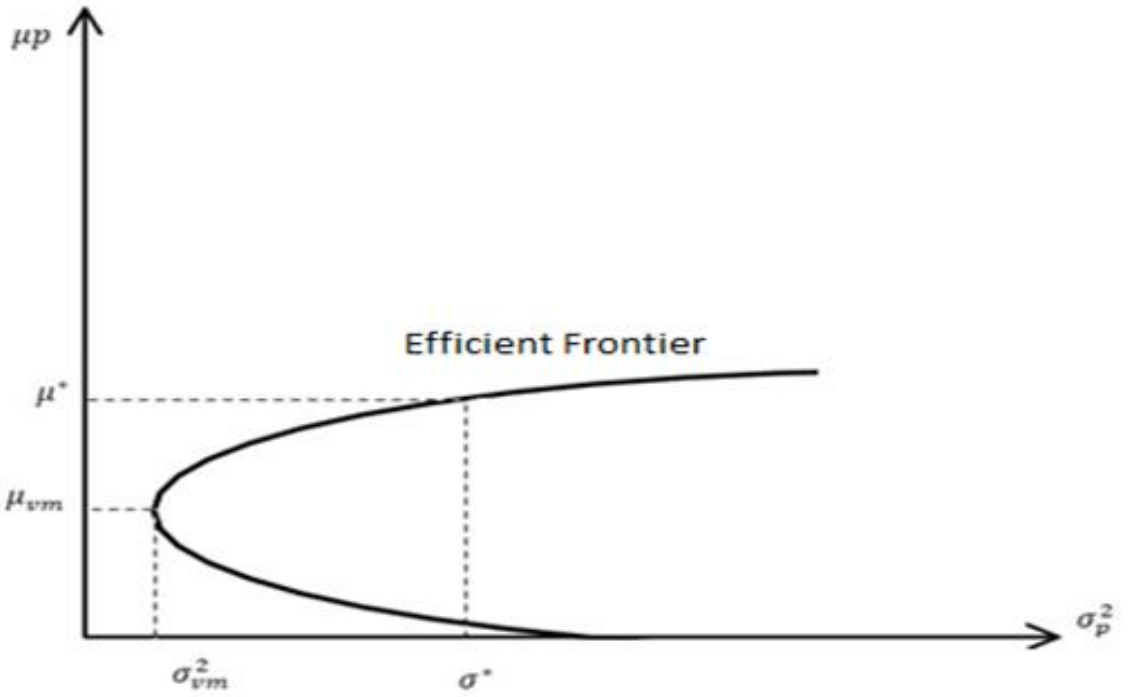
1.5 THE EFFICIENT FRONTIER

We proceed with the analysis, combining multiple securities with different risks and returns. In this way, it is possible to get a portfolio that presents as the average return, the weighted average of the individual stocks' returns, and as risk, the variance of the portfolio return, with the only constraint that the sum of the weights in the portfolio has to be equal to one[5].

Depending on the weight held by each activity we can find countless portfolios. In this manner, the most risk-averse agent will purchase more securities with low standard deviation and consequently lower return, compared with the less risk-averse agent who purchases riskier stocks, but with higher returns.

Combining investments, in individual securities, we can get a wider selection of risk-return ratios. With the aim to maximize the return and minimize the risk, we can get to determine the frontier of efficient portfolios.

[5] Banfi A. '*I mercati e gli strumenti finanziari'*. Seventh Edition. 2013, Isedi.



The Efficient Frontier*: The efficient frontier is the set of all efficient portfolios, where the risky assets come with different weights outlining the curve*[6].

These portfolios are clearly the best that can be create by combining the various activities, as there are no other portfolios that have the same performance and the same variance, any other else will have or lower returns or higher variance, the consequences for these portfolios will be the "*domination*" by the portfolios that are in the efficient frontier.

A portfolio *p* is not dominated, if and only if does not exist a portfolio *p'* providing the same expected return, namely $\mu_p=\mu_{p'}$, and a variance of the return $\sigma_{p'}^2 < \sigma_p^2$, but then *p* is not dominated if, and only if, among all the portfolios that offer an expected return equal to μ_p, it is the only one whose variance is minimal.

Among all the efficient portfolios we can identify the minimum variance portfolio.

$$\omega_{vm} = arg_{\omega} \min \omega^T V_R \omega$$

$$\text{Subject to: } \omega^T = 1$$

where the superscript T denotes the "transposed" of the vector of weights ω, while V_R indicates the variance-covariance matrix.

[6] Richard Brealy, Stewart Myers, Franklin Allen, Sandro Sandri, *"Principi di Finanza Aziendale"*, Sixth Edition, Mc Graw-Hill, Milan 2011.

This portfolio is, by definition, also the one with the least standard deviation possible, and would be chosen by the individual totally risk averse. No investor would want to hold a portfolio whose return is lower than the minimum variance one, all portfolios that are lower, in fact, are dominated.

Working with more than two securities, so with a very large number, but finished, we can determine the value of the global minimum variance portfolio leading the previous constrained maximization, using matrices and vectors.
In particular, this corresponds to solving the following *constrained optimization problem:*

$$min_{\omega} \frac{1}{2}\sigma_p^2 \equiv \frac{1}{2}\omega' V \omega$$
$$\omega' z = E_p$$
$$\omega' 1 = 1$$

Notice that we want to minimize a quadratic form and since the variance-covariance matrix is positive, we can assume this problem has a unique solution, obtained by solving the system of its first order conditions. This system is derived as follows.
Define the Lagrangian associated with the optimization problem as:

$$min_{\omega,\lambda,\gamma} L \equiv \frac{1}{2}\omega' V \omega + \lambda(E_p - \omega' z) + \gamma(1 - \omega' 1)$$

The first order conditions:

$$\frac{dL}{d\omega} = V\omega - \lambda z - \gamma 1 = 0$$
$$\frac{dL}{d\lambda} = E_p - \omega' z = 0$$
$$\frac{dL}{d\gamma} = 1 - \omega' 1 = 0$$

To solve this system, we can pre-multiply the first equation (of the F.O.C.) by the inverse of the covariance matrix, V^{-1}, to obtain:

$$\omega = \lambda(V^{-1} z) + \gamma(V^{-1} 1)$$

Then, pre-multiplying by z' we get:

$$z'\omega = \lambda(z'V^{-1}z) + \gamma(z'V^{-1}1)$$

which corresponds to Ep (see first order conditions). Pre-multiplying the system (the equation obtained above) by 1' we find instead that:

$$1'\omega = \lambda(1'V^{-1}z) + \gamma(1'V^{-1}1)$$

Readjusting, we have a new system to solve:

$$E_p = B\lambda + A\gamma$$
$$1 = A\lambda + C\gamma$$

Where

$$B = z'V^{-1}z, \quad A = 1'V^{-1}z, \quad C = 1'V^{-1}1 \quad \text{with } D = BC - A^2$$

Plugging λ and γ in the first equation and isolating the terms in Ep, gives the following optimal solution:

$$\omega_p = g - hE_p$$

where the vectors g and h are defined as follows

$$g = \frac{1}{D}[B(V^{-1}1) - A(V^{-1}z)]$$
$$h = \frac{1}{D}[C(V^{-1}z) - A(V^{-1}1)]$$

The vectors g and g + h correspond to two particular portfolios. They are the solution to the optimal choice problem for respectively Ep = 0 and Ep = 1.

Inserting the expected return Ep into equation of ω_p gives the desired portfolio's composition. An important property of the portfolio frontier is that is a convex set, in that when defining a convex combination of several frontier portfolios we still find a frontier of portfolios.

The portfolio variance associated with the optimal portfolio *p* is obtained by inserting the expression for ω_p in ω'Vω. Therefore, to the expected return Ep corresponds the following variance:

$$\sigma_p^2 = \frac{1}{D}(CE_p^2 - 2AE_p + B)$$

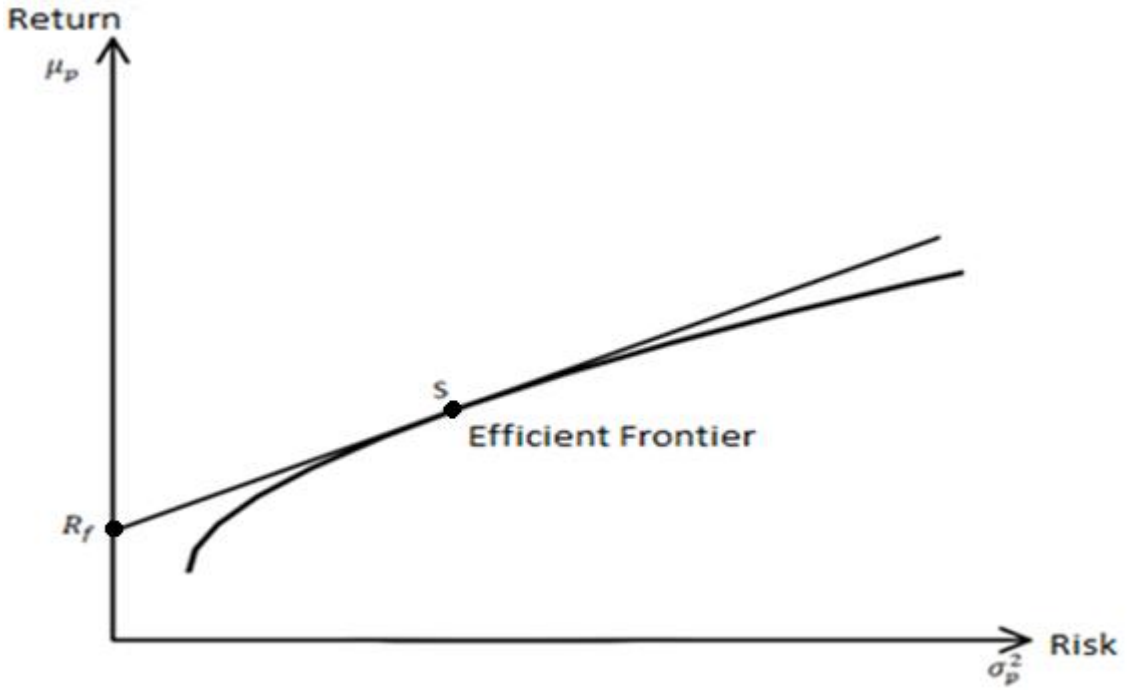
Varying Ep we obtain different values of the variance.
The weights of the global minimum variance portfolio are given by the equation:

$$\omega_p = g - h\frac{A}{C}$$

A portfolio *p*, on the frontier portfolio with expected return, Ep, larger than or equal to the expected return of the minimum variance portfolio, Ep ≥ A/C, is an efficient portfolio. The set of the efficient portfolios is the efficient frontier.

1.6 BORROWING OR INVESTING IN RISK FREE RATE

It should also consider the possibility of taking (debt) or give (investment) to borrow part of the initial allocation to a certain rate of interest R_f risk-free. If we invest in government bonds (T-bills) and in a portfolio of shares *S*, we can get any combination of risk and expected return on the straight line joining R_f to *S*. The efficient frontier, therefore, changes shape becoming a straight line. This line is called the *Market Line of Capital* or *Capital Market Line* (CML) and is tangent to the efficient frontier of risky securities.
The more vertical is the line, the better is the trade-off between return and risk.



Capital Market Line: *straight line that describes the new investment opportunities after the introduction of the risk-free asset*[7].

The risk-free asset has a return which variance is null; so it is also the global minimum variance portfolio and this is an efficient portfolio. The risky portfolio, necessarily composed by the only existing risky activity, it is also an efficient portfolio; In fact, we cannot find any other portfolio that offers the same average return with lower standard deviation. The efficient frontier will still be generated by the combination of two efficient portfolios: one is the global minimum variance, or the portfolio composed only by the risk free asset, and the other is the risky portfolio.

Now, we can try to evaluate the function of the new efficient frontier in the case of N risky assets and one risk-free. Even in this situation, we can describe a portfolio by a particular choice of the vector ω for the proportions of initial wealth invested in the N risky assets. However, now we do not need to impose the restriction that the sum of its element is equal to 1.

[7] Custom Publishing*, "Analisi e copertura dei fabbisogni finanziari",* Mc Graw-Hill Education, Milan 2013.

In fact, given the vector ω of weights for the risky assets we have a proper portfolio insofar the proportion of the investor's wealth invested in the risk-free asset is 1-ω'1. In this way the investor completely exhausts her resources, in that

$$\omega'1 + (1 - \omega'1) = 1$$

Notice that the proportion of wealth invested in the risk-free asset, 1-ω'1, can be negative, indicating that by short-selling the risk-free asset the investor borrows at the risk-free rate R_f. On the contrary, if 1-ω'1 is positive our investor lends at the risk-free rate R_f. In brief, introducing the risk-free asset corresponds to assuming that investor can borrow and lend freely at some certain interest rate.

Hence, let us reconsider the optimization problem we solved to identify the portfolio frontier. Considering that a portfolio which invests the proportions of wealth ω into the N risky assets and the proportion 1-ω'1 into the risk-free one possesses expected return equal to $\omega'z + (1 - \omega'1)r_f$, we solve the following problem:

$$min_{\omega} \ \frac{1}{2}\omega' V \omega$$

$$\text{Subject to } \omega'z + (1 - \omega'1)r_f = E_p$$

Introducing a new Lagrangian associated with this optimization problem, we consider the following optimization

$$min_{\omega,X} \ \Lambda \equiv \frac{1}{2}\omega' V \omega + X[E_p - \ \omega'z + (1 - \omega'1)r_f]$$

The FOCs w.r.t., ω, are

$$V\omega - X(z - 1r_f) = 0$$

where (z - 1rf) is the vector of expected excess returns of the risky assets on the risk-free one. This vector indicates the premium the risky assets pay on average over the risk-free rate. The first order condition with respect to the Lagrangian multiplier, X, is

$$r_f + \omega'(z - 1r_f) = E_p$$

Then, substituting this one into the previous, we find that

$$X = \frac{E_p - r_f}{H}$$

where H = (z -1rf)'V^{-1}(z -1rf), so that

$$\omega_p = V^{-1}(z - 1r_f)\frac{E_p - r_f}{H}$$

This expression implies that the vector of weights for the risky assets, which solves the optimization problem, can be decomposed into two terms:

- a N*1 vector function of the covariance matrix, V, and of the vector of the expected excess returns, (z -1rf),
- and a scalar which depends on the required expected return, Ep.

Such result indicates that if we isolate the investment into the N risky assets, the relative proportions remain unchanged when we choose a different expected return, Ep.
In other words, if an investor requires a larger expected return, Ep, the scalar term (Ep - rf)/H indicates that he will have to invest proportionally more into all risky assets, so that the relative weights do not change.
Inserting the vector ω_p into the expression for the variance of the portfolio return, we find that

$$\sigma_p^2 = \omega'_p = \frac{1}{H}(E_p - r_f)^2$$

Taking the square root of this expression we find that

$$E_p = r_f \pm \sqrt{H}\sigma_p$$

This equation is the straight line that defines the Capital Market Line, with slope $\sqrt{H}$. The slope of the capital market line (CML) indicates the trade-off between risk and return along the efficient frontier. This is defined as the ratio between the expected excess return of any efficient portfolio on its standard deviation, (Ep- r_f)/σ_p.
We can, now, identify with λ the weight with which the risky activity comes in the portfolio, so (1-λ) is the weight at which the risk-free asset enters in the portfolio, where the risky portfolio is the tangent portfolio S. The performance of the portfolio is R:

$$R = \lambda R_S + (1 - \lambda)R_f$$

The portfolio expected return is μ, and is equal to:

$$\mu = \lambda\mu_S + (1 - \lambda)R_f$$

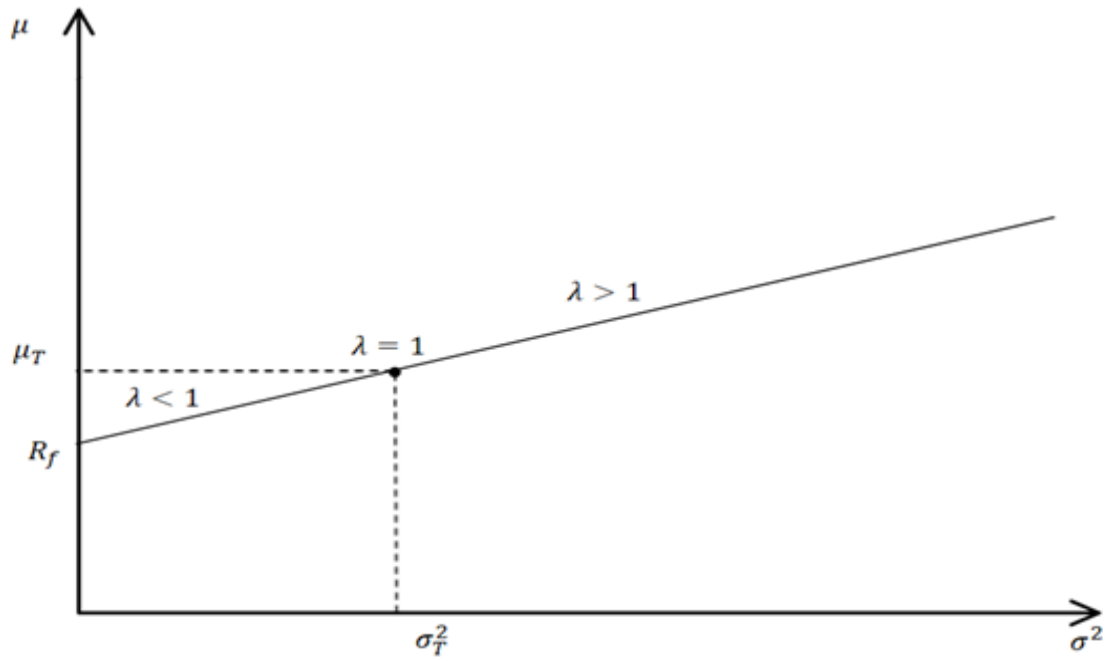
And the standard deviation is σ, equal to:

$$\sigma = \lambda\sigma_S$$

where μ_S and σ_S denote, respectively, the average return and the standard deviation of the return of the risky activity of tangency. By solving the equation of the standard deviation to λ, and substituting into the expected return, it has:

$$\mu_p = R_f + \frac{(\mu_S - R_f)}{\sigma_S}\sigma$$

This is exactly the equation of the CML efficient frontier described above: it is a straight line with an intercept R_f and slope (μ-R_f)/σ_S. While μ-R_f is called risk premium[8].



The New Portfolio Frontier: ***E****xtension of the field of investment opportunities. The straight line is the equation of the efficient frontier having r_f as intercept and as slope the Sharpe ratio, which is the trade-off between return and risk that the market offers, if λ>1 "we go short" on the risk-free activity*[9].

[8] Banfi A. '*I mercati e gli strumenti finanziari'*. Seventh Edition. 2013, Isedi.

[9] E. Agliardi, G. Chiesa, "*Economia dei Mercati Finanziari*", Carocci Editore 2003.

When we lend, we place ourselves at a point between R_f and S, and $\lambda > 0$. Whereas when we borrow at risk-free rate, we are able to extend the investment opportunity beyond the point S, then $(1-\lambda) > 1$ and $\lambda < 0$.

This result defines what financial economists call the *principle of separation*.

In other words, the investment decision is composed of two separate phases. The first is the determination of the efficient frontier, independent of the degree of risk aversion. The second is to combine the risk-free asset with the tangency portfolio. The choice of how to combine the activities is determined by personal characteristics.

The relationship between the risk premium and the standard deviation is called *Sharpe Ratio*.

$$Sharpe\ Ratio = \frac{(\mu_S - R_f)}{\sigma_S}$$

Following the footsteps of the Sharpe ratios, it is possible to measure the performance (risk-adjusted) obtained from the management of a portfolio.

Financial economists often imagine a world in which all investors possess the same estimations of expected returns, variances and covariances. Although, that fact can never be realized to the letter, it can be considered a useful simplifying assumption in a world in which investors have access to similar sources of information. This hypothesis is called *homogeneous expectations*. In this case the capital market line (CML) would be the same for all individuals. In other words, all investors would draw the same efficient frontier of risky assets, because they would work on the same data. Since the risk-free rate would be the same for everyone, every investor would consider the tangency portfolio as the portfolio of risky assets to be held.

If all investors would hold the same portfolio of risky assets, it would be possible to determine it. Common sense suggests that it is a weighted portfolio of the market value of all existing securities. It is, consequently, the market portfolio.

From this, we can afford that:

- The market portfolio lies on the CML;
- The market portfolio only contains risky assets.

In practice the market portfolio is not observable, because it would contain all stocks in the world. To overcome this problem, the financial economists use some market indexes,

e. g. FTSE 100, Dow Jones Euro Stoxx 50 or the Standard & Poor's (S&P) 500, as a representative sample of the market portfolio, depending on the country that they are analyzing. Of course, in reality not all investors hold the same portfolio.

1.7 ALTERNATIVE OPTIMIZATIONS

It describes, here, two allocation models known as Black-Litterman and Robust Optimization, as they are variant of the mean-variance model defined by Markowitz. Nevertheless, then, in the development of the analysis will be used the Portfolio Theory, as basic model, because it is faster and easier to apply, even though it can be less accurate in some points.

1) THE BLACK-LITTERMAN MODEL

In the first years of the nineties Fischer Black and Robert Litterman developed, inside Goldman Sachs, a model for the computation of the optimal weights of portfolio. The Black-Litterman asset allocation model is a sophisticated portfolio construction method that overcomes the problems of highly-concentrated portfolios, input sensitivity, and estimation error maximization. These three are the main problems with the mean-variance optimization, in which the return is maximized for a given level of risk.

In fact, one of the Black-Litterman model's strengths is that the weights of the final portfolio deviates from the market one (considered as neutral, of equilibrium) according to the extremism of the opinions and to the trust that the investor puts in them. Intuitively, therefore, if the manager has no personal views, he would possess a slice of the market portfolio, given that the equilibrium condition is computed through the Capital Asset Pricing Model and the reverse optimization technique.

The Black-Litterman model use a Bayesian approach to combine the subjective views of an investors regarding the expected returns of one or more assets with the market equilibrium vector of expected returns to form a new mixed estimate of expected returns. The model uses equilibrium returns as a neutral starting point. Equilibrium returns are the set of returns that clear the markets. The equilibrium returns are derived using a reverse optimization method in which the vector of implied excess equilibrium returns is obtained from knows information using equation:

$$\Pi = \lambda V \omega_{mkt}$$

Where Π is the implied excess equilibrium return vector (N*1), λ is the risk-aversion coefficient, V is the variance-covariance matrix and ω_{mkt} is the market capitalization weight.

The risk aversion coefficient characterizes the expected risk-return tradeoff. In a reverse optimization process the coefficient acts as a scaling factor for the evaluation of excess returns. The weighted reverse optimized excess returns is to equal the specified market risk premiuim[10].

As previously mentioned, in the Black-Litterman model is possible to create an optimal portfolio that has as its basis the market portfolio. This portfolio can be modify adding new data, according to the information that the investor possesses and the trust that he puts in. Therefore, one of the major innovation of the model is that it allows to express two types of views[11]:

- Absolute views, namley the opinion of the investor about a greater or lower return given by an asset, compared for example to return recorded the prevoius year by the same (this is the only kind of views that an investor can insert in the Markowitz process);
- Relative views, namely the opinion of the investor about a greater or lower return given by an asset compared to another asset.

Thereafter, the inputs to insert, for what concern the views, are the opinions that the investor has for the future movement of the assets and a matrix that contains his trust towards the market. The views can be, even, in conflict by each other because there is then the process of the model that combines the views and the equilibrium, with the respective errors, in order to set up the optimal allocation. Assuming that the investor has

[10] Satchell S., "*Quantitative Finance: Forecasting expected Returns in the Financial Markets*", Kidlington Academic Press. April 2011.

[11] Black F., Litterman R. (1991), "*Asset Allocation: combining investor views with market equilibrium*", The Journal of Fixed Income, September, pp. 7-18.

k different opinions on a total of N assets, the views are represented in the form of linear combinations of expected returns:

$$P * \mu = Q + \Omega V$$

Where P is a matrix (k*N), that containd the weight of every views of the investor, every row of P represents a view, where their value is different to 0, if the asset price is subject to an opinion, otherwise it is equal to 0. If the view is relative then the sum of the weights will be equal to 0, if the view is absolute then the sum of the weights will be equal to 1.

Therefore, the Black-Litterman formulas are:

$$MBL = [(rV)^{-1} + P'\Omega^{-1}P]^{-1}[(rV)^{-1}\Pi + P'\Omega^{-1}P]$$

$$VBL = [(rV)^{-1} + P'\Omega^{-1}P]^{-1}$$

Where MBL is the new vector of returns, *r* is a scalar, V is the variance-covariance matrix of excess returns, P is the matrix that identifies the assets involved in the views, Ω is a diagonal covariance matrix of errors terms, Π is the implied excess equilibrium.
It follows that the conditional distribution (post) of μ, that is the expected returns of assets, is a normal and can be specified as:

$$\mu_{post} \sim N(MBL, VBL)$$

MBL and VBL, are the outputs to which Black and Litterman arrive in their own model and, represent respectively the vector of expected returns and the variance-covariance matrix of the assets take in consideration by the investor for his portfolio, putting together both his own views and implicit returns. How the MBL returns will approach (or, in the same way, deviate) to one of the two inputs, depends on the variance of the market portfolio and the confidence that the operator has placed in his forecasts[12].

[12] Black F., Litterman R. (1992), "*Global Portfolio Optimization*", Financial Analysts Journal, Vol. 48, No. 5, pp. 28-43.

2) THE ROBUST OPTIMIZATION

In 2002 Reha Tütüncü and Matt Koenig presented an innovative approach to asset allocation problem under data uncertainty, the Robust Optimization, as opposed to the classical approach where, once estimated inputs, returns and covariances to the asset allocation problem, these are treat as certain[13].

This model overcomes the portfolio theory as the Markowitz optimization process results with the set of efficient portfolios that tend to be concentrated in a small number of securities which do not appear to be well-diversified, and efficient portfolios are too sensitive to initial variations of the parameters[14].

Therefore, the approach of Tütüncü and Koenig tries to overcome these limitations by using the robust optimization, that seeks to find an optimal asset allocation strategy, whose behavior is subjected to worst possible realizations of the uncertain inputs. The Robust optimization is an evolving branch of the field of optimization that seeks to offer vehicles to incorporate estimation risk into the decision making process in portfolio choice/asset allocation.

The two scholars apply the approach to optimize the worst case of realization of the data and evaluate the corresponding objective value to the portfolio selection problem using a judicious choice of the uncertainty set. This problem can be resolved in some cases with a standard quadratic programming, in other cases this is not possible, therefore is formulated the robust optimization as a saddle-point problem.

The optimal portfolio selection problem can be a mathematic formulation as quadratic optimization problem. The convex set of portfolios refers to minimizing a convex quadratic function (or, equivalently, maximizing a concave quadratic function) subject to linear equality and inequality constraints. The solution of a convex function associated with an asset allocation problem generates an efficient portfolio on the efficient frontier. To generate the entire efficient frontier, the function has to be parametrized and this can be done in three essentially equivalent ways:

- Maximize expected return subject to an upper limit on the variance.

[13] R. Tütüncü and M. Koenig, "*Robust Asset Allocation",* September 18, 2003.

[14] Satchell S., "*Quantitative Finance: Forecasting expected Returns in the Financial Markets*", Kidlington Academic Press. April 2011.

- Minimize the variance subject to a lower limit on the expected return.
- Maximize the risk-adjusted expected return.

These three problems are parametrized by the variance limit, expected return limit, and the risk-aversion parameter, respectively. The last two of these are:

$$min_{\omega \in R^n}\ \omega^T V \omega$$
$$s.t.\ \mu^T \omega \geq R, \omega \in \mathrm{X}$$

And

$$max_{\omega \in R^n}\ \mu^T \omega - \lambda \omega^T V \omega$$
$$s.t.\ \omega \in \mathrm{X}$$

Where μ is the vector of expected return, V is the variance-covariance matrix, ω is the proportion of the portfolio to be invested in security *i, X* represents the polyhedral set of feasible portfolios, R is the lower limit on the expected return one would like to achieve and finally λ represents the risk-aversion parameter. The objective function of problem represents a risk-adjusted expected return function. Since the covariance matrix is always positive semidefinite, the convex quadratic programming problems is solvable in polynomial time. By solving the two equations for different values of R and λ, one can generate a sequence of optimal portfolios on the efficient frontier ranging from the portfolio with the smallest overall variance to the portfolio with the highest expected return.

The strategy is to represent all the available information on the unknown input parameters in the form of an uncertainty set, namely a set that contains most of the possible values for these parameters.

The two scholars assessed that the robust optimization provides a valuable asset allocation vehicle to conservative investors.

Chapter 2

THE CAPITAL ASSET PRICING MODEL (CAPM)

Summary: 2.1 The Beta – 2.2 The Expected Return – 2.3 The Security Market Line (SML) – 2.4 Three aspects concerning the Capital Asset Pricing Model – 2.5 Roll's Critique – 2.6 Validity of the Model – 2.7 Limitations of the CAPM

The mean-variance model, in which agents choose the optimal portion of the securities, leads to an equilibrated model of expected returns, known as *Capital Asset Pricing Model* (CAPM).

The CAPM is a model according to which the expected returns increase linearly with the beta of an asset. In this way the expected return is positively related to its risk[15].

This equilibrium model of financial markets was proposed by William Sharpe in a historic contribution of 1964, "*Capital asset prices: a theory of market equilibrium under conditions of risk*", Journal of Finance[16]. Came to similar results, simultaneously and independently, John Lintner in 1965, in *"The valuation of risky assets and selection of risky investments in stock portfolios and capital budgets*", Review of Economics and Statistics[17].

The core of the CAPM is the relationship between the expected return on any security and the performance of the market portfolio.

The model is based on the assumptions that the agents in the economy:

- Evaluate the lotteries based on the statistical moments of the mean and the variance.
- They have access to the same set of activities, and among them one is risky.
- They have the same information about all activities.
- They face the same prices of the assets and these are determined competitively, agents are price takers, and the markets are in equilibrium, the demand is equal to supply.

[15] E. Agliardi, G. Chiesa, "*Economia dei Mercati Finanziari*", Carocci Editore 2003.

[17] W. Sharpe, *"Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk",* Journal of Finance, 19, pp. 425-442, September 1964; John Lintner, *"The valuation of risky assets and selection of risky investments in stock portfolios and capital budgets*", Review of Economics and Statistics, 47, pp.13-37, February 1965.

From these assumptions it follows that the set of possible choices is identical between individuals, they eliminate from the cluster the dominated portfolios and the efficient frontier is the same for everyone and is defined by the equation:

$$\mu = R_f + \frac{(\mu_T - R_f)}{\sigma_T}\sigma$$

All individuals build portfolios that are linear combinations of the non-risky activity and the only efficient risky portfolio. These portfolios are not necessarily identical: agents with different degrees of risk aversion will choose portfolios in which the non-risky activity enters with a different weight, but the risky portfolio will be the same for everyone.

In addition to these new assumptions, remain valid even the assumptions previously introduced:

- There exists, N risky assets with random returns, r, which present a vector of expected values μ and a covariance matrix V.
- There exists a risk-free asset which pays a certain return r_f.
- Markets are frictionless, in that there are no transaction costs or constraints to trading. Investors can purchase and sell unlimited quantities of all assets; they can freely short-sell assets and lend and borrow at the risk-free rate r_f.
- Information is fully and readily available, so that investors possess symmetric information on asset returns.
- Investors have preferences which uniquely depend on the mean and variance (standard deviation) of the return on the portfolio of assets they possess.
- Assets are infinitely divisible.

2.1 THE BETA

The risk of a well-diversified portfolio depends on the systematic risk of the securities included in the portfolio, and the Beta is the measure of systematic risk. Increasing the number of securities in the portfolio, the risk decreases until all the specific risk of every stock is eliminated and only the systematic risk persists.

The Beta measures the sensitivity of a stock to the variations occurring in the market portfolio. This change is detected by the covariance of the asset's return with that of the market portfolio, and is also called non-diversifiable risk of the security.

$$\beta = \frac{cov(R_i, R_T)}{\sigma_T^2}$$

where $cov(R_i, R_T)$ is the covariance between the return of the asset *i* and the return of the market or tangency portfolio, while σ_T^2 is the variance of the market or tangency portfolio. A particularly important property is that the average Beta of all the securities, when it is weighted by the portion of market value of each stock compared to the market portfolio, is equal to 1[18].

$$\sum_{i=1}^{n} \omega_i \beta_i = 1$$

where ω_i is the market share *i* of the security relatives to the entire market, while N is the number of securities in the market. This means that if we ponder all securities according to their market value, the portfolio that results, is the market portfolio.

By definition, the Beta of the market portfolio is 1. Stocks with a Beta greater than 1 tend to amplify the global movements of the market (the activity is more risky than the market): in general it is believed that companies with aggressive business policies or with high levels of debt present values of the Beta higher than the other. These securities are called *"aggressive securities".*

Conversely, stocks with a value of Beta between 0 and 1 tend to move in the same direction of the market, but not with the same intensity (the business is less risky than the market): these are generally securities issued by companies operating in the fields of traditional economy and are called *"defensive securities"*. Therefore, investors can use the Beta to evaluate the relative risk of various stocks.

The β_i of securities, therefore, measure the incremental effect of the share *i* to the risk of the portfolio. A stock with $\beta_i=0$, when added to the portfolio, has an additional influence on the portfolio variance equal to zero, while securities with $\beta_i<0$ reduce the portfolio

[18] Richard Brealy, Stewart Myers, Franklin Allen, Sandro Sandri, *"Principi di Finanza Aziendale"*, Sixth Edition, Mc Graw-Hill, Milan 2011.

variance. Of course, without altering other conditions, the greater is the amount of the stock *i* held (namely the greater is the absolute value of ω_i) the greater will be the impact of β_i on the total portfolio variance. Since a security with a low value of β_i reduces the total variance of a risky portfolio, it will be held even if it has an expected return relatively low. All investors face a trade-off between risk, who do not like, and a return, that love. The activities that reduce total risk of portfolio are, therefore, held in equilibrium even though they have a relatively low return.

The Beta of an individual portfolio of N stocks is:

$$\beta_p = \sum_{i}^{n} \omega_i \beta_i$$

it is simply the weighted average of the Betas of the individual stocks in the portfolio. Researchers have shown that the best measure of the risk of a security included in a large portfolio is the Beta of the stock itself.

2.2 THE EXPECTED RETURN

Starting from the results previously obtained, considering a set of N securities and an asset without risk, we can try to determine mathematically what is the relationship between the return of a security and its risk in the market.

We restart from the vector of weights of an efficient portfolio *p* lying in the *Capital Market Line*:

$$\omega_p = XV^{-1}(z - 1r_f)$$

where X is the corresponding Lagrangian multiplier. Among these efficient portfolios we isolate the tangent portfolio, T, through the condition $1'\omega_T = 1$, in that this portfolio contains only risky assets. Imposing this condition, we find that:

$$\omega_T = \frac{1}{1'V^{-1}(z - 1r_f)} V^{-1}(z - 1r_f)$$

This portfolio will play a pivotal role in the derivation of the CAPM. Reformulating the equation of the weights of the efficient portfolio *p* as follow:

$$V\omega_p = X(z - 1r_f)$$

This expression possesses a very simple interpretation, in that on the right side we have the vector of expected premium paid by the N risky assets on the risk-free rate, while on the left side we have the vector of the covariances of the excess returns of the N risky assets, z - $1r_f$, with the excess return on portfolio *p*, μp - r_f. To realize this the following condition holds:

$$cov\left[\left(z - 1r_f\right), \left(\mu_p - 1r_f\right)\right] = cov[\left(z - 1r_f\right), \left(z - 1r_f\right)'\omega_p]$$

where we have used the fact that $\mu_p = r'\omega_p + (1 + 1'\omega_p)r_f$. Then, exploiting the linearity of the covariance operator, we find that:

$$cov\left[\left(z - 1r_f\right), \left(\mu_p - 1r_f\right)\right] = V'\omega_p$$

$$\text{Where } V = cov[\left(z - 1r_f\right), \left(z - 1r_f\right)']$$

Loosely speaking, the condition which identifies the efficient portfolio *p*, can be reformulated as:

$$z - 1r_f = kV\omega_p$$

for some positive constant k. This optimality condition can be read as follows. The portfolio *p* is efficient in that the average expected excess returns on the N risky assets are proportional to their covariances with the excess return on portfolio *p*.
Importantly, this property is shared by all efficient portfolios that lie in the CML and are represented by a point. This means that such property will be satisfied by the tangent portfolio, an implication which is central to the proof of the CAPM.

However, such property is not satisfied only by the tangent portfolio. This is important for a fundamental criticism to the empirical analysis of the CAPM put forward by Roll (1977). Pre-multiplying the left side in the last equation by w_p, we find that:

$$\omega'_p\left(z - 1r_f\right) = E_p - r_f$$

$$\text{in that } \omega'_p\left(z - 1r_f\right) = \mu_p - r_f.$$

On the other hand, Pre-multiplying the right side in the equation by w'$_{p,}$ we have that:

$$\omega'_p kV\omega_p = k\sigma_p^2$$

Thus, we conclude that the following holds:

$$E_p - r_f = k\sigma_p^2 \qquad \text{or} \qquad k = \frac{E_p - r_f}{\sigma_p^2}$$

Substituting out k into the previous equation, we have:

$$z - 1r_f = \frac{E_p - r_f}{\sigma_p^2} V\omega_p \qquad \text{or equivalently} \qquad z - 1r_f = \beta^p (E_p - r_f)$$

$$\text{where } \beta^p \equiv \frac{V\omega_p}{\sigma_p^2}$$

Recalling that Vw$_p$ is equal to the vector of covariances between the excess returns on the N risky assets and the excess return on portfolio, p, $cov\left[\left(z - 1r_f\right), \left(\mu_p - 1r_f\right)\right]$, we can re-write the optimality condition in scalar form defining that for any risky asset *i* its expected excess return on the risk-free rate is proportion to its correlation with any efficient portfolio *p*, in that:

$$E_i - 1r_f = \beta^p (E_p - r_f)$$

$$\text{where } \beta^p \equiv \frac{cov[r_i, r_p]}{\sigma_p^2}$$

In other words, the equation links the expected excess returns on the risky assets to their co-movement with the return on the generic efficient portfolio *p*.

This is the CAPM formulation and it is an equilibrium condition since we have introduced that the markets are in equilibrium, for any asset the total supply equals aggregate demand.

From this analysis, we can say that the CAPM provides an elegant model of the determinants of the expected return equilibrium μ of each single risky activity on the market. It affords that the excess of expected return on every single risky activity (μ-R_f) is directly related to the expected excess returns of the market portfolio (R_m-R_f), with the constant of proportionality given by Beta of the single risky activity:

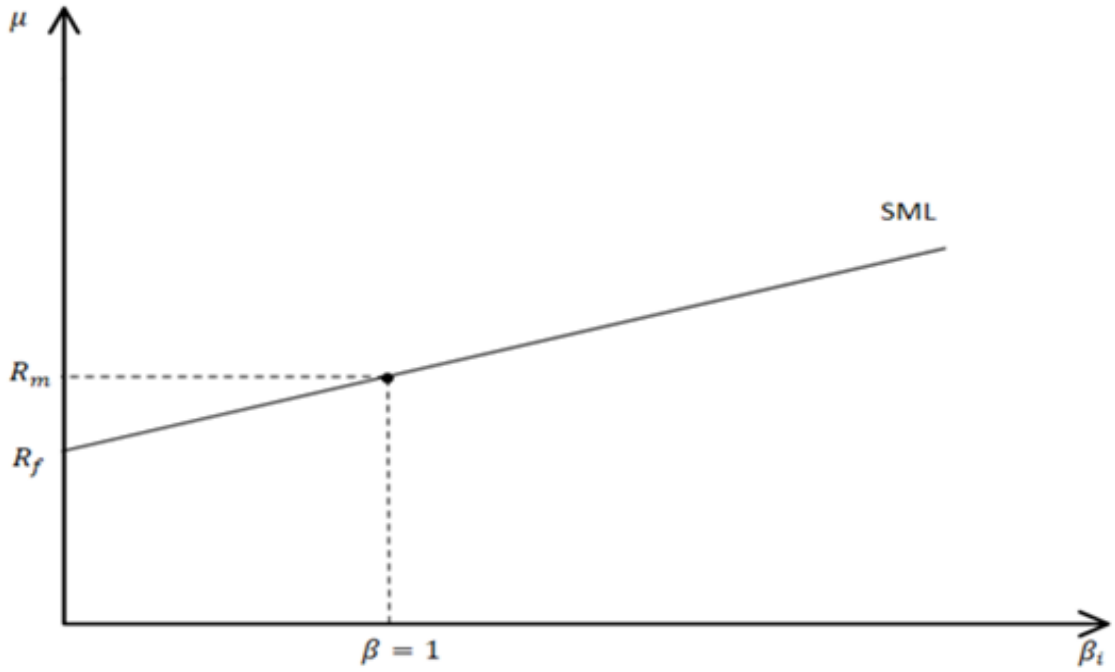
$$\mu - R_f = \beta\left(R_m - R_f\right)$$

This formula implies that the expected return on a security is linearly correlated with its Beta. Since the average market return was higher for long periods at the risk-free rate, (R_m-R_f) is presumably positive.
Once determine the expected return μ, we can use it in order to determine the net present value of a financial asset defining its correct price, or alternatively, given any valuation model, it is possible to solve the equation for the evaluation of financial asset depending on the discount rate, assuming the observed market price as fair. If the resulting discount rate is lower or higher than that implied by the CAPM, the financial asset can be overpriced or underpriced.

The CAPM explains the excess of the expected return on the stock *i* given the excess of expected market returns. This equation is not predictive of the performance of the security *i*, since both variables, dependent and independent, are considered at time *t*. Rather, the CAPM implies that contemporary movements in the (μ-R_f) are connected to contemporary changes in the excess of the market return (R_m-R_f).
In general terms, if the ex post returns (or effective) approximates on average the expected returns ex ante, it is possible to think that the *Capital Asset Pricing Model* is able to explain the average returns of the security *i*. The model implies that it should receive a compensation based on how the security contributes to the overall portfolio risk, given by the Beta.
The risk premium (R_m-R_f) is presumably always positive. Otherwise no risk averse agent would hold the market portfolio of risky assets when he could earn more money, for sure, investing all its richness in the activity without risk (R_f). The starting point for determining the premium prize for future risk is the average premium prize of the past risk. In Italy the premium for the medium-risk, calculated from 1860 to 1994, was equal to 5.7%. Nowadays it is considered reasonable a risk premium of the market of 5-5.5%.

2.3 THE SECURITY MARKET LINE (SML)

The equation of the CAPM can be represented graphically by an upward straight line. The straight line starts at the point R_f and rises up to the height of R_m when the Beta is equal to 1, and then continues. This straight line is called *the Security Market Line* (SML).



The Security Market Line: *The SML has as intercepts R_f, the risk-free rate, and as slope R_m-R_f,, the risk premium*[19]*.*

The line will have a positive slope as long as the expected return of the market will be greater than the risk-free rate. Since the market portfolio is a risky activity, the theory indicates that, the expected return should be higher than the risk-free rate.

Along the line, the excess of the average return on each security (μ-R_f) should be proportional to the Beta activity, so a stock with a high risk should, therefore, provide a higher average return.

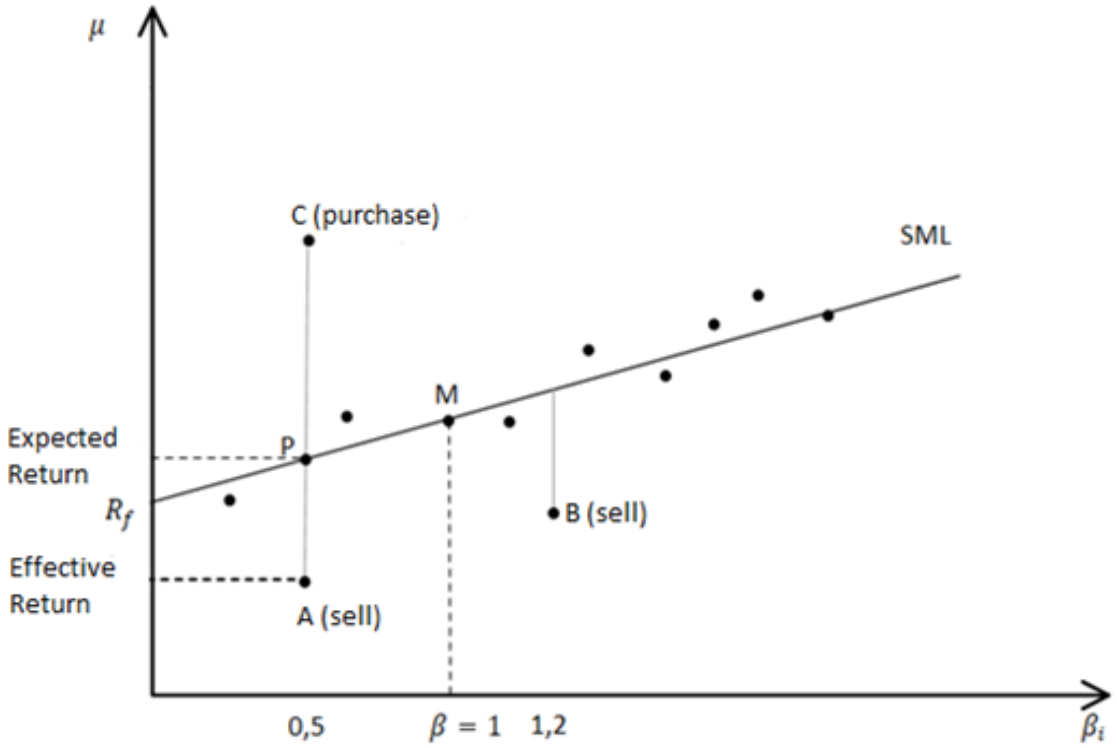
$$\frac{(\mu_i - R_f)}{(\mu_j - R_f)} = \frac{\beta_i}{\beta_j}$$

It is possible, then, to use the SML in order to identify overvalued or undervalued stocks. If we consider, for example, a stock A with a Beta of 0.5. We can replicate the Beta of the stock A by purchasing a portfolio P consisting of the 50% of a risk-free asset (with β_i=0) and the remaining 50%, invested in a stock with a Beta equal to the unit (i.e. β_p=0,5(0) + 0,5(1) =0,5). However, this portfolio has a higher expected return than A by definition. So A should be sold, given that its actual performance is less than the

[19] Custom Publishing*, "Analisi e copertura dei fabbisogni finanziari",* Mc Graw-Hill Education, Milan 2013.

equilibrium return given by its Beta. If A is sold, its current price would fall by increasing its expected return, so that A will tend towards P.

Similarly, we could duplicate an asset B with β_i=1,2 by borrowing 20% of our wealth at the risk-free rate, and using those funds, together with our wealth, to invest in a stock with β_i=1 (M).



***The predictive ability of the SML**: the graphic explains how the SML carries out its predictive role, determining whether an asset is overvalued or undervalued*[20]**.**

Alternatively, if we consider a security C with β_i=0,5 too, but which actually presents a higher average return than that indicated by the SML. An investor should purchase C. In fact, securities such as A and C do not have the correct price and a speculator might short sell A and use the funds collected to buy C. If the error is corrected, then the price of the stock C will increase a bit at a time until that someone will try to buy the stock because of its abnormal high average return. On the contrary, anyone who tries to short sell A, will make decrease its market price. This error can afford to make considerable profits for the first few investors who realize it, but this also brings back the securities to their equilibrium point, along the Security Market Line.

[20] Richard Brealy, Stewart Myers, Franklin Allen, Sandro Sandri, *"Principi di Finanza Aziendale"*, Sixth Edition, Mc Graw-Hill, Milan 2011.

2.4 THREE ASPECTS CONCERNING THE CAPM

We highlight, finally, three specific key aspects that distinguish the portfolio theory from the model for the determination of the expected return of a stock.

1. *Linearity*. The basic intuition that is about a positively sloped curve is quite clear. Since the Beta is the appropriate measure of risk, stocks with high Beta should have an expected return greater than those who have a low value of Beta. Therefore, the curve, in this case, is a straight line because all the securities that are positioned below the line would not be held by anyone, and their prices would drop. This price adjustment process would increase the expected returns of the securities, and would continue until the shares would arise in the Security Market Line. The reverse process takes place for the securities that are located above the SML. In equilibrium the prices fit along the straight line, if the line had a curvilinear course, many securities would present anomalies in their price. This process allows us, to predict the performance of the securities, and opens to revenue opportunities.
2. *Portfolios and Securities*. The CAPM applies to both the securities and the portfolios. In particular, the expected return of a portfolio is given by the weighted average of the expected returns of the asset that compose the portfolio itself, while the Beta is a weighted average of the Betas of securities in the portfolio, where the weights are the proportion with which the securities enter in the portfolio.
3. *The difference between the CML and the SML*. The Capital Market Line and the Security Market Line are two entirely different lines. The first (CML) defines the efficient frontier of portfolios both composes from risky assets and asset without risk. Each point in the line represents an entire portfolio. The variables considered are the expected return and standard deviation, the individual stocks do not lie along the CML. The second line (SML), instead, links the expected return of a security with its Beta. The straight line operates for all individual securities and for all the possible portfolios, while the CML is only worth of efficient portfolios.

In addition to these three aspects, it is important to underline that the Capital Asset Pricing Model is one of the most important progress produced from the financial economy. It is a model, clearly effective, for investment purposes, because it shows how the expected return of an activity is correlated with its Beta. However, it should specify that, like any other model, it needs to be tested empirically to give some information about reality. And like every other theoretical model, the CAPM can only provide guidance on the real world. Any operator will never admit to be convinced that the CAPM fully explains the performance of the stocks, of the investments or of the financial securities portfolios.

2.5 ROLL'S CRITIQUE

Before examining the empirical tests, it is important to understand if the CAPM can be effectively tested. Richard Roll, in the *Journal of Financial Economics*, said that *it is almost impossible to build a portfolio that includes all of the existing shares (the true market portfolio), any test carried out on the CAPM with the use of a representative market index (e.g. FTSE 100, DAX, S&P 500 and CAC40) will test that particular portfolio and not the true market portfolio*. This means that, in reality, the CAPM is impossible to test empirically because the underlying market portfolio is undetectable. Whatever test on the CAPM that uses indexes representative of the market will be subject to this criticism.

Scholars have devoted much time to discuss the merits of the Roll's critique (1977). Nevertheless, the use of the model in the corporate finance and investment is widespread. The financial sites such as Yahoo!Finance, Bloomberg and Morningstar frequently provide Beta estimation of the Wall Street's firms.

In front of the unquestioned popularity of the CAPM, and taking into account of the weaknesses that are associated with any empirical testing, it is important to understand if this model is able to explain at least a part of the variations that determines the performance of securities.

2.6 VALIDITY OF THE MODEL

The first empirical tests on CAPM were conducted more than 30 years ago, and have been favorable to the model. Using data on US equity markets relative to the period between the thirties and sixties of the last century, researchers have shown that the average return of an equity portfolio was positively correlated to the Beta of portfolio, a consistent finding towards the CAPM validity. Whereas some evidence emerged in these studies were less consistent with the CAPM, for example, indicated that the average return of a portfolio, whose Beta is zero, is higher than the risk-free rate (an incoherent discovery with the CAPM). Financial economists have, nevertheless, adopted the model immediately.

One of the first works on the empirical validity was conducted by Eugene Fama and James MacBeth, "*Risk, Return and Equilibrium: Empirical Tests*", Journal of Political Economy (1973), performing an empirical analysis of the data in the period between 1926 and 1968. The two economists start with some basic observations on the expected return and the Beta:

$$\mu - R_f = \beta\left(R_m - R_f\right)$$

$$\beta = cov\frac{(R_i, R_m)}{\sigma_m^2}$$

$$\text{therefore } \mu_i = R_f(1 - \beta_i) + \beta_i\mu_m$$

They have noted that for any activity:

1. The relationship between excess return and risk is linear;
2. The Beta is a complete measure of risk, no other risk factor is present in the equation

$$\mu = R_f(1 - \beta_i) + \beta_i\mu_M;$$

3. With risk aversion, higher risks have to match higher expected returns (μ_m-R_f>0).

The third implication is obvious, the first and the second are less obvious. If these were not worth (if they were not validated by the data), it means that some securities would be

systematically undervalued or overvalued, if compared to the requirements of the relationship between expected return and risk defined by

$$\mu = R_f(1 - \beta_i) + \beta_i \mu_M.$$

Their methodology is in two stages since the Beta cannot be directly observed:

- First phase: Beta estimated through *time series* linear regression model with the form:

$$R_i = \alpha + \beta_i R_m$$

- Second phase: they used the estimations for the Beta as observations of the regressors in the *cross-section* regression with the form:

$$R_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 \beta_i^2 + \gamma_3 s_i + \eta_i$$

  where s_i is a risk factor of the activity *i* that is not correlated with β_i. Comparing the equation with $\mu = R_f(1 - \beta_i) + \beta_i \mu_M$

  This is validated by the data if:
  $E(\gamma_1) = \mu_i - R_f$;
  $E(\gamma_2) = 0$: the relationship between return and risk is linear;
  $E(\gamma_3) = 0$: The Beta is a complete measure of risk. The results do not fully confirm the first and the third hypothesis.

Despite the large number of empirical studies developed, the CAPM has not been seriously put under discussion until the nineties, when two papers of Eugene Fama and Kenneth French, "*The cross-section of expected returns*" Journal of Finance (1992), which revisit and it deepen the article of Fama-MacBeth (1972), and "*Common risk factors in the returns on stocks and bonds*" Journal of Financial Economics (1993)[21], have presented inconsistent evidences with the model. Their work has received a great deal of attention, both in academic circles and in the popular press, so that the newspapers were

[21] E. Fama and K. French, "*Common risk factors in the return on stocks and bonds*" Journal of Financial Economics, 1993, 33, pp. 3-56; E. Fama and K. French, "*The cross-section of expected returns*" Journal of Finance, June 1992.

publishing titles such as "*Beta Is Dead!*"[22]. The articles of Fama and French contain two closely interconnected observations.

First, they conclude that for US companies, the relationship between average return and Beta is weak between 1941 and 1990, and virtually non-existent between 1963 and 1990. Secondly, they say that the average return of a share is negatively correlated with both the price-earnings ratio (P/E) of the enterprise, and with the ratio of market value-book value (P/BV, Price/Book Value) of the enterprise.

Other researches have provided convincing evidence against the CAPM by demonstrating that the most profitable securities in the recent past tend to have higher returns in the near future (the "momentum" factor theorized by Carhart in 1997). In addition, Ang, Hodrick, Xing and Zhang (2009) found that the stocks characterized by greater sensitivity to the volatility of market returns have lower returns than comparable enterprises that have stock with the same Beta of the market. It may be that these results are applicable only for the US market; however, an insufficient predictive value of the Beta was detected by Fama and French (1998) even in other countries.

These criticisms, if confirmed by further research, would be extremely harmful for the CAPM. After all, the CAPM says that the expected returns of equity securities should be correlated only with the Beta, and not with other factors such as price-earnings ratio, the ratio of market value and accounting value, the momentum or the market volatility.

On the other hand, even at the researches, such as those conducted by Fama and French, have been done several criticisms. First, although Fama and French, Carhart and Ang and colleagues cannot exclude the hypothesis that average returns would not be closely related to the Beta, we cannot reject the hypothesis that the average returns are correlated with Beta exactly as specifies the CAPM; in other words, even though 50 years of data may appear many, they may not be enough to adequately test the CAPM. Second, the results may be due to a statistical error known as "*distortion of hindsight*". Third, P/E, P/BV, momentum, and market volatility are four of the infinite factors that we might consider: therefore, the relationship between performance of the stocks and these variables could be the result of a surface effect caused by the mode of *data mining*. Fourth, the average returns of US securities actually have a positive correlation with the Beta between 1927 and today; so, there are no particular reasons to emphasize a period shorter than this. Fifth,

[22] C. Wallace, "*Is Beta Dead?*", Institutional Investor, pp.23-30, 1980.

the average returns are still positively correlated with the Beta for shorter periods, when estimating the coefficient using annual data instead of monthly.

2.7 LIMITATIONS OF THE CAPM

The *Capital Asset Pricing Model* is based on various assumptions that have not been fully explained. For example, it was assumed that investing in Treasury Bonds (T-bills) is completely without risk. It is true that the risk of default is highly unlikely, however this does not guarantee a real return and the risk of inflation remains. Another hypothesis is that investors can borrow at the same interest rate at which they can invest. In general, deposit rates are higher than lending rates and in the market is not present a single risk-free interest rate.

Beyond that, the basic assumptions of the model are excessively restrictive, and extremely far away from the reality. It is implausible that all agents have the same utility function and are distinguished only by their risk aversion.

It also considers the market as perfectly competitive where agents are price-takers and they are all in possession of the same information, with the absence of transaction costs.

Moreover, there are the assumptions of the mean-variance model, which affords that the distribution of risky returns is characterized only by the random variables of the mean and variance.

In addition, although the returns of the securities from 1927 to today are positively related to their Beta, its critics point out that the slope of the line is "particularly flat" than expected by the CAPM. Critics also point out that the model returns have not grown in recent years according to the Beta, but according to other measures[23].

Finally, the CAPM underestimates the majority shareholder's influence and their importance in the decision-making process for the destination of the cash flows of a company.

[23] F. Black, *"Beta and Return"*, Journal of Portfolio Management, 20, pp. 8-18. 1993.

Chapter 3

THE PORTFOLIO CONSTRUCTION AND THE PASSIVE ALLOCATION

Summary: 3.1 The Stocks' selection – .32 The First phase: The CAPM – 3.3 The Second phase: The historical Sharpe Ratio – 3.4 The 1/N Allocation

3.1 THE STOCKS' SELECTION

In this section will be build the portfolio of securities to be tested. Initially, are taken into account thirty shares ending up to select only twenty among those that will then enter in a final portfolio. This portfolio will be tested for the period that runs from the 1 January 2016 to the 31 December 2016.

The shares will be rejected in two phases, in the first phase are rejected the stocks which do not satisfy the equilibrium condition defined by the CAPM; while in the second phase, are rejected the under-performing shares, using as a criterion of performance the *Sharpe Ratio*.

The price of the securities used, is downloaded from the time series database of Yahoo!Finance, for which is used a monthly time frequency in the period that goes from the 1 December 2010 to the 31 January 2015.

The criteria for the selection of the first thirty securities are two:

- The individual securities must have a positive monthly average return in the period in question (01/01/2010 - 31/12/2015). It is used this period in order to prevent, in the development of the analysis on historical returns, entry the period of the "2008 crisis" and subsequent. Thus the performance of the shares is not distorted by this factor.
- The thirty securities must belong to different sectors and geographical areas, so that the portfolio that will result is well diversified, in this way we can better test the effect of macro events, and can also eliminate the specific risk component of each stock, which it may be subject to change due to unexpected announcements or scandals (e. g. Dieselgate) that may relate to companies to which they refer.

The stocks considered are the following[24]:

Table 1. The selected stocks

NR.	STOCK	CODE	SECTOR	GEOGRAPHIC AREA
1	Nike	NKE	Sportswear	Global-US
2	The Boeing Company	BA	Aerospace	Global-US
3	Apple	AAPL	Information Technology	Global-US
4	Mac Donald	MCD	Fast Food	Global-US
5	Johnson and Johnson	JNJ	Pharmaceutical	Global-US
6	Airbus Group SE	AIR.PA	Aerospace	Global-France
7	The Procter & Gamble Company	PG	Consumer Goods	Global-US
8	The Coca-Cola Company	KO	Beverage	Global-US
9	AT & T	T	Telecommunication	US-Texas
10	Chevron	CVX	Oil and Gas	US-California
11	Walmart	YMT	Super Centers	US-Arkansas
12	Bank of America	BAC	Financial Services	US-North Carolina
13	General Electric Company	GE	Industrial Conglomerates	US-Massachusetts
14	Intel	INTC	Semiconductors	US-California
15	IBM	IBM	Consulting and Services	US-New York
16	Caterpillar Inc.	CAT	Construction Industries	US-Illinois
17	The Travelers Companies, Inc.	TRV	Insurance	US-Minnesota
18	Fiat Chrysler Automobiles	FCA	Automotive	Europe-Italy
19	Barclays PLC	BARC.L	Banking	Europe-England
20	HSBC Holdings	HSBA.L	Financial Services	Europe-England
21	Volkswagen Aktiengesellschaft	VOW.DE	Automotive	Europe-Germany
22	Allianz SE	ALV.DE	Insurance	Europe-Germany
23	Deutsche Lufthansa Aktiengesellschaft	LHA.DE	Air Transport	Europe-Germany
24	Bayer Aktiengesellschaft	BAYN.DE	Pharmaceutical	Europe-Germany
25	Michelin	ML.PA	Autoparts	Europe-France
26	Pernod Ricard SA	RI.PA	Wine and Spirits	Europe-France
27	Buzzi Unicem	BZU.MI	Construction and Materials	Europe-Italy
28	Valsoia SPA	VLS:MI	Foods	Europe-Italy
29	Banca Generali	BGN:MI	Financial Services	Europe-Italy
30	Repsol SA	REP.MC	Oil and Gas	Europe-Spain

[24] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

After selecting the shares, we proceed to download the prices on a monthly basis for the period (01/01/2010 - 31/12/2015)[25]. We can then calculate the average return, the variance and the standard deviation for the period in question, for each stock [26].

Table 2. The Return, Variance and Standard Deviation of the stocks

	MEAN	VARIANCE	STANDARD DEVIATION
NKE	2,2475%	0,3744%	6,1184%
BA	1,6110%	0,3602%	6,0014%
AAPL	2,2636%	0,5075%	7,1237%
MCD	1,2423%	0,1315%	3,6259%
JNJ	1,0401%	0,1451%	3,8089%
AIR.PA	2,5527%	0,7049%	8,3960%
KO	0,9815%	0,1585%	3,9812%
PG	0,6894%	0,1461%	3,8228%
T	0,9592%	0,1604%	4,0055%
CVX	0,7735%	0,3257%	5,7067%
WMT	0,5134%	0,2162%	4,6495%
BAC	0,6755%	0,9571%	9,7831%
GE	1,4261%	0,4205%	6,4846%
INTC	1,2927%	0,3964%	6,2964%
IBM	0,4456%	0,1984%	4,4537%
CAT	0,9212%	0,7030%	8,3845%
TRV	1,4791%	0,2576%	5,0753%
FCA	2,6129%	1,5145%	12,3063%
BARC.L	0,4767%	0,9092%	9,5351%
HSBA.L	0,2631%	0,2601%	5,0997%
VOW.DE	1,7716%	0,8501%	9,2199%
ALV.DE	1,6083%	0,4178%	6,4641%
LHA.DE	0,8262%	0,7494%	8,6568%
BAYN.DE	1,5522%	0,3308%	5,7517%
ML.PA	1,1897%	0,5260%	7,2526%
RI.PA	1,1611%	0,3324%	5,7653%
BZU.MI	1,1106%	0,9079%	9,5283%
VLS.MI	2,9046%	0,9823%	9,9109%
BGN.MI	2,6505%	0,6519%	8,0739%
REP.MC	0,2179%	0,5611%	7,4905%

[25] Stock prices have been downloaded from https://it.finance.yahoo.com/, see Appendix A for all values.

[26] See Appendix A for the table of prices and the variance-covariance matrix.

From the table, we can see that all the stocks considered present a positive average return in that period. Obviously, every share has a different risk profile, due to many factors both endogenous and exogenous, however the trade-off between risk and return is always maintained on average. In fact, to a riskier share, or a higher variance, corresponds basically a higher return in almost all the cases with the only exception for the stocks HSBA and REP, which have a very low return compared to their risk.

In general, the securities are distributed around a straight line with a positive slope that describe the trade off between risk and return.

Trade off Risk and Return: *In the horizontal axis is measured the standard deviation of the stocks, while in the vertical one is measured the return.*

3.2 THE FIRST PHASE: THE CAPM

After having described the characteristics of the thirty shares taken into account, we pass to the first phase of the effective selection, in which will be evaluated if the equilibrium condition defined by the CAPM is satisfied by the securities. The stocks, that do not comply with the requirement will be discarded. To test the condition of equilibrium defined by CAPM for these stocks we must effectuate a simple linear regression, which

analysis is used to explain the relationship between a continuous variable z_i, called response or dependent variable, and an explanatory variable, called independent.
The linear regression assumes the following formulation[27]:

$$\alpha z_{i,t} = \beta_i z_{M,t} + \varepsilon$$
$$\text{Where } t = 1, 2, \dots, T.$$

The first part of the equation represents the explanation of the predictor variable to the response variable, while the second component represents the portion of variability of the response variable called accidental, casual or erroneous component. The relationship between the two variables is a straight line which presents as intercepts the coefficient α, while the angular coefficient is measured by β_i[28].
Before making the regressions, we define the assumptions of the simple linear regression model, which mainly relate to the error component and are as follows:

- $\varepsilon \sim N_n\ (0, \sigma^2)$
- $E(\varepsilon) = 0$
- $E(\varepsilon\varepsilon') = \sigma^2 I_n$

The assumptions refer to the distribution of errors, which is a type of multivariate normal, with zero mean and constant variance (homoscedasticity); in addition, the errors are uncorrelated, two by two. The dependent variable on which is focused the purpose of this study is the excess of return of each security on the risk free asset, while as the independent variable, and therefore as regressor, it is used the excess of the market index compared to the risk free asset.

In the specific case of the CAPM, the value referred to the α coefficient indicates the gain (or loss) generated by the security that does not depend on systematic risk and thus, if is significant, it indicates the company's ability to create or destroy value in the capital market (Jensen's Alpha). A positive Alpha, in fact, indicates that the stock has presented higher returns than those required for the systematic risk reward, and thus it is said that

[27] C. Harvey, "*The Econometrics Analysis of time series*", second edition. The MIT Press. 1990.

[28] Norman Darped and Harry Smith, "*Applied Regression Analysis*", Third Edition, John Wiley and Sons Inc. 2014.

the company has created value. A negative Alpha indicates instead that the company produces lower returns than those forecasted for the compensation of systematic risk and therefore destroys value.

For the assumptions of the model, however, it is said that the market pays only the systematic risk and the Alpha would have no reason to exist, if not in the short term, and then tend to zero for the law of supply and demand, according to the equilibrium condition of the CAPM, already described in the preceding paragraphs[29].

Under this set of assumptions, it must be true that in the linear model the following restrictions are respected by the coefficients:

$$\alpha_i = \mu_i - \beta_i \mu_M \qquad \text{and} \qquad \beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}$$

As a consequence, if the Sharpe and Lintner's standard CAPM applies, the following holds

$$\alpha_i = 0$$

as

$$\alpha_i = \mu_i - \beta_i \mu_M = \big(E[r_i] - r_f\big) - \beta_i \big(E[r_i] - r_f\big) = 0$$

The OLS method identifies the straight line in the space ($z_{M,t}$; $z_{i,t}$) which fits these data best, by minimizing the mean squared error between the actual values of the dependent variable, $z_{i,t}$, and the corresponding fitted ones, $\tilde{z}_{i,t} \equiv \hat{\alpha}_i - \hat{\beta}_i \hat{z}_{M,t}$. The intercept of this straight line corresponds to the estimated alpha, $\hat{\alpha}_i$.

A possible way to verify the CAPM is to run a test of significance for the coefficient $\hat{\alpha}_i$ (H_0: $\alpha_i = 0$ vs. $\alpha_i \neq 0$). This test will allow to check the implication of the CAPM[30].

[29] See chapter 2 and Richard Brealy, Stewart Myers, Franklin Allen, Sandro Sandri, *"Principi di Finanza Aziendale"*, Sixth Edition, Mc Graw-Hill, Milan 2011.

[30] Norman Darped and Harry Smith, "*Applied Regression Analysis*", Third Edition, John Wiley and Sons Inc. 2014.

In other words, we check whether the asset *i* lies on the SML. To do so we can consider the appraisal ratio. This is given by the ratio of the estimated alpha of assets on its estimated standard deviation:

$$t_{\hat{\alpha}_i} \equiv \frac{\hat{\alpha}_i}{\hat{\sigma}_{\hat{\alpha}_i}}$$

Under the assumption of normality (i.e., if we assume that $z_{i,t}$ and $z_{M,t}$ are jointly normally distributed), we can prove that the joint distribution of the estimated coefficients α_i and β_i is also a bivariate-normal.

Hence if the excess returns are generated by a bivariate distribution, under the null hypothesis, H_0: $\alpha_i = 0$, the appraisal ratio $t_{\hat{\alpha}_i}$ is distributed as a t-student random variable with T-2 degrees of freedom. This implies that an exact, finite sample test for the significance of the coefficient α_i is immediately available.[31]

Once defined the theory of the simple linear regression, we pass to the effective calculation, therefore are downloaded the time series, for the same period of the securities, of the price and the return of the market index Standard and Poor's 500 (S&P 500) and the return of the risk free asset 3 months T-Bill, between the January 2010 and December 2015, with monthly frequency[32].The average return of the market index and of the risk free asset is:

S&P 500	**0,98%**	**3 months T-Bill**	**0,068%**

Once obtained the returns of these new securities, we can calculate the excess return between the market and the risk-free rate and between the stocks and the risk-free rate [33], so after determining those we can set the regressions, using the econometric software.

It is executed a linear regression for each stock, through the OLS model (ordinary least squares) and for each regression, it is reported the coefficient of the α parameter, the coefficient of the β parameter, the P-value associated with them and finally the value of

[31] C. Harvey, "*The Econometrics Analysis of time series*", second edition. The MIT Press. 1990.

[32] See Appendix B for the table of returns. The price of the S&P 500 is downloaded from https://fred.stlouisfed.org/series/SP500, while the return of the T-Bill is downloaded from https://fred.stlouisfed.org/series/DGS3MO

[33] See Appendix B for the excess of returns.

the Adjusted R squared. The econometric software GRETL gives us the P-value, which is the smallest level of significance in relation to which it is possible to reject the null hypothesis. If the P-value is less than the significance level, then we reject the null hypothesis.

$$\text{If P-value} > 10\% \rightarrow \text{the CAPM holds, } \alpha = 0$$

$$\text{If P-value} < 10\% \rightarrow \text{the CAPM does not hold, } \alpha \neq 0$$

While the level of R squared demonstrates how the adaptation of the observed data is good to explain the model. The software also provides the value of the test T, in which if the value of T calculated proves greater than the T found on the table of Student's T at a significance level of 5% or 10% (5% is considered a good level) and with n-2 degrees of freedom (n = number of observations), then it must refuse the null hypothesis with a probability of error inferior of the respectively 5% or 10%. Otherwise it is accepted the null hypothesis. However, this value of the T test is not reported because the information on the acceptability or rejection of the null hypothesis is already given by the P-value on the coefficients. Finally, it is provided below the table summarizing the output of regression where in the last right column it is shown if the share is consistent with the assumptions of the CAPM, and so if the equilibrium condition is satisfied.

Table 3. The CAPM Regression Results[34]

STOCK	CODE	α (Alfa)	β (Beta)	P-Value (α)	P-Value (β)	Adj R squared	CAPM
Nike	NKE	0,0150632	0,73793	0,0294	7,91E-05	0,19186	×
The Boeing Company	BA	0,0055007	1,08835	0,318	8,10E-11	0,45223	✓
Apple	AAPL	0,0134238	0,93528	0,0855	1,27E-05	0,230195	×
Mac Donald	MCD	0,0077747	0,43510	0,0566	8,38E-05	0,190563	×
Johnson and Johnson	JNJ	0,0042807	0,59634	0,2674	8,13E-08	0,333404	✓
Airbus Group SE	AIR.PA	0,0180475	0,74539	0,0703	0,0046	0,097482	×
The Procter & Gamble Company	PG	0,0018655	0,47673	0,6576	4,09E-05	0,206434	✓
The Coca-Cola Company	KO	0,0044612	0,51236	0,3051	2,06E-05	0,221405	✓
AT & T	T	0,0049608	0,43314	0,2773	0,0005	0,152256	✓
Chevron	CVX	-0,003555	1,16311	0,438	9,33E-15	0,577484	✓
Walmart	YMT	0,0010535	0,37285	0,8484	0,0109	0,077064	✓
Bank of America	BAC	-0,008017	1,54462	0,4156	5,90E-08	0,339403	✓
General Electric company	GE	0,0009378	1,38588	0,8463	5,85E-17	0,634775	✓
Intel	INTC	0,0034909	0,95984	0,5879	2,08E-07	0,315446	✓
IBM	IBM	-0,001639	0,59368	0,7322	9,03E-06	0,239097	✓
Caterpillar Inc.	CAT	-0,006340	1,63018	0,376	6,17E-13	0,523517	✓
The Travelers Companies, Inc.	TRV	0,0054694	0,94721	0,2291	1,33E-11	0,479738	✓
Fiat Chrysler Automobiles	FCA	0,0105936	1,62831	0,4265	1,12E-05	0,234457	✓
Barclays PLC	BARC	-0,00924	1,46098	0,3433	1,74E-07	0,318943	✓
HSBC Holdings	HSBA	-0,004565	0,71427	0,3991	2,69E-06	0,264403	✓
Volkswagen Aktiengesellschaft	VOW	0,0053452	1,28145	0,5851	3,24E-06	0,260591	✓
Allianz SE	ALV	0,0053587	1,10101	0,3884	2,27E-09	0,39768	✓
Deutsche Lufthansa Aktiengesellschaft	LHA	9,56E-05	0,82067	0,9924	0,0024	0,112451	✓
Bayer Aktiengesellschaft	BAYN	0,0082963	0,71758	0,1927	4,08E-05	0,206535	✓
Michelin	ML.PA	0,0040085	0,79018	0,6267	0,0004	0,154259	✓
Pernod Ricard SA	RI.PA	0,0041579	0,74241	0,6092	2,12E-05	0,220804	✓
Buzzi Unicem	BZU	-0,002524	1,41952	0,7975	4,35E-07	0,30109	✓
Valsoia SPA	VLS	0,0237109	0,51029	0,0534	0,1077	0,023116	×
Banca Generali	BGN	0,0163508	1,03848	0,0665	2,13E-05	0,2207	×
Repsol SA	REP	0,0005207	0,10733	0,9553	0,6562	-0,011563	✓

From the output of the regression we note that the CAPM is valid for twenty-four of the thirty stocks, and observing the P-value on the Alpha we note that do not satisfy the

[34] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

Processing performed through the econometric software.

equilibrium condition the securities NKE, AAPL, MCD, AIR.PA, VLS.MI and BGN.MI; for all these shares the null hypothesis H_0: $\alpha_i = 0$ is rejected, since the value of the t-statistic falls outside the acceptance region. To perform the test was chosen a level of significance, that defines the acceptance region for the null hypothesis, bounded by the corresponding critical values of the student's T. If the statistic value falls outside the acceptance area, we reject the null hypothesis. In this case, setting a significance level of the 10%, the corresponding critical value is 1.294, given the sample size of 71 data (therefore 69 degrees of freedom).

By controlling the Betas, instead, we note that the securities BA, CVX, ALV.DE, TRV and INTC move together with the market, while the other securities analysed presents various Betas. The Beta of the stocks JNJ, PG, KO, T, YMT, IBM, HSBC, LHA.DE, BAYN.DE, MI.PA and RI.PA is less than 1 (Beta of the market index), so they do not amplify the market movement. Thus, these stocks tend to move in the same direction of the market, but not with the same intensity (the assets are less risky than the market). These securities may be considered as "*defensive stocks*". No one security has a negative Beta, namely with totally opposite trend to that of the market.

While the securities BAC, GE, CAT, FCA, BARC.L, VOW.DE and BZU.MI have a Beta higher than one, this means that they amply the market movement. Namely, the stocks tend to move more than the market, they will have a bigger return during the growth period, but on the other hand they will have a very bad performance during the crisis period. These securities are more dangerous than the market, so that these stocks are defined as "*aggressive stocks*". Among these riskier securities, CAT is distinguished from the others to be the more risky (Beta=1,63018).

Finally, the value of the R squared measures the goodness of adaptation of the model to the observed data, namely how much of the observed variability is attributed to the variability of the explanatory variables.

$$R^2 = \frac{ESS}{TSS}$$

Where ESS is the *Explained Sum of Squares* and TSS is the *Total Sum of Squares*[35].

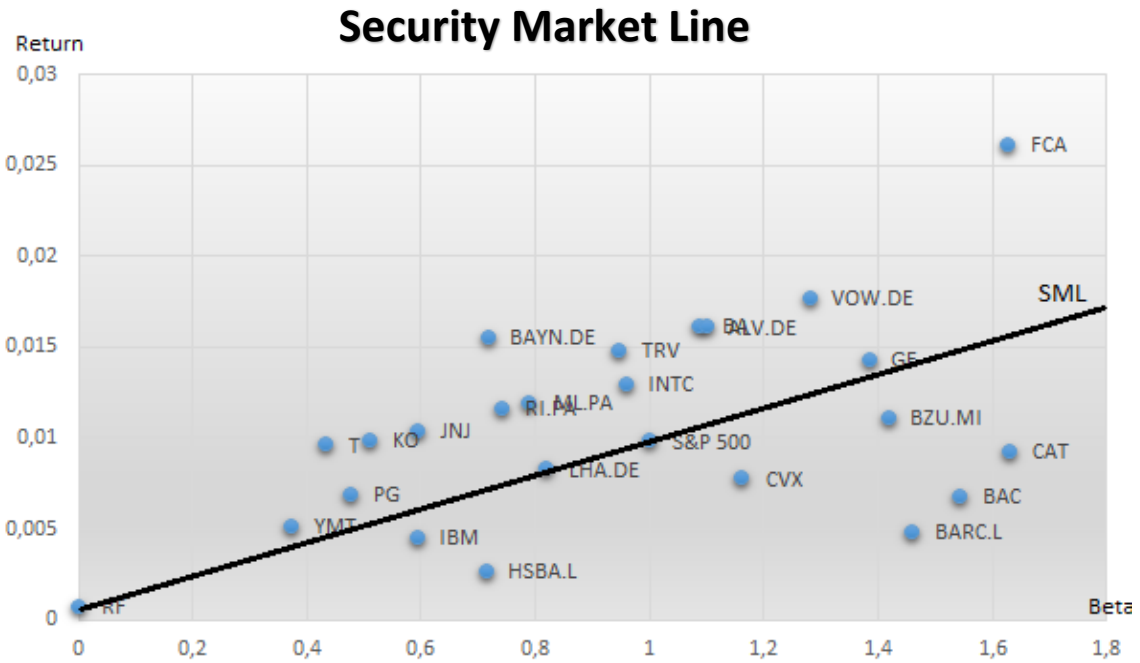
The larger is the R squared (nearest to the unit), better the model explains the data. In this

[35] Palomba, G. "*Elementi di statistica per l'econometria*". CLUA libri, Ancona. Second Edition. 2010.

case we have that the levels of R squared are relatively low, however, this was quite predictable because we are using a regression with a single regressor.

Interestingly, from the analysis of the R squared emerges that the value of REP.MC is negative. A value of R squared negative means that the market is not able to explain the variations of the stock, for this reason the stock REP.MC is excluded from the panel of stocks.

The securities that meet the equilibrium condition defined by the CAPM are therefore twenty-three of the thirty initials.



Security Market Line: *In the graph there are the selected securities with their risk-return relationship compared with the market. In the horizontal axis is measured the standard deviation of the stocks, while in the vertical one is measured the return.*

3.3 THE SECOND PHASE: THE HISTORICAL SHARPE RATIO

Once completed the first phase of stock selection, we pass to the second phase which will allow us to reduce the number of securities from twenty-three to twenty, and this time the selection criteria will be determined by the index of Sharpe. Following the historical traces of Sharpe ratios, it is possible to measure the performance, corrected for the risk, computed by a stock.

Through this index we can, therefore, identify which are the securities that pay a higher risk premium given the risk.

$$Sharpe\ Ratio = \frac{(r_i - r_f)}{\sigma_i}$$

Where $(r_i - r_f)$ is the risk premium and σ_i is the standard deviation of the single stock. To compute the calculations, it is used the average return of the risk-free rate (3 months T-Bill)[36], and the values of standard deviation and average return already considered.

Table 4. The Return and Standard Deviation of the 23 stocks

	MEAN	VARIANCE	STANDARD DEVIATION
BA	1,6110%	0,3602%	6,0014%
JNJ	1,0401%	0,1451%	3,8089%
KO	0,9815%	0,1585%	3,9812%
PG	0,6894%	0,1461%	3,8228%
T	0,9592%	0,1604%	4,0055%
CVX	0,7735%	0,3257%	5,7067%
YMT	0,5134%	0,2162%	4,6495%
BAC	0,6755%	0,9571%	9,7831%
GE	1,4261%	0,4205%	6,4846%
INTC	1,2927%	0,3964%	6,2964%
IBM	0,4456%	0,1984%	4,4537%
CAT	0,9212%	0,7030%	8,3845%
TRV	1,4791%	0,2576%	5,0753%
FCA	2,6129%	1,5145%	12,3063%
BARC.L	0,4767%	0,9092%	9,5351%
HSBA.L	0,2631%	0,2601%	5,0997%
VOW.DE	1,7716%	0,8501%	9,2199%
ALV.DE	1,6083%	0,4178%	6,4641%
LHA.DE	0,8262%	0,7494%	8,6568%
BAYN.DE	1,5522%	0,3308%	5,7517%
ML.PA	1,1897%	0,5260%	7,2526%
RI.PA	1,1611%	0,3324%	5,7653%
BZU.MI	1,1106%	0,9079%	9,5283%

[36] See Appendix B for the returns of the risk free asset.

The values for the Sharpe ratios of each security are [37]:

Table 5. The Sharpe Ratio of every stock

STOCK	Stock's Return	Sharpe Ratio
TRV	1,48%	28%
BAYN.DE	1,55%	26%
BA	1,61%	26%
JNJ	1,04%	26%
ALV.DE	1,61%	24%
KO	0,98%	23%
T	0,96%	22%
GE	1,43%	21%
FCA	2,61%	21%
INTC	1,29%	19%
RI.PA	1,16%	19%
VOW.DE	1,77%	18%
PG	0,69%	16%
ML.PA	1,19%	15%
CVX	0,77%	12%
BZU.MI	1,11%	11%
CAT	0,92%	10%
YMT	0,51%	10%
LHA.DE	0,83%	9%
IBM	0,45%	8%
BAC	0,68%	6%
BARC.L	0,48%	4%
HSBA.L	0,26%	4%

Where the average return of the risk free asset is: **Risk free** 0,068%

From the observation of the table, we can see that the securities present a historical Sharpe ratio which is around to 15% on average. The stocks with the lowest value are the two English shares of BARC.L and HSBA.L, while the security with the highest value is linked to "*the Travelers Companies, Inc*." (TRV), the US insurance company.

The data are reported in order according to the Sharpe ratio, from the largest to the smallest, and so it would be logical to exclude the three stocks with lower value.

[37] In the table the order of the securities is given by the index of Sharpe, in particular, in descending order.

However in the process of exclusion it must also be considered that the portfolio should be well diversified both geographically and from the sector point of view, so we are going to take away from the basket of the shares the titles BAC, LHA.DE and CAT[38]:

Table 6. The 20 Final stocks

STOCK	CODE	Sharpe Ratio	SECTOR
The Boeing Company	BA	26%	Aerospace
Johnson and Johnson	JNJ	26%	Pharmaceutical
The Procter & Gamble Company	PG	16%	Consumer Goods
The Coca-Cola Company	KO	23%	Beverage
AT & T	T	22%	Telecommunication
Chevron	CVX	12%	Oil and Gas
Walmart	YMT	10%	Super Centers
General Electric company	GE	21%	Industrial Conglomerates
Intel	INTC	19%	Semiconductors
IBM	IBM	8%	Consulting and Services
The Travelers Companies, Inc.	TRV	28%	Insurance
Fiat Chrysler Automobiles	FCA	21%	Automotive
Barclays PLC	BARC.L	4%	Banking
HSBC Holdings	HSBA.L	4%	Financial Services
Volkswagen Aktiengesellschaft	VOW.DE	18%	Automotive
Allianz SE	ALV.DE	24%	Insurance
Bayer Aktiengesellschaft	BAYN.DE	26%	Pharmaceutical
Michelin	ML.PA	15%	Autoparts
Pernod Ricard SA	RI.PA	19%	Wine and Spirits
Buzzi Unicem	BZU.MI	11%	Construction and Materials

3.4 THE 1/N ALLOCATION

Once determined the most suitable set of securities in order to develop the analysis, we proceed with the construction of the passive allocated portfolio, or 1/N portfolio, in which all the twenty selected securities become part of the portfolio with a weight equal to 1/N, namely each stock enters in the portfolio with identical weight, and none of them is short sold ($\omega_i < 0$) in order to extend the investment opportunities.

[38] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

Given a basket of twenty securities, the weight equivalent of each stock is $\omega_i = 0{,}05$, exactly equal to 1/20, without considering the risk-free asset in the composition of the portfolio, being that we only use risky shares.

Table 7. The 1/N Weights

STOCK	WEIGHT
BA	0,05
JNJ	0,05
KO	0,05
PG	0,05
T	0,05
CVX	0,05
YMT	0,05
GE	0,05
INTC	0,05
IBM	0,05

STOCK	WEIGHT
TRV	0,05
FCA	0,05
BARC.L	0,05
HSBA.L	0,05
VOW.DE	0,05
ALV.DE	0,05
BAYN.DE	0,05
ML.PA	0,05
RI.PA	0,05
BZU.MI	0,05

After specifying the weight of securities, it is possible to download the price of securities for the period that goes from 1 January 2016 to the 31 December 2016, on a daily basis. Once retrieved the prices, we can calculate the daily returns of the securities and the variance-covariance matrix[39], so that it can be weighted the stocks for their weight and compute the average return, the variance and the standard deviation for the passive allocated portfolio.

Table 8. The 1/N Portfolio performance

MEAN	1,224%
STANDARD DEVIATION	9,17%
VARIANCE	0,084%

The passive allocated portfolio shows a positive return, and a relatively low risk position. The construction of this portfolio does not involve any kind of effort from the point of view of the mathematical elaborations and the application of financial theories. It could be the uninformed investor's portfolio that merely seek to build a portfolio of securities

[39] See Appendix C for the prices and the variance-covariance matrix, with daily rate, downloaded between the 1 January 2016 and the 31 December 2016 from https://it.finance.yahoo.com/, while Appendix D for the portfolio return.

geographically and sectorally diverse between them. Obviously this portfolio is not an efficient portfolio, and we expect to get a better result through the next processing.
From the efficiency point of view, we can calculate the Sharpe ratio for this portfolio, using as a risk-free rate the performance of the 3 months T-Bill during the 2016[40].

Sharpe Ratio of the Portfolio 28,79%

The value of the Sharpe ratio shows us how this portfolio has a higher value than a single stock can do of its own, and thus the performance of a set of grouped securities is much more advantageous than a single one.

[40] See Appendix C for the daily return of the 3 months T-Bill.

Chapter 4

THE EFFICIENT FRONTIER AND

THE ACTIVE ALLOCATION

Summary: 4.1 The Efficient Frontier – .42 The Global Minimum Variance Portfolio – 4.3 The Tangent Portfolio – 4.4 The Mean-Variance Allocation

The analysis continues with the determination of the portfolio actively allocated, after the definition of the passive allocation. Before doing so we must, however, find the weights of the best efficient risky portfolio, the tangent portfolio.

To do that, we have to recover the data of the time series, of the twenty selected securities, already downloaded for the period from 1 January 2010 to 31 December 2015, and it observes for the best distribution of the securities that can compose the portfolio, in order to maximize the return and minimize the risk.

At first, it will determine the efficient frontier by following the steps already explained in chapter 1 of the Harry Markowitz's mean-variance model, and then it will determine the global minimum variance portfolio; to finally introduce in the analysis a risk free asset r_f (3 months T-Bill), and then to determine the weights, the performance and the risk of the tangent portfolio.

With the weights of the portfolio of tangency we can set the active allocation for the following period, namely the year 2016, and evaluating the performance, of course both from the efficiency and risk point of view. With the weights of the tangent portfolio it is expected to achieve a greater trade-off between risk and return than the passive allocation portfolio.

4.1 THE EFFICIENT FRONTIER

To determine the efficient frontier, we need: the vector of the average returns z, its transposed z ', the vector 1, its transposed 1', the variance and covariance matrix V and its inverse V^{-1}.

Here, we summarize the values[41].

The vector of the average returns z, the vector 1 and The variance and covariance matrix V of the twenty stocks.

Table 9. The vector of the Average Returns.

STOCK	z
BA	1,61%
JNJ	1,04%
KO	0,98%
PG	0,69%
T	0,96%
CVX	0,77%
YMT	0,51%
GE	1,43%
INTC	1,43%
IBM	1,29%

STOCK	z
TRV	0,45%
FCA	2,61%
BARC.L	1,48%
HSBA.L	2,61%
VOW.DE	0,48%
ALV.DE	0,26%
BAYN.DE	1,77%
ML.PA	1,61%
RI.PA	1,16%
BZU.MI	1,55%

Table 10. The Variance-Covariance matrix.

	BA	JNJ	KO	PG	T	CVX	YMT	GE	INTC	IBM	TRV	FCA	BARC	HSB	VOW	ALV	BAYN	ML	RI	BZU
BA	0,004	0,001	0,001	0,001	0,001	0,001	0,001	0,002	0,001	0,001	0,002	0,002	0,002	0,001	0,002	0,002	0,002	0,002	0,002	0,003
JNJ	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,000	0,001	0,001	0,001	0,001	0,001	0,001	0,001
KO	0,001	0,001	0,002	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,001	0,000
PG	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,000	0,000	0,001	0,000	0,001	0,001	0,000	0,001	0,000	0,000	0,001	0,000
T	0,001	0,001	0,001	0,000	0,002	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,001
CVX	0,001	0,001	0,001	0,001	0,001	0,003	0,000	0,003	0,002	0,001	0,002	0,002	0,002	0,001	0,002	0,001	0,001	0,001	0,001	0,002
YMT	0,001	0,001	0,001	0,001	0,001	0,000	0,002	0,001	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,000	0,000	0,001
GE	0,002	0,001	0,001	0,001	0,001	0,003	0,001	0,004	0,002	0,001	0,002	0,003	0,003	0,001	0,002	0,002	0,001	0,001	0,001	0,003
INTC	0,001	0,001	0,001	0,000	0,001	0,002	0,001	0,002	0,004	0,001	0,001	0,002	0,002	0,001	0,002	0,002	0,001	0,000	0,001	0,001
IBM	0,001	0,000	0,000	0,000	0,000	0,001	0,001	0,001	0,001	0,002	0,001	0,001	0,002	0,001	0,001	0,001	0,000	0,001	0,001	0,001
TRV	0,002	0,001	0,001	0,001	0,001	0,002	0,001	0,002	0,001	0,001	0,003	0,002	0,002	0,001	0,002	0,002	0,001	0,001	0,001	0,001
FCA	0,002	0,001	0,000	0,000	0,001	0,002	0,001	0,003	0,002	0,001	0,002	0,015	0,005	0,002	0,006	0,005	0,003	0,003	0,003	0,006
BARC	0,002	0,000	0,000	0,001	0,000	0,002	0,001	0,003	0,002	0,002	0,002	0,005	0,009	0,003	0,003	0,004	0,002	0,002	0,001	0,004
HSBA	0,001	0,001	0,000	0,001	0,000	0,001	0,000	0,001	0,001	0,001	0,001	0,002	0,003	0,003	0,002	0,002	0,001	0,002	0,001	0,002
VOW	0,002	0,001	0,000	0,000	0,000	0,002	0,001	0,002	0,002	0,001	0,002	0,006	0,003	0,002	0,009	0,003	0,003	0,003	0,002	0,003
ALV	0,002	0,001	0,000	0,001	0,000	0,001	0,001	0,002	0,002	0,001	0,002	0,005	0,004	0,002	0,003	0,004	0,002	0,003	0,002	0,004
BAYN	0,002	0,001	0,001	0,000	0,000	0,001	0,001	0,001	0,001	0,000	0,001	0,003	0,002	0,001	0,003	0,002	0,003	0,002	0,002	0,002
ML	0,002	0,001	0,000	0,000	0,000	0,001	0,000	0,001	0,000	0,001	0,001	0,003	0,002	0,002	0,003	0,003	0,002	0,005	0,002	0,003
RI	0,002	0,001	0,001	0,001	0,001	0,001	0,000	0,001	0,001	0,001	0,001	0,003	0,001	0,001	0,002	0,002	0,002	0,002	0,003	0,002
BZU	0,003	0,001	0,000	0,000	0,001	0,002	0,001	0,003	0,001	0,001	0,001	0,006	0,004	0,002	0,003	0,004	0,002	0,003	0,002	0,009

[41] See Appendix A for the prices of securities between 2010 and 2015.

In order to determine the efficient frontier, we must set the equation where the dependent variable is the standard deviation, while the independent variable is the expected return, as result of the constrained maximization already seen in chapter 2, which seeks to minimize the risk given the constraints of the return of the portfolio and the sum of the weights equal to one [42].

The equation that relates the minimum standard deviation and the expected return of portfolio is the following:

$$\sigma_p = \sqrt{\frac{(CE_p^2 - BE_p + A)}{DELTA}}$$

Where $A = z'V^{-1}z$, $B = z'V^{-1}1$, $C = 1'V^{-1}1$, $DELTA = AC - B^2$

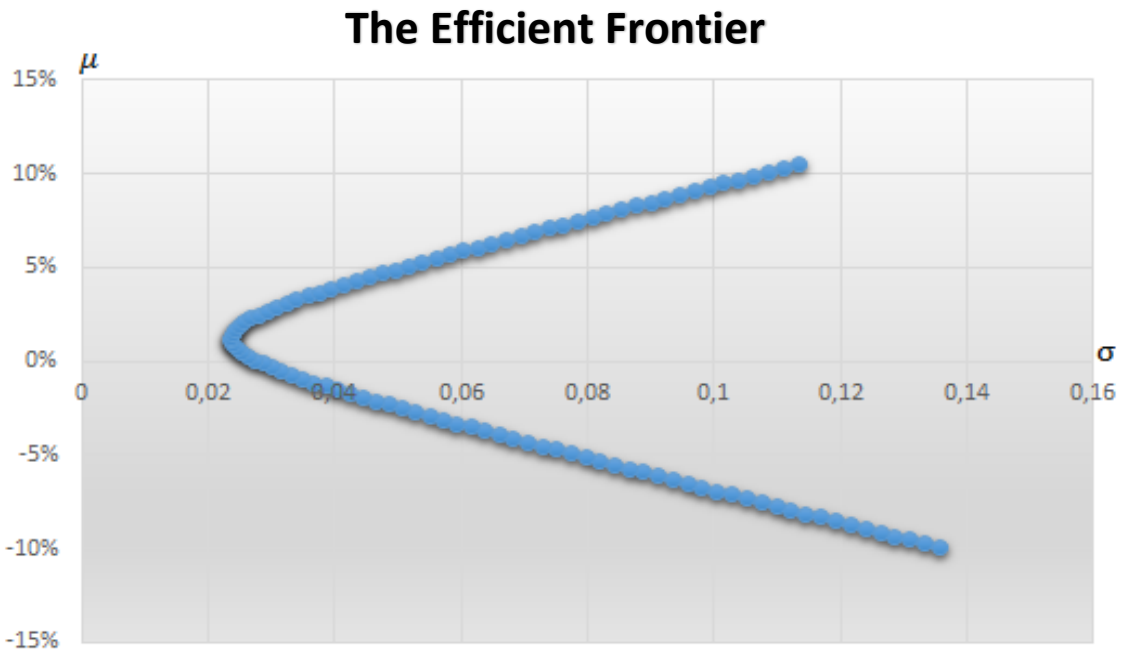
So we calculate the values for A, B, C and DELTA, multiplying the vectors and the matrix, respectively:

Table 11. The values of A, B, C and Delta.

A	0,9292643
B	20,363866
C	1765,8667
DELTA	1226,2698

Once have determined the three values, it is possible to plot the graph that associate the expected return, with a range of values that goes from -10% to + 10%, and the corresponding value of standard deviation. The set of points which is obtained is the efficient frontier.

[42] See chapter 1, paragraph 1.5 "*The Efficient Frontier*".



***The Efficient Frontier*:** it is *the set of efficient portfolios, where the twenty selected securities enter with different weights describing the curve of the portfolios*[43].

4.2 THE GLOBAL MINIMUM VARIANCE PORTFOLIO

Now we pass to the computation of the return, of the standard deviation and of the weights of the Global Minimum Variance portfolio.

For the calculation of the return and the standard deviation, it is started with the previous formula obtained thanks to the optimization process:

$$\sigma_p = \sqrt{\frac{(CE_p^2 - BE_p + A)}{AC - B^2}}$$

It raises to the square both the left and right side of the equation, obtaining:

$$\sigma_p^2 = \frac{CE_p^2 - BE_p + A}{AC - B^2}$$

And taking the derivative respect to E_p we have:

$$2E_pC - 2B = 0 \qquad E_p = \frac{B}{C}$$

[43] Personnel data processing.

Substituting this value in σ_p^2, we obtain:

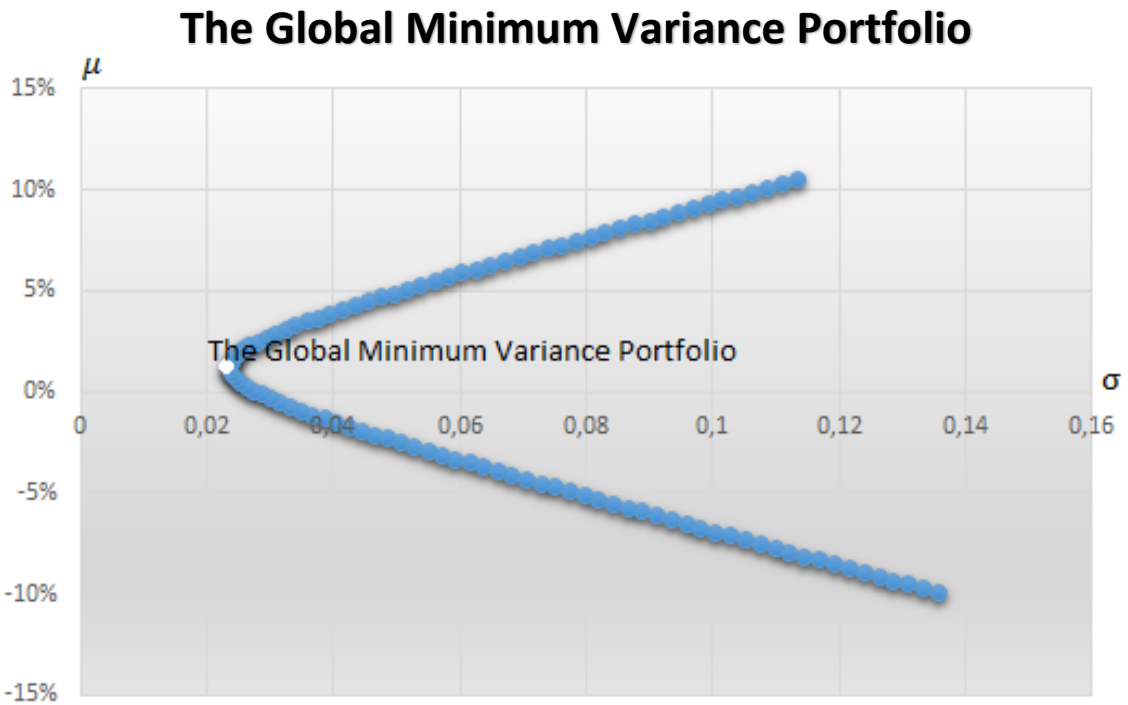
$$\sigma_p^2 = \frac{1}{C}$$

Substituting the values of B and C, with those found previously, we can determine the main values of risk and return for the Global Minimum Variance portfolio.

Table 12. The Global Minimum Variance Portfolio performance

EXPECTED RETURN	1,153%
STANDARD DEVIATION	2,38%
VARIANCE	0,0006

The values found are consistent with those described by the efficient frontier, in fact, the Global Minimum Variance portfolio is the efficient portfolio with the lowest risk and all portfolios with lower return to this are dominated.
Graphically the Global Minimum Variance portfolio is the vertex of the parabola drawn by the efficient portfolios.



***The Global Minimum Variance portfolio**: it is the portfolio with the lowest risk composed of risky securities and is in the summit of the curve drawn by the efficient portfolios*[44].

[44] Personnel data processing.

After determining the values for risk and return of the portfolio, we can calculate the vector of weights, by which every individual stocks enter in the portfolio.
The formula of the vector of weights is obtained from the constrained minimization with the Lagrangian multiplier, and has as result the following equation.

$$\omega_p = g - hE_p$$

where the vectors g and h are defined as follows

$$g = \frac{1}{D}[B(V^{-1}1) - A(V^{-1}z)]$$

$$h = \frac{1}{D}[C(V^{-1}z) - A(V^{-1}1)]$$

Substituting the values found in the formulas, we determine the values of h and g for each stock that compose the portfolio.

Table 13. The values of h and g

STOCK	H			g		
BA	-23,99	202,56	**-226,55**	17564,78	-525,70	**18090,47**
JNJ	-15,69	-266,58	**250,88**	-23116,25	-343,84	**-22772,41**
KO	251,75	113,40	**138,34**	9833,86	5516,80	**4317,07**
PG	523,56	259,05	**264,51**	22463,36	11473,23	**10990,12**
T	390,20	-21,65	**411,85**	-1877,65	8550,77	**-10428,42**
CVX	-50,11	-93,97	**43,86**	-8149,08	-1098,14	**-7050,94**
YMT	89,49	-111,01	**200,50**	-9625,96	1961,13	**-11587,09**
GE	-286,03	-10,77	**-275,26**	-933,65	-6268,03	**5334,38**
INTC	117,76	120,35	**-2,59**	10436,24	2580,67	**7855,57**
IBM	323,48	136,92	**186,56**	11872,94	7088,66	**4784,28**
TRV	-75,56	-74,29	**-1,26**	-6442,52	-1655,74	**-4786,78**
FCA	2,73	91,27	**-88,54**	7914,81	59,88	**7854,93**
BARC.L	-30,48	-39,89	**9,42**	-3459,49	-667,86	**-2791,63**
HSBA.L	134,86	353,08	**-218,22**	30617,52	2955,32	**27662,20**
VOW.DE	-74,36	-76,68	**2,31**	-6649,08	-1629,56	**-5019,52**
ALV.DE	49,23	-383,22	**432,45**	-33231,26	1078,85	**-34310,11**
BAYN.DE	125,15	245,15	**-120,00**	21258,12	2742,57	**18515,55**
ML.PA	174,97	99,13	**75,84**	8595,71	3834,20	**4761,51**
RI.PA	8,91	-112,97	**121,87**	-9796,13	195,18	**-9991,31**
BZU.MI	5,09	-15,18	**20,27**	-1316,41	111,47	**-1427,87**

So that, using the expected return already found, equal to 1,153%, we are able to determine the vector of weights of the Global Minimum Variance Portfolio [45].

Table 14. The Optimal Weights of GMVP

	Optimal Weights of GMVP
BA	-1,46%
JNJ	-0,96%
KO	15,34%
PG	31,91%
T	23,78%
CVX	-3,05%
YMT	5,45%
GE	-17,43%
INTC	7,18%
IBM	19,71%

	Optimal Weights of GMVP
TRV	-4,60%
FCA	0,17%
BARC.L	-1,86%
HSBA.L	8,22%
VOW.DE	-4,53%
ALV.DE	3,00%
BAYN.DE	7,63%
ML.PA	10,66%
RI.PA	0,54%
BZU.MI	0,31%

This is the portfolio with the lowest risk, and it would be held by the individual more risk averse.

From the efficiency point of view, we can calculate the Sharpe ratio for this portfolio, using as a risk-free rate the performance of the 3 months T-Bill during the 2016[46].

Sharpe Ratio of the GMVP 45,59%

The value of the Sharpe ratio shows us a good performance of the portfolio, greater compared with the one that would be obtained with another type of weight composition.

4.3 THE TANGENT PORTFOLIO

At this point of the analysis we include the risk-free asset in order to extend the set of investment opportunities. In this way, it is possible to calculate the vector of weights for the tangent portfolio and, consequently, its return and variance.

[45] It is the result of the equation: $\omega_p = g - hE_p$

[46] See Appendix B for the return of the 3 months T-Bill.

The formula of the weights of the tangency portfolio[47] is the following one:

$$\omega_p = V^{-1}(z - 1r_f)\frac{E_p - r_f}{H}$$

$$\text{where } H = (z - 1r_f)'V^{-1}(z - 1r_f)$$

So to calculate the weights we simply replace the values, where the vector of average returns and the variance-covariance matrix is the same as before, while the risk-free rate is still the 3 months T-Bill. Once determine the weights we can, then, compute the average return and the standard deviation:

$$E_T = \frac{H}{1'V^{-1}(z-1r_f)} \quad \text{and} \quad \sigma_T = \frac{\sqrt{H}}{1'V^{-1}(z-1r_f)}$$

The components for the calculation of the weights are the following:

Table 15. The matrix product to compute the Optimal Weights

	V^-1*1	V^-1*Z	1rf	Z-1rf	(V^-1) * (Z-1rf)
BA	-25,815	9,947	0,068%	1,543%	9,964
JNJ	-16,885	-13,091	0,068%	0,972%	-13,079
KO	270,911	5,569	0,068%	0,914%	5,385
PG	563,411	12,721	0,068%	0,622%	12,338
T	419,899	-1,063	0,068%	0,891%	-1,348
CVX	-53,926	-4,615	0,068%	0,706%	-4,578
YMT	96,304	-5,451	0,068%	0,446%	-5,517
GE	-307,801	-0,529	0,068%	1,358%	-0,320
INTC	126,728	5,910	0,068%	1,358%	5,824
IBM	348,100	6,724	0,068%	1,225%	6,487
TRV	-81,308	-3,648	0,068%	0,378%	-3,593
FCA	2,941	4,482	0,068%	2,545%	4,480
BARC.L	-32,796	-1,959	0,068%	1,411%	-1,937
HSBA.L	145,126	17,339	0,068%	2,545%	17,240
VOW.DE	-80,022	-3,765	0,068%	0,409%	-3,711
ALV.DE	52,979	-18,819	0,068%	0,195%	-18,855
BAYN.DE	134,678	12,038	0,068%	1,704%	11,947
ML.PA	188,285	4,868	0,068%	1,540%	4,740
RI.PA	9,585	-5,547	0,068%	1,093%	-5,554
BZU.MI	5,474	-0,745	0,068%	1,484%	-0,749

[47] See chapter 1, paragraph 1.6 "*Borrowing or Investing in risk free rate*".

Therefore, the tangency weights are:

Table 16. The Optimal Weights od Tangency

	Optimal Weights of TANG		Optimal Weights of TANG
BA	52%	**TRV**	-19%
JNJ	-68%	**FCA**	23%
KO	28%	**BARC.L**	-10%
PG	64%	**HSBA.L**	90%
T	-7%	**VOW.DE**	-19%
CVX	-24%	**ALV.DE**	-98%
YMT	-29%	**BAYN.DE**	62%
GE	-2%	**ML.PA**	25%
INTC	30%	**RI.PA**	-29%
IBM	34%	**BZU.MI**	-4%

having now available the vector of weights, we compute the average return, the standard deviation and the variance of the tangency portfolio:

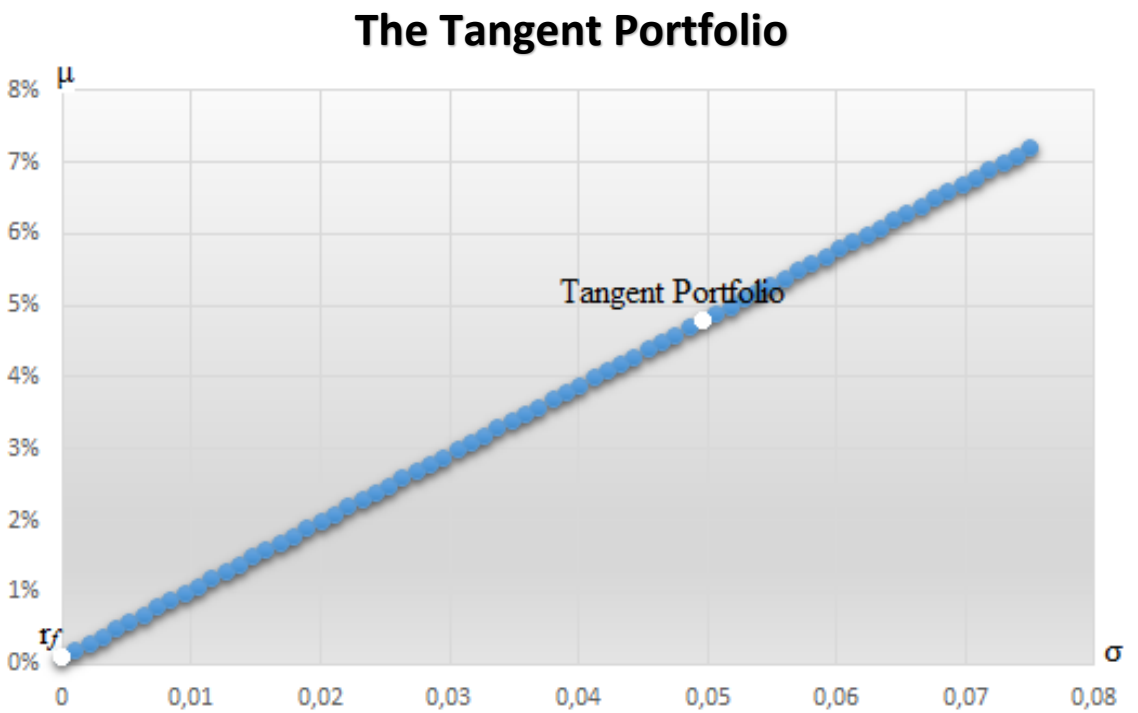
Table 17. The Tangent Portfolio Performance

EXPECTED RETURN	4,777%
STANDARD DEVIATION	4,98%
VARIANCE	0,0003

Graphically, to insert the risk-free asset to the set of portfolios, allows us to change the efficient frontier moving from a curve to a straight line and thus the minimum variance portfolio, now, becomes the risk-free asset.

In addition, the new frontier, the Capital Market Line, allows borrowing at the rate r_f and consequently it permits to increase the investment opportunities. The new frontier is described by the following function:

$$\mu_p = R_f + \frac{(\mu_T - R_f)}{\sigma_T}\sigma_p$$



The Tangent Portfolio: *having entered a risk free asset in the analysis makes changing the curve of efficient portfolios in a straight line, all the points on the line are a combination of the tangent portfolio and the risk free asset. The line has as intercepts Rf and as slope* $(\mu_T - R_f)/\sigma_T$ *.In the horizontal axis is measured the standard deviation of the portfolios, while in the vertical one is measured the return*[48].

The tangent portfolio is the best portfolio for definition, it is the one that presents the most performances considering both the risk and the return. Observing the Sharpe ratio[49], it has a very high value.

Sharpe Ratio of the TANG 94,53%

The value of the ratio is more than twice of the value of the portfolio at minimum variance.

4.4 THE MEAN-VARIANCE ALLOCATION

This chapter concludes by setting a portfolio with mean-variance allocation, or active allocation, for the year following the period used on the historical series analyzes.
Therefore, the twenty selected stock are tested in this new period using the weights of the tangency portfolio, the best portfolio among the risky. For this reason, we expect that this

[48] Personnel data processing.

[49] See Appendix B for the return of the 3 months T-Bill.

portfolio can achieve a better performance than the one with passive allocation, both in terms of performance and risk.

Here, are reported the weights with which the securities will make up the active allocated portfolio:

Table 18. The Mean-Variance Portfolio Weights

STOCK	WEIGHT	STOCK	WEIGHT
BA	52,0%	**TRV**	-18,7%
JNJ	-68,2%	**FCA**	23,4%
KO	28,1%	**BARC.L**	-10,1%
PG	64,4%	**HSBA.L**	90,0%
T	-7,0%	**VOW.DE**	-19,4%
CVX	23,9%	**ALV.DE**	-98,4%
YMT	-28,8%	**BAYN.DE**	62,3%
GE	-1,7%	**ML.PA**	24,7%
INTC	30,4%	**RI.PA**	-29,0%
IBM	33,8%	**BZU.MI**	3,9%

These are the weights of the securities, computed through the maximization process in previous paragraph. Now, we use the share performance for the period that goes from 1 January 2016 to the 31 December 2016, on a daily basis, in order to calculate and evaluate the global portfolio's performance in that year.

This portfolio, however, only initially foresees an optimal allocation, and then always proceed with the same weight for each stock, regardless to the fact that these can record a negative performance during a period, and then it would be preferable to replace them with others stocks (no fine tuninng).

Once retrieved the daily returns of the securities and the variance-covariance matrix[50], we can find these values for the mean-variance portfolio, using the formulas of the average return and of the variance. In particular, we have that the mean-variance allocated portfolio presents a return, a variance and a standard deviation equal to:

Table 19. The Mean-Variance Portfolio performance

MEAN	2,040%
STANDARD DEVIATION	2,282%
VARIANCE	0,052%

[50] See Appendix C for the prices, with daily rate, downloaded between the 1 January 2016 and the 31 December 2016 from https://it.finance.yahoo.com/, while Appendix D for the portfolio return.

The active allocated portfolio shows a positive return, and a relatively greater risk position than the passive one. The mean-variance portfolio is more efficient than the previous one, as we expected. This portfolio could also record better performance if it were re-weighted periodically, but this dependent to its management. From an efficiency point of view, we calculate the Sharpe ratio, using as risk-free rate the performance of the 3 months T-Bill during the 2016[51].

Sharpe Ratio of the Portfolio 47,30%

The value of the Sharpe ratio shows us how this portfolio has performed in the 2016, the value is greater than the previous portfolio with passive allocation, the latter in fact had a value equal to 28,79%. The active allocated portfolio could be the portfolio of an informed investor who knows how to maximize its performance, without rebalancing the weights during the period in question.

[51] See Appendix C for the daily return of the 3 months T-Bill.

Chapter 5

THE BEHAVIORAL FINANCE

Summary: 5.1 The Efficiency of the Markets – 5.2 The Theory of Expected Utility – 5.3 The Prospect Theory – 5.4 Theoretical Principles of Behaviour – 5.5 A behavioural Perspective of Themes – 5.6 Investor Sentiment and Stock Market prices.

The behavioural finance is a descriptive theory that seeks to attribute meanings to the behaviours in the financial sector, its birth is due to the inadequacy shown by the traditional financial and valuation of investments model towards the actual operative world.

This theory applies the results of cognitive psychology to the process of the decisions and choices in economy and finance, showing how emotional and instinctive factors guide the behavior of people, which are not perfectly rational, thus denying the expected utility theory that is the foundation of the classical model.

Hereafter, are presented the foundations of the traditional model, hesitating on the analysis of the assumptions underlying the efficient markets and the expected utility theory, highlighting their main limitations and consequently the birth of the behavioural finance.

5.1 THE EFFICIENCY OF THE MARKETS

The traditional finance uses models in which the agents are subjects entirely rational. The rationality can mean two things:

- First, when agents receive new information they need to update their beliefs on the basis of these.
- Second, it expected that given the beliefs the agents make choices that are normatively accepted, namely that are consistent with what is contained in the Subjective Expected Utility (SEU)[52].

[52] Barberis N., Thaler R. (2002), "*A survey of survey of behavioural finance",* University of Chicago.

One of the fundamentals models of financial economy is represented by the Efficient Market Hypothesis (EMH), which states that market prices fully reflect all available information. Developed by Paul A. Samuelson, and independently by Eugene F. Fama in the 1960s, this idea has been applied extensively to theoretical models and empirical studies of financial securities prices, generating considerable controversy as well as fundamental insights into the price-discovery process[53]. Nevertheless, the markets are not always efficient, but they can also appear in other forms, as the theory of efficient markets assumes that, all subjects are rational and there are no clutches in the stock price, or which is equal to its fundamental value, that is the sum of the net present value of all the discounted future cash flows, wherever investors in developing their expectations, properly examine all the information available to them, and where the discount rate is consistent with a specific preference.

In this situation, we note that prices of securities reflect the information available at that time in the market. All operators make their choices with the same information and no one would be able to beat the market trying to achieve an expected average returns higher than those of equilibrium, so that the stock prices are considered to follow a *random walk* path[54]. No one earns consistently superior returns in such a market. Collecting more information does not help to achieve greater returns, because all available information is already impounded in today's stock prices.

Economists define three levels of market efficiency, which are distinguished by the degree of information reflected in security prices. In the first level, prices reflect the information contained in the recollection of past prices. This is called *weak market efficiency.* If markets are efficient in the weak sense, it is impossible to make consistently superior profits by studying past returns, prices will follow a random walk.

The second level of efficiency requires that prices reflect not just past prices but all the other public information, for example from internet or the financial press. This is known as *semistrong market efficiency.* If markets are semistrong efficient, then prices will adjust

[53] L. Blume and S. Durlauf, The New Palgrave: A Dictionary of Economics, Second Edition, 2007. New York: Palgrave McMillan. And Fama E. (1998), *"Market Efficiency, Long-Term Returns and Behavioural Finance"*, Journal of Financial Economics, vol. 22.

[54] Kendall, M. G., "*The Analysis of Economic Time-series, part I. Prices*", in Journal of the Royal Statistical Society, 96, pp. 11-25, 1953.

immediately to public information, such as the announcement of the last quarter's earnings, a new issue of stock, or a proposal to merge two companies.

With *strong market efficiency*, prices reflect all the information that can be acquired by careful analysis of the company and the economy. In such a market we would observe lucky and unlucky investors, but we wouldn't find any superior investment managers who can consistently beat the market[55]. The strong form of efficiency requires that also the knowledge of confidential information relating to a stock, therefore the unknown market news, does not allow to take profit opportunities. While the two previous forms of efficiency seem to be more or less confirmed in the reality, things that can hardly be said for the third. The existence of laws that regulate the trade of securities by the manager of the same company, (insider trading laws managers) can be considered as aimed to achieve that level of efficiency. Only in the weak form of efficiency, professional investors who manage to read public information are more likely to achieve excess returns, while those with only past information cannot do this. If efficiency is semi-strong only those in possession of inside information may achieve higher returns, whereas, in an efficient market with strong form, no extra profit can be realized. It notices, in addition, that the strong form of efficiency implies the semi-strong form and consequently the weak one, and that of semi-strong implies only the weak one.

Moreover, in an efficient market in the strong sense is hard that the stock prices would be correct in every single moment. For this reason, we must treat the strong efficiency in a more elastic way, it is true that the value of securities should reflect immediately and correctly all the information, but we must always take into account a sufficient time interval necessary to ensure the adjustment of prices. Therefore, if in a given market, none is able to realize extra profits, then this market is efficient. This is generally the main criterion used to determine whether a market is efficient or not.

From the empirical point of view were conducted numerous studies with the aim to test the efficiency of markets. To test the hypothesis of the weak form, researchers measured the profitability of some of the trading rules used by those investors who claim to find

[55] Fama E. (1998), *"Market Efficiency, Long-Term Returns and Behavioural Finance"*, Journal of Financal Economics, vol. 22.

patterns in security prices. It appears that throughout the world there are few patterns in day-to-day returns.

To analyze the semistrong form of the efficient-market hypothesis, researchers have measured how rapidly security prices respond to different news, such as earnings or dividend announcements, news of a takeover, or macroeconomic information. To do this has been set a model for the abnormal return.

$$Abnormal\ stock\ return = actual\ stock\ return - expected\ stock\ return$$
$$= \tilde{r} - (\alpha + \beta \tilde{r}_m)$$

This abnormal return should only reflect firm-specific news[56]. The stock-price adjustment is immediate and complete. After the price moves on the public announcement day, the run-up is over, and there is no significant further drift in the stock price, either upward or downward[57].

Tests of the hypothesis of the strong form have examined the recommendations of professional security analysts and have looked for mutual funds or pension funds that could predictably outperform the market. Some researchers have found a slight persistent outperformance, but just as many have concluded that professionally managed funds fail to recoup the costs of management[58].

5.2 THE THEORY OF EXPECTED UTILITY

The Expected Utility theory is the other pillar on which the traditional finance is based. This theory was developed in 1947 by von Neumann and Morgenstern, and has been applied for decades as one of the main models of economic behavior, until it is born the Prospect Theory, proposed by Tversky and Kahneman in the seventies, that is, a new model more reliable with the economic reality.

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[56] Abnormal returns are also often calculated using the Fama-French three-factor model, available at Kenneth French Library, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[57] Partell, J.M. and Wolfson, M.A., "*The Intraday Speed of Adjustment of Stock Prices to Earnings and Dividend Announcements*", Journal of Financial Economics, 13, pp. 223-252, June 1984.

[58] M. Carhart, "On Persistence in Mutual Fund Performance," Journal of Finance, 52, pp. 57–82. March 1997.

This Expected Utility theory has as its key point that individuals are entirely rational, and act according to predetermined patterns, where the starting point is the assumption that people prefer to have a greater economic return compared to a minor one, and then that the marginal utility of wealth is always positive (hypothesis of non-satiation).
The expected utility theory has three basic characteristics:

- is based on mathematical models,
- it is normative,
- considers the full rationality of the behavior of individuals.

More specifically, this theory tries to describe the preferences arisen from individuals under risk condition, like a particular agent who knows what events may occur and has to make a decision without knowing ex-ante what will be the future result, and he will then search to give a certain probability of occurring to each event[59].
The expected utility is defined by mathematical expectations of elementary utility (associated utilities to the individual values of the resources available to the agent), that is the weighted average of the utility associated with individual values of the resources available to the agents, where the weights are the probabilities with which these values will be realized.
The utility theory defines that economic actors, in front of a choice, prefer to move towards an economic combination associated with the highest utility, that is the event which is associated to a greater probability of obtaining gains and to a lower probability of losses[60].

[59] Von Neumann J., Morgenstern O. (1944), *"Theory of Games and Economic Behaviour",* Princeton University Press.

[60] Agnoli P., Piccolo F., *"Probabilità e scelte razionali. Una introduzione alla scienza delle decisioni",* Armando Editore. 2008.

The utility function can be used to associate a certain measure of utility to each choice, this formulation has two properties[61]:

1. The utility function is increasing respects to the players' rationality requirement.
2. The utility can be used to sort risky alternatives and is expressed as a function of the possible outcomes and their probabilities of occurrence.

Finally, this function can take different forms according to the attitude, or aversion, to the risk of each subject. A subject is risk averse if for all probability distributions P:

$$CE_p < E(\tilde{x}|P)$$

Where CE_p is the certain equivalent of a lottery, while $E(\tilde{x}|P)$ is the expected value of the same lottery. In this case the curve expressing this relationship is of a concave type with strictly decreasing marginal utility.
For the subject neutral to the risk, instead, that it would have that:

$$CE_p = E(\tilde{x}|P)$$

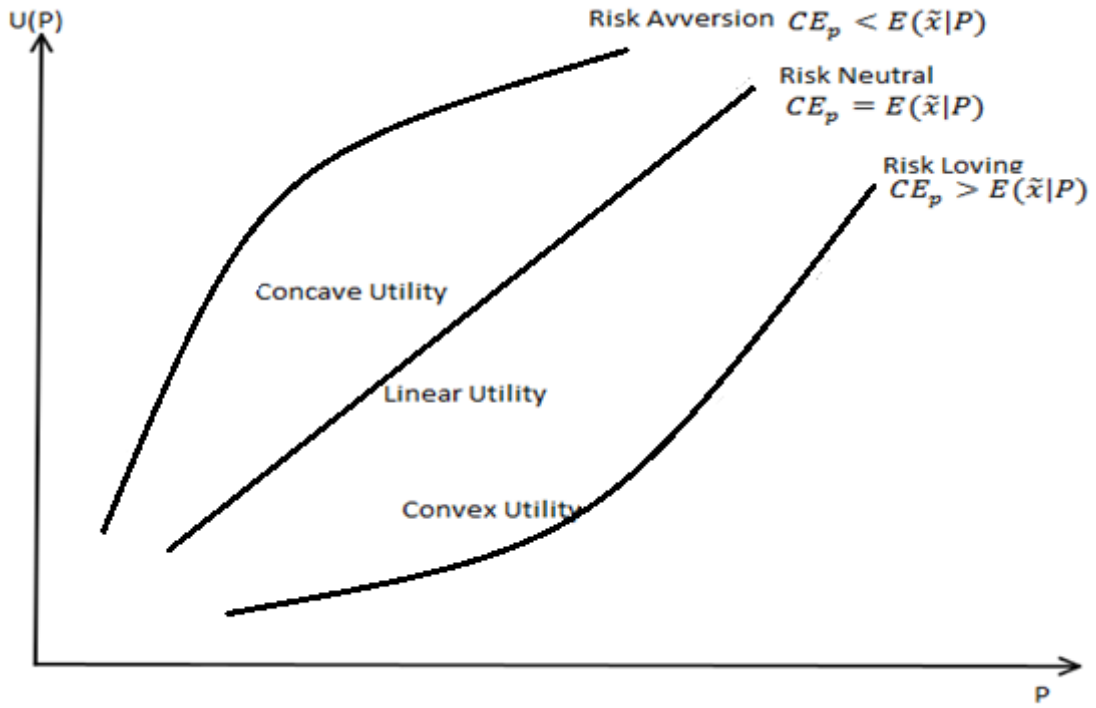
or that the certainty equivalent is equal to its probability of realization, and thus the utility curve connected to it will be a straight line with zero marginal utility.
Finally, the subject lover of risk will require a certainty equivalent greater than the probability of lottery realization:

$$CE_p > E(\tilde{x}|P)$$

The utility curve associated to this type of function is convex with positive marginal utility[62].

[61] Von Neumann J., Morgenstern O. (1944), *"Theory of Games and Economic Behaviour",* Princeton University Press. And E. T. Jaynes, G. Larry Bretthorst, "*Probability Theory: The Logic of Science*", Cambridge University Press, 2003.

[62] Legrenzi P., *"Psicologia e investimenti finanziari. Come la finanza comportamentale aiuta a capire le scelte di investimento".* Bologna Il Mulino, 2006. And E. Agliardi, G. Chiesa, "*Economia dei Mercati Finanziari*", Carocci Editore 2003.



Form of the expected utility function: *the utility function may be concave, straight or convex depending on the level of risk aversion of each subject*[63]**.**

Within the theory it has been introduced the concept of the risk premium, namely the maximum price that a person is willing to pay in order to eliminate the uncertainty contained in the choice and to obtain in this way the expected gain. It is defined as the difference between the expected value of the lottery and the certainty equivalent of the lottery.

$$q(P) = E(\tilde{x}|P) - CE_p$$

Where $q(P)$ is the risk premium, CE_p is the certainty equivalent of a lottery, while $E(\tilde{x}|P)$ is the expected value of the lottery itself.

[63] Legrenzi P., *"Psicologia e investimenti finanziari. Come la finanza comportamentale aiuta a capire le scelte di investimento".* Bologna Il Mulino, 2006.

The classical utility theory developed by Von Neumann and Morgestern is based on five axioms, thanks to which the logic behind the decision behavior becomes much simpler[64], that are:

1. *Comparability*: an individual is always able to compare, establishing an order of preference, different uncertain results which tend to exclude each other.
2. *Transitivity*: if he prefers *A* to *B* and *B* to *C*, then *A* is preferred to *C*. If a person is indifferent between *A* and *B* and between *B* and *C*, then it is also indifferent between *A* and *C*.
3. *Strong Independence*: when comparing different games, it has to focus the attention only on the results that are not common.
4. *Measurability*: if *A* is preferred to *B* which it is consequently preferred to *C* then there may be a probability α for which, the game compose by the outcomes *A* and *C*, is equivalent to *B*. This axiom therefore requires that for each game there is only one certain result equivalent that is between the extreme results of the game.
5. *Orderability*: states that two games, which present similar results, are equivalent if they have the same chances, otherwise should prefer the one with the greatest chances of getting the preferred outcome.

These are the five main axioms upon which the expected utility theory is based, however subsequent studies have shown that the real decision-makers sometimes act in violation of the axioms just seen, so not in an entirely rational manner.

The expected utility theory refers to an ideal subject, to which is attributed a rationality through the comply with the axioms, and this does not take into account the characteristics and limitations of the human rationality, nor any emotional issues that may affect the actual choices; thus provides a role model to follow. Nevertheless, if such a pattern has to be applied to the real choices forecast, it will probably have to give up the strong axiomatization that characterizes it, to leave room for factors which play a significant role in the actual behavior of individuals.

[64] Varian A. "*Microeconomic Analysis*", Norton, New York. 1984.

5.3 THE PROSPECT THEORY

Since the '50s there have been numerous studies, both at theoretical and practical level, that have tried to shift the focus from the traditional theories, strongly centered on the theme of rationality, to more realistic theories that try to investigate the factors driving the behavior.

The behavioral finance is defined as a descriptive theory that attempts to analyze the behavior of the agents in the financial sector, examining the psychological factors involved in the various decision-making processes[65].

It is clear that this discipline has gradually moved away from the traditional theory, trying to interpret the irrational behavior of investors. We have to note that the behavioral finance is not the result of a single theory, but is seen as an interdisciplinary approach in which it is easy to find elements of economy, psychology, finance and sociology, which influence the choices of individuals under uncertainty.

Over the years, many scholars have analyzed the behavior of individuals noting that, as conditioning elements of the decisions, there are: the past experiences, the beliefs, the context, the information presentation and the incomplete information.

Later, to develop the matter in a very wide way, were Amos Tversky and Daniel Kahneman, who managed to change the opinion about the validity of the traditional model, based on the maximization of wealth and the perfect rationality of the subject, managing to give the behavioral finance a sufficient visibility in academic circles, so that this could be considered a theory at all effects. They, in fact, presented a new theory no more normative but descriptive, the Prospect Theory, whose goal is the description of the behavior of individuals in risky circumstances[66], where the choices of individuals examined in risky circumstances are: if they consume all the income or they save it; they save for future purposes or save money without an intended purpose; to manage their own savings or entrust them to an expert.

The first criticism of the expected utility theory came from the French economist Maurice Allais who showed that the subjects tend to overestimate reliable results and

[65] Rigoni U. (2006)*, "Finanza comportamentale e gestione del risparmio".* Giappichelli Editore. 2006.

[66] Kahneman, D. and Tversky, A., "*Prospect Theory: An Analysis of Decision under Risk*", Econometrica, 47, pp. 263-291, 1979.

underestimate those that are considered probable or uncertain. This is absolutely in contrast with what is stated by the classical theory, according to which the utility of a random event is equal to the linear combination of the probability of each result[67].

The Kahneman and Tversky, taking a cue from that experiment of Allais, showed that there are two factors that contrast with the expected utility theory:

1) *Attitude toward risk*. The two psychologists observed that individuals, when they have to take risky decisions, show a clear reluctance to suffer damage. It seems that investors are focused not only on the current value of the securities held, but also consider whether their investments are presenting a profit or a loss. In particular there are three effects to be considered:
    - The effect-certainty: in which individuals show a preference for a sure event with respect to a possible one. This clashes with the claims of the expected utility theory where an individual, in front of a choice which involves the same level of utility, appears to be indifferent between choosing an alternative or the other. According to Kahneman and Tversky however, it can occur situations in which the winnings are possible and not only probable and therefore individuals express a preference towards the situation that presents a greater potential payout. It notices how people always prefer to get some certain gain although lower, than a higher but likely[68].
    - The effect-reflection: presenting the same experiment, however, and in this case considering the losses, the two psychologists were able to show how the behavior of individuals, typically risk averse in the case of a gaining situation, corresponds to a behavior of risk appetite when considering the area of the losses. As in the case of certainty effect also in the reflection effect, it is violated the principles of the expected utility, where to a sure result is attributed more importance than to an uncertain

[67] M. Allais. "*Le Comportement de l'Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l'Ecole Americaine*". Econometrica. 21, pp. 503-546. October 1953.

[68] Thaler R. and Johnson E., "*Gambling with the House Money and Trying to Break Even: The Effects of Priori Outcomes on Risky Choice*". Management Science, 36, pp. 643-660. 1990.

one. In this context of negative domain are detected behaviors of risk propension (*risk seeking*).

- The effect-isolation: in order to simplify the whole process of choosing, it proceeds to the breakdown of each alternative, taking into account only some elements of the alternative and not the whole. This process, however, as Kahneman and Tversky pointed out on an experimental basis, can lead to incoherent preferences, as people would overlook the shared components between different alternatives for focusing solely on those distinctive.

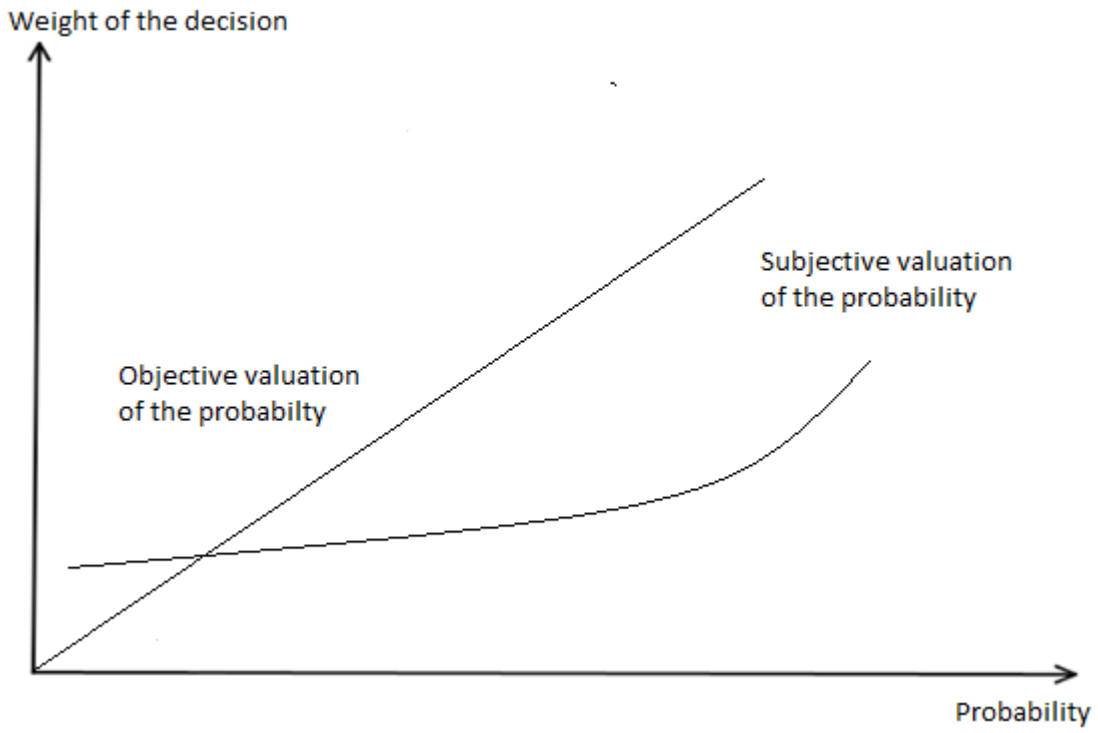
Each of these factors is a criticism against the expected utility theory that claim which the whole decision-selection process takes place in a completely rational manner.

The idea of the two psychologists Kahneman and Tversky is demonstrated by of the existence of the *framing* effect. This assumes that, if an individual is acting in a rational manner should consider equivalent two options even though they are presented differently. The two scholars have shown, conversely, that if the options are perfectly equal, but proposed in a different way, we may notice the differences in the preferences of the individuals, this is the so-called *framing effect*[69].
All of this independently from the utility function and the probability distribution.

2) *Beliefs on the probabilities*. Psychologists have also found that when people trying to predict possible future results, tend to be based on what has already happened in similar circumstances. Consequently, they are inclined to give excessive weight to a limited sequence of recent events.

[69] Kahneman, D. and Tversky, A., "*The framing of Decisions and Psychology of Choice*". Science. 211, pp. 453-458. 1981.

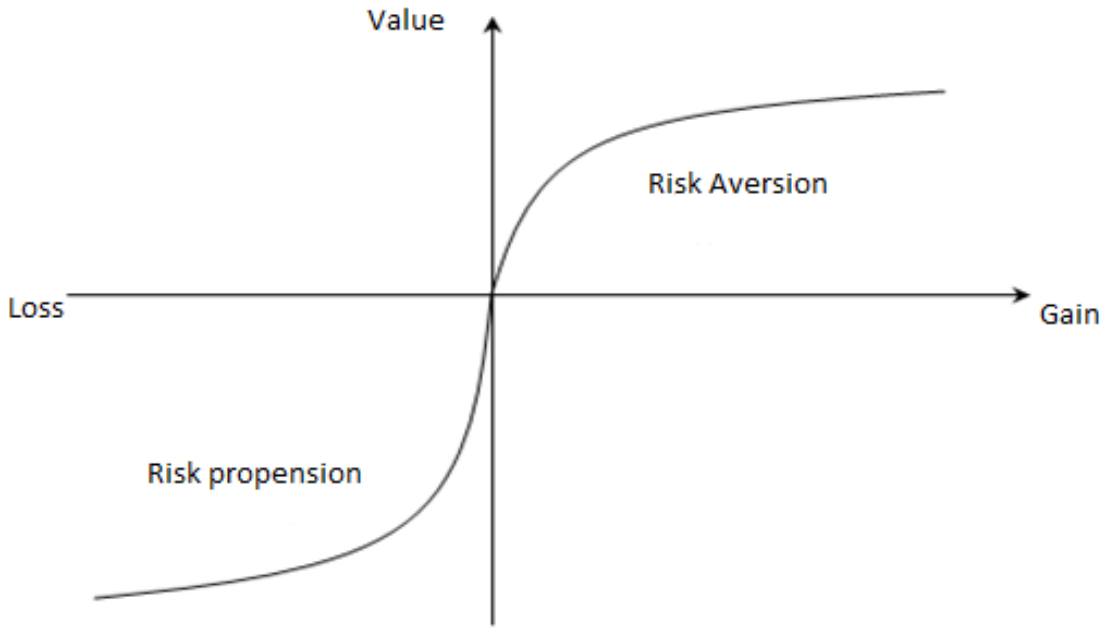
Almost all individuals are too conservative or too slow to update their beliefs in front of new evidences. Even when they tend to correct their personal beliefs, they do so with a breadth and a depth smaller than those required by the principle of rationality.



Probability function: *The probability function with respect to the effective probability of each subject*[70].

If we regard to the probability function considering the change in wealth instead that the realization of an event, there would have a concave function in the area of the gains and a convex one in the area of the losses, then we are witnessing a decrease in the marginal value of earnings and losses as they increase their size.

[70] Legrenzi P., *"Psicologia e investimenti finanziari. Come la finanza comportamentale aiuta a capire le scelte di investimento".* Bologna Il Mulino, 2006.



Probability function in case of Wealth: *the probability function with respect to wealth is a curve with sinusoidal trend, but with steeper trend in losses (loss aversion).*

One of the most important consequences of this feature is that according to the different contexts, the subjects may experience a risk adverse or risk loving behavior. If there are elements that make it fall within the revenue, then they adopt a risk-averse behavior, contrary if they fall to the loss area they will be more inclined to risk.
These arguments were supported by numerous empirical tests, but it is appropriate also to consider the conditions prevailing a particular subject because it is possible to witness a change in the personal value function[71].
From all these inefficiencies has origin the behavioral finance, not with the aim to replace the classical theories but with to integrate them and to intervene in their inefficiencies.

In addition to the Prospect Theory, Kahneman and Tversky also developed the Theory of Cumulative Prospective, which in a sense tries to recover the logic followed by the expected utility models. This differs from the classical Prospect Theory for the ways in which it tries to weight the probabilities, namely it tries to consider the cumulative distribution of the same and to be able to apply the functions of values that are different depending on whether there are gains or losses.

[71] Kahneman D., Tversky A. (1974), "*Judgment Under Uncertainty: Heuristics and Biases",* Science, vol.185.

One of the great discoveries, that we were able to note within this theory, lies in different deviation of decision weights to be applied to uncertain outcomes. In fact, while in the previous theory the different decision weights depend on the probability of single outcome, this is not the case in the cumulative function, where the decision weights depend on the cumulative distribution and on the probability to obtain this, it has to use different weighting functions to earnings and losses.
Another important thing to consider in the Cumulative Prospective Theory is that this can interpret behaviors towards risk which are much more diverse than they did previously. In general, it can be said that subjects adopt a risk-averse behavior when they are dealing with situations that present a quite possible result and with an amount of gain not excessively high. They adopt an attitude inclined to the risk when identify high-value of gains to be realized, however unlikely this is the typical case of lotteries. The behavior of the subjects will be the opposite when considering the domain of losses[72].

5.4 THEORETICAL PRINCIPLES OF BEHAVIOR

We pass to analyse what are the theoretical principles that lie at the basis of behavioral finance, in which we can identify elements related to psychology, sociology and finally anthropology. It is, therefore, known that this is not an only one well-identified topic but it has several interdisciplinary elements.

In this section, we examine the investment choices and their future projection, identifying a prediction of what will be the random events. The operation to be able to identify what are the events that will occur and their chances can be particularly risky and difficult, for this reason we tend to divide this into three distinct phases, that are: the information capture, the processing of these and finally the emission of a probability of the occurrence of such event[73].

[72] Tversky A., Kahneman D. (1986), *"Rational Choice and the Framing of Decisions",* Journal of Business, vol. 59.

[73] Barberis N., Thaler R. (2002), "*A survey of survey of behavioural finance",* University of Chicago.

The subjects try to overcome the problem to assign a probability to a set of events by using the heuristic, which consists in a group of rules that are used to solve, in a simplified way, particularly complex choices.

The main feature of these heuristics is to be able to relate the difficulty of the task to the ability, albeit limited, of collecting and processing the available information. Heuristics is an answer to a particular problem, which is not necessarily the best one, but simply a good one, given specific constraints.

Certain heuristics are involved in the behavior of individuals when they are dealing with particular situations that can help to understand why the most used choices differ from the theories that normally are based on assumptions of rational and optimal behavior, in this case we speak of *bias*[74].

The important thing to take into account is that, on one hand the heuristics simplify the work of the mind, and on the other hand can lead to erroneous conclusions excessively simple.

These distortions can, therefore, determine:

- *Errors*, when it has to make decisions for individual investments, in fact, these can lead to an insufficient return or an excessive risk exposure;
- *Collective biases*, in fact it can create the phenomena of "contagion". It is no longer a single condition linked to a single individual investment but becomes a collective phenomenon;
- *Various market inefficiencies*, that can be for example wrong prices or anomalies in returns between different activities, and so on.

The behavioral finance seeks to understand these anomalies and in a sense to exploit them in a way such as to be able to identify the optimal investment strategy.

So the existence of these heuristics shows that the human brain uses cognitive filters to be able to simplify and to select the really interesting information, failing to take them into account because of the physiological limits[75].

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[74] Kahneman D., Tversky A. (1974), "*Judgment Under Uncertainty: Heuristics and Biases",* Science, vol.185.

[75] Legrenzi P., *"Psicologia e investimenti finanziari. Come la finanza comportamentale aiuta a capire le scelte di investimento".* Bologna Il Mulino, 2006. And Alemanni B., Franzosi A. (2006)*, "Portafoglio e psicologia dei trader online", IlSole24Ore.*

The main heuristics are:

1) *Representativeness,* which states that individuals establish their opinions on stereotypes and habits, so do not evaluate events subjectively, with the risk of hitting errors as overestimate the probability of joint events and underestimate that of disjoint events.
2) *Availability,* in acquiring information, persons are conditioned by how these are recalled to their minds, as events that occurred recently or personally experienced, with the risk to consider as more numerous, the easiest events to remember.
3) *Anchor*, individuals in making decisions start with an initial data, which serves as an anchor, and proceed on an adjustment based with the new information they receive, of course, the mistake lies in the unbalance towards the initial data.
4) *Overconfidence and Underconfidence,* the subjects overestimate or underestimate their capabilities.
5) *Conservatism,* resistance of part of individuals to change their ideas and beliefs even if it is proved that they are incorrect.
6) *Overreaction and Underreaction,* excessive response of prices when they receive new information.
7) *Loss Aversion,* aversion to the subject of the loss, the risk of having more wealth decreases.
8) *Aversion to ambiguity,* hostile attitude of subjects towards all those situations of uncertainty, they prefer situations where the probabilities are known.
9) *Hindsight,* make judgments in retrospect, overestimating the probability to predict the event.
10) *Supplied effect,* the subjects attribute different values to an asset depending on whether they own it or they should buy it.
11) *Regret,* the subjects try to avoid the displeasure caused by a wrong choice.
12) *Mental Accounting,* the mental process by which individuals are able to organize and account for all economic transactions, of course, the risk is to follow mental rules.

5.5 A BEHAVIORAL PERSPECTIVE OF THE THEMES

The scope of theoretical principles of behavioral finance varies depending on the topics considered. Considering the objective to set a sentiment based allocated portfolio, we evaluate which are the behavioral principles, among those considered, and how they interact with the theme of building a portfolio.

We consider, in particular:

1) THE EXUBERANCE OF THE AGENTS IN THE MARKETS,
2) RISK ASSESSMENT,
3) THE DIVERSIFICATION,
4) THE RISK FREE RATE AND THE EQUITY RISK PREMIUM.

1) THE EXUBERANCE OF THE AGENTS IN THE MARKETS

   The belief in efficient markets has come under assault from within the academy. The notion that markets make systematic mistakes and fail to reflect the true value often is now backed up not only by evidence but has also been linked to well-documented strangenesses in human nature[76].

   In a survey article on the topic, Barberis and Thaler list the following characteristics that skew investor behavior[77]:

   a) *Overconfidence.* Investors are over confident in their own judgments, as evidenced by their inability to estimate confidence intervals for quantities and probabilities of event occurring.

   b) *Optimism and wishful thinking.* Individuals have unrealistically optimistic views of their own capabilities. For instance, the 90% of people, when characterizing their own skills, describe themselves as above the average.

   c) *Representativeness.* Individuals show systematic biases in how they classify data and evaluate. One manifestation of this bias is that they ignore sample sizes, when judging likelihood, treating a 60% success rate in a sample of 10 and the

[76] Shiller, R., "*Irrational Exuberance"*, Princeton University Press. 2000

[77] Barberis N., Thaler R. (2002), "*A survey of survey of behavioural finance",* University of Chicago. And Damodaran A., "*Applied Corporate Finance*", Fourth Edition, Stern school of Business, New York University, Wiley.

same success rate in a sample of 1,000 equivalently, even though the latter should convey more information.

d) *Conservatism and belief perseverance.* Individuals seem to attach to much weight to their prior beliefs about data and to not react sufficiently to new information. Once they form an opinion, they are reluctant to search for evidence that may contradict that opinion and when faced with such evidence, they view it with excessive skepticism. In some cases, in what is called the confirmation bias, they actually look at contradictory evidence as supportive of their beliefs.

e) *Anchoring.* When forming estimates, individuals start with an initial and often arbitrary value and adjust this value insufficiently.

f) Availability biases. When assessing the likelihood of an event, individuals looking for relevant information often overweight more recent events and events that affect them personally more than they should in making their judgments.

Given that these characteristics are widespread and perhaps universal, we should not be surprised that markets reflect them. The overconfidence and over optimism feed into price bubbles in individual stocks as well as the entire market, and those who question the rationality of the bubbles are often ignored (belief perseverance). Anchoring and availability biases can distort how we value individual companies, again leading to significant differences between market prices and true values.

2) RISK ASSESSMENT

The mean–variance framework for assessing risk is focused on measuring risk quantitatively, with one number especially, the standard deviation. While this focus is understandable, because it introduces discipline into the process and makes it easier for us to follow up and measure expected returns, it may not fully capture the complicated relationship that we have with risk. Behavioral finance scholars present three aspects of risk assessment that interact with the mean–variance model view of risk:

a) *Loss aversion.* In experiments with human subjects, there is evidence that individuals are affected far more negatively by a loss than they are helped by an equivalent gain (see graph on page 99, *Probability function in case of Wealth*),

and that they generally measure losses in money terms rather than percentage terms. Put another way, investors are loss averse rather than risk averse. Consequently, investments where there is even a small chance of a significant loss in wealth will be viewed as risky, even if they have only a small standard deviation.

b) *Familiarity bias.* Individuals seem to perceive less risk with investments that they are familiar with than with unfamiliar investments. Thus, they see less risk in a domestic company with a long provenance than they do in an emerging market firm. This may explain why there is a "home bias" in portfolios, where investors overinvest in investments in their domestic market and underinvest in foreign investments. In an extension of this bias, the risk that individuals perceive in an activity or investment is inversely proportional to the difficulty they face in understanding it.

c) *Emotional factors* There is an emotional component to risk that quantitative risk measures cannot capture. This component can have both a positive effect, where gains accentuate positive effects (happiness and optimism), and a negative effect, where losses feed into negative effects (worry and anxiety). More generally, investor moods can affect risk perceptions with investments that are viewed as relatively safe in buoyant times becoming risky when investor moods shift.

In recent years, there have been attempts to build composite risk measures that bring these behavioral components into the analysis. While no consensus has emerged, it may explain why quantitative measures of risk (such as standard deviation) for firm may deviate from the many qualitative risk measures that often exist for the same firm.

3) THE DIVERSIFICATION

The argument that investors should diversify is logical, at least in a mean–variance world full of rational investors. The reality, though, is that most investors do not diversify. In one of the earliest studies of this phenomenon, Blume, Crockett, and Friend examined the portfolios of individual investors and reported that 34% of

the investors held only one stock in their portfolios, 55% held between one and ten stocks, and only 11% held more than 10 stocks[78].

While these investors could be granted the excuse that mutual funds were neither as prevalent nor as accessible as they are today, Goetzmann and Kumar looked at 60,000 investors at a discount brokerage house between 1991 and 1996 and concluded that there has been little improvement on the diversification front and that the absence of diversification cannot be explained away easily with transactions costs.

While some researchers have tried to find explanations within the conventional finance framework, behavioral economists offer three possible reasons:

a) *The gambling instinct.* One possible explanation is that investors construct their portfolios, as layered pyramids, with the bottom layer designed for downside protection and the top layer for risk seeking and upside potential. Investing in one or a few stocks in the top layer may not yield efficient risk-taking portfolios, but they offer more upside. In a sense, these investments are closer to lottery tickets than to financial investments[79].

b) *Over confidence.* Investors who overweight specific industries or stock characteristics tend to be less diversified than the universe of investors. They argue that this is consistent with investors being overconfident in their own abilities to find winners and thus not diversifying[80].

c) *Narrow framing and estimation biases.* Investors who frame their investment decisions narrowly (looking at pieces of their portfolio rather than the whole) or misestimate correlations (by assuming that individual stocks are more highly correlated with each other than they really are) will hold less diversified portfolios.

In summary, many individual investors and some institutional investors seem to ignore the lessons of diversification and choose to hold narrow portfolios. Their

[78] Blume, Crockett and Friend. "*Stockownership in United States: Characteristics and Trends*". Survey of Current Business, 54, pp. 16-40. 1974.

[79] H. Shefrin, and M. Statman. "*Behavioural Portfolio Theory*", Journal of Financial and Quantitative Analysis. 35, pp. 127–151. 2000.

[80] W. Geotzmann. and A. Kumar. "*Equity Portfolio Diversification*", Review of Finance. 12, pp. 433-463. August 2008.

perspective on risk may vary from more diversified investors in the same companies.

4) THE RISK FREE RATE AND THE EQUITY RISK PREMIUM

Risk-free rates and equity risk premiums vary over time and managers often are confronted with numbers that they believe are “not normal.” This was the case in early 2009, when managers saw the U.S. ten-year T. Bond at 2.3% and equity risk premiums at close to 7%. Faced with these unusual numbers, many analysts and corporate treasurers decided to override them and go with what they believed were more normal values. While this push toward normalization has an empirical basis, there is also a behavioral spin that we can put on it. As we noted, there is significant evidence that individuals anchor their estimates to arbitrary starting values. In the case of CFOs, those starting values may very well be the risk-free rates and equity risk premiums that they have become familiar with over their working lifetime, leading to very different definitions of what comprises normal. In addition, firms that have been using the same equity risk premiums for long periods find it hard to abandon these estimates, even in the face of substantial evidence to the contrary.

5.6 INVESTORS SENTIMENT AND STOCK MARKET PRICES

Individual investors are all those subjects that act in the financial markets in a completely autonomous way. Which in addition to experience feelings and emotions, already described, and generate the resulting distortive effect, they are also incline to everything that happens in the outside world and they prove reactions that can be illogical. These reactions, when considered on a social level, can create massive distortion of the global market.

A relatively large number of studies shows that sentiment may drive agents' investment decisions. In this respect, the behavioral finance literature provides strong evidence supporting the existence of a strong relation between stock returns and investor sentiment. Investor sentiment is typically defined as a belief about future cash flows and investment risks that is not justified by the facts at hand.

Certain events may create either a positive or a negative sentiment that strongly affects investors' decisions during the investment process and thus related stock market prices, as for example: St. Patrick Day, Yom Kippur War, Rosh Hashanah, International Sporting Games, Macro Political events and so on.

Early studies observe, for example, that sunshine, which is well known to be a driver of peoples' mood, tends to comove positively with daily stock returns[81].While, other scholars find that lunar phases may affect stock market returns[82]

More recent studies find that international sporting games (soccer games particularly) heavily affect investors' sentiment, and in turn, stock market returns.

And, finally, Kaplansky and Levy show that major aviation disasters tend to generate a negative sentiment within two days after the event.

For our analysis, we consider three of the main events that may affect the price of securities in the stock market.

In detail they are:

1) THE FIFA WORLD CUP
2) THE AVIATION DISASTERS
3) GLOBALLY DANGEROUS DISEASES

1) THE FIFA WORLD CUP

It is observed that there is a strong association between results of soccer games and local stock returns. In the spirit of their work, Kaplansky and Levy propose a novel approach to exploit this effect on the aggregate international level with the following two unique features[83]:

- The aggregate effect does not depend on the games' results;
- The aggregate effect is based on many games.

[81] D. Hirshleifer and T. Shumway, "*Good Day Sunshine: Stock Returns and the Weather*". Journal of Finance. 58, pp. 1009-1032. 2003."

[82] K. Yuan, L. Zheng, Q. Zhu, "*Are Investors Moonstrack? Lunar Phases and Stock Returns*". Journal of Empirical Finance. 13, pp. 1-23. 2006.

[83] G. Kaplansky and H. Levy, "*Exploitable predictable irrationality: The FIFA World Cup effect on the U.S: stock market*". Journal of Financial and Quantitative Analysis. 45, pp. 535-553. 2010.

The main hypothesis is that during the World Cup period is a global negative effect, induced by all losing countries' fans at an international level. This hypothesis is based on the relatively large proportion of investors in any given country who invest internationally, mainly in the U.S. market. The study is also motivated by the fact that about one-third of the transactions in the U.S. market are involved with non-U.S. investors.

Therefore, the World Cup effect can lead to profitable strategies. The benefic effect, on average, is from being out of the market during the World Cup period. One can employ a very simple investment strategy to exploit the event effect and to obtain abnormal returns. The strategy is very simple: decrease the equity exposure during the of event period effect days (EPED).

2) THE AVIATION DISASTERS

Kaplansky and Levy examine large-scale aviation disasters. Their study is based on the hypothesis that that aviation disasters affect people mood and increase their anxiety which negatively affects the investment in risky assets. Therefore, we expect to observe negative rates of return in the stock market following aviation disasters.

The two scholars provide evidence that aviation disasters negatively affect stock prices for a short period of a few days. In addition, they find a mean-reverting reversal effect two days after the event.

As aforementioned, the main hypothesis tested is that a large-scale aviation disaster increases investors' fear and anxiety, which in turn negatively affects stock prices. This hypothesis relies on the observed relations between the media coverage of disasters, the fear and anxiety that aviation disasters provoke, and the reduced willingness of investors to take risks when fear and anxiety increase[84].

Thus, we require three crucial elements for the event effect to exist:

- The widespread media coverage of the disasters;
- The coverage that is sufficiently emotionally compelling to provoke fear and anxiety;

[84] G. Kaplansky and H. Levy, "*Sentiment and Stock Prices: The case of Aviation Disasters*". Journal of Financial Economics. 95, pp. 174-201. 2010.

- With increased levels of fear and anxiety, investors are more pessimistic regarding stock prices, tend to take less risks, or both.

The authors argue that all these elements are present in aviation disasters. Regressions results reveal that all-disasters first day is negative and large in absolute, however, the third-day effect is always positive confirming the presence of a reversal effect.

3) GLOBALLY DANGEROUS DISEASES

Donadelli, Kizys, and Riedel, examine whether investors' mood, driven by World Health Organization (WHO) alerts and media news on globally dangerous diseases, is priced in pharmaceutical companies' stocks in the United States.

Therefore, differently from previous studies, the scholars focus on a specific sector (i.e., the pharmaceutical one). The authors rely on the following four globally dangerous diseases: SARS, Influenza A(H1N1), Polio and Ebola. The principal hypothesis here is that disaster response networks (DRN) will positively affect investment in pharmaceutical stocks.

The studies suggest that DRNs have a positive and significant sentiment effect among investors (in Wall Street). The effect is stronger (weaker) for small (large) companies, who are less (more) likely to engage in the development of new vaccines in the wake of DRNs.

The authors show also that a potential negative mood (in Main Street) does not alter the positive sentiment effect. These findings give rise to profitable trading strategies leading to sizable out-performances over the market portfolio. In addition, and more importantly, they show that returns on DRN-based trading strategies are economically and statistically greater than returns on randomly drawn strategies[85].

[85] M. Donadelli, R. Kizys, and M. Riedel, "*Globally Dangerous Diseases: Bad News for Main Street, Good News for Wall Street?*". Journal of Financial Market. 2016.

Chapter 6

THE SENTIMENT BASED ALLOCATION

Summary: 6.1 The Considered Events – 6.2 The Speculative Operations – 6.3 The Sentiment based Allocation

We have seen in the previous chapters which events may affect the behavior of the individuals, that feel emotions as anxiety and fear towards the future. This mass behavior, in a cascade process, can affect the price of the shares in the market.
In this section, therefore, we try to verify, for the year 2016 how the main events may have influenced the market. To do so, it will be tested the portfolio of twenty securities built in the previous chapters (Chapter 4), verifying which performance it is capable to record, using the previously described speculative tactics, and following the empirical evidence provided by many scholars in the field of behavioral finance in the recent years. For each event happened, we will try to use speculative tactics, which in the case of international sporting events are concretized by reducing the exposure of our portfolio to such events, for the aviation disaster we will try to reduce the portfolio's exposure during the event period effect days (EPED) and then, increases it by the third day after the disaster, finally, for the globally dangerous diseases, we would increase the exposure of pharmaceutical stocks in our portfolio. While, everything related to the events of other kinds, such as macro events of political nature, terroristic, and generally of great market instability, it will try to reduce the exposure of the securities geographically or sectorally closer to the event in question, in order not to be affected much from the instability that can be generated.

It uses, therefore, these speculative tactics in order to observe how the portfolio is sensitive to the mass behavior of individuals and how, through these techniques, can be obtained a gain due to the benefic effect of macro events.
The previously constructed portfolio is already well diversified, both by geographic area and by sector. This allows us to reduce the exposure from one geographic area to another, and if it was required, from one sector to another.

The stocks considered are the following[86]:

Table 20. The 20 selected stocks

STOCK	CODE	SECTOR	GEOGRAPHIC AREA
The Boeing Company	BA	Aerospace	Global-US
Johnson and Johnson	JNJ	Pharmaceutical	Global-US
The Procter & Gamble Company	PG	Consumer Goods	Global-US
The Coca-Cola Company	KO	Beverage	Global-US
AT & T	T	Telecommunication	US-Texas
Chevron	CVX	Oil and Gas	US-California
Walmart	YMT	Super Centers	US-Arkansas
General Electric company	GE	Industrial Conglomerates	US-Massachusetts
Intel	INTC	Semiconductors	US-California
IBM	IBM	Consulting and Services	US-New York
The Travelers Companies, Inc.	TRV	Insurance	US-Minnesota
Fiat Chrysler Automobiles	FCA	Automotive	Europe-Italy
Barclays PLC	BARC.L	Banking	Europe-England
HSBC Holdings	HSBA.L	Financial Services	Europe-England
Volkswagen Aktiengesellschaft	VOW	Automotive	Europe-Germany
Allianz SE	ALV	Insurance	Europe-Germany
Bayer Aktiengesellschaft	BAYN	Pharmaceutical	Europe-Germany
Michelin	ML.PA	Autoparts	Europe-France
Pernod Ricard SA	RI.PA	Wine and Spirits	Europe-France
Buzzi Unicem	BZU.MI	Construction and Materials	Europe-Italy

The portfolio will be composed of all twenty stocks and the analysis will be conducted by applying the techniques ex ante, without knowing the actual return that the portfolio might have.

The type of security has been selected in order to be very sensitive to the events that occur in the course of 2016, so we can exploit better the sentiment effect that could be generate.

[86] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

6.1 THE CONSIDERED EVENTS

The considered events, occurred during the whole 2016, are a total of sixteen, divided by sector and geographic area. These events were selected among the main events that have marked this year and include international sports competitions, terroristic attacks, political events, natural disasters, and so on. Are not taken into account, in the analysis, minor behavioural factors, as the presence of sunny days, the moon phases, the "Monday" effect and others.

The events considered are the following[87]:

Table 21. The main events of the 2016

NR	DATE	ECONOMIC EVENTS	GEOGRAPHICAL AREA	TYPE
1	Jan-14	**Virus Zika**	North/South America	Global Disease
2	Mar-19	**The Disappearance of FlyDubai 981**	Europe-Eastern Europe	Aviation Disaster
3	Mar-22	**Attacks in Brussels**	Europe-Western Europe	Terroristic Attack
4	May-19	**Falling Egypt Air 804**	Global-Southern Europe	Aviation Disaster
5	May-28	**Champions League Final**	Europe-Western Europe	FIFA Cup
6	June-13	**Orlando Massacre**	US-North America/Florida	Terroristic Attack
7	June-23	**Brexit**	Europe-Western Europe/UK	Political Event
8	July-10	**Final FIFA European Championship**	Europe-Western Europe/France	FIFA Cup
9	July-14	**Nice Massacre**	Europe-Western Europe/France	Terroristic Attack
10	Aug-05	**Start of the Olympic Games**	Global North/South America	World Cup
	Aug-21	**Finish of the Olympic Games**	Global North/South America	World Cup
11	Aug-24	**The Earthquake in Center Italy**	Europe-Western Europe/Italy	Natural Disaster
12	Sept-16	**Deutsche Bank Scandal**	Europe-Western Europe/Germany	Financial Scandal
13	Nov-08	**Presidential Elections in USA**	US-North America	Political Event
14	Dec-04	**Constitutional Referendum in Italy**	Europe-Western Europe/Italy	Political Event
15	Dec-19	**Attack in Berlin**	Europe-Western Europe/Germany	Terroristic Attack
16	Dec-25	**The Disappearance of TU-154**	Europe-Eastern Europe	Aviation Disaster

[87] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

The events considered in the course of 2016, in chronological order, are[88]:

January 14th **THE SPREAD OF THE ZIKA VIRUS**

The Zika virus causes infection in humans similar to the virus of yellow fever or Japanese encephalitis, the disease is viral, and is transmitted by the bite of infected mosquitoes. Psychosis is very high.

The media coverage is very strong and it is going to continue for several months, with the peak in February 2016, when the World Health Organization declared that the epidemic by Zika virus is a Public Health Emergency of International Concern (PHEIC).

March 19th **THE DISAPPEARANCE OF THE FLYDUBAI 981**

FlyDubai flight 981 crashes in Rostov on Don. The victims on board of the Boeing 737-800 are 66.

March 22nd **THE TERRORISTIC ATTACKS IN BRUSSELS**

The attacks of 22 March 2016 in Brussels take place at about 8:00 in Brussels, Belgium: two at Brussels airport and one at the Maalbeek subway station. The attacks were claimed, after 15:00, by the Islamic State, commonly known as ISIS, through the press agency Amaq. The victims are 32 in total.

May 19th **THE FALLING OF THE EGYPT AIR 804**

The Airbus A320-232, direct from Paris to Cairo of the Egypt Air company falls into the Mediterranean Sea shortly before reaching Egypt. The victims are 66 passengers and the flight crew.

May 28th **THE CHAMPIONS LEAGUE FINAL**

The final of the 61st edition of the Champions League is played at the Stadio Giuseppe Meazza in Milan between the two biggest teams of

[88]News available at the newspaper "La Repubblica", http://www.repubblica.it

Madrid, Atletico Madrid and Real Madrid, with the latter coming out winner from the clash lasted until the penalty shootout.

June 13th **THE ORLANDO MASSACRE**

The Orlando (FL) massacre is a mass shooting occurred inside the nightclub Pulse, in deep night. The homophobic attack has killed 49 people and wounded 53. It is the terroristic event with more deaths in the United States after the September 11, 2001.

June 23th **THE BREXIT**

The referendum on the permanence of the United Kingdom in the European Union, also known as referendum on Brexit, is an advisory and non-binding referendum to test the support for the continuation of the perpetuity of the UK in the European Union. The referendum ended with a vote in favor of leaving the EU with 51.9%, against 48.1% who voted to stay in the EU.

July 10th **THE EUROPEAN CHAMPIONSHIP FINAL**

The final of the European Football Championship 2016 is held on July 10, 2016 at the Stade de France in Saint-Denis between the National Portuguese and the French one. The Portuguese national team beat the tournament host team to one goal to zero, scored in the extra time.

July 14th **THE NICE MASSACRE**

The massacre of Nice has been a terrorist attack in the department of Alpes-Maritimes in Nice, in France, when a man, driving a truck, has voluntarily invested in speed the crowd attending public festivities on the occasion of the French national holiday on to the promenade des Anglais. The victims are 87, while the injured are 302, the Islamic State (ISIS) has claimed the responsibility for the attack.

August 5th /21st **THE OLYMPIC GAMES**

The Games of the XXXI Olympiad (*Jogos da XXXI Olimpíada*) are held in Rio de Janeiro. Brazil becomes the first South American country to host an edition of the Summer Olympic Games.

August 24th **THE EARTHQUAKE IN CENTRAL ITALY**

The quake had a magnitude of 6.0, with epicenter situated along the Valle del Tronto, between the towns of Accumoli (RI) and Arquata del Tronto (AP). The earthquake caused the death of 299 people, while the total damage suffered are quantifiable in a sum not less than 4 billion euro.

September 16th **THE DEUSTCHE BANK SCANDAL**

The US government has imposed a fine of 14 billion to Deutsche Bank in order to close the scandal over toxic mortgage-related securities. A figure among the highest so far inflicted to a bank for misleading investors about the risks of these securities, understate them.

November 8tt **THE PRESIDENTAL ELECTIONS IN USA**

The presidential elections in the United States have seen the entrepreneur and TV personality of New York, Donald Trump, candidate of the Republican Party, defeating the candidate of the Democratic Party Hillary Clinton, subverting surprisingly the initial forecasts.

December 4th **THE CONSTITUTIONAL REFERENDUM IN ITALY**

The third constitutional referendum in the history of the Italian Republic has seen the majority of voters reject the text of the law finally adopted by the Camera in April 12, 2016 and entitled "*Provisions for exceeding the equal bicameralism, reducing the number of MPs, containment of the institutions' operating costs, the elimination of the CNEL and the revision of title V of part II of the Constitution*".

December 19th **THE ATTACK IN BERLIN**

The Berlin attack is a terroristic attack that takes place in December 19, 2016, which results in 12 dead and 56 wounded among the visitors of a Christmas market in Berlin.

A long track with Polish license plate, from Italy, has invested the crowd at the Christmas market in the district of Berlin Breitscheidplatz, near the Kaiser Wilhelm Memorial Church in Charlottenburg.

The responsibility for the attack was claimed by the so-called Islamic State, with a propaganda video released through the Amaq press agency.

December 25th **THE DISAPPEARANCE OF THE FLIGHT TU-154**

The airplane Tu-154 disappeared, carrying 92 passengers, the flight is departed from Sochi headed to Syria, and crashed on the Black Sea. It is assumed a fault or a human error.

The events considered are distributed between the European and American area, the same of the selected securities.

6.2 THE SPECULATIVE OPERATIONS

The events considered are of various kinds and types, each of them, consequently, will create a different effect on investors, which in sequence is reflected in the price of certain stocks, more susceptible to the type of event. For this reason, we consider a speculative tactic for each event and it identifies the securities involved in the portfolio, capable to generate greater profit from the mass behaviour.

The main strategies to be adopted by type of event are:

- **Global Disease:** in the case of epidemics, the theory suggests that we should increase the exposure of stocks of companies that operate in the pharmaceutical sector, in particular of small size. In the case of Zika virus, this has as the epicenter the South America and the media impact on the contagious is presented in a very strong manner. For this reason, in the portfolio it will be increased the portion of shares of the US pharmaceutical company Johnson and Johnson (JNJ), from the

day of the disclosure of the news. Later, it will also increase the portion of the security Bayer Aktiengesellschaft (BAYN.DE), when the psychosis will also have reached the European area. After that the two stocks will be carry back to the tangency weight, once gone the media effect.

During this period, it will be, also, reduced the exposure of the portfolio to the American market.

- **Aviation Disaster:** in the case of air disaster events, we have seen that these cause effects of anxiety and fear among investors, resulting in the reduced performance of the shares for the two days following the disaster, then they rise generating an opposite effect from the third day.

  In the course of 2016 there were three air crashes of big portions, two in Eastern Europe area, while another in the Mediterranean Sea. In all three events it will be first reduced and then increased exposure of the stock of Boeing Company (BA), the only company operating in the aerospace sector in the portfolio. Then, it will be reduced the exposure of the companies geographically closer to the place of the accident, which may be affected by a fear effect.

- **Terroristic Attack and Natural Disaster:** in the event of terrorist attacks and natural disasters, we tend to use the same speculative tactic in both cases, by reducing the exposure of the closest securities to the attack sites or natural disaster in question. The portfolio is then rebalanced whenever it presents an event of this kind through the European geographical area to the American, and vice versa. The securities considered as “global” will be held constant as the "base" of the portfolio.

- **Political Event and Financial Scandal:** with regard to the political events and the financial scandals, it is mainly used the strategy to reduce the exposure of the securities geographically closer to the event, however, will then be increased a few days after, trying to exploit the relaunch effect due to the exuberance of the agents present in these markets, to rebalance the overhang generated by the event in question.

- **FIFA and World Cup:** finally, in international sports competitions, the theory suggests that we should reduce the exposure of the portfolio securities during event period effect days. In all cases there will be a general negative trend. For this reason, during the FIFA competitions, the exposure of European stocks is reduced in favour of the American ones, and vice versa in the world competitions.

6.3 THE SENTIMENT BASED ALLOCATION

The thesis concludes with the development of sentiment based allocated portfolio, and then the evaluation of this, looking its performance and risk. The events considered are of various kinds and types, each of them therefore will create a different effect on investors, and for each one of them will be reallocated the portfolio in order to realize more gain.

Having available the daily returns of the securities and the variance-covariance matrix[89], we can find these values variance, standard deviation and return[90] for the sentiment based portfolio, equal to:

Table 22. The Sentiment based Portfolio performance

MEAN	3,599%
STANDARD DEVIATION	2,333%
VARIANCE	0,0548%

The sentiment based portfolio shows a positive return, and a relatively riskier position than the mean-variance one. This is a very great return and, this means that there is an effect caused by the human beings. The portfolio is able to exploit the events, generating a return of almost the double of the mean-variance allocation portfolio, presenting approximately the same risk. Exploiting the human psyche and the mass behavior of individuals, the portfolio has generated an extra return.

[89] See Appendix C for the prices, the returns and the variance-covariance matrix, with daily rate, downloaded between the 1 January 2016 and the 31 December 2016 from https://it.finance.yahoo.com/

[90] See Appendix D, for the return of portfolio during 2016.

Here, we summarize the weights for each period, of the first ten stocks, for 2016[91]:

Table 22. The Weights of the first ten stocks

Periood	Weights	BA	JNJ	KO	PG	T	CVX	YMT	GE	INTC	IBM
Jan - 1/ Jan - 14	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Jan - 14/ Mar - 14	**Virus Zika**	-20%	90%	-20%	-10%	-20%	20%	-10%	-25%	-35%	-30%
Mar - 14/ Mar - 20	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Mar - 20/ Mar - 22	**Aviation Disaster**	-20%	-68%	40%	50%	20%	11%	35%	20%	40%	40%
Mar - 22/ Apr - 1	**Terroristic Attack**	10%	-40%	30%	20%	25%	5%	10%	35%	35%	60%
Apr - 1/ May - 19	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
May - 19/ May - 27	**Aviation Disaster**	-20%	-68%	40%	50%	20%	11%	35%	20%	40%	40%
May - 27/ June 1	**FIFA Cup**	6%	17%	30%	10%	55%	20%	23%	10%	10%	30%
June - 1/ June - 13	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
June - 13/ June - 23	**Terroristic Attack**	15%	-20%	-18%	30%	23%	-5%	17%	15%	18%	-5%
June - 23/ July - 10	**Brexit**	10%	60%	25%	35%	27%	15%	15%	5%	8%	12%
July - 10/ July - 14	**FIFA Cup**	6%	17%	30%	10%	55%	20%	23%	10%	10%	30%
July - 14/ July - 22	**Terroristic Attack**	75%	35%	10%	10%	10%	5%	15%	6%	-5%	33%
July - 22/ Aug - 5	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Aug - 5/ Aug - 21	**World Cup**	5%	-30%	-10%	5%	-25%	12%	8%	12%	4%	-20%
Aug - 21/ Aug - 24	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Aug - 24/ Aug - 31	**Natural Disaster**	25%	30%	30%	20%	25%	5%	35%	25%	5%	40%
Aug - 31/ Sept - 16	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Sept - 16/ Sept 30	**Financial Scandal**	10%	40%	15%	25%	20%	15%	10%	5%	8%	12%
Sept - 30/ Nov - 8	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Nov - 8/ Nov - 20	**Political Event**	10%	-10%	-10%	5%	-5%	5%	-45%	-15%	15%	10%
Nov- 20/ Dec - 4	**Tangency**	52%	-68%	28%	64%	-7%	-24%	-29%	-2%	30%	34%
Dec - 4/ Dec - 19	**Political Event**	20%	60%	30%	20%	25%	15%	15%	25%	15%	40%
Dec - 19/ Dec - 25	**Terroristic Attack**	10%	40%	15%	25%	20%	15%	10%	5%	8%	12%
Dec - 25/ Dec - 31	**Aviation Disaster**	-20%	38%	30%	15%	20%	31%	35%	20%	10%	10%

[91] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

The weights of the second ten stocks[92]:

Table 23. The Weights of the second ten stocks

Period	Weights	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
Jan - 1/ Jan - 14	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Jan - 14/ Mar - 14	Virus Zika	-10%	15%	5%	5%	20%	10%	60%	25%	10%	20%
Mar - 14/ Mar - 20	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Mar - 20/ Mar - 22	Aviation Disaster	35%	-15%	-25%	10%	20%	-50%	-20%	10%	-29%	-4%
Mar - 22/ Apr - 1	Terroristic Attack	-10%	-10%	-25%	-10%	-20%	-20%	10%	-10%	10%	-5%
Apr - 1/ May - 19	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
May - 19/ May - 27	Aviation Disaster	35%	-15%	-25%	10%	20%	-50%	-20%	10%	-29%	-4%
May - 27/ June 1	FIFA Cup	20%	-20%	-20%	-10%	-5%	-5%	-45%	-10%	-16%	0%
June - 1/ June - 13	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
June - 13/ June - 23	Terroristic Attack	-15%	-15%	17%	25%	15%	-18%	20%	19%	-8%	-10%
June - 23/ July - 10	Brexit	20%	15%	-26%	-14%	-40%	-7%	-10%	-25%	-10%	-15%
July - 10/ July - 14	FIFA Cup	20%	-20%	-20%	-10%	-5%	-5%	-45%	-10%	-16%	0%
July - 14/ July - 22	Terroristic Attack	-16%	-6%	10%	-10%	-14%	-10%	-13%	-22%	-8%	-5%
July - 22/ Aug - 5	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Aug - 5/ Aug - 21	World Cup	-25%	10%	25%	20%	5%	25%	25%	14%	20%	20%
Aug - 21/ Aug - 24	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Aug - 24/ Aug - 31	Natural Disaster	-10%	-30%	-25%	-10%	-20%	-20%	10%	-10%	-10%	-15%
Aug - 31/ Sept - 16	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Sept - 16/ Sept 30	Financial Scandal	30%	15%	-16%	-14%	-13%	-12%	-20%	-15%	-10%	-5%
Sept - 30/ Nov - 8	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Nov - 8/ Nov - 20	Political Event	-10%	15%	27%	8%	25%	14%	10%	10%	6%	35%
Nov- 20/ Dec - 4	Tangency	-19%	23%	-10%	90%	-19%	-98%	62%	25%	-29%	-4%
Dec - 4/ Dec - 19	Political Event	40%	-20%	5%	-30%	-20%	-45%	-20%	-10%	-10%	-55%
Dec - 19/ Dec - 25	Terroristic Attack	30%	35%	-16%	-44%	-13%	-12%	-10%	-15%	-10%	-5%
Dec - 25/ Dec - 31	Aviation Disaster	31%	-15%	-15%	-20%	20%	-15%	-20%	-30%	-29%	4%

The various weights were changed depending on the event, according to the tactics already shown. In detail, it is possible to see all the periods of the year, as they were considered and the associated weights.

[92] The geographical areas are divided with the following colors:

Global Market	US Market	EU Market

The sentiment based portfolio is the most efficient portfolio among the three studied, as we expected. From an efficiency point of view we calculate the Sharpe ratio, using as risk-free rate the performance of the 3 months T-Bill during the 2016[93].

Sharpe Ratio of the Portfolio 113%

The value of the Sharpe ratio shows how the portfolio has performed in the 2016. This value grater then 100% means that is very remunerative, considering both the risk and the return that it faces. Obviously, this value is the bigger among the three portfolios analyzed, but it is important to note that the values of the return and the Sharpe ratio do not consider the costs.

[93] See Appendix C for the daily return of the 3 months T-Bill.

CONCLUSIONS

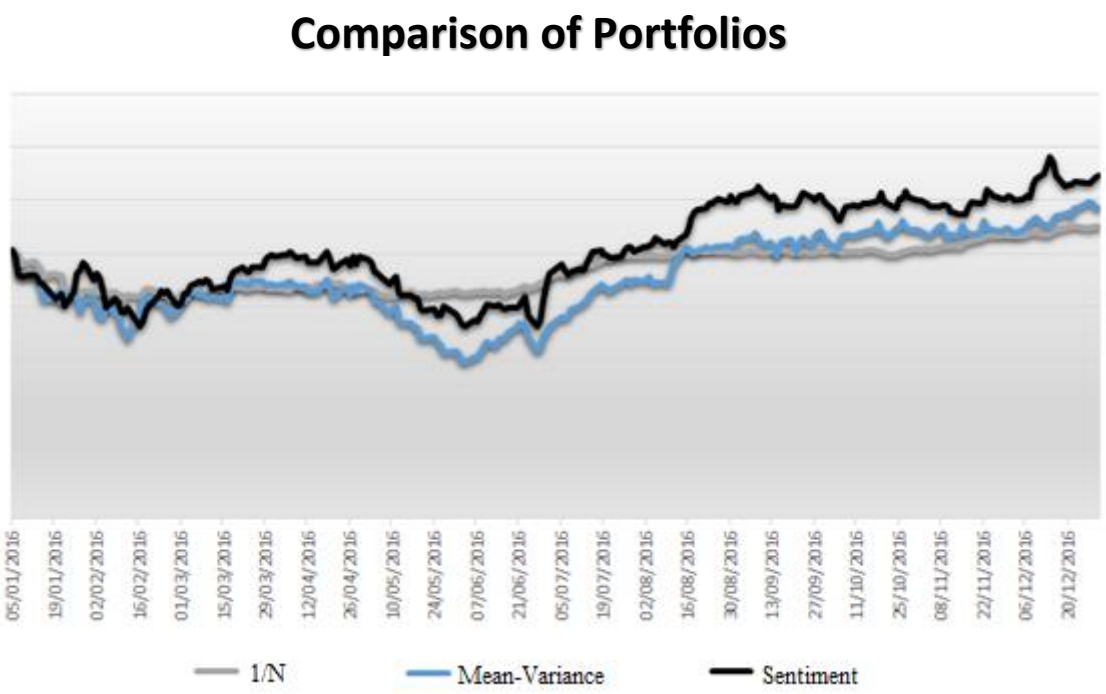
With this thesis, I wanted to compare the classic financial model and the one defined by the school of thought of the behavioral finance, building a portfolio of twenty securities allocated in three different ways: Passive, Active and Sentiment Based.
In this way, I tried to provide a proof that including in the analysis also the mass behavior of individuals it is possible to record an extra performance.

For the 1/N allocated portfolio, the analysis was only based on the assessment of securities that were going to compose the portfolio which is simply allocated in the same way between all the stocks, using an equal weight for each security. What has been achieved is a very balanced portfolio, but with a low return, not particularly far from the one provided by the risk-free rate (T-Bill at three months). This is, anyway, a good result, but due more to the fortuity than to an allocative capacity, since all the stocks considered had positive performances during the year, without undergoing big variations.
On the other hand, it should be emphasized that this is not an efficient portfolio and no technical analysis have been done on it.
In fact, if we tried to calculate the same portfolio for the prior period respect the one of the case study, namely between January 2010 and December 2015, there would be a dominated portfolio, even lower than the minimum variance one, with almost zero return and variance equal to the tangent portfolio.
The passively allocated portfolio is in fact the portfolio of a scarce informed investor who has not the desire to manage his activities directly. In fact, the investor does not need to conduct special studies on the data analysis to compose it, and its realization is very fast. But, on the other hand, it could be much more subject to systematic variation of the market and in the event of a crisis would no longer be able to recover its return, recording a negative performance.
To neglect the past trend of a share is superficial, because if on one hand, the historical performance cannot provide an accurate prediction of the future, on the other hand, it is always a good starting point to set a portfolio, where it is expected that if a security, and the company that manages it, have always react well to market changes, it will continue to do.

The Mean-Variance allocation portfolio has been calculated, conversely, following the development of Harry Markowitz portfolio's theory, passing through the minimum variance portfolio, to arrive, then, to the one of tangency. In this way, it was possible to obtain the weights of the tangent portfolio, and then set the portfolio with mean-variance allocation, which shows a discrete performance and a good variance. This portfolio is riskier than the passive allocated one, however, the tradeoff between the risk and the return is respected among the two, because, observing the Sharpe Ratio for the mean-variance allocated portfolio, this is higher and better than the other one. This portfolio does not require special handling, it is only set on a deep study that seeks to increase the performance and decrease the risk. This is the portfolio of the investor who wants to maximize its performance without continuously reallocate its portfolio and, in this way, he is able to pay a lower amount of banking fees.

Finally, the sentiment based portfolio starts from the weights and the theories of the mean-variance allocation portfolio, and then overcomes it in its operational phase, when it is reallocated immediately after a macro event, allowing to have an extra and very high performance while keeping unchanged the risk. In this case, the events that alternate themselves during the year 2016 make the difference because they have disturbed the investors' choices.

All events detected had a high-emotional and media impact, and they are the basic factor to increase the anxiety and the fear of the investors, generating the perfect field to adopt speculative strategies. For this reason, the sentiment based portfolio had a greater performance compared to the other two portfolios. Nevertheless, if the year of study has not been characterized by a large number of macro events, the difference between the portfolios would be much thinner. Paradoxically, it is probably the mean-variance allocated portfolio to come defeated, because the weights used are the result of analysis of time series ranging from 2010 to 2015, on a monthly basis. With the wider frequency of the time series with respect to the daily, and for an overall period of tranquility, the weights used are not able to detect the variations due to particular events. Variations, however, that are captured by the sentiment based portfolio and that allow it to make a very high return.



Comparison of Portfolios: *the graph registers the trend that took the three portfolios, putting them in a comparison in the year considered for the analysis*[94]**.**

The high performance of the sentiment based portfolio should be well contextualized with the model to which it refers, as the high return of the portfolio is also due to the force that the underlying assumption are able to guarantee, of course this have to be also applied to the other two portfolios. However, if we try to take into account certain transaction costs and bank fees in the portfolio return calculation, it would get a net value. Considering a bank fee of 0.05% for the daily return of the portfolio, and a transaction cost of 0.1% for each time the sentiment based portfolio is reallocated to meet the various events, it would get an average return of 2.642%. This value, when compared to the gross return of the portfolio equal to 3,599%, represents its 73%. This means that about the 27% would be spent on fees and commission costs.

It should be emphasized, also, that these costs are on the returns of the portfolio, while in the operative world are presented fixed costs, which, however, have a little impact on the value of the portfolio where the greater is the amount invested, less the portfolio of securities is affected by this factor. Nevertheless, even after expenses, which would be considered for all portfolios, the sentiment based portfolio is the best.

[94] Personnel data processing.

The sentiment based portfolio, taking advantage from the errors made by investors when they have to perform certain investment decisions, conquests a very high return by taking simple speculative strategies. In addition, it must be underlined that these errors are general, systematic and persistent, as investors do not adopt a perfectly rational behavior, but find, from one side, limits of cognitive and informative type and, from the other, preferences that are completely unstable. For this reason, the portfolio sentiment based is the best and the theory of the behavioral finance will always have more future and credibility. The classical theory, because of its unlikely hypotheses excessively distant from the reality, is destined to leave space to the behavioral finance and to further theories that rely on cognitive perception of investors in reference to certain events.

APPENDIX A

The stocks' price between the January 2010 and December 2015, monthly series.

Date	NKE	BA	AAPL	MCD	JNJ	AIR.PA	KO	PG
04/01/2010	14,582	51,32	25,119	50,2996	50,739	12,868	22,1781	50,0504
01/02/2010	15,463	53,85	26,762	51,8810	51,242	13,733	21,5526	51,4572
01/03/2010	16,880	61,91	30,735	54,2130	53,031	13,497	22,6688	51,4490
01/04/2010	17,433	61,76	34,148	57,3575	52,299	12,714	22,0299	50,9354
03/05/2010	16,623	55,04	33,597	54,7914	47,852	14,761	21,1850	50,0587
01/06/2010	15,571	53,81	32,897	53,9720	48,476	15,287	20,8322	49,1491
01/07/2010	16,975	58,44	33,646	57,1348	47,680	16,474	22,9063	50,5082
02/08/2010	16,136	52,74	31,795	60,3116	47,243	15,736	23,2263	49,2777
01/09/2010	18,543	57,41	37,111	61,5086	51,336	16,578	24,5079	49,5255
01/10/2010	18,843	60,95	39,365	64,1998	52,810	17,113	25,6805	52,9021
01/11/2010	19,929	55,35	40,695	65,1433	51,435	15,64	26,6381	50,8216
01/12/2010	19,834	56,65	42,187	63,8620	51,685	15,804	27,7345	53,5346
03/01/2011	19,152	60,31	44,379	61,2912	49,947	19,075	26,5032	52,9254
01/02/2011	20,673	62,87	46,196	63,4783	51,807	18,993	26,9544	52,8583
01/03/2011	17,639	64,55	45,581	63,8222	49,960	18,613	28,1777	51,6427
01/04/2011	19,182	69,65	45,793	65,6843	55,416	18,93	28,6534	54,8605
02/05/2011	19,678	68,49	45,492	68,9035	57,232	20,733	28,3773	56,6357
01/06/2011	21,046	64,89	43,902	71,2526	56,577	21,117	28,7881	53,7363
01/07/2011	21,086	61,85	51,071	73,0779	55,106	22,142	29,0962	52,4038
01/08/2011	20,267	59,08	50,332	76,9156	56,463	20,207	30,1401	54,2702
01/09/2011	20,073	53,47	49,873	74,7127	54,653	19,402	29,1008	53,8441
03/10/2011	22,617	58,13	52,941	78,9914	55,253	19,553	29,4281	54,9790
01/11/2011	22,577	61,08	49,988	81,8742	56,052	20,308	29,1690	55,4773
01/12/2011	22,707	65,23	52,970	85,990	56,797	22,096	30,3578	57,3160
03/01/2012	24,502	65,97	59,703	84,8999	57,083	23,496	29,2992	54,5954
01/02/2012	25,428	67,04	70,945	85,6948	56,862	24,947	30,3101	58,5619
01/03/2012	25,636	66,52	78,415	84,6762	57,631	28,094	32,3458	58,2068
02/04/2012	26,447	68,70	76,378	84,1152	56,879	27,289	33,3554	55,5812
01/05/2012	25,660	62,63	75,561	77,7187	55,077	24,768	32,6605	54,4022
01/06/2012	20,821	66,85	76,381	77,0140	59,603	26,012	34,4060	53,4939
02/07/2012	22,142	66,50	79,881	77,7361	61,068	27,241	35,5545	56,8562
01/08/2012	23,178	64,63	87,380	78,4618	60,029	28,227	32,9143	59,1907
04/09/2012	22,595	63,00	87,624	80,4433	61,347	22,963	33,6077	61,1024
01/10/2012	21,754	63,76	78,196	76,1033	63,047	25,518	32,9438	61,4975
01/11/2012	23,206	67,65	77,228	76,9990	62,626	24,103	33,8293	62,0216
03/12/2012	24,674	68,63	70,220	78,0340	62,958	27,464	32,3394	60,2985

Date	NKE	BA	AAPL	MCD	JNJ	AIR.PA	KO	PG
02/01/2013	25,846	67,27	60,102	84,2973	66,389	32,217	33,2226	67,2968
01/02/2013	26,143	70,48	58,583	85,5213	68,902	36,476	34,5430	68,2101
01/03/2013	28,327	78,69	58,750	88,9011	73,809	36,96	36,3386	68,9980
01/04/2013	30,530	83,78	58,766	91,0859	77,159	37,337	38,0369	69,2434
01/05/2013	29,698	91,23	60,083	86,7935	76,781	41,951	35,9342	69,2344
03/06/2013	30,671	94,38	52,976	88,9775	78,313	38,757	36,2912	69,4418
01/07/2013	30,305	96,83	60,457	88,1506	85,282	42,362	36,2640	72,9698
01/08/2013	30,358	96,18	65,522	85,4924	79,409	41,154	34,5449	70,7798
03/09/2013	35,103	108,7	64,114	87,1685	79,666	44,448	34,5228	68,6898
01/10/2013	36,611	120,7	70,293	87,4494	85,106	47,765	36,0630	73,9524
01/11/2013	38,244	124,7	75,217	88,9532	87,597	49,322	36,8846	77,1303
02/12/2013	38,122	126,7	75,887	88,6426	84,757	52,673	37,9125	74,5569
02/01/2014	35,315	116,3	67,714	86,0298	81,869	49,653	34,7095	70,7009
03/02/2014	38,073	120,4	71,608	87,6656	85,860	50,389	35,0583	72,5834
03/03/2014	35,914	117,2	73,037	90,3191	91,555	49,067	35,7616	74,3735
01/04/2014	35,472	120,5	80,297	93,4056	94,407	46,708	37,7319	76,7814
01/05/2014	37,516	127,0	86,617	94,2049	95,225	50,364	37,8423	75,1444
02/06/2014	37,828	119,5	89,013	93,5641	98,191	46,85	39,4789	73,0989
01/07/2014	37,623	113,1	91,570	87,8243	93,939	41,509	36,6177	72,4914
01/08/2014	38,431	119,8	98,668	87,7955	98,014	44,806	38,8824	77,9198
02/09/2014	43,642	120,3	96,983	88,8166	100,71	47,693	40,0481	78,5105
01/10/2014	45,486	118,0	103,962	87,8048	101,84	45,568	39,3159	82,4476
03/11/2014	48,578	127,7	114,980	91,4919	102,95	46,903	42,3762	85,433
01/12/2014	47,177	123,5	106,714	88,5527	99,451	39,584	39,9090	86,0565
02/01/2015	45,264	138,1	113,269	87,3619	95,238	45,185	38,9165	80,1986
02/02/2015	47,792	144,2	124,683	94,2793	98,174	52,967	40,9299	80,9978
02/03/2015	49,371	143,5	120,772	92,8875	96,345	57,859	38,6485	77,9627
01/04/2015	48,638	137,1	121,471	92,0391	95,004	59,353	38,6580	76,2591
01/05/2015	50,167	135,2	126,978	92,2428	96,604	59,353	39,0393	75,1849
01/06/2015	53,300	133,5	122,251	91,4159	94,019	56,815	37,6989	75,0411
01/07/2015	56,853	138,7	118,226	96,0217	96,671	63,062	39,4767	74,1664
03/08/2015	55,141	126,5	110,400	92,1727	91,355	56,707	37,7853	68,3356
01/09/2015	60,831	126,8	107,991	95,5776	90,743	51,65	38,8881	69,5636
01/10/2015	64,818	143,4	116,998	108,886	98,208	61,852	41,0497	74,5208
02/11/2015	65,437	141,7	116,319	111,607	99,138	66,811	41,6282	73,0183
01/12/2015	61,985	140,9	103,498	115,498	100,58	60,524	41,9603	77,4770

Date	T	CVX	WMT	BAC	GE	INTC	IBM	CAT
04/01/2010	17,381863	56,380	44,985	14,43262	12,870	15,503	104,699	43,686
01/02/2010	17,004889	57,061	45,523	15,839753	12,935	16,537	109,266	47,709
01/03/2010	17,710857	59,847	47,075	16,981482	14,658	17,955	110,202	52,559
01/04/2010	18,151405	64,275	45,416	16,962454	15,190	18,398	110,847	57,296
03/05/2010	16,925524	58,845	43,057	14,974146	13,168	17,376	108,185	51,128
01/06/2010	16,848906	54,057	40,936	13,679666	11,687	15,778	106,647	50,548
01/07/2010	18,384142	60,709	43,593	13,365519	13,065	16,710	110,897	59,084
02/08/2010	19,156645	59,564	42,948	11,861422	11,736	14,443	106,872	55,196
01/09/2010	20,269331	65,168	45,844	12,480693	13,268	15,694	116,428	66,649
01/10/2010	20,505976	66,415	46,400	10,908697	13,080	16,388	124,639	66,948
01/11/2010	19,981102	65,661	46,332	10,432335	12,925	17,431	123,327	72,058
01/12/2010	21,124317	73,997	46,450	12,720964	15,051	17,324	127,948	79,775
03/01/2011	20,074903	76,982	48,293	13,092866	16,574	17,678	141,235	83,017
01/02/2011	20,702244	84,767	44,771	13,62688	17,335	17,836	141,689	88,083
01/03/2011	22,328953	87,823	45,145	12,72056	16,614	16,764	142,730	95,288
01/04/2011	23,02346	89,416	47,686	11,718565	16,945	19,232	149,304	99,174
02/05/2011	23,348984	86,377	48,212	11,212796	16,274	18,847	148,521	90,916
01/06/2011	23,23801	84,673	46,396	10,467824	15,756	18,554	150,824	91,483
01/07/2011	21,945724	85,645	46,020	9,273957	14,963	18,696	159,880	85,251
01/08/2011	21,360704	82,032	46,775	7,812735	13,626	17,019	151,799	78,528
01/09/2011	21,390705	76,845	45,641	5,852379	12,837	18,042	154,413	63,720
03/10/2011	22,322277	87,186	49,879	6,531332	14,094	20,747	163,031	81,965
01/11/2011	22,070951	86,009	51,796	5,212376	13,419	21,247	166,674	84,933
01/12/2011	23,03056	89,004	52,881	5,327354	15,253	20,684	163,021	78,616
03/01/2012	22,727386	86,260	54,297	6,831661	15,934	22,535	170,752	95,110
01/02/2012	23,639263	91,979	52,279	7,645929	16,367	23,111	175,093	99,546
01/03/2012	24,133841	90,369	54,518	9,180871	17,244	24,177	185,702	92,843
02/04/2012	25,791573	89,821	52,478	7,780236	16,823	24,417	184,304	89,954
01/05/2012	26,779034	83,613	59,031	7,060629	16,402	22,378	172,403	76,693
01/06/2012	27,946749	89,728	62,529	7,857952	18,108	23,079	174,798	74,321
02/07/2012	30,088017	93,198	66,753	7,051023	18,030	22,256	175,156	74,179
01/08/2012	29,072388	96,156	65,464	7,675432	17,995	21,691	174,889	75,166
04/09/2012	29,913456	99,928	66,546	8,492977	19,884	19,796	186,199	75,791
01/10/2012	27,764425	94,510	67,646	8,964275	18,439	18,896	174,602	75,159
01/11/2012	27,395197	91,391	64,941	9,483665	18,501	17,272	171,346	75,540
03/12/2012	27,058073	93,509	61,865	11,17815	18,546	18,199	172,680	79,870
02/01/2013	28,284611	99,571	63,425	10,898937	19,686	18,570	183,065	87,696
01/02/2013	29,195181	102,089	64,177	10,822008	20,684	18,627	181,808	82,330
01/03/2013	29,829329	103,544	68,287	11,737495	20,595	19,483	193,097	77,517
01/04/2013	30,820131	106,324	70,924	11,862772	19,855	21,365	183,356	75,952

Date	T	CVX	WMT	BAC	GE	INTC	IBM	CAT
01/05/2013	28,787945	107,831	68,706	13,163726	20,773	21,864	189,200	76,966
03/06/2013	29,125273	103,957	68,385	12,402073	20,822	21,819	173,819	73,996
01/07/2013	29,3874	110,590	71,552	14,080114	21,882	21,017	177,394	74,892
01/08/2013	28,187574	106,667	67,405	13,617206	20,777	19,986	166,608	74,557
03/09/2013	28,179241	107,615	68,310	13,317946	21,616	20,841	169,268	75,334
01/10/2013	30,566849	106,251	70,887	13,482008	23,652	22,250	163,811	75,825
01/11/2013	29,730904	109,358	74,822	15,267385	24,122	21,880	165,122	76,953
02/12/2013	29,688684	111,564	73,102	15,035793	25,567	23,826	172,373	82,602
02/01/2014	28,510252	99,703	69,377	16,175308	22,922	22,523	162,366	85,980
03/02/2014	27,3209	103,923	69,395	15,962856	23,435	22,942	171,101	88,782
03/03/2014	30,007639	107,149	71,461	16,619802	23,821	23,914	177,864	90,979
01/04/2014	30,947826	113,105	74,527	14,629292	24,741	24,730	181,542	97,068
01/05/2014	30,748444	111,590	72,223	14,629292	24,649	25,531	171,344	94,149
02/06/2014	30,653086	118,642	70,624	14,86109	24,380	28,877	168,472	100,080
01/07/2014	31,251572	117,451	69,222	14,745063	23,331	31,671	178,138	93,372
01/08/2014	30,698368	118,642	71,496	15,55725	24,101	32,850	179,780	101,083
02/09/2014	30,944239	109,358	72,415	16,536282	23,968	32,756	177,471	91,778
01/10/2014	30,996576	109,935	72,225	16,642967	24,146	31,994	153,697	94,700
03/11/2014	31,477007	100,706	82,898	16,526585	24,782	35,272	152,650	93,944
01/12/2014	29,884472	103,767	81,780	17,401707	23,863	34,363	151,021	85,474
02/01/2015	29,703882	94,841	80,923	14,736492	22,560	31,286	144,310	75,291
02/02/2015	31,183666	99,649	79,923	15,378479	24,769	31,710	153,502	78,049
02/03/2015	29,460264	98,061	78,794	15,016753	23,644	29,822	152,137	75,347
01/04/2015	31,705231	103,741	74,770	15,543656	25,808	31,043	162,365	82,475
01/05/2015	31,613703	97,171	71,598	16,099833	25,989	33,102	162,028	80,994
01/06/2015	32,510677	91,010	68,378	16,657032	25,537	29,221	155,352	80,519
01/07/2015	32,220116	83,472	69,390	17,498692	25,085	27,809	154,712	75,332
03/08/2015	30,791821	77,369	62,827	15,991534	23,855	27,643	142,418	73,234
01/09/2015	30,216793	75,353	62,934	15,296833	24,456	29,193	139,606	62,619
01/10/2015	31,524132	86,817	55,558	16,475023	28,044	32,796	134,897	70,710
02/11/2015	31,67465	88,303	57,110	17,113211	29,033	33,914	135,521	70,381
01/12/2015	32,370796	86,987	59,997	16,570635	30,433	33,602	133,771	65,837

Date	TRV	FCA	BARC.L	HSBA.L	VOW.DE	ALV.DE	LHA.DE
04/01/2010	42,711	2,32937	208,964	451,694	55,9506	55,04	10,342
01/02/2010	44,329	1,9711	242,628	479,904	56,0474	58,04	9,751
01/03/2010	45,747	2,45815	279,741	449,752	62,8708	63,53	10,925
01/04/2010	43,033	2,70521	262,621	449,752	63,3607	59,21	11,165
03/05/2010	41,956	2,42653	237,674	426,171	62,6727	58,54	9,6
01/06/2010	42,083	2,31505	210,759	417,617	62,503	58,88	10,133

Date	TRV	FCA	BARC.L	HSBA.L	VOW.DE	ALV.DE	LHA.DE
01/07/2010	43,109	2,67258	259,252	438,525	64,9688	64,1	11,099
02/08/2010	41,861	2,51354	236,337	440,297	64,1022	58,23	11,081
01/09/2010	44,841	3,07768	234,149	441,186	72,2144	59,8	11,997
01/10/2010	47,509	3,30606	214,61	443,991	84,3022	64,77	13,679
01/11/2010	46,467	3,46648	201,074	448,843	93,004	60,79	14,586
01/12/2010	48,264	4,19511	205,392	450,782	94,6122	63,98	14,551
03/01/2011	48,741	4,62571	230,59	471,206	99,1686	72,98	13,652
01/02/2011	51,920	4,38758	253,446	469,405	98,2752	75,1	13,181
01/03/2011	51,847	4,16902	219,928	452,189	96,7564	71,24	13,305
01/04/2011	55,159	4,80147	223,692	462,347	107,5666	76,47	13,63
02/05/2011	54,113	4,92142	220,14	454,214	105,957	72,3	14,023
01/06/2011	51,231	5,04471	204,066	442,48	115,2834	72,41	13,917
01/07/2011	48,379	4,60156	177,449	425,379	116,1933	68,45	13,013
01/08/2011	44,281	2,88689	136,789	391,026	96,3121	53,92	10,925
01/09/2011	43,127	2,72828	129,259	362,096	84,7929	53,27	9,036
03/10/2011	51,641	2,95885	156,456	397,074	103,5458	61,05	9,144
01/11/2011	49,783	2,54835	145,361	365,329	102,3629	57,87	8,905
01/12/2011	52,758	2,36575	141,973	362,78	94,3104	55,56	8,507
03/01/2012	51,982	3,05615	171,409	391,334	112,3717	63,19	9,786
01/02/2012	51,688	2,89355	200,497	410,247	115,7383	68,43	9,651
01/03/2012	53,167	2,93753	192,519	416,918	110,006	67,25	9,721
02/04/2012	57,765	2,43106	178,606	417,143	120,7407	63,28	9,106
01/05/2012	56,122	2,53235	145,048	387,422	113,6868	58,08	8,075
01/06/2012	57,771	2,64431	133,982	426,658	111,0245	62,85	8,675
02/07/2012	56,694	2,66563	138,219	405,975	120,3119	64,42	9,758
01/08/2012	58,586	2,88689	151,781	420,113	120,9733	69,33	9,339
04/09/2012	62,215	2,76693	177,955	440,232	121,6272	73,55	10,043
01/10/2012	64,657	2,5057	188,432	476,613	140,4971	75,99	11,224
01/11/2012	64,548	2,37241	204,77	499,238	146,1021	79,4	12,147
03/12/2012	65,869	2,52569	218,422	506,44	152,034	83,25	13,556
02/01/2013	71,959	2,99884	250,552	561,084	158,8533	83,69	13,923
01/02/2013	73,756	2,73894	258,901	572,593	147,4566	83,18	14,708
01/03/2013	77,652	2,7656	245,534	565,762	137,0875	84,17	14,503
01/04/2013	78,777	3,02682	241,233	566,568	140,9784	89,01	14,451
01/05/2013	77,219	4,08175	271,384	590,307	157,9431	98,82	15,831
03/06/2013	74,165	3,57861	235,815	554,607	143,3019	92,63	14,846
01/07/2013	77,533	3,99178	243,818	608,116	163,7804	96,71	14,327
01/08/2013	74,146	3,80519	240,881	554,613	160,9574	89,45	12,856
03/09/2013	79,156	3,92515	244,339	549,199	160,4311	95,89	13,723
01/10/2013	80,585	3,85851	242,59	565,299	171,8666	102,24	13,58

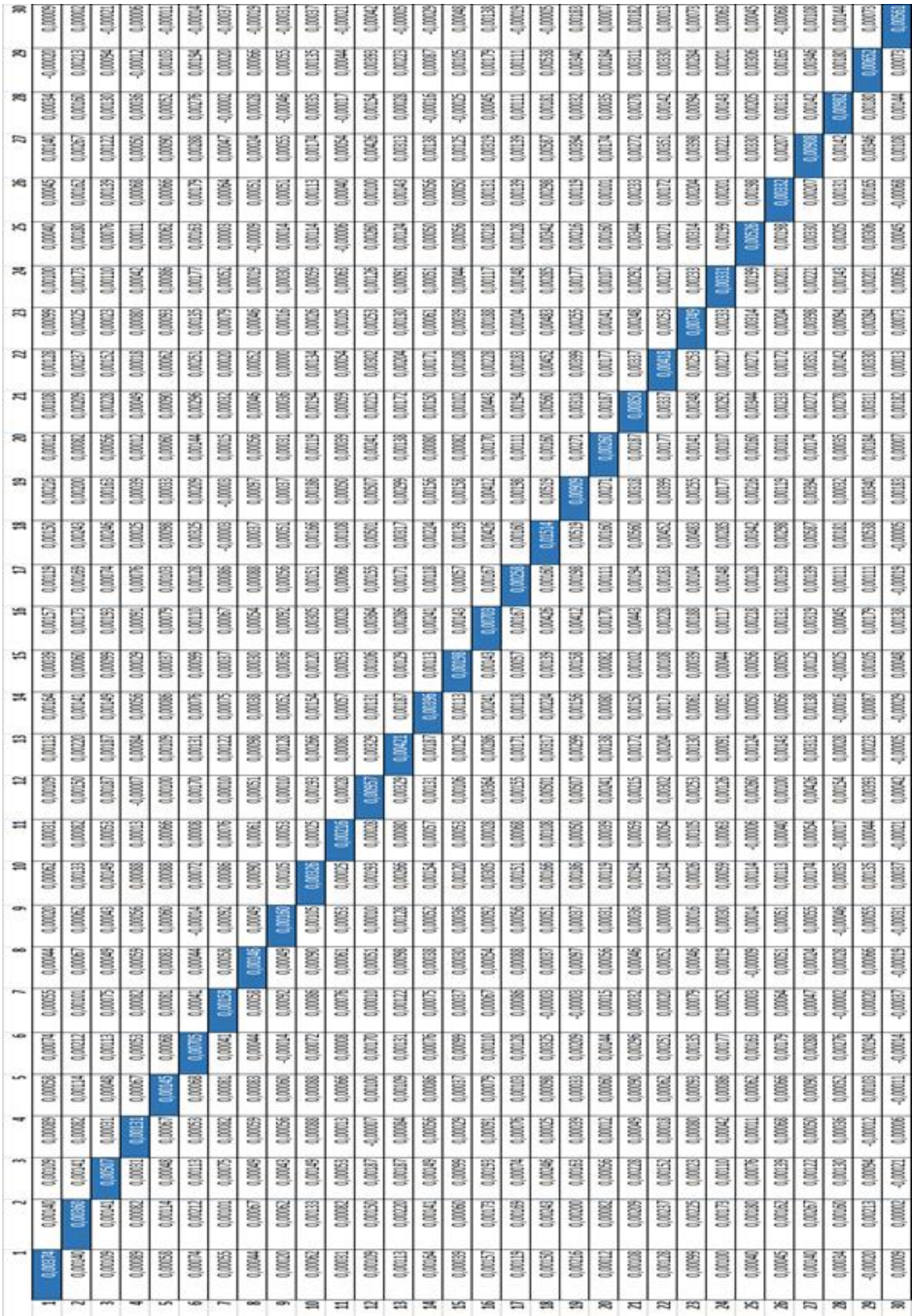
Date	TRV	FCA	BARC.L	HSBA.L	VOW.DE	ALV.DE	LHA.DE
01/11/2013	84,731	3,89183	251,166	565,133	183,3977	105,5	15,217
02/12/2013	85,028	3,9618	251,397	548,973	188,4216	107,56	14,679
02/01/2014	76,331	4,9281	251,905	519,634	172,5843	102,16	16,817
03/02/2014	78,735	5,05471	236,211	521,872	177,1298	107,03	17,887
03/03/2014	80,395	5,63115	218,689	512,866	176,0771	101,25	18,102
01/04/2014	85,572	5,78443	236,305	509,995	184,2111	103,02	17,639
01/05/2014	88,284	5,10802	232,342	536,21	188,9699	107,18	18,879
02/06/2014	89,387	4,80481	200,172	505,275	184,6184	104,86	15,298
01/07/2014	85,102	4,82813	212,306	542,346	169,2173	107,61	12,952
01/08/2014	89,995	4,95808	212,086	560,972	166,7237	111,84	12,854
02/09/2014	89,788	5,09137	214,921	538,688	160,6611	110,59	12,2
01/10/2014	96,344	5,93437	227,536	555,821	165,9415	109,16	11,503
03/11/2014	99,833	6,68408	232,623	553,649	177,8712	119,29	14,005
01/12/2014	101,703	6,39752	231,057	528,965	176,1111	118,34	13,493
02/01/2015	98,792	7,79698	222,185	529,834	192,7835	126,14	14,688
02/02/2015	103,231	9,18978	243,773	501,76	216,4475	128,89	12,781
02/03/2015	104,427	10,09607	233,312	510,565	234,7333	139,45	12,761
01/04/2015	97,648	8,89656	245,526	576,476	222,2657	131,48	12,098
01/05/2015	97,657	9,69625	260,708	562,92	219,3151	128,71	12,488
01/06/2015	93,927	8,75661	251,535	515,37	207,3752	125,96	11,283
01/07/2015	103,120	9,52964	279,006	524,048	183,895	134,44	12,049
03/08/2015	96,735	8,35676	253,351	474,327	166,5596	128,26	10,591
01/09/2015	97,323	7,737	236,587	456,038	104,8616	126,46	12,122
01/10/2015	110,387	8,9432	224,813	470,23	125,9938	143,68	13,103
02/11/2015	112,030	9,00985	217,22	490,518	141,181	151,16	13,234
01/12/2015	110,953	8,61	213,036	496,725	142,1801	147,46	14,21

Date	BAYN.DE	ML.PA	RI.PA	BZU.MI	VLS:MI	BGN:MI	REP.MC
04/01/2010	42,557	44,1512	50,9571	10,3702	3,6568	5,3998	11,82
01/02/2010	41,864	40,1681	48,3529	8,7667	3,5815	4,7578	11,47
01/03/2010	43,077	42,779	54,9421	8,9539	3,6222	5,7468	12,08
01/04/2010	41,352	43,0221	55,8859	10,8023	3,5489	5,6288	12,22
03/05/2010	40,311	43,5241	54,0245	8,7013	3,7286	5,4668	11,43
01/06/2010	40,754	46,1035	55,9121	8,0634	3,6105	5,8253	11,49
01/07/2010	39,084	46,6866	53,0009	8,1462	3,4502	6,3096	11,49
02/08/2010	42,686	46,8063	54,4394	6,9727	3,6021	6,317	12,79
01/09/2010	45,317	46,1275	54,0511	7,5473	3,3954	6,6164	12,73
01/10/2010	47,505	47,235	56,2219	8,0342	3,5114	6,8751	13,35
01/11/2010	49,57	43,0777	56,0575	7,0019	3,3743	6,0989	14,08
01/12/2010	48,994	44,3836	62,8158	8,3166	3,543	6,7014	13,13

Date	BAYN.DE	ML.PA	RI.PA	BZU.MI	VLS:MI	BGN:MI	REP.MC
03/01/2011	47,718	43,9455	62,1641	8,9496	3,6928	7,3371	14,73
01/02/2011	49,773	48,7641	59,6464	9,9819	3,5852	7,5404	16,63
01/03/2011	48,409	49,26	58,834	10,0111	3,7995	7,9396	17,61
01/04/2011	52,582	55,9299	60,5838	10,1767	3,9226	8,3758	17,5
02/05/2011	51,764	55,3273	62,6283	9,4124	4,0955	8,2121	17,45
01/06/2011	50,391	57,2896	60,6821	9,3831	3,9973	7,4203	17,14
01/07/2011	50,837	49,8311	62,3231	8,0057	3,7869	6,4695	17,33
01/08/2011	40,802	43,3155	56,3541	6,8188	3,7378	5,3246	16,3
01/09/2011	37,675	38,4054	53,0449	5,9591	3,6396	5,5109	14,85
03/10/2011	42,079	44,7511	60,8624	6,5062	3,6116	5,6157	14,79
01/11/2011	44,32	40,0789	63,8829	6,6478	3,5941	5,6429	16,21
01/12/2011	44,901	38,8005	65,3605	6,6039	3,5064	5,5885	16,58
03/01/2012	48,664	44,4283	66,9384	7,9227	3,5064	6,6713	17,57
01/02/2012	50,446	43,9696	70,833	8,289	3,3311	7,2884	15,93
01/03/2012	47,937	47,427	71,5079	8,5918	3,3749	7,8084	14,85
02/04/2012	49,842	47,9282	71,5171	7,5075	3,3714	7,1681	14,27
01/05/2012	47,903	41,8551	72,0552	6,6714	3,2211	6,3404	10,96
01/06/2012	53,186	45,6021	76,8528	7,2864	3,3026	7,5501	9,16
02/07/2012	57,991	49,0923	80,505	8,0539	3,1651	7,7212	10,02
01/08/2012	57,767	50,5362	78,8123	8,1031	3,3695	7,8421	10,31
04/09/2012	62,6	53,9997	80,321	8,477	3,3424	8,6931	11,6
01/10/2012	62,937	58,6946	76,3836	9,0083	3,384	9,4606	11,96
01/11/2012	65,148	63,3452	80,9183	9,2101	3,4057	10,3866	12,22
03/12/2012	67,34	63,416	81,2715	10,3711	3,4202	10,7704	12,93
02/01/2013	68,08	60,7054	85,6957	10,5483	3,5053	11,7966	12,51
01/02/2013	71,058	60,6257	92,3785	11,857	3,4383	11,1709	13,42
01/03/2013	75,377	57,7999	90,3523	11,7979	3,6084	12,3472	13,31
01/04/2013	75,985	56,8078	87,3687	11,424	3,7459	13,0563	12,93
01/05/2013	79,505	62,2277	86,1511	12,7065	3,9154	14,7399	14,52
03/06/2013	78,584	63,0723	79,1802	11,4023	3,7456	14,538	14,39
01/07/2013	83,783	69,2969	83,9406	10,9577	4,2248	16,6361	13,26
01/08/2013	80,608	66,46	82,3837	10,0585	5,7315	15,0208	14,71
03/09/2013	83,6	75,2553	86,0883	11,3825	6,8354	14,6609	14,36
01/10/2013	87,801	70,7108	83,0027	12,6176	8,1563	16,9171	14,99
01/11/2013	94,189	73,4007	79,0004	12,4694	10,6517	18,4358	16,18
02/12/2013	97,786	70,9219	78,4134	12,9535	9,4913	19,7615	15,81
02/01/2014	93,892	71,9043	75,4117	13,3883	9,9724	19,9458	15,38
03/02/2014	98,697	81,3146	80,7617	14,9198	10,3781	20,7623	14,59
03/03/2014	94,17	83,3436	80,0136	13,3784	10,614	21,0256	15,29
01/04/2014	95,916	80,7913	81,9169	12,6176	11,2272	19,8931	15,55

Date	BAYN.DE	ML.PA	RI.PA	BZU.MI	VLS:MI	BGN:MI	REP.MC
01/05/2014	101,767	85,4046	85,1743	12,6616	14,1036	19,7123	16,29
02/06/2014	98,937	82,4381	83,0437	12,1952	14,4307	18,4975	17,35
01/07/2014	94,88	77,5254	79,9983	12,0563	12,7952	19,2521	16,49
01/08/2014	97,882	79,4999	85,7712	11,441	13,5071	19,0313	15,94
02/09/2014	106,371	70,5532	85,6756	10,7266	13,5937	19,2614	16,17
01/10/2014	108,817	65,3666	86,813	10,6869	12,3142	19,4638	16,1
03/11/2014	116,01	69,8636	91,9179	11,9372	13,9015	19,6111	15,25
01/12/2014	108,385	71,1106	88,9765	10,4289	14,5846	21,2123	15,45
02/01/2015	122,82	81,9374	102,8061	10,7961	14,5172	22,7676	13,71
02/02/2015	126,657	81,2288	102,1792	12,5226	16,8358	23,2829	13,84
02/03/2015	134,234	87,4924	106,2298	13,8424	22,4252	26,8444	15,24
01/04/2015	124,595	94,1623	107,0495	14,3187	23,1468	27,5714	15,29
01/05/2015	125,945	94,4817	108,5443	13,8696	25,0757	30,7131	16,24
01/06/2015	122,435	91,015	99,9129	12,7147	22,8935	29,9543	15,3
01/07/2015	130,967	86,3766	106,0047	15,4826	24,5691	28,4177	14,31
03/08/2015	117,949	83,5587	90,9084	15,1242	21,6465	25,3445	13,9
01/09/2015	111,61	78,8428	87,6617	14,8753	18,0225	23,9122	11,59
01/10/2015	118,339	87,7323	104,2064	15,3233	22,4064	26,5965	9,46
02/11/2015	123,166	91,8962	105,6698	16,9363	22,309	27,3458	10,43
01/12/2015	112,927	85,1177	103,2169	16,5281	20,5262	27,6778	11,22

The variance-covariance matrix of the thirty stocks.



APPENDIX B

The price and the return of the market index Standard and Poor's 500 (S&P 500) and the return of the risk free asset 3 months T-Bill, between the January 2010 and December 2015, with monthly series.

S&P 500

Date	Price	Return
04/01/2010	1073,87	
01/02/2010	1104,49	0,028513689
01/03/2010	1169,43	0,058796426
01/04/2010	1186,69	0,01475923
03/05/2010	1089,41	-0,081975842
01/06/2010	1030,71	-0,053882442
01/07/2010	1101,6	0,06877785
02/08/2010	1049,33	-0,047449184
01/09/2010	1141,2	0,087551103
01/10/2010	1183,26	0,036855994
01/11/2010	1180,55	-0,00229025
01/12/2010	1257,64	0,06530004
03/01/2011	1286,12	0,022645574
01/02/2011	1327,22	0,031956564
01/03/2011	1325,83	-0,001047313
01/04/2011	1363,61	0,02849538
02/05/2011	1345,2	-0,013500953
01/06/2011	1320,64	-0,018257461
01/07/2011	1292,28	-0,021474426
01/08/2011	1218,89	-0,056791107
01/09/2011	1131,42	-0,071761988
03/10/2011	1253,3	0,107723039
01/11/2011	1246,96	-0,005058715
01/12/2011	1257,6	0,008532764
03/01/2012	1312,41	0,043583062
01/02/2012	1365,68	0,040589464
01/03/2012	1408,47	0,031332315
02/04/2012	1397,91	-0,007497453
01/05/2012	1310,33	-0,062650726
01/06/2012	1362,16	0,039554982
02/07/2012	1379,32	0,012597574
01/08/2012	1406,58	0,01976337
04/09/2012	1440,67	0,024236154
01/10/2012	1412,16	-0,01978941

3 months T-Bill

Date	Return
04/01/2010	
01/02/2010	0,040%
01/03/2010	0,085%
01/04/2010	0,145%
03/05/2010	0,150%
01/06/2010	0,080%
01/07/2010	0,140%
02/08/2010	0,145%
01/09/2010	0,130%
01/10/2010	0,150%
01/11/2010	0,115%
01/12/2010	0,105%
03/01/2011	0,135%
01/02/2011	0,145%
01/03/2011	0,150%
01/04/2011	0,110%
02/05/2011	0,090%
01/06/2011	0,120%
01/07/2011	0,080%
01/08/2011	0,035%
01/09/2011	0,050%
03/10/2011	0,030%
01/11/2011	0,025%
01/12/2011	0,010%
03/01/2012	0,090%
01/02/2012	0,015%
01/03/2012	0,050%
02/04/2012	0,090%
01/05/2012	0,070%
01/06/2012	0,070%
02/07/2012	0,100%
01/08/2012	0,090%
04/09/2012	0,060%
01/10/2012	0,040%

S&P 500		
Date	**Price**	**Return**
01/11/2012	1416,18	0,002846717
03/12/2012	1426,19	0,00706823
02/01/2013	1498,11	0,050428097
01/02/2013	1514,68	0,011060649
01/03/2013	1569,19	0,035987724
01/04/2013	1597,57	0,018085768
01/05/2013	1630,74	0,020762812
03/06/2013	1606,28	-0,014999302
01/07/2013	1685,73	0,04946208
01/08/2013	1632,97	-0,031298019
03/09/2013	1681,55	0,029749523
01/10/2013	1756,54	0,044595753
01/11/2013	1805,81	0,028049472
02/12/2013	1848,36	0,023562792
02/01/2014	1782,59	-0,035582906
03/02/2014	1859,45	0,04311703
03/03/2014	1872,34	0,006932166
01/04/2014	1883,95	0,006200789
01/05/2014	1923,57	0,02103028
02/06/2014	1960,23	0,019058332
01/07/2014	1930,67	-0,015079831
01/08/2014	2003,37	0,037655295
02/09/2014	1972,29	-0,015513837
01/10/2014	2018,05	0,023201461
03/11/2014	2067,56	0,024533589
01/12/2014	2058,9	-0,004188588
02/01/2015	1994,99	-0,031040806
02/02/2015	2104,5	0,054892511
02/03/2015	2067,89	-0,017396107
01/04/2015	2085,51	0,00852082
01/05/2015	2107,39	0,010491382
01/06/2015	2063,11	-0,021011672
01/07/2015	2103,84	0,01974203
03/08/2015	1972,18	-0,062580818
01/09/2015	1920,03	-0,026442832
01/10/2015	2079,36	0,082983118
02/11/2015	2080,41	0,000504869
01/12/2015	2043,94	-0,017530185

3 months T-Bill	
Date	**Return**
01/11/2012	0,090%
03/12/2012	0,070%
02/01/2013	0,070%
01/02/2013	0,100%
01/03/2013	0,090%
01/04/2013	0,060%
01/05/2013	0,040%
03/06/2013	0,050%
01/07/2013	0,040%
01/08/2013	0,040%
03/09/2013	0,020%
01/10/2013	0,050%
01/11/2013	0,070%
02/12/2013	0,070%
02/01/2014	0,040%
03/02/2014	0,050%
03/03/2014	0,050%
01/04/2014	0,030%
01/05/2014	0,030%
02/06/2014	0,040%
01/07/2014	0,030%
01/08/2014	0,030%
02/09/2014	0,020%
01/10/2014	0,020%
03/11/2014	0,020%
01/12/2014	0,030%
02/01/2015	0,030%
02/02/2015	0,020%
02/03/2015	0,030%
01/04/2015	0,020%
01/05/2015	0,020%
01/06/2015	0,020%
01/07/2015	0,030%
03/08/2015	0,070%
01/09/2015	0,020%
01/10/2015	0,020%
02/11/2015	0,120%
01/12/2015	0,230%

The excess of return of the market index (S&P 500) on the risk-free asset (3 months T-Bill) and the excess of return of the thirty stocks on the risk-free asset (3 months T-Bill).

E(Market)	E(NKE)	E(BA)	E(AAPL)	E(MCD)	E(JNJ)	E(AIR.PA)	E(KO)
2,81%	6,00%	4,90%	6,50%	3,10%	0,95%	6,68%	-2,86%
5,79%	9,08%	14,88%	14,76%	4,41%	3,41%	-1,80%	5,09%
1,33%	3,13%	-0,39%	10,96%	5,66%	-1,53%	-5,95%	-2,96%
-8,35%	-4,80%	-11,03%	-1,76%	-4,62%	-8,65%	15,95%	-3,99%
-5,47%	-6,41%	-2,31%	-2,16%	-1,58%	1,22%	3,48%	-1,75%
6,74%	8,88%	8,45%	2,13%	5,72%	-1,78%	7,62%	9,82%
-4,89%	-5,09%	-9,89%	-5,65%	5,42%	-1,06%	-4,62%	1,25%
8,63%	14,79%	8,72%	16,59%	1,85%	8,53%	5,22%	5,39%
3,54%	1,47%	6,01%	5,92%	4,23%	2,72%	3,08%	4,63%
-0,34%	5,64%	-9,29%	3,26%	1,35%	-2,72%	-8,72%	3,61%
6,43%	-0,58%	2,23%	3,56%	-2,07%	0,38%	0,94%	4,01%
2,13%	-3,58%	6,33%	5,06%	-4,16%	-3,50%	20,56%	-4,57%
3,05%	7,80%	4,10%	3,95%	3,42%	3,58%	-0,57%	1,56%
-0,25%	-14,82%	2,52%	-1,48%	0,39%	-3,71%	-2,15%	4,39%
2,74%	8,64%	7,80%	0,35%	2,81%	10,81%	1,59%	1,58%
-1,44%	2,50%	-1,77%	-0,75%	4,81%	3,19%	9,43%	-1,05%
-1,95%	6,83%	-5,37%	-3,62%	3,29%	-1,26%	1,73%	1,33%
-2,23%	0,11%	-4,76%	16,25%	2,48%	-2,68%	4,77%	0,99%
-5,71%	-3,92%	-4,51%	-1,48%	5,22%	2,43%	-8,77%	3,55%
-7,23%	-1,01%	-9,55%	-0,96%	-2,91%	-3,26%	-4,03%	-3,50%
10,74%	12,65%	8,70%	6,12%	5,70%	1,07%	0,75%	1,09%
-0,53%	-0,20%	5,05%	-5,60%	3,62%	1,42%	3,84%	-0,91%
0,84%	0,56%	6,77%	5,96%	5,03%	1,32%	8,79%	4,07%
4,27%	7,82%	1,04%	12,62%	-1,37%	0,41%	6,25%	-3,58%
4,04%	3,76%	1,62%	18,82%	0,92%	-0,40%	6,16%	3,44%
3,08%	0,77%	-0,82%	10,48%	-1,24%	1,30%	12,56%	6,67%
-0,84%	3,07%	3,18%	-2,69%	-0,75%	-1,39%	-2,96%	3,03%
-6,34%	-3,05%	-8,90%	-1,14%	-7,67%	-3,24%	-9,31%	-2,15%
3,89%	-18,93%	6,67%	1,02%	-0,98%	8,15%	4,95%	5,27%
1,16%	6,25%	-0,62%	4,48%	0,84%	2,36%	4,62%	3,24%
1,89%	4,59%	-2,91%	9,30%	0,84%	-1,79%	3,53%	-7,52%
2,36%	-2,58%	-2,58%	0,22%	2,47%	2,13%	-18,71%	2,05%
-2,02%	-3,76%	1,17%	-10,80%	-5,44%	2,73%	11,09%	-2,02%
0,19%	6,59%	6,01%	-1,33%	1,09%	-0,76%	-5,64%	2,60%
0,64%	6,26%	1,38%	-9,14%	1,27%	0,46%	13,87%	-4,47%
4,97%	4,68%	-2,05%	-14,48%	7,96%	5,38%	17,24%	2,66%

E(Market)	E(NKE)	E(BA)	E(AAPL)	E(MCD)	E(JNJ)	E(AIR.PA)	E(KO)
1,01%	1,05%	4,67%	-2,63%	1,35%	3,69%	13,12%	3,87%
3,51%	8,26%	11,55%	0,20%	3,86%	7,03%	1,24%	5,11%
1,75%	7,72%	6,42%	-0,03%	2,40%	4,48%	0,96%	4,61%
2,04%	-2,77%	8,84%	2,20%	-4,75%	-0,53%	12,32%	-5,57%
-1,55%	3,23%	3,40%	-11,88%	2,47%	1,95%	-7,66%	0,94%
4,91%	-1,23%	2,56%	14,08%	-0,97%	8,86%	9,26%	-0,11%
-3,17%	0,13%	-0,71%	8,34%	-3,06%	-6,93%	-2,89%	-4,78%
2,95%	15,61%	13,05%	-2,17%	1,94%	0,30%	7,98%	-0,08%
4,41%	4,25%	11,01%	9,59%	0,27%	6,78%	7,41%	4,41%
2,73%	4,39%	3,18%	6,94%	1,65%	2,86%	3,19%	2,21%
2,29%	-0,39%	1,60%	0,82%	-0,42%	-3,31%	6,72%	2,72%
-3,60%	-7,40%	-8,27%	-10,81%	-2,99%	-3,45%	-5,77%	-8,49%
4,26%	7,76%	3,45%	5,70%	1,85%	4,82%	1,43%	0,95%
0,64%	-5,72%	-2,71%	1,95%	2,98%	6,58%	-2,67%	1,96%
0,59%	-1,26%	2,78%	9,91%	3,39%	3,09%	-4,84%	5,48%
2,07%	5,73%	5,39%	7,84%	0,83%	0,84%	7,80%	0,26%
1,87%	0,79%	-5,97%	2,73%	-0,72%	3,07%	-7,02%	4,28%
-1,54%	-0,57%	-5,34%	2,84%	-6,16%	-4,36%	-11,43%	-7,28%
3,74%	2,12%	5,85%	7,72%	-0,06%	4,31%	7,91%	6,15%
-1,57%	13,54%	0,44%	-1,73%	1,14%	2,74%	6,42%	2,98%
2,30%	4,21%	-1,96%	7,18%	-1,16%	1,10%	-4,48%	-1,85%
2,43%	6,78%	8,17%	10,58%	4,18%	1,07%	2,91%	7,76%
-0,45%	-2,91%	-3,29%	-7,22%	-3,24%	-3,43%	-15,63%	-5,85%
-3,13%	-4,09%	11,81%	6,11%	-1,37%	-4,27%	14,12%	-2,52%
5,47%	5,56%	4,39%	10,06%	7,90%	3,06%	17,20%	5,15%
-1,77%	3,28%	-0,54%	-3,17%	-1,51%	-1,89%	9,21%	-5,60%
0,83%	-1,51%	-4,51%	0,56%	-0,93%	-1,41%	2,56%	0,00%
1,03%	3,12%	-1,36%	4,51%	0,20%	1,66%	-0,02%	0,97%
-2,12%	6,23%	-1,30%	-3,74%	-0,92%	-2,70%	-4,30%	-3,45%
1,94%	6,64%	3,90%	-3,32%	5,01%	2,79%	10,97%	4,69%
-6,33%	-3,08%	-8,85%	-6,69%	-4,08%	-5,57%	-10,15%	-4,35%
-2,66%	10,30%	0,19%	-2,20%	3,67%	-0,69%	-8,94%	2,90%
8,28%	6,53%	13,05%	8,32%	13,90%	8,21%	19,73%	5,54%
-0,07%	0,83%	-1,28%	-0,70%	2,38%	0,83%	7,90%	1,29%
-1,98%	-5,50%	-0,82%	-11,25%	3,26%	1,23%	-9,64%	0,57%

E(PG)	E(T)	E(CVX)	E(WMT)	E(BAC)	E(GE)	E(INTC)	E(IBM)
2,77%	-2,21%	1,17%	1,16%	9,71%	0,46%	6,63%	4,32%
-0,10%	4,07%	4,80%	3,32%	7,12%	13,24%	8,49%	0,77%

E(PG)	E(T)	E(CVX)	E(WMT)	E(BAC)	E(GE)	E(INTC)	E(IBM)
-1,14%	2,34%	7,25%	-3,67%	-0,26%	3,48%	2,32%	0,44%
-1,87%	-6,90%	-8,60%	-5,34%	-11,87%	-13,46%	-5,71%	-2,55%
-1,90%	-0,53%	-8,22%	-5,00%	-8,72%	-11,32%	-9,28%	-1,50%
2,63%	8,97%	12,16%	6,35%	-2,44%	11,65%	5,77%	3,84%
-2,58%	4,06%	-2,03%	-1,62%	-11,40%	-10,32%	-13,71%	-3,77%
0,37%	5,68%	9,28%	6,61%	5,09%	12,92%	8,53%	8,81%
6,67%	1,02%	1,76%	1,06%	-12,75%	-1,57%	4,28%	6,90%
-4,05%	-2,67%	-1,25%	-0,26%	-4,48%	-1,30%	6,25%	-1,17%
5,23%	5,62%	12,59%	0,15%	21,83%	16,35%	-0,72%	3,64%
-1,27%	-5,10%	3,90%	3,83%	2,79%	9,98%	1,91%	10,25%
-0,27%	2,98%	9,97%	-7,44%	3,93%	4,45%	0,75%	0,18%
-2,45%	7,71%	3,45%	0,69%	-6,80%	-4,31%	-6,16%	0,59%
6,12%	3,00%	1,70%	5,52%	-7,99%	1,89%	14,61%	4,50%
3,15%	1,32%	-3,49%	1,01%	-4,41%	-4,05%	-2,09%	-0,61%
-5,24%	-0,60%	-2,09%	-3,89%	-6,76%	-3,30%	-1,67%	1,43%
-2,56%	-5,64%	1,07%	-0,89%	-11,49%	-5,12%	0,69%	5,92%
3,53%	-2,70%	-4,25%	1,61%	-15,79%	-8,97%	-9,01%	-5,09%
-0,84%	0,09%	-6,37%	-2,48%	-25,14%	-5,84%	5,96%	1,67%
2,08%	4,33%	13,43%	9,26%	11,57%	9,76%	14,97%	5,55%
0,88%	-1,15%	-1,37%	3,82%	-20,22%	-4,81%	2,38%	2,21%
3,30%	4,34%	3,47%	2,08%	2,20%	13,65%	-2,66%	-2,20%
-4,84%	-1,41%	-3,17%	2,59%	28,15%	4,38%	8,86%	4,65%
7,25%	4,00%	6,61%	-3,73%	11,90%	2,70%	2,54%	2,53%
-0,66%	2,04%	-1,80%	4,23%	20,03%	5,30%	4,56%	6,01%
-4,60%	6,78%	-0,70%	-3,83%	-15,35%	-2,53%	0,91%	-0,84%
-2,19%	3,76%	-6,98%	12,42%	-9,32%	-2,57%	-8,42%	-6,53%
-1,74%	4,29%	7,24%	5,86%	11,22%	10,34%	3,06%	1,32%
6,19%	7,56%	3,77%	6,66%	-10,37%	-0,53%	-3,66%	0,10%
4,02%	-3,47%	3,08%	-2,02%	8,77%	-0,28%	-2,63%	-0,24%
3,17%	2,83%	3,86%	1,59%	10,59%	10,43%	-8,80%	6,41%
0,61%	-7,22%	-5,46%	1,61%	5,51%	-7,31%	-4,59%	-6,27%
0,76%	-1,42%	-3,39%	-4,09%	5,70%	0,24%	-8,68%	-1,96%
-2,85%	-1,30%	2,25%	-4,81%	17,80%	0,17%	5,30%	0,71%
11,54%	4,46%	6,41%	2,45%	-2,57%	6,08%	1,97%	5,94%
1,26%	3,12%	2,43%	1,09%	-0,81%	4,97%	0,21%	-0,79%
1,07%	2,08%	1,34%	6,31%	8,37%	-0,52%	4,51%	6,12%
0,30%	3,26%	2,62%	3,80%	1,01%	-3,65%	9,60%	-5,10%
-0,05%	-6,63%	1,38%	-3,17%	10,93%	4,58%	2,29%	3,15%
0,25%	1,12%	-3,64%	-0,52%	-5,84%	0,19%	-0,26%	-8,18%
5,04%	0,86%	6,34%	4,59%	13,49%	5,05%	-3,71%	2,02%

E(PG)	E(T)	E(CVX)	E(WMT)	E(BAC)	E(GE)	E(INTC)	E(IBM)
-3,04%	-4,12%	-3,59%	-5,84%	-3,33%	-5,09%	-4,95%	-6,12%
-2,97%	-0,05%	0,87%	1,32%	-2,22%	4,02%	4,26%	1,58%
7,61%	8,42%	-1,32%	3,72%	1,18%	9,37%	6,71%	-3,27%
4,23%	-2,80%	2,85%	5,48%	13,17%	1,92%	-1,73%	0,73%
-3,41%	-0,21%	1,95%	-2,37%	-1,59%	5,92%	8,82%	4,32%
-5,21%	-4,01%	-10,67%	-5,14%	7,54%	-10,39%	-5,51%	-5,85%
2,61%	-4,22%	4,18%	-0,02%	-1,36%	2,19%	1,81%	5,33%
2,42%	9,78%	3,05%	2,93%	4,07%	1,60%	4,19%	3,90%
3,21%	3,10%	5,53%	4,26%	-12,01%	3,83%	3,38%	2,04%
-2,16%	-0,67%	-1,37%	-3,12%	-0,03%	-0,40%	3,21%	-5,65%
-2,76%	-0,35%	6,28%	-2,25%	1,54%	-1,13%	13,06%	-1,72%
-0,86%	1,92%	-1,03%	-2,01%	-0,81%	-4,33%	9,65%	5,71%
7,46%	-1,80%	0,98%	3,26%	5,48%	3,27%	3,69%	0,89%
0,74%	0,78%	-7,85%	1,26%	6,27%	-0,57%	-0,31%	-1,30%
4,99%	0,15%	0,51%	-0,28%	0,63%	0,72%	-2,35%	-13,42%
3,60%	1,53%	-8,42%	14,76%	-0,72%	2,61%	10,22%	-0,70%
0,70%	-5,09%	3,01%	-1,38%	5,27%	-3,74%	-2,61%	-1,10%
-6,84%	-0,63%	-8,63%	-1,08%	-15,35%	-5,49%	-8,99%	-4,47%
0,98%	4,96%	5,05%	-1,26%	4,34%	9,77%	1,34%	6,35%
-3,78%	-5,56%	-1,62%	-1,44%	-2,38%	-4,57%	-5,98%	-0,92%
-2,21%	7,60%	5,77%	-5,13%	3,49%	9,13%	4,07%	6,70%
-1,43%	-0,31%	-6,35%	-4,26%	3,56%	0,68%	6,61%	-0,23%
-0,21%	2,82%	-6,36%	-4,52%	3,44%	-1,76%	-11,74%	-4,14%
-1,20%	-0,92%	-8,31%	1,45%	5,02%	-1,80%	-4,86%	-0,44%
-7,93%	-4,50%	-7,38%	-9,53%	-8,68%	-4,97%	-0,67%	-8,02%
1,78%	-1,89%	-2,63%	0,15%	-4,36%	2,50%	5,59%	-1,99%
7,11%	4,31%	15,19%	-11,74%	7,68%	14,65%	12,32%	-3,39%
-2,14%	0,36%	1,59%	2,68%	3,75%	3,41%	3,29%	0,34%
5,88%	1,97%	-1,72%	4,82%	-3,40%	4,59%	-1,15%	-1,52%

E(IBM)	E(CAT)	E(TRV)	E(FCA)	E(BARC)	E(HSBA)	E(VOW)	E(ALV)	E(LHA)
4,32%	9,17%	3,75%	-15,42%	16,07%	6,21%	0,13%	5,41%	-5,75%
0,77%	10,08%	3,11%	24,62%	15,21%	-6,37%	12,09%	9,37%	11,95%
0,44%	8,87%	-6,08%	9,91%	-6,26%	-0,15%	0,63%	-6,94%	2,05%
-2,55%	-10,92%	-2,65%	-10,45%	-9,65%	-5,39%	-1,24%	-1,28%	-14,17%
-1,50%	-1,22%	0,22%	-4,67%	-11,40%	-2,09%	-0,35%	0,50%	5,47%
3,84%	16,75%	2,30%	15,30%	22,87%	4,87%	3,81%	8,73%	9,39%
-3,77%	-6,73%	-3,04%	-6,10%	-8,98%	0,26%	-1,48%	-9,30%	-0,31%
8,81%	20,62%	6,99%	22,31%	-1,06%	0,07%	12,53%	2,57%	8,14%

E(IBM)	E(CAT)	E(TRV)	E(FCA)	E(BARC)	E(HSBA)	E(VOW)	E(ALV)	E(LHA)
6,90%	0,30%	5,80%	7,27%	-8,49%	0,49%	16,59%	8,16%	13,87%
-1,17%	7,52%	-2,31%	4,74%	-6,42%	0,98%	10,21%	-6,26%	6,52%
3,64%	10,60%	3,76%	20,91%	2,04%	0,33%	1,62%	5,14%	-0,34%
10,25%	3,93%	0,85%	10,13%	12,13%	4,40%	4,68%	13,93%	-6,31%
0,18%	5,96%	6,38%	-5,29%	9,77%	-0,53%	-1,05%	2,76%	-3,60%
0,59%	8,03%	-0,29%	-5,13%	-13,37%	-3,82%	-1,70%	-5,29%	0,79%
4,50%	3,97%	6,28%	15,06%	1,60%	2,14%	11,06%	7,23%	2,33%
-0,61%	-8,42%	-1,99%	2,41%	-1,68%	-1,85%	-1,59%	-5,54%	2,79%
1,43%	0,50%	-5,45%	2,39%	-7,42%	-2,70%	8,68%	0,03%	-0,88%
5,92%	-6,89%	-5,65%	-8,86%	-13,12%	-3,94%	0,71%	-5,55%	-6,58%
-5,09%	-7,92%	-8,51%	-37,30%	-22,95%	-8,11%	-17,15%	-21,26%	-16,08%
1,67%	-18,9%	-2,66%	-5,54%	-5,55%	-7,45%	-12,01%	-1,26%	-17,34%
5,55%	28,60%	19,71%	8,42%	21,01%	9,63%	22,09%	14,57%	1,17%
2,21%	3,60%	-3,62%	-13,90%	-7,12%	-8,02%	-1,17%	-5,23%	-2,64%
-2,20%	-7,45%	5,97%	-7,18%	-2,34%	-0,71%	-7,88%	-4,00%	-4,48%
4,65%	20,89%	-1,56%	29,09%	20,64%	7,78%	19,06%	13,64%	14,94%
2,53%	4,65%	-0,58%	-5,34%	16,95%	4,82%	2,98%	8,28%	-1,39%
6,01%	-6,78%	2,81%	1,47%	-4,03%	1,58%	-5,00%	-1,77%	0,68%
-0,84%	-3,20%	8,56%	-17,33%	-7,32%	-0,04%	9,67%	-5,99%	-6,42%
-6,53%	-14,8%	-2,92%	4,10%	-18,86%	-7,19%	-5,91%	-8,29%	-11,39%
1,32%	-3,16%	2,87%	4,35%	-7,70%	10,06%	-2,41%	8,14%	7,36%
0,10%	-0,29%	-1,96%	0,71%	3,06%	-4,95%	8,27%	2,40%	12,38%
-0,24%	1,24%	3,25%	8,21%	9,72%	3,39%	0,46%	7,53%	-4,38%
6,41%	0,77%	6,13%	-4,22%	17,18%	4,73%	0,48%	6,03%	7,48%
-6,27%	-0,87%	3,89%	-9,48%	5,85%	8,22%	15,47%	3,28%	11,72%
-1,96%	0,42%	-0,26%	-5,41%	8,58%	4,66%	3,90%	4,40%	8,13%
0,71%	5,66%	1,98%	6,39%	6,60%	1,37%	3,99%	4,78%	11,53%
5,94%	9,73%	9,18%	18,66%	14,64%	10,72%	4,42%	0,46%	2,64%
-0,79%	-6,22%	2,40%	-8,77%	3,23%	1,95%	-7,27%	-0,71%	5,54%
6,12%	-5,94%	5,19%	0,88%	-5,25%	-1,28%	-7,12%	1,10%	-1,48%
-5,10%	-2,08%	1,39%	9,39%	-1,81%	0,08%	2,78%	5,69%	-0,42%
3,15%	1,29%	-2,02%	34,81%	12,46%	4,15%	11,99%	10,98%	9,51%
-8,18%	-3,91%	-4,01%	-12,38%	-13,16%	-6,10%	-9,32%	-6,31%	-6,27%
2,02%	1,17%	4,50%	11,51%	3,35%	9,61%	14,25%	4,36%	-3,54%
-6,12%	-0,49%	-4,41%	-4,71%	-1,24%	-8,84%	-1,76%	-7,55%	-10,31%
1,58%	1,02%	6,74%	3,13%	1,42%	-1,00%	-0,35%	7,18%	6,72%
-3,27%	0,60%	1,75%	-1,75%	-0,77%	2,88%	7,08%	6,57%	-1,09%
0,73%	1,42%	5,07%	0,79%	3,47%	-0,10%	6,64%	3,12%	11,98%
4,32%	7,27%	0,28%	1,73%	0,02%	-2,93%	2,67%	1,88%	-3,61%
-5,85%	4,05%	-10,27%	24,35%	0,16%	-5,38%	-8,45%	-5,06%	14,53%

E(IBM)	E(CAT)	E(TRV)	E(FCA)	E(BARC)	E(HSBA)	E(VOW)	E(ALV)	E(LHA)
5,33%	3,21%	3,10%	2,52%	-6,28%	0,38%	2,58%	4,72%	6,31%
3,90%	2,42%	2,06%	11,35%	-7,47%	-1,78%	-0,64%	-5,45%	1,15%
2,04%	6,66%	6,41%	2,69%	8,03%	-0,59%	4,59%	1,72%	-2,59%
-5,65%	-3,04%	3,14%	-11,72%	-1,71%	5,11%	2,55%	4,01%	7,00%
-1,72%	6,26%	1,21%	-5,98%	-13,89%	-5,81%	-2,34%	-2,20%	-19,01%
5,71%	-6,73%	-4,82%	0,46%	6,03%	7,31%	-8,37%	2,59%	-15,37%
0,89%	8,23%	5,72%	2,66%	-0,13%	3,40%	-1,50%	3,90%	-0,79%
-1,30%	-9,23%	-0,25%	2,67%	1,32%	-3,99%	-3,66%	-1,14%	-5,11%
-13,4%	3,16%	7,28%	16,54%	5,85%	3,16%	3,27%	-1,31%	-5,73%
-0,70%	-0,82%	3,60%	12,61%	2,22%	-0,41%	7,17%	9,26%	21,73%
-1,10%	-9,05%	1,84%	-4,32%	-0,70%	-4,49%	-1,02%	-0,83%	-3,69%
-4,47%	11,94%	-2,89%	21,85%	-3,87%	0,13%	9,44%	6,56%	8,83%
6,35%	3,64%	4,47%	17,84%	9,70%	-5,32%	12,25%	2,16%	-13,00%
-0,92%	-3,49%	1,13%	9,83%	-4,32%	1,72%	8,42%	8,16%	-0,19%
6,70%	9,44%	-6,51%	-11,90%	5,22%	12,89%	-5,33%	-5,74%	-5,22%
-0,23%	-1,82%	-0,01%	8,97%	6,16%	-2,37%	-1,35%	-2,13%	3,20%
-4,14%	-0,61%	-3,84%	-9,71%	-3,54%	-8,47%	-5,46%	-2,16%	-9,67%
-0,44%	-6,47%	9,76%	8,80%	10,89%	1,65%	-11,35%	6,70%	6,76%
-8,02%	-2,86%	-6,26%	-12,38%	-9,27%	-9,56%	-9,50%	-4,67%	-12,17%
-1,99%	14,52%	0,59%	-7,44%	-6,64%	-3,88%	-37,06%	-1,42%	14,44%
-3,39%	12,90%	13,40%	15,57%	-5,00%	3,09%	20,13%	13,60%	8,07%
0,34%	-0,59%	1,37%	0,63%	-3,50%	4,19%	11,93%	5,09%	0,88%
-1,52%	-6,69%	-1,19%	-4,67%	-2,16%	1,04%	0,48%	-2,68%	7,14%

E(BAYN.)	E(ML.PA)	E(RI.PA)	E(BZU.MI)	E(VLS.MI)	E(BGN.MI)	E(REP.MC)
-1,67%	-9,06%	-5,15%	-15,50%	-2,10%	-11,93%	-3,00%
2,81%	6,41%	13,54%	2,05%	1,05%	20,70%	5,23%
-4,15%	0,42%	1,57%	20,50%	-2,17%	-2,20%	1,01%
-2,67%	1,02%	-3,48%	-19,60%	4,91%	-3,03%	-6,61%
1,02%	5,85%	3,41%	-7,41%	-3,25%	6,48%	0,44%
-4,24%	1,12%	-5,35%	0,89%	-4,58%	8,17%	-0,14%
9,07%	0,11%	2,57%	-14,55%	4,26%	-0,03%	11,17%
6,03%	-1,58%	-0,84%	8,11%	-5,87%	4,61%	-0,60%
4,68%	2,25%	3,87%	6,30%	3,27%	3,76%	4,72%
4,23%	-8,92%	-0,41%	-12,96%	-4,02%	-11,41%	5,35%
-1,27%	2,93%	11,95%	18,67%	4,89%	9,77%	-6,85%
-2,74%	-1,12%	-1,17%	7,48%	4,09%	9,35%	12,05%
4,16%	10,82%	-4,20%	11,39%	-3,06%	2,63%	12,75%
-2,89%	0,87%	-1,51%	0,14%	5,83%	5,14%	5,74%

E(BAYN.)	E(ML.PA)	E(RI.PA)	E(BZU.MI)	E(VLS.MI)	E(BGN.MI)	E(REP.MC)
8,51%	13,43%	2,86%	1,54%	3,13%	5,38%	-0,73%
-1,65%	-1,17%	3,28%	-7,60%	4,32%	-2,04%	-0,38%
-2,77%	3,43%	-3,23%	-0,43%	-2,52%	-9,76%	-1,90%
0,81%	-13,10%	2,62%	-14,76%	-5,34%	-12,89%	1,03%
-19,77%	-13,11%	-9,61%	-14,86%	-1,33%	-17,73%	-5,98%
-7,71%	-11,39%	-5,92%	-12,66%	-2,68%	3,45%	-8,95%
11,66%	16,49%	14,71%	9,15%	-0,80%	1,87%	-0,43%
5,30%	-10,47%	4,94%	2,15%	-0,51%	0,46%	9,58%
1,30%	-3,20%	2,30%	-0,67%	-2,45%	-0,97%	2,27%
8,29%	14,41%	2,32%	19,88%	-0,09%	19,29%	5,88%
3,65%	-1,05%	5,80%	4,61%	-5,01%	9,24%	-9,35%
-5,02%	7,81%	0,90%	3,60%	1,26%	7,08%	-6,83%
3,88%	0,97%	-0,08%	-12,71%	-0,19%	-8,29%	-4,00%
-3,96%	-12,74%	0,68%	-11,21%	-4,53%	-11,62%	-23,27%
10,96%	8,88%	6,59%	9,15%	2,46%	19,01%	-16,49%
8,93%	7,55%	4,65%	10,43%	-4,26%	2,17%	9,29%
-0,48%	2,85%	-2,19%	0,52%	6,37%	1,48%	2,80%
8,31%	6,79%	1,85%	4,55%	-0,86%	10,79%	12,45%
0,50%	8,65%	-4,94%	6,23%	1,20%	8,79%	3,06%
3,42%	7,83%	5,85%	2,15%	0,55%	9,70%	2,08%
3,29%	0,04%	0,37%	12,54%	0,36%	3,63%	5,74%
1,03%	-4,34%	5,37%	1,64%	2,42%	9,46%	-3,32%
4,27%	-0,23%	7,70%	12,31%	-2,01%	-5,40%	7,17%
5,99%	-4,75%	-2,28%	-0,59%	4,86%	10,44%	-0,91%
0,75%	-1,78%	-3,36%	-3,23%	3,75%	5,68%	-2,91%
4,59%	9,50%	-1,43%	11,19%	4,48%	12,85%	12,26%
-1,21%	1,31%	-8,14%	-10,31%	-4,39%	-1,42%	-0,95%
6,58%	9,83%	5,97%	-3,94%	12,75%	14,39%	-7,89%
-3,83%	-4,13%	-1,89%	-8,25%	35,62%	-9,75%	10,90%
3,69%	13,21%	4,48%	13,14%	19,24%	-2,42%	-2,40%
4,98%	-6,09%	-3,63%	10,80%	19,27%	15,34%	4,34%
7,21%	3,73%	-4,89%	-1,24%	30,52%	8,91%	7,87%
3,75%	-3,45%	-0,81%	3,81%	-10,96%	7,12%	-2,36%
-4,02%	1,35%	-3,87%	3,32%	5,03%	0,89%	-2,76%
5,07%	13,04%	7,04%	11,39%	4,02%	4,04%	-5,19%
-4,64%	2,45%	-0,98%	-10,38%	2,22%	1,22%	4,75%
1,82%	-3,09%	2,35%	-5,72%	5,75%	-5,42%	1,67%
6,07%	5,68%	3,95%	0,32%	25,59%	-0,94%	4,73%
-2,82%	-3,51%	-2,54%	-3,72%	2,28%	-6,20%	6,47%
-4,13%	-5,99%	-3,70%	-1,17%	-11,36%	4,05%	-4,99%

E(BAYN.)	E(ML.PA)	E(RI.PA)	E(BZU.MI)	E(VLS.MI)	E(BGN.MI)	E(REP.MC)
3,13%	2,52%	7,19%	-5,13%	5,53%	-1,18%	-3,37%
8,65%	-11,27%	-0,13%	-6,26%	0,62%	1,19%	1,42%
2,28%	-7,37%	1,31%	-0,39%	-9,43%	1,03%	-0,45%
6,59%	6,86%	5,86%	11,68%	12,87%	0,74%	-5,30%
-6,60%	1,75%	-3,23%	-12,67%	4,88%	8,13%	1,28%
13,29%	15,20%	15,51%	3,49%	-0,49%	7,30%	-11,29%
3,10%	-0,88%	-0,63%	15,97%	15,95%	2,24%	0,93%
5,95%	7,68%	3,93%	10,51%	33,17%	15,27%	10,09%
-7,20%	7,60%	0,75%	3,42%	3,20%	2,69%	0,31%
1,06%	0,32%	1,38%	-3,16%	8,31%	11,37%	6,19%
-2,81%	-3,69%	-7,97%	-8,35%	-8,72%	-2,49%	-5,81%
6,94%	-5,13%	6,07%	21,74%	7,29%	-5,16%	-6,50%
-10,01%	-3,33%	-14,31%	-2,38%	-11,97%	-10,88%	-2,94%
-5,39%	-5,66%	-3,59%	-1,67%	-16,76%	-5,67%	-16,64%
6,01%	11,25%	18,85%	2,99%	24,30%	11,21%	-18,40%
3,96%	4,63%	1,28%	10,41%	-0,55%	2,70%	10,13%
-8,54%	-7,61%	-2,55%	-2,64%	-8,22%	0,98%	7,34%

APPENDIX C

The stocks' price between the 1 January 2016 and the 31 December 2016, daily frequency.

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
04/01/2016	136	98	41	76	32	85	60	27	33	131
05/01/2016	136	98	41	76	33	86	61	27	33	131
06/01/2016	134	98	41	75	32	82	62	27	32	130
07/01/2016	129	96	40	75	32	79	63	27	31	128
08/01/2016	126	95	40	74	32	79	62	27	31	127
11/01/2016	126	95	40	74	32	77	62	28	31	128
12/01/2016	128	96	41	74	32	79	62	28	32	128
13/01/2016	124	94	41	73	32	78	60	27	31	126
14/01/2016	125	96	41	74	33	82	61	28	32	128
15/01/2016	122	94	40	73	32	80	60	27	29	125
19/01/2016	122	95	41	74	33	78	61	27	29	123
20/01/2016	118	93	40	74	32	76	59	27	29	117
21/01/2016	119	93	40	75	33	78	60	28	29	118
22/01/2016	121	94	41	76	34	80	61	27	29	118
25/01/2016	120	94	41	75	33	77	62	27	29	118
26/01/2016	124	98	41	77	34	81	62	28	29	118
27/01/2016	113	99	41	77	34	80	62	29	29	117
28/01/2016	114	99	41	78	34	82	62	28	29	118
29/01/2016	116	102	42	80	34	83	64	28	30	120
01/02/2016	118	102	42	79	35	82	66	27	30	120
02/02/2016	114	101	41	78	34	78	65	28	29	118
03/02/2016	118	101	41	79	35	81	64	29	29	120
04/02/2016	120	101	41	79	35	81	65	28	29	123
05/02/2016	119	98	41	79	35	79	65	28	28	124
08/02/2016	116	99	41	81	35	82	65	28	28	124
09/02/2016	115	99	42	81	35	79	64	29	28	121
10/02/2016	114	99	41	80	35	80	64	28	28	117
11/02/2016	106	99	41	78	35	79	63	29	28	115
12/02/2016	106	99	42	79	35	82	64	29	28	118
16/02/2016	110	100	42	80	35	82	64	29	28	119
17/02/2016	114	100	42	81	35	86	64	28	29	123
18/02/2016	115	101	42	80	35	84	62	29	29	129
19/02/2016	112	102	42	80	35	84	63	28	28	130
22/02/2016	115	103	43	80	35	86	64	29	29	130
23/02/2016	114	102	42	80	35	82	65	28	28	129

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
24/02/2016	113	103	43	80	35	83	65	27	29	129
25/02/2016	114	104	43	80	36	83	66	29	29	131
26/02/2016	115	104	42	79	35	82	65	28	29	128
29/02/2016	115	103	42	78	35	81	64	28	29	128
01/03/2016	117	105	42	79	36	84	65	29	30	131
02/03/2016	117	105	42	81	36	84	64	29	30	133
03/03/2016	117	104	43	81	36	85	64	29	30	134
04/03/2016	118	104	43	82	36	85	65	29	30	134
07/03/2016	120	105	43	81	36	88	66	28	30	136
08/03/2016	119	104	43	81	36	86	66	29	30	135
09/03/2016	120	105	43	81	36	90	66	29	30	137
10/03/2016	120	105	44	80	37	91	66	29	31	136
11/03/2016	122	106	44	80	37	92	66	29	31	139
14/03/2016	123	105	44	79	37	91	66	29	31	139
15/03/2016	123	106	44	79	37	91	67	28	31	139
16/03/2016	125	105	44	79	37	92	67	28	31	141
17/03/2016	128	105	45	81	37	94	66	28	31	143
18/03/2016	131	105	45	81	37	95	66	28	32	143
21/03/2016	133	105	45	81	37	94	67	29	32	145
22/03/2016	132	105	44	81	37	93	66	28	32	144
23/03/2016	130	106	44	81	37	91	66	29	31	141
24/03/2016	129	106	44	81	37	92	67	29	31	144
28/03/2016	128	106	45	81	37	92	67	29	31	144
29/03/2016	128	107	45	81	38	92	67	29	32	145
30/03/2016	126	107	45	81	38	92	67	29	32	144
31/03/2016	124	106	45	80	37	92	67	28	32	147
01/04/2016	124	107	46	82	37	91	68	29	32	148
04/04/2016	123	106	46	81	38	91	68	29	31	148
05/04/2016	124	107	45	81	37	90	67	29	31	146
06/04/2016	125	107	46	82	37	92	68	29	31	146
07/04/2016	124	107	45	81	37	92	67	29	31	144
08/04/2016	125	107	46	81	37	93	67	29	31	145
11/04/2016	125	107	45	81	37	92	66	30	31	145
12/04/2016	126	107	46	81	37	95	67	29	31	146
13/04/2016	128	108	45	81	37	95	68	29	31	147
14/04/2016	128	108	45	81	37	95	67	30	31	147
15/04/2016	128	108	45	81	37	94	68	29	31	148
18/04/2016	129	109	45	82	37	96	68	28	31	148
19/04/2016	130	110	45	82	38	97	68	29	31	140
20/04/2016	127	111	43	80	37	99	68	29	31	142

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
21/04/2016	128	111	43	80	37	98	67	29	31	145
22/04/2016	128	111	43	80	37	99	67	29	31	145
25/04/2016	128	111	44	80	37	98	68	29	31	145
26/04/2016	130	111	43	78	37	99	68	30	31	145
27/04/2016	134	110	44	79	37	101	68	29	31	146
28/04/2016	132	110	44	79	37	99	67	29	30	143
29/04/2016	132	110	44	79	37	99	65	29	30	142
02/05/2016	131	110	44	80	38	100	66	29	30	141
03/05/2016	129	110	44	80	38	98	66	29	30	140
04/05/2016	128	110	44	80	38	98	66	29	29	140
05/05/2016	129	111	44	80	37	98	66	30	29	143
06/05/2016	130	110	44	81	38	99	67	29	30	145
09/05/2016	129	111	44	81	38	97	68	30	29	145
10/05/2016	132	112	45	81	38	98	67	29	30	147
11/05/2016	131	112	44	81	38	98	65	30	30	146
12/05/2016	132	112	45	81	38	99	66	30	29	146
13/05/2016	130	111	44	80	38	98	64	29	29	145
16/05/2016	132	112	45	80	38	99	65	29	30	147
17/05/2016	130	112	44	79	38	99	64	30	30	145
18/05/2016	129	111	43	79	37	98	62	30	30	145
19/05/2016	126	111	43	79	37	98	68	29	29	142
20/05/2016	125	111	43	79	37	98	69	29	30	145
23/05/2016	126	111	43	79	37	97	69	29	30	144
24/05/2016	125	111	43	80	37	98	69	30	31	146
25/05/2016	127	112	43	80	37	100	70	29	31	149
26/05/2016	127	111	44	80	38	99	70	29	31	150
27/05/2016	127	112	44	80	38	100	70	30	31	150
31/05/2016	124	111	44	80	38	99	70	29	31	151
01/06/2016	124	111	44	81	37	99	70	29	31	150
02/06/2016	125	113	44	81	38	98	70	29	31	151
03/06/2016	125	113	44	81	38	99	70	29	31	150
06/06/2016	130	114	44	81	38	99	70	29	31	150
07/06/2016	130	114	44	81	38	101	70	30	31	151
08/06/2016	131	114	44	81	38	101	70	30	31	151
09/06/2016	131	115	45	82	39	101	70	29	31	151
10/06/2016	129	115	45	82	39	100	70	29	32	150
13/06/2016	128	115	44	81	39	100	70	29	32	149
14/06/2016	128	116	44	82	39	99	70	30	32	148
15/06/2016	128	115	44	82	39	99	70	30	31	148
16/06/2016	127	115	45	82	39	99	70	29	31	148

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
17/06/2016	128	114	44	82	39	100	70	29	31	149
20/06/2016	131	115	44	82	39	101	70	30	32	151
21/06/2016	129	115	44	82	40	101	70	29	32	151
22/06/2016	130	115	44	82	40	100	71	30	32	150
23/06/2016	131	116	44	83	40	102	71	30	32	153
24/06/2016	125	114	43	81	40	100	71	30	31	144
27/06/2016	121	115	43	80	41	98	71	30	30	141
28/06/2016	122	117	43	81	41	100	71	29	31	143
29/06/2016	125	118	44	83	41	101	71	30	31	146
30/06/2016	128	120	45	83	42	103	72	29	32	149
01/07/2016	128	120	44	83	42	102	72	29	32	150
05/07/2016	125	121	45	84	42	101	72	31	32	149
06/07/2016	125	121	45	84	42	102	73	30	32	150
07/07/2016	125	121	44	84	41	101	73	30	33	150
08/07/2016	128	121	45	84	42	103	73	30	33	152
11/07/2016	130	121	45	84	42	103	73	30	34	153
12/07/2016	129	121	45	84	41	105	72	30	34	154
13/07/2016	128	121	45	85	42	104	73	31	34	155
14/07/2016	129	122	45	85	42	105	73	30	35	157
15/07/2016	130	121	45	85	42	105	73	30	35	157
18/07/2016	131	121	45	85	42	104	73	31	35	157
19/07/2016	133	124	45	85	42	104	73	30	35	157
20/07/2016	133	123	45	85	42	103	73	31	35	159
21/07/2016	131	123	45	85	42	103	73	30	34	158
22/07/2016	131	123	45	85	42	104	73	30	34	159
25/07/2016	131	123	45	85	42	101	73	31	34	160
26/07/2016	133	123	44	85	41	101	73	31	35	159
27/07/2016	134	123	43	84	42	100	72	30	34	159
28/07/2016	131	123	43	84	42	100	72	30	34	159
29/07/2016	132	124	43	85	42	100	72	30	34	158
01/08/2016	131	124	43	86	42	97	73	30	34	159
02/08/2016	130	123	43	86	42	98	72	30	34	158
03/08/2016	130	122	43	85	42	99	72	30	34	158
04/08/2016	129	122	43	85	42	98	72	30	34	159
05/08/2016	130	123	43	85	42	98	73	30	35	161
08/08/2016	130	122	43	85	42	99	72	30	35	161
09/08/2016	131	122	43	85	42	99	73	30	35	160
10/08/2016	131	122	43	86	42	98	73	30	34	161
11/08/2016	132	122	43	86	42	99	73	30	34	162
12/08/2016	132	122	43	86	42	100	73	30	34	160

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
15/08/2016	134	121	44	86	42	101	73	30	35	160
16/08/2016	134	119	43	86	41	101	72	30	35	159
17/08/2016	134	120	43	86	41	101	72	30	35	159
18/08/2016	134	119	43	87	40	103	74	31	35	160
19/08/2016	133	119	43	87	40	101	72	30	35	159
22/08/2016	134	118	43	86	40	101	72	30	35	159
23/08/2016	133	119	43	87	40	101	71	30	35	159
24/08/2016	132	118	43	87	40	101	72	30	35	158
25/08/2016	132	118	43	87	40	101	71	31	35	157
26/08/2016	131	118	43	87	40	100	71	30	35	157
29/08/2016	132	119	43	88	40	101	71	31	35	158
30/08/2016	130	119	43	87	40	101	71	30	35	158
31/08/2016	128	119	43	87	40	100	71	31	36	157
01/09/2016	129	118	43	88	40	99	72	31	36	158
02/09/2016	130	119	43	88	40	100	72	30	36	158
06/09/2016	132	119	43	88	40	101	72	30	36	159
07/09/2016	132	119	43	87	40	102	72	31	36	160
08/09/2016	132	119	43	87	40	103	71	30	36	158
09/09/2016	128	117	42	86	39	100	70	30	35	154
12/09/2016	129	118	42	88	40	101	71	31	36	157
13/09/2016	128	117	42	86	39	98	71	31	35	154
14/09/2016	127	117	42	86	39	97	71	30	35	153
15/09/2016	127	118	42	87	40	99	72	31	36	154
16/09/2016	126	117	42	87	39	97	72	30	37	152
19/09/2016	127	117	42	88	39	97	72	31	37	153
20/09/2016	127	117	42	88	39	97	71	31	37	153
21/09/2016	130	118	42	87	40	99	72	31	37	154
22/09/2016	131	119	43	88	40	99	72	31	37	155
23/09/2016	131	118	42	87	40	98	72	31	37	154
26/09/2016	130	117	42	87	40	98	71	31	36	153
27/09/2016	130	118	42	88	40	98	72	31	37	155
28/09/2016	131	119	42	89	40	101	71	31	37	157
29/09/2016	130	116	42	88	40	100	70	31	37	157
30/09/2016	131	117	42	89	40	102	72	31	37	157
03/10/2016	131	118	42	88	40	101	71	30	37	156
04/10/2016	131	118	41	88	39	100	71	31	37	155
05/10/2016	134	118	41	88	39	101	71	31	38	156
06/10/2016	133	118	41	89	39	101	69	31	38	155
07/10/2016	133	118	41	89	38	101	68	31	38	154
10/10/2016	135	119	41	88	39	103	67	31	38	156

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
11/10/2016	133	117	41	88	39	102	67	31	37	153
12/10/2016	132	117	41	88	39	101	67	31	37	153
13/10/2016	132	117	41	88	39	100	68	31	37	152
14/10/2016	132	117	41	88	39	100	68	31	37	153
17/10/2016	133	118	41	87	39	100	68	31	37	153
18/10/2016	134	115	42	87	39	101	68	31	37	149
19/10/2016	135	114	42	86	39	101	68	31	35	150
20/10/2016	135	114	42	85	38	101	68	31	35	150
21/10/2016	135	113	42	84	37	100	68	31	35	148
24/10/2016	136	113	42	84	36	100	69	31	35	149
25/10/2016	138	113	42	87	36	100	69	31	35	150
26/10/2016	144	114	42	87	36	100	69	31	35	150
27/10/2016	142	115	42	87	36	99	69	31	35	152
28/10/2016	142	115	42	87	36	103	69	31	34	151
31/10/2016	141	115	42	87	36	104	70	31	35	152
01/11/2016	141	115	42	87	36	105	69	31	34	151
02/11/2016	140	114	42	87	36	104	69	31	34	151
03/11/2016	139	114	42	87	36	104	69	31	34	151
04/11/2016	138	114	41	85	36	104	69	32	34	151
07/11/2016	142	116	42	87	36	106	69	31	35	154
08/11/2016	142	116	43	87	37	106	69	31	35	155
09/11/2016	145	119	42	86	37	107	71	31	35	155
10/11/2016	148	119	41	83	36	107	71	31	35	160
11/11/2016	149	118	41	84	36	106	71	31	35	161
14/11/2016	150	116	41	83	36	106	70	31	34	158
15/11/2016	148	116	41	84	36	108	71	31	35	159
16/11/2016	146	116	41	83	37	108	71	32	35	159
17/11/2016	145	116	41	83	37	108	69	31	35	160
18/11/2016	146	115	41	82	37	109	68	31	35	160
21/11/2016	147	115	41	83	37	110	69	32	35	163
22/11/2016	150	113	41	83	38	111	70	32	35	163
23/11/2016	150	113	41	83	38	111	70	31	35	162
25/11/2016	150	114	41	83	39	111	71	31	35	163
28/11/2016	150	113	41	83	39	111	71	32	36	165
29/11/2016	152	112	41	83	39	109	71	32	35	164
30/11/2016	151	111	40	82	38	112	70	32	35	162
01/12/2016	152	111	40	82	38	113	70	31	34	160
02/12/2016	152	112	40	82	38	113	70	31	34	160
05/12/2016	152	112	41	83	38	113	69	32	34	160
06/12/2016	152	112	41	83	39	113	70	32	35	160

Date	BA	JNJ	KO	PG	T	CVX	WMT	GE	INTC	IBM
07/12/2016	154	111	41	84	40	114	71	32	36	165
08/12/2016	155	111	41	84	40	115	70	31	36	165
09/12/2016	156	112	42	84	40	116	70	32	36	167
12/12/2016	157	115	42	85	41	117	72	32	36	166
13/12/2016	157	116	42	85	41	117	72	32	37	168
14/12/2016	154	115	41	84	41	116	71	32	37	169
15/12/2016	154	116	42	85	41	117	71	31	37	168
16/12/2016	155	116	42	85	41	118	71	32	36	167
19/12/2016	156	116	42	85	42	118	72	32	37	167
20/12/2016	156	116	42	85	42	118	72	32	37	168
21/12/2016	157	115	42	84	42	118	71	32	37	167
22/12/2016	157	115	42	84	42	119	70	32	37	167
23/12/2016	158	116	42	85	42	118	70	32	37	167
27/12/2016	157	116	42	85	42	118	70	32	37	167
28/12/2016	156	115	41	84	42	118	69	32	37	166
29/12/2016	156	115	42	84	42	118	69	32	37	167
30/12/2016	156	115	41	84	42	118	69	32	36	166

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
04/01/2016	107	9	209	482	142	147	113	86	104	17
05/01/2016	108	8	209	484	138	141	108	83	100	16
06/01/2016	107	8	206	471	130	141	108	82	100	17
07/01/2016	104	8	200	465	130	142	107	80	100	16
08/01/2016	104	8	195	462	123	139	104	79	98	16
11/01/2016	104	7	194	461	125	137	101	79	97	15
12/01/2016	106	7	197	468	127	137	100	79	97	15
13/01/2016	104	8	196	468	129	139	102	80	98	14
14/01/2016	105	7	192	465	130	139	102	80	99	15
15/01/2016	103	7	187	450	126	137	100	79	97	14
19/01/2016	103	7	183	444	120	135	98	77	95	14
20/01/2016	101	6	185	453	118	134	99	77	95	13
21/01/2016	100	7	177	432	122	135	100	78	97	14
22/01/2016	101	6	181	437	116	130	98	76	95	13
25/01/2016	100	7	186	444	122	131	101	77	98	13
26/01/2016	101	7	177	439	125	134	103	79	101	14
27/01/2016	101	7	178	440	123	133	103	79	101	14
28/01/2016	102	7	177	445	125	133	103	82	102	14
29/01/2016	105	7	174	442	126	133	102	83	102	14
01/02/2016	105	6	181	456	121	130	99	81	102	13
02/02/2016	104	6	178	449	120	134	101	81	106	14

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
03/02/2016	104	6	169	433	120	133	100	82	107	14
04/02/2016	105	6	161	416	118	130	98	80	105	14
05/02/2016	104	6	169	426	114	127	96	80	104	13
08/02/2016	103	6	169	424	115	127	96	80	102	13
09/02/2016	104	6	160	406	119	124	94	80	99	13
10/02/2016	103	5	152	401	112	122	92	78	98	12
11/02/2016	101	5	155	409	109	118	91	78	96	12
12/02/2016	105	6	144	389	111	120	92	79	98	13
16/02/2016	105	5	153	408	109	114	91	76	91	12
17/02/2016	107	5	156	414	111	116	92	77	92	12
18/02/2016	106	6	157	415	119	119	94	79	94	13
19/02/2016	106	6	164	423	118	118	93	82	92	12
22/02/2016	107	6	157	416	123	121	95	82	95	13
23/02/2016	105	6	157	417	125	122	97	82	95	13
24/02/2016	106	6	161	413	119	121	96	83	95	13
25/02/2016	107	6	157	409	123	123	97	83	97	14
26/02/2016	106	6	153	402	121	122	95	82	97	13
29/02/2016	105	6	160	416	117	120	92	80	95	13
01/03/2016	108	6	165	433	117	122	93	80	96	14
02/03/2016	108	6	167	426	122	124	95	80	96	14
03/03/2016	107	6	154	427	129	124	94	81	96	14
04/03/2016	108	7	158	437	131	126	96	83	99	14
07/03/2016	108	7	162	431	134	128	96	84	96	14
08/03/2016	109	7	170	434	135	128	95	84	96	14
09/03/2016	109	7	169	431	138	127	96	84	97	14
10/03/2016	110	7	167	429	136	127	96	85	97	14
11/03/2016	112	7	164	431	130	127	95	84	97	15
14/03/2016	111	7	159	426	130	128	96	84	98	14
15/03/2016	112	7	165	432	125	125	93	81	97	14
16/03/2016	113	7	162	433	130	130	96	84	99	15
17/03/2016	114	7	163	433	132	132	98	84	101	15
18/03/2016	114	7	163	437	128	132	98	85	101	15
21/03/2016	114	7	160	430	131	132	97	87	100	15
22/03/2016	114	7	161	429	130	131	96	87	100	15
23/03/2016	114	7	160	429	130	131	97	87	100	15
24/03/2016	113	7	156	428	129	131	100	86	100	15
28/03/2016	114	7	155	429	133	131	101	87	100	15
29/03/2016	114	7	152	419	130	131	102	87	102	15
30/03/2016	115	7	152	419	126	129	100	86	98	15
31/03/2016	115	7	150	419	126	129	100	86	98	15

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
01/04/2016	115	7	151	414	126	129	100	86	98	15
04/04/2016	116	7	149	418	126	129	100	86	97	15
05/04/2016	114	7	149	415	129	130	101	88	98	15
06/04/2016	114	7	149	413	127	129	101	87	96	15
07/04/2016	113	7	146	412	123	126	99	86	94	15
08/04/2016	113	7	149	399	124	126	100	85	96	15
11/04/2016	113	6	146	402	120	123	98	83	95	15
12/04/2016	113	6	150	398	120	124	99	84	95	15
13/04/2016	114	6	153	400	120	123	98	84	95	15
14/04/2016	113	6	155	400	120	125	99	85	95	15
15/04/2016	114	7	166	402	121	126	100	86	95	16
18/04/2016	114	6	168	429	121	128	100	88	95	16
19/04/2016	114	7	166	431	126	132	103	90	97	16
20/04/2016	114	7	167	431	128	133	103	90	98	16
21/04/2016	107	7	167	429	125	133	103	89	98	16
22/04/2016	109	7	170	434	125	134	104	90	99	16
25/04/2016	108	7	171	445	129	136	107	91	100	17
26/04/2016	108	7	170	451	135	140	107	91	102	17
27/04/2016	108	7	169	446	139	140	108	92	97	17
28/04/2016	107	7	173	440	137	140	108	93	95	17
29/04/2016	108	7	174	449	134	139	107	93	95	16
02/05/2016	109	7	173	446	136	138	106	93	94	16
03/05/2016	108	7	170	447	141	137	103	94	94	17
04/05/2016	108	7	170	433	142	137	102	93	94	17
05/05/2016	109	7	164	433	138	134	98	88	93	16
06/05/2016	109	7	161	426	137	138	100	89	94	16
09/05/2016	109	7	160	418	133	137	98	88	92	16
10/05/2016	110	7	161	418	131	135	97	87	92	16
11/05/2016	110	7	157	413	130	133	98	87	93	16
12/05/2016	111	7	162	411	132	139	98	87	94	16
13/05/2016	111	7	163	414	134	141	100	87	95	16
16/05/2016	110	7	162	412	139	142	100	89	95	17
17/05/2016	109	7	164	405	138	139	100	88	94	17
18/05/2016	109	7	163	411	135	137	95	88	93	17
19/05/2016	108	7	163	410	136	138	96	88	94	17
20/05/2016	110	7	169	416	136	138	96	87	94	17
23/05/2016	109	6	170	417	132	137	96	87	93	17
24/05/2016	112	6	175	415	135	138	96	87	93	17
25/05/2016	112	6	175	418	135	136	89	86	92	17
26/05/2016	112	6	180	416	135	138	90	88	93	17

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
27/05/2016	112	6	185	423	134	138	84	88	93	17
31/05/2016	112	6	184	434	137	142	87	90	95	17
01/06/2016	113	6	185	434	138	145	87	91	96	18
02/06/2016	113	6	185	436	139	147	86	92	97	18
03/06/2016	113	6	181	436	140	147	85	91	96	18
06/06/2016	112	6	179	433	141	147	86	92	97	18
07/06/2016	113	6	179	433	139	147	86	91	96	18
08/06/2016	113	6	179	434	137	146	86	93	96	18
09/06/2016	113	6	179	433	140	145	88	94	96	18
10/06/2016	113	6	179	436	135	143	89	93	96	18
13/06/2016	112	6	177	435	137	143	89	93	97	18
14/06/2016	111	6	175	435	139	144	92	94	97	18
15/06/2016	111	6	168	432	137	143	92	93	96	18
16/06/2016	111	6	164	422	134	140	90	92	95	17
17/06/2016	111	6	159	419	132	135	88	91	93	16
20/06/2016	111	6	160	414	129	131	88	88	92	16
21/06/2016	111	6	158	414	127	129	87	87	90	16
22/06/2016	111	6	165	415	129	129	88	87	91	16
23/06/2016	113	6	176	420	125	129	88	86	91	16
24/06/2016	110	6	179	426	129	130	88	86	90	16
27/06/2016	109	6	181	428	134	136	90	89	93	16
28/06/2016	113	6	186	432	132	136	92	89	94	16
29/06/2016	115	6	153	442	130	138	92	88	94	16
30/06/2016	118	6	126	436	135	142	93	90	95	17
01/07/2016	118	6	131	426	128	127	88	86	93	16
05/07/2016	117	5	137	432	120	120	87	82	93	14
06/07/2016	117	5	138	441	123	124	89	84	93	15
07/07/2016	116	5	139	453	120	127	89	84	96	15
08/07/2016	117	5	136	457	120	128	90	85	98	16
11/07/2016	117	6	135	451	126	127	91	88	99	16
12/07/2016	117	5	131	452	122	126	91	86	98	16
13/07/2016	117	5	134	449	119	122	90	84	98	16
14/07/2016	117	5	138	452	118	119	88	83	97	15
15/07/2016	117	5	145	458	117	120	89	83	98	15
18/07/2016	116	6	148	466	122	122	91	85	98	16
19/07/2016	116	6	144	464	124	125	93	86	99	16
20/07/2016	116	6	147	462	127	127	93	88	98	17
21/07/2016	115	6	149	464	125	127	93	88	98	17
22/07/2016	116	6	150	466	128	129	93	89	99	17
25/07/2016	116	6	149	472	128	128	94	88	99	17

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
26/07/2016	116	6	150	474	128	127	93	89	99	17
27/07/2016	115	6	151	482	127	126	92	88	98	17
28/07/2016	116	6	151	479	131	127	92	89	99	17
29/07/2016	115	6	150	480	131	125	92	88	99	17
01/08/2016	115	6	148	477	130	126	92	88	100	18
02/08/2016	116	6	149	481	132	126	93	89	100	17
03/08/2016	117	6	146	484	133	127	94	90	101	18
04/08/2016	117	6	154	478	137	129	95	92	100	18
05/08/2016	117	6	150	482	131	127	94	91	101	18
08/08/2016	117	6	145	476	133	128	96	91	101	18
09/08/2016	117	6	147	470	132	128	96	92	101	18
10/08/2016	117	5	148	491	127	127	94	91	100	17
11/08/2016	117	6	151	503	127	127	94	91	99	18
12/08/2016	117	6	156	516	129	128	94	92	101	18
15/08/2016	117	6	160	523	132	126	96	94	102	18
16/08/2016	116	6	163	526	131	130	96	94	101	18
17/08/2016	116	6	163	530	134	134	99	95	103	19
18/08/2016	116	6	163	538	134	136	97	95	103	19
19/08/2016	116	6	164	538	135	136	98	97	105	19
22/08/2016	116	6	162	540	133	135	98	96	105	20
23/08/2016	115	6	160	536	134	135	99	96	105	20
24/08/2016	116	6	161	536	131	133	98	95	105	19
25/08/2016	116	6	160	537	128	131	96	94	104	19
26/08/2016	116	6	160	536	129	132	97	95	103	19
29/08/2016	117	6	165	533	128	129	96	95	104	19
30/08/2016	117	6	167	535	128	129	96	94	104	19
31/08/2016	117	6	164	534	130	131	97	95	103	19
01/09/2016	117	6	166	536	130	130	97	96	102	19
02/09/2016	117	6	169	540	128	129	96	95	102	19
06/09/2016	118	6	172	540	130	130	97	95	102	19
07/09/2016	118	6	172	550	128	129	97	95	102	19
08/09/2016	116	6	174	557	131	133	97	95	102	20
09/09/2016	114	6	173	561	130	134	96	95	102	19
12/09/2016	115	6	170	573	130	134	94	95	104	19
13/09/2016	114	6	170	574	132	136	95	96	106	19
14/09/2016	113	6	173	561	132	136	94	96	106	19
15/09/2016	114	6	175	560	131	135	96	96	106	19
16/09/2016	114	6	170	566	133	135	97	98	106	19
19/09/2016	114	6	171	571	132	136	96	98	105	18
20/09/2016	114	6	170	558	132	137	95	97	103	18

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
21/09/2016	115	6	170	552	131	135	94	95	103	18
22/09/2016	116	6	165	554	130	132	93	95	102	17
23/09/2016	115	6	167	565	130	131	94	94	101	18
26/09/2016	113	6	166	559	130	133	91	95	102	18
27/09/2016	114	6	172	574	127	130	91	94	101	18
28/09/2016	114	6	173	576	129	131	92	95	103	18
29/09/2016	113	6	171	581	128	132	92	96	103	18
30/09/2016	114	6	168	570	128	134	92	96	104	18
03/10/2016	112	6	166	567	130	137	92	97	105	18
04/10/2016	112	6	166	562	129	137	91	97	105	18
05/10/2016	112	6	167	570	127	132	89	96	105	18
06/10/2016	113	5	168	564	124	130	89	97	105	18
07/10/2016	114	5	168	571	126	132	89	99	104	18
10/10/2016	114	6	169	571	127	131	88	98	104	18
11/10/2016	113	6	172	580	129	132	89	98	104	18
12/10/2016	115	6	174	589	129	132	89	98	105	18
13/10/2016	114	6	174	593	131	133	90	99	107	18
14/10/2016	114	6	172	595	132	135	90	99	105	18
17/10/2016	114	6	172	612	132	136	90	99	104	18
18/10/2016	115	6	168	617	131	136	90	98	102	18
19/10/2016	116	6	167	611	133	137	91	99	104	18
20/10/2016	109	6	170	609	133	137	90	98	104	18
21/10/2016	108	6	170	601	132	139	90	98	103	18
24/10/2016	108	6	174	612	131	136	89	97	102	18
25/10/2016	108	6	177	606	132	138	91	97	103	18
26/10/2016	108	6	183	615	131	138	89	96	104	18
27/10/2016	107	6	183	617	132	139	91	96	105	18
28/10/2016	108	6	184	622	132	139	90	96	107	18
31/10/2016	108	6	184	626	134	140	90	97	109	18
01/11/2016	106	6	182	624	134	140	90	98	109	18
02/11/2016	105	6	191	626	135	142	91	99	108	18
03/11/2016	105	6	191	623	135	143	92	99	107	18
04/11/2016	104	6	190	628	134	143	90	99	107	18
07/11/2016	105	7	187	625	136	143	91	98	108	18
08/11/2016	107	7	182	616	136	142	91	98	107	18
09/11/2016	106	7	185	613	136	142	90	99	107	18
10/11/2016	109	7	181	604	134	141	91	98	107	17
11/11/2016	110	6	185	597	129	138	89	97	105	18
14/11/2016	112	6	186	595	128	138	89	96	104	18
15/11/2016	111	6	193	622	128	136	88	95	104	18

Date	TRV	FCA	BARC	HSBA	VOW	ALV	BAYN	ML	RI	BZU
16/11/2016	111	6	201	620	128	141	89	96	104	18
17/11/2016	111	6	202	622	127	141	89	96	105	18
18/11/2016	111	6	212	625	126	146	93	96	104	19
21/11/2016	111	7	212	620	127	149	94	95	100	20
22/11/2016	112	7	210	635	129	151	94	95	99	19
23/11/2016	112	7	213	638	129	152	94	95	99	20
25/11/2016	113	7	212	629	130	153	95	97	101	20
28/11/2016	113	7	211	632	128	152	91	97	101	20
29/11/2016	113	7	212	634	129	152	92	98	102	21
30/11/2016	113	7	213	632	128	151	91	99	102	20
01/12/2016	115	7	215	639	130	150	91	100	101	20
02/12/2016	115	7	216	643	130	152	90	101	100	20
05/12/2016	115	7	212	640	132	151	88	101	99	20
06/12/2016	116	7	214	641	133	151	89	101	97	20
07/12/2016	118	7	216	633	133	151	89	101	98	20
08/12/2016	118	7	219	632	131	149	88	101	98	20
09/12/2016	119	7	213	635	130	150	89	101	98	20
12/12/2016	121	7	217	630	129	150	89	101	99	20
13/12/2016	121	7	227	627	128	149	87	101	97	20
14/12/2016	120	7	235	627	128	148	87	101	97	20
15/12/2016	120	8	239	654	130	151	88	101	97	21
16/12/2016	122	8	233	680	131	154	88	102	98	21
19/12/2016	122	8	228	676	134	156	90	105	98	22
20/12/2016	122	8	230	674	136	158	93	105	98	22
21/12/2016	123	8	222	652	135	157	95	106	101	23
22/12/2016	122	8	229	660	135	155	95	105	102	22
23/12/2016	122	8	228	659	137	156	96	104	102	22
27/12/2016	122	8	222	669	137	155	96	105	101	21
28/12/2016	122	8	228	667	138	157	98	105	101	22
29/12/2016	122	8	229	658	140	157	98	105	103	22
30/12/2016	122	9	227	655	140	156	98	105	103	22

The return of the risk free asset, 3 months T-Bill, between the 1 January 2016 and the 31 December 2016, with daily frequency.

3 months T-Bill

Date	Return	Date	Return	Date	Return
05/01/2016	0,022%	05/05/2016	0,021%	02/09/2016	0,033%
06/01/2016	0,020%	06/05/2016	0,019%	06/09/2016	0,033%
07/01/2016	0,021%	09/05/2016	0,020%	07/09/2016	0,033%

Date	Return		Date	Return		Date	Return
08/01/2016	0,020%		10/05/2016	0,019%		08/09/2016	0,032%
11/01/2016	0,020%		11/05/2016	0,024%		09/09/2016	0,034%
12/01/2016	0,021%		12/05/2016	0,024%		12/09/2016	0,035%
13/01/2016	0,021%		13/05/2016	0,026%		13/09/2016	0,035%
14/01/2016	0,022%		16/05/2016	0,027%		14/09/2016	0,037%
15/01/2016	0,025%		17/05/2016	0,029%		15/09/2016	0,036%
19/01/2016	0,024%		18/05/2016	0,028%		16/09/2016	0,033%
20/01/2016	0,026%		19/05/2016	0,028%		19/09/2016	0,029%
21/01/2016	0,026%		20/05/2016	0,030%		20/09/2016	0,030%
22/01/2016	0,028%		23/05/2016	0,031%		21/09/2016	0,030%
25/01/2016	0,031%		24/05/2016	0,033%		22/09/2016	0,030%
26/01/2016	0,031%		25/05/2016	0,035%		23/09/2016	0,022%
27/01/2016	0,031%		26/05/2016	0,035%		26/09/2016	0,018%
28/01/2016	0,032%		27/05/2016	0,033%		27/09/2016	0,018%
29/01/2016	0,035%		31/05/2016	0,031%		28/09/2016	0,025%
01/02/2016	0,033%		01/06/2016	0,032%		29/09/2016	0,026%
02/02/2016	0,035%		02/06/2016	0,034%		30/09/2016	0,027%
03/02/2016	0,034%		03/06/2016	0,030%		03/10/2016	0,026%
04/02/2016	0,033%		06/06/2016	0,029%		04/10/2016	0,029%
05/02/2016	0,029%		07/06/2016	0,030%		05/10/2016	0,032%
08/02/2016	0,030%		08/06/2016	0,028%		06/10/2016	0,033%
09/02/2016	0,032%		09/06/2016	0,028%		07/10/2016	0,032%
10/02/2016	0,030%		10/06/2016	0,024%		10/10/2016	0,033%
11/02/2016	0,031%		13/06/2016	0,026%		11/10/2016	0,033%
12/02/2016	0,028%		14/06/2016	0,026%		12/10/2016	0,035%
16/02/2016	0,030%		15/06/2016	0,027%		13/10/2016	0,037%
17/02/2016	0,030%		16/06/2016	0,027%		14/10/2016	0,030%
18/02/2016	0,030%		17/06/2016	0,026%		17/10/2016	0,032%
19/02/2016	0,030%		20/06/2016	0,027%		18/10/2016	0,034%
22/02/2016	0,031%		21/06/2016	0,027%		19/10/2016	0,034%
23/02/2016	0,033%		22/06/2016	0,028%		20/10/2016	0,035%
24/02/2016	0,032%		23/06/2016	0,027%		21/10/2016	0,035%
25/02/2016	0,033%		24/06/2016	0,027%		24/10/2016	0,034%
26/02/2016	0,032%		27/06/2016	0,031%		25/10/2016	0,033%
29/02/2016	0,033%		28/06/2016	0,027%		26/10/2016	0,034%
01/03/2016	0,033%		29/06/2016	0,027%		27/10/2016	0,033%
02/03/2016	0,033%		30/06/2016	0,026%		28/10/2016	0,030%
03/03/2016	0,036%		01/07/2016	0,026%		31/10/2016	0,030%
04/03/2016	0,028%		05/07/2016	0,026%		01/11/2016	0,034%
07/03/2016	0,029%		06/07/2016	0,028%		02/11/2016	0,035%
08/03/2016	0,032%		07/07/2016	0,028%		03/11/2016	0,037%
09/03/2016	0,029%		08/07/2016	0,027%		04/11/2016	0,038%

Date	Return
10/03/2016	0,030%
11/03/2016	0,032%
14/03/2016	0,033%
15/03/2016	0,034%
16/03/2016	0,034%
17/03/2016	0,034%
18/03/2016	0,031%
21/03/2016	0,029%
22/03/2016	0,030%
23/03/2016	0,031%
24/03/2016	0,030%
28/03/2016	0,030%
29/03/2016	0,030%
30/03/2016	0,029%
05/01/2016	0,023%
01/04/2016	0,020%
04/04/2016	0,021%
05/04/2016	0,023%
06/04/2016	0,023%
07/04/2016	0,023%
08/04/2016	0,023%
11/04/2016	0,023%
12/04/2016	0,023%
13/04/2016	0,023%
14/04/2016	0,022%
15/04/2016	0,023%
18/04/2016	0,022%
19/04/2016	0,022%
20/04/2016	0,022%
21/04/2016	0,021%
22/04/2016	0,023%
25/04/2016	0,023%
26/04/2016	0,023%
27/04/2016	0,025%
28/04/2016	0,024%
29/04/2016	0,024%
02/05/2016	0,022%
03/05/2016	0,022%
04/05/2016	0,022%

Date	Return
11/07/2016	0,029%
12/07/2016	0,028%
13/07/2016	0,031%
14/07/2016	0,029%
15/07/2016	0,031%
18/07/2016	0,032%
19/07/2016	0,032%
20/07/2016	0,032%
21/07/2016	0,031%
22/07/2016	0,032%
25/07/2016	0,032%
26/07/2016	0,033%
27/07/2016	0,032%
28/07/2016	0,031%
29/07/2016	0,031%
01/08/2016	0,025%
02/08/2016	0,028%
03/08/2016	0,029%
04/08/2016	0,029%
05/08/2016	0,028%
08/08/2016	0,026%
09/08/2016	0,028%
10/08/2016	0,031%
11/08/2016	0,029%
12/08/2016	0,028%
15/08/2016	0,028%
16/08/2016	0,029%
17/08/2016	0,031%
18/08/2016	0,027%
19/08/2016	0,030%
22/08/2016	0,030%
23/08/2016	0,030%
24/08/2016	0,029%
25/08/2016	0,030%
26/08/2016	0,031%
29/08/2016	0,033%
30/08/2016	0,034%
31/08/2016	0,033%
01/09/2016	0,033%

Date	Return
07/11/2016	0,038%
08/11/2016	0,041%
09/11/2016	0,043%
10/11/2016	0,045%
11/11/2016	0,048%
14/11/2016	0,055%
15/11/2016	0,051%
16/11/2016	0,047%
17/11/2016	0,044%
18/11/2016	0,044%
21/11/2016	0,046%
22/11/2016	0,049%
23/11/2016	0,051%
25/11/2016	0,049%
28/11/2016	0,048%
29/11/2016	0,048%
30/11/2016	0,048%
01/12/2016	0,048%
02/12/2016	0,049%
05/12/2016	0,049%
06/12/2016	0,049%
07/12/2016	0,052%
08/12/2016	0,051%
09/12/2016	0,054%
12/12/2016	0,051%
13/12/2016	0,054%
14/12/2016	0,055%
15/12/2016	0,051%
16/12/2016	0,051%
19/12/2016	0,052%
20/12/2016	0,052%
21/12/2016	0,052%
22/12/2016	0,051%
23/12/2016	0,052%
27/12/2016	0,051%
28/12/2016	0,053%
29/12/2016	0,047%
30/12/2016	0,051%

APPENDIX D

The portfolios' returns between the January 2016 and December 2016, daily series of the three portfolios, Passive, Active and Sentiment based.

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
05/01/2016	0,0114	0,0113	0,0113
06/01/2016	-0,0188	-0,0228	-0,0228
07/01/2016	-0,0176	-0,0702	-0,0702
08/01/2016	-0,0163	-0,0056	-0,0056
11/01/2016	0,0066	0,0033	0,0033
12/01/2016	0,0035	0,0041	0,0041
13/01/2016	-0,0078	-0,0097	-0,0097
14/01/2016	-0,0192	-0,0233	-0,0233
15/01/2016	-0,0275	-0,0625	-0,0040
19/01/2016	0,0095	0,0075	-0,0524
20/01/2016	-0,0123	0,0190	0,0063
21/01/2016	0,0081	-0,0396	0,0124
22/01/2016	-0,0857	0,0224	-0,0472
25/01/2016	0,0124	0,0233	0,0533
26/01/2016	-0,0016	-0,0291	0,0689
27/01/2016	-0,0067	-0,0525	0,0094
28/01/2016	0,0129	0,0414	0,0302
29/01/2016	0,0083	0,0042	-0,0118
01/02/2016	-0,0080	0,0110	-0,0515
02/02/2016	0,0046	-0,0639	0,0241
03/02/2016	-0,0079	-0,0086	-0,0159
04/02/2016	0,0105	-0,0052	-0,0481
05/02/2016	-0,0085	0,0454	-0,0641
08/02/2016	0,0012	-0,0242	0,0355
09/02/2016	-0,0071	-0,0164	-0,0198
10/02/2016	-0,0050	-0,0433	-0,0336
11/02/2016	-0,0102	-0,0126	-0,0043
12/02/2016	0,0123	-0,0267	0,0187
16/02/2016	-0,0054	0,1082	-0,0677
17/02/2016	0,0131	0,0388	0,0147
18/02/2016	0,0158	0,0109	0,0429
19/02/2016	0,0017	0,0038	0,0201
22/02/2016	-0,0149	-0,0287	0,0333
23/02/2016	-0,0063	-0,0044	0,0215
24/02/2016	-0,0026	-0,0124	-0,0107

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
05/07/2016	0,0104	0,0340	0,0324
06/07/2016	-0,0094	-0,0059	-0,0221
07/07/2016	0,0022	0,0052	-0,0148
08/07/2016	0,0095	0,0338	0,0107
11/07/2016	0,0195	0,0081	0,0085
12/07/2016	0,0076	0,0130	-0,0048
13/07/2016	0,0038	0,0087	0,0233
14/07/2016	-0,0071	0,0233	0,0237
15/07/2016	0,0044	0,0106	0,0240
18/07/2016	0,0134	0,0261	0,0055
19/07/2016	0,0069	-0,0100	-0,0144
20/07/2016	0,0096	0,0028	-0,0035
21/07/2016	-0,0063	-0,0178	-0,0094
22/07/2016	0,0090	0,0073	-0,0029
25/07/2016	0,0006	0,0226	0,0073
26/07/2016	0,0004	0,0108	0,0226
27/07/2016	0,0066	0,0025	0,0108
28/07/2016	-0,0098	-0,0164	0,0025
29/07/2016	0,0012	0,0184	-0,0164
01/08/2016	-0,0009	-0,0050	0,0184
02/08/2016	0,0003	0,0067	-0,0050
03/08/2016	0,0030	0,0084	0,0067
04/08/2016	0,0050	-0,0297	0,0084
05/08/2016	-0,0075	0,0065	0,0200
08/08/2016	0,0008	0,0043	-0,0220
09/08/2016	-0,0004	-0,0090	0,0054
10/08/2016	0,0047	0,0377	-0,0006
11/08/2016	0,0099	0,0437	-0,0132
12/08/2016	0,0047	0,0044	0,0206
15/08/2016	0,0083	0,0579	0,0241
16/08/2016	-0,0008	-0,0073	0,0284
17/08/2016	-0,0085	-0,0124	0,0298
18/08/2016	-0,0036	-0,0079	0,0252
19/08/2016	0,0046	0,0055	0,0124
22/08/2016	-0,0018	0,0168	0,0055

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
25/02/2016	-0,0013	-0,0118	0,0115
26/02/2016	-0,0182	-0,0206	-0,0250
29/02/2016	0,0078	0,0215	-0,0328
01/03/2016	0,0185	0,0362	0,0118
02/03/2016	-0,0121	-0,0029	0,0352
03/03/2016	0,0157	0,0170	-0,0027
04/03/2016	0,0134	0,0263	0,0335
07/03/2016	-0,0123	-0,0171	0,0171
08/03/2016	0,0030	0,0027	-0,0101
09/03/2016	0,0070	-0,0104	0,0158
10/03/2016	-0,0003	-0,0023	-0,0095
11/03/2016	0,0008	0,0049	-0,0256
14/03/2016	-0,0012	-0,0072	0,0041
15/03/2016	0,0048	0,0043	0,0043
16/03/2016	-0,0120	-0,0132	-0,0132
17/03/2016	0,0115	0,0417	0,0417
18/03/2016	0,0003	0,0274	0,0274
21/03/2016	0,0035	0,0011	0,0166
22/03/2016	-0,0037	-0,0096	-0,0106
23/03/2016	-0,0002	-0,0035	-0,0054
24/03/2016	0,0007	0,0177	0,0153
28/03/2016	0,0041	0,0007	0,0032
29/03/2016	0,0000	-0,0171	0,0281
30/03/2016	-0,0094	0,0013	0,0125
31/03/2016	0,0034	0,0017	0,0008
01/04/2016	-0,0003	-0,0039	-0,0039
04/04/2016	0,0003	0,0065	0,0065
05/04/2016	-0,0075	0,0015	0,0015
06/04/2016	0,0028	0,0089	0,0089
07/04/2016	0,0125	-0,0107	-0,0107
08/04/2016	0,0021	-0,0127	-0,0127
11/04/2016	-0,0095	0,0067	0,0067
12/04/2016	0,0065	-0,0225	-0,0225
13/04/2016	-0,0022	0,0001	0,0001
14/04/2016	0,0073	-0,0018	-0,0018
15/04/2016	0,0065	-0,0010	-0,0010
18/04/2016	-0,0069	0,0460	0,0460
19/04/2016	0,0117	-0,0311	-0,0311
20/04/2016	0,0011	-0,0386	-0,0386
21/04/2016	-0,0089	0,0076	0,0076
22/04/2016	0,0078	0,0153	0,0153

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
23/08/2016	-0,0003	-0,0028	0,0168
24/08/2016	0,0053	0,0020	-0,0007
25/08/2016	-0,0042	0,0051	0,0105
26/08/2016	0,0011	0,0013	0,0041
29/08/2016	0,0018	0,0052	-0,0092
30/08/2016	-0,0020	-0,0091	0,0238
31/08/2016	-0,0047	-0,0075	-0,0186
01/09/2016	0,0014	0,0245	-0,0075
02/09/2016	-0,0009	0,0137	0,0245
06/09/2016	0,0078	0,0083	0,0137
07/09/2016	-0,0019	0,0156	0,0083
08/09/2016	0,0048	-0,0162	0,0156
09/09/2016	-0,0145	-0,0331	-0,0162
12/09/2016	0,0097	0,0136	-0,0331
13/09/2016	-0,0033	-0,0187	0,0136
14/09/2016	-0,0045	-0,0322	-0,0187
15/09/2016	0,0076	0,0233	-0,0322
16/09/2016	0,0025	0,0285	0,0163
19/09/2016	-0,0038	0,0094	-0,0087
20/09/2016	-0,0045	-0,0269	0,0025
21/09/2016	0,0025	-0,0183	0,0079
22/09/2016	-0,0023	0,0351	0,0224
23/09/2016	0,0008	0,0147	0,0217
26/09/2016	-0,0026	-0,0321	-0,0180
27/09/2016	0,0019	0,0432	-0,0097
28/09/2016	0,0078	0,0101	0,0143
29/09/2016	-0,0051	0,0034	0,0014
30/09/2016	0,0026	-0,0280	-0,0238
03/10/2016	0,0025	-0,0299	-0,0280
04/10/2016	-0,0044	-0,0133	-0,0299
05/10/2016	-0,0024	0,0337	-0,0133
06/10/2016	-0,0058	0,0200	0,0337
07/10/2016	0,0056	0,0047	0,0200
10/10/2016	0,0004	-0,0058	0,0047
11/10/2016	0,0002	0,0089	-0,0058
12/10/2016	0,0020	0,0065	0,0089
13/10/2016	0,0015	-0,0020	0,0065
14/10/2016	0,0020	0,0063	-0,0020
17/10/2016	0,0019	0,0082	0,0063
18/10/2016	-0,0031	0,0249	0,0082
19/10/2016	0,0004	-0,0319	0,0249

Portfolio Allocation			
	Passive	**Active**	**Sentiment**
25/04/2016	-0,0209	0,0171	0,0171
26/04/2016	-0,0064	-0,0247	-0,0247
27/04/2016	0,0039	0,0325	0,0325
28/04/2016	-0,0075	-0,0248	-0,0248
29/04/2016	0,0030	0,0289	0,0289
02/05/2016	0,0004	-0,0159	-0,0159
03/05/2016	-0,0020	-0,0073	-0,0073
04/05/2016	-0,0027	-0,0369	-0,0369
05/05/2016	-0,0091	-0,0039	-0,0039
06/05/2016	-0,0051	-0,0129	-0,0129
09/05/2016	-0,0070	-0,0310	-0,0310
10/05/2016	0,0009	0,0151	0,0151
11/05/2016	0,0054	0,0131	0,0131
12/05/2016	0,0071	-0,0608	-0,0608
13/05/2016	-0,0035	-0,0073	-0,0073
16/05/2016	0,0112	0,0012	0,0012
17/05/2016	-0,0076	-0,0136	-0,0136
18/05/2016	-0,0090	-0,0008	-0,0008
19/05/2016	0,0033	-0,0408	-0,0408
20/05/2016	0,0051	-0,0042	-0,0042
23/05/2016	-0,0089	0,0065	0,0065
24/05/2016	0,0087	-0,0002	-0,0002
25/05/2016	-0,0025	-0,0240	-0,0240
26/05/2016	0,0059	-0,0026	-0,0026
27/05/2016	-0,0024	-0,0312	0,0371
31/05/2016	0,0078	0,0072	-0,0328
01/06/2016	-0,0060	-0,0149	-0,0149
02/06/2016	-0,0032	-0,0256	-0,0256
03/06/2016	-0,0002	-0,0026	-0,0026
06/06/2016	0,0055	0,0197	0,0197
07/06/2016	-0,0009	-0,0034	-0,0034
08/06/2016	0,0012	0,0210	0,0210
09/06/2016	0,0033	0,0185	0,0185
10/06/2016	-0,0052	0,0233	0,0233
13/06/2016	-0,0020	-0,0169	-0,0046
14/06/2016	0,0086	0,0280	0,0085
15/06/2016	-0,0078	-0,0005	-0,0117
16/06/2016	-0,0077	-0,0032	-0,0102
17/06/2016	0,0119	0,0235	0,0055
20/06/2016	0,0031	0,0229	0,0041
21/06/2016	0,0051	0,0130	0,0015

Portfolio Allocation			
	Passive	**Active**	**Sentiment**
20/10/2016	-0,0057	-0,0047	-0,0319
21/10/2016	-0,0073	-0,0211	-0,0047
24/10/2016	-0,0012	0,0352	-0,0211
25/10/2016	0,0074	0,0032	0,0352
26/10/2016	0,0044	0,0262	0,0032
27/10/2016	0,0010	-0,0221	0,0262
28/10/2016	0,0037	-0,0108	-0,0221
31/10/2016	0,0042	-0,0009	-0,0108
01/11/2016	-0,0016	-0,0054	-0,0009
02/11/2016	0,0026	-0,0056	-0,0054
03/11/2016	-0,0004	-0,0111	-0,0056
04/11/2016	-0,0014	-0,0005	-0,0111
07/11/2016	0,0101	0,0353	-0,0005
08/11/2016	0,0012	0,0036	0,0065
09/11/2016	0,0028	-0,0372	-0,0045
10/11/2016	-0,0055	-0,0119	-0,0035
11/11/2016	0,0065	0,0134	-0,0188
14/11/2016	-0,0039	-0,0056	-0,0129
15/11/2016	0,0061	0,0409	0,0008
16/11/2016	0,0082	-0,0260	0,0030
17/11/2016	-0,0010	-0,0030	0,0264
18/11/2016	0,0088	-0,0053	0,0197
21/11/2016	0,0064	0,0060	-0,0053
22/11/2016	0,0062	0,0452	0,0060
23/11/2016	0,0015	-0,0248	0,0452
25/11/2016	0,0047	-0,0092	-0,0248
28/11/2016	-0,0005	-0,0027	-0,0092
29/11/2016	0,0028	0,0118	-0,0027
30/11/2016	-0,0052	-0,0038	0,0118
01/12/2016	0,0004	-0,0097	-0,0038
02/12/2016	0,0023	-0,0085	-0,0097
05/12/2016	-0,0006	0,0155	0,0030
06/12/2016	0,0050	0,0108	0,0116
07/12/2016	0,0068	0,0169	-0,0045
08/12/2016	-0,0039	0,0070	0,0490
09/12/2016	0,0040	0,0130	0,0214
12/12/2016	-0,0074	-0,0321	0,0180
13/12/2016	0,0018	-0,0004	0,0290
14/12/2016	-0,0035	-0,0093	0,0346
15/12/2016	0,0111	0,0272	-0,0178
16/12/2016	0,0069	0,0174	-0,0551

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
22/06/2016	0,0048	-0,0098	0,0184
23/06/2016	0,0059	0,0109	0,0186
24/06/2016	-0,0077	-0,0509	-0,0695
27/06/2016	0,0082	-0,0539	-0,0415
28/06/2016	0,0122	0,0136	0,0253
29/06/2016	-0,0016	0,0445	0,0771
30/06/2016	0,0065	0,0067	0,0589
01/07/2016	0,0224	0,0283	0,0402

Portfolio Allocation			
Date	**Passive**	**Active**	**Sentiment**
19/12/2016	0,0078	0,0065	-0,0367
20/12/2016	0,0063	0,0055	0,0016
21/12/2016	-0,0024	-0,0107	0,0025
22/12/2016	0,0012	0,0295	0,0133
23/12/2016	-0,0012	-0,0023	-0,0039
27/12/2016	-0,0014	0,0250	-0,0044
28/12/2016	0,0046	-0,0073	0,0100
29/12/2016	0,0012	-0,0147	0,0129
30/12/2016	-0,0010	-0,0027	0,0096

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