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**Dynamic Contracting in Long Term Care (LTC)
Insurance Markets**

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Contents

1	Introduction	1
1.1	Related Literature	3
2	Theory	5
3	The Model	6
3.1	Environment and Contracts	6
3.2	Solving the Model	8
3.3	Characterization of Equilibrium Contracts	9
3.4	Numerical Illustration	11
4	Persistent health risk	13
4.1	Persistence in Risk Class i	13
4.2	Solving the Model	14
4.3	Characterization of Equilibrium Contracts	15
4.4	Complete Persistence in Risk Class i	17
5	Conclusion	17
	References	23

Dynamic Contracting in Long Term Care (LTC) Insurance Markets

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We study the role of dynamic contracting frictions in limiting the size of the private market for long term care (LTC) insurance. Building on [Harris and Holmstrom \[1982\]](#) and [Hendel and Lizzeri \[2003\]](#), we develop a multi period model of the LTC insurance market with symmetric learning and one-sided commitment. Insurers can commit to offering future premia but consumers may lapse to contracts offered by competitors after underwriting. We consider a set of market features and characterize the equilibrium contracts for each. All equilibrium contracts feature front loading: higher early premia finance later premium caps that insure against the possibility of entering a higher risk class and receive higher premia (reclassification risk). We show that longer term contracts provide better insurance against reclassification risk, but at the cost of heavier early period loading. A numerical illustration confirms these results. We then show that when insurers are allowed to adjust premia based on past health realizations, specifically because health class assignment is assumed to be persistent, buyers with a history of sickness remain in riskier pools and receive less protection against future reclassification risk.

1. INTRODUCTION

The market for private long term care (LTC) insurance, designed to cover late life health expenditures such as nursing home costs, has received growing attention in recent years. The United States provides a useful benchmark for evaluating the state of LTC insurance markets, since LTC spending as a share of GDP is broadly comparable across aging OECD economies, unlike acute health care, where cross country variation is larger due to the availability of universal health care programs [[Brown and Finkelstein, 2011](#)]. In 2008, U.S. LTC expenditures reached \$203 billion, about 8.7% of total health spending and roughly 1.4% of GDP [[Centers for Medicare and Medicaid Services, 2010](#)]. Furthermore, because LTC utilization rises sharply with age [[Brown and Finkelstein, 2011](#)], the projected doubling of the 80+ population between 2010 and 2050 [[OECD Health Policy Studies, 2011](#)] suggests a significant rise in future care needs. On an individual level, the average nursing

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home in the US charges nearly \$6,000 per month [MetLife Mature Market Institute, 2009]. Between 35% and 50% of 65-year-olds will enter a nursing home at some point; among those who do, 10–20% remain for more than five years, implying a sizable right tail of expenditures [Brown and Finkelstein, 2009]. Despite these risks, private coverage remains limited: only about 14% of individuals aged 60 and over hold LTC insurance according to the 2008 Health and Retirement Study. Additionally, coverage among 60–69 year-olds rose from roughly 10% in the late 1990s to just over 13% by [(AHIP) America's Health Insurance Plans, 2007, Brown and Finkelstein, 2011].

To conclude, as populations age and longevity increases, long-term care (LTC) expenditures constitute a major, largely uninsured financial risk for older households. Particularly, despite large and uncertain costs, private coverage remains limited. This combination of substantial exposure and low take-up motivate a deeper analysis of the market for LTC insurance.

The objective of this paper is to develop a theoretical framework for analyzing the structure of long-term insurance contracts in LTC markets, with the aim of explaining the persistently small size of this market. The literature has proposed a variety of explanations for this so-called “LTC puzzle.” Building on the findings of Hendel and Lizzeri [2003] and Finkelstein et al. [2005], we explore the role of dynamic contracting problems in limiting the size of the private LTC insurance market.

Research in dynamic contracting has shown that commitment is a key element shaping long term contracts structure. For instance, in long-term insurance markets, consumers benefit not only from one period insurance, but also from insurance against the possibility of being reclassified into a higher risk group and facing sharply increasing premia, referred to as reclassification risk [Hirshleifer, 1971, Finkelstein et al., 2005]. Full insurance against reclassification risk implies equal marginal utility of consumption across time and risk groups. However, without consumer commitment, any attempt to provide long-term reclassification risk insurance is undermined by profitable offers from competitors in the future. Front loading, namely increasing premiums in early periods allows for full insurance against reclassification risk.

The private long-term care (LTC) insurance market is of particular interest for studying dynamic contracting issues. Four features make it an ideal laboratory. First, as health worsens over years, LTC needs become more likely, making reclassification risk quantitatively important. Second, both sides of the market learn from public information (ageing, mandatory medical diagnoses), so there is symmetric learning. Third, there is one-sided commitment by the insurance sellers. Consumers are allowed to lapse to competitive contracts at any time after underwriting. Lastly, the market is largely unregulated [Brown and Finkelstein, 2009], unlike the health and life insurance markets.

Although much attention has already been devoted to exploring the effects of a lack of commitment in two-period models of dynamic contracting [Hendel, 2017], less attention has been devoted to settings with longer time horizons or where past health status affects future reclassification probabilities. This gap is particularly relevant for the market for long-term care (LTC) insurance, where contracts cover decades of risk for the elderly and, in the absence of regulation, insurers may adjust premia based on health histories. Understanding how front loading and reclassification-risk insurance

interact in environments with longer horizons and with persistent health states is crucial for evaluating the design of health insurance contracts in light of growing demand for long-term care.

We therefore expand the analysis in two directions. Our first innovation is to extend the horizon from two to three periods. We show that longer contracts provide stronger protection against reclassification risk, particularly in later periods, but require front loading in all earlier periods.

Our second innovation is to relax the assumption that health is independent across periods and instead allow reclassification probabilities to depend on earlier sickness realizations. A history of sickness increases the likelihood of entering a higher risk class, which worsens the risk pool in later periods. This assumption motivates the key mechanism driving the novel results of the model: insurers can adjust third period premia based on the health class realized in period 2. As a result, equilibrium contracts imply higher capped premia in the third period for buyers with a history of higher risk. In the extreme case where health class is fully persistent after period two, reclassification risk at period three vanishes, so the optimal contract reduces to spot coverage from period two onward, with front loading only in the first period.

The remainder of the paper is structured as follows. In Section 2 we summarize the existing findings on dynamic contracting and how our work contributes to this literature. In Section 3 we adapt the framework of [Hendel and Lizzeri \[2003\]](#) for the life insurance market. Section 3.2 characterizes the equilibrium contracts. Section 3.4 complements the theoretical analysis in Section 3 with a numerical illustration of the equilibrium contracts derived. This exercise clearly illustrates the trade off between front loading and insurance against reclassification risk. Furthermore, it underscores the testable implications of model. In Section 4, we examine how persistence in health status affects the extent of reclassification risk insurance.

1.1 *Related Literature*

This paper relates to two strands of work: research on the limited size of the private long term care (LTC) insurance market, and theoretical research on dynamic contracting under symmetric learning and one-sided commitment.

The need for private insurance is motivated by the substantial risk of requiring long-term care in old age. Some scholars have suggested that the prospect of such expenses helps to explain why many elderly households retain considerable assets until very late in life [[De Nardi et al., 2010](#)]. Substantial empirical evidence shows that the private LTC insurance coverage among the elderly is extremely limited despite sizable expenditure risk, particularly in the United States. Moreover, private insurance covers a minor share of total expected LTC spending, leaving many of the expenses to be paid out-of-pocket [[Brown and Finkelstein, 2009, 2011](#)].

Several demand and supply side explanations for this phenomenon have been proposed. On the demand side, [Pauly \[1990\]](#) first hypothesized that this phenomenon might be rational and explainable by the presence of means tested public coverage. Later, [Brown and Finkelstein \[2008\]](#) established that in the US, Medicaid (means tested public health-care) implicitly taxes private policies, reducing their value. Limited consumer knowledge

or rationality (as found by [Lusardi and Mitchell \[2007\]](#)) could play an important role in reducing LTC coverage. Bequest motives may further reduce demand by increasing the value of precautionary saving and self insurance [[Lockwood, 2018](#)]. State-dependent preferences provide another explanation: if buyers have lower marginal utility of consumption when in care, they may rationally choose not to purchase insurance [[De Donder and Leroux, 2021](#)].

On the supply side, evidence suggests that insurers charge extremely high loads on LTC contracts [[Brown and Finkelstein, 2007](#)]. Such pricing could be driven by high transaction costs or by imperfect competition in the market, though, to our knowledge, the role of market structure has not yet been examined in detail. Asymmetric information is another important factor. [Finkelstein and McGarry \[2006\]](#) show that there are more levels of private information in the LTC insurance market: individuals differ not only in risk type but also in insurance preferences. They find that people with higher insurance coverage are not necessarily higher risk. Furthermore, there is evidence of moral hazard in the LTC market [[Konetzka et al., 2019](#)], and [Courbage and Zweifel \[2011\]](#) develop a model to understand how moral hazard can reduce insurance uptake. Lastly, dynamic contracting problems might help explain the LTC puzzle. Individuals who learn that their health is better than expected lapse from long term contracts, thus raising the average risk of the remaining pool and hence the premium. [Finkelstein et al. \[2005\]](#) show that individuals who exit long term contracts are ex post lower risk, as predicted by theoretical models [[Hendel and Lizzeri, 2003](#)].

There has been a sustained effort to develop theoretical models, or to adapt existing ones, to the private LTC setting. Research on dynamic adverse selection was pioneered by [Harris and Holmstrom \[1982\]](#). Their work has spurred interest in understanding the role of commitment and learning in long term contracts. The relevant literature can be reviewed in [Hendel \[2017\]](#). For our purposes, we build most directly on the work of [Hendel and Lizzeri \[2003\]](#), who analyze dynamic adverse selection in life insurance markets.

Theoretical models of dynamic contracting with, one sided commitment and symmetric learning share several common features. Symmetric and gradual learning introduces the risk of lapsation in the future, whereas the absence of consumer commitment prevents full insurance. As a result, the optimal long term contracts must feature front loading of premia that finance delayed "discounts" on premia for the future. The highest risk types prefer staying in the long term contract, whereas lowest risk types are indifferent between staying in the contract and lapsing to competing spot contracts. [Hendel and Lizzeri \[2003\]](#) develop a model of this kind for the life insurance market and test it empirically, whereas [Finkelstein et al. \[2005\]](#) examine the same mechanism in the context of LTC insurance.

We extend this literature by deriving the optimal contract in the LTC setting when the time horizon is lengthened and when premia can be conditioned on health history. The first extension clarifies how the trade-off between front loading and insurance shapes equilibrium contracts. In particular, longer time horizons increase the degree of front loading required in early periods to finance higher sickness risks in later life. The second extension shows the effects of allowing premia to depend on past health status. Since

individuals with a history of sickness are more likely to be reclassified into higher-risk groups. As a result, equilibrium contracts require additional front loading and can only sustain higher capped premia for previously high-risk buyers in later periods.

2. THEORY

Before introducing the model, we briefly discuss its main features and how it departs from the framework of [Hendel and Lizzeri \[2003\]](#), which serves as our starting point. As in their model, our setting assumes symmetric learning. Buyers and sellers acquire information simultaneously over time, which is typical in insurance markets. We further assume one sided commitment: insurers can credibly commit to the contracts they offer, but consumers cannot be forced to remain in a contract if cheaper alternatives become available in the future. Lastly, buyers differ in their income process, hence in their ability to front load. Note that there is no asymmetric information on either side of the market.

Consider a baseline two period model. At the start of the second period new information about sickness risk is revealed to all parties. Once risk classes are known, high-risk buyers face the prospect of sharply higher premia in period two. A contract that fully insures reclassification risk sets second period premia independent of the realized health state. If consumer commitment were possible, insurers would offer future pooled premia reflecting the average risk of the pool. In the absence of such commitment, healthier types lapse to more favorable spot contracts. Therefore period two premia must not exceed the actuarially fair premia; otherwise, healthy types would lapse to spot contracts offered by competing firms. However, such a contract yields negative profits in the second period, which must be financed by pricing above the actuarially fair premium in the period one, creating the need for front loading in equilibrium. However, anticipating losses is costly, which limits the ability of buyers to fully insure reclassification risk. There might be several reasons why borrowing is costly such as liquidity constraints or credit constraints. Buyers with a lower cost of front loading are better able to insure reclassification risk by shifting resources from the first period to the second. We model this heterogeneity in the spirit of [Hendel and Lizzeri \[2003\]](#). Note that liquidity limitations lead to less than full insurance even in a competitive setting with no asymmetry of information.

We build on their work in two ways. First we extend their model by lengthening the time horizon from two periods to three periods. At the start of the second and third period new information about sickness risk, completely independent on previous periods, is revealed to all parties. Moreover, health worsens over time, thereby increasing the risk of sickness in each period. For this reason actuarially fair premia greatly increase between periods. Extending the model to three periods exacerbates the role of front loading. As usual, in any equilibrium contract expected losses from the cap applied in the latter periods must be funded through front loading in the preceding periods. Moreover, low-risk states pay their spot price, while higher-risk states share a common cap below spot. Just like in the baseline two period model, limits on liquidity in period one and two might prevent buyers from fully front loading. What sets this new model apart from the two-period case is that partial insurance against reclassification risk results in more classes paying a capped premium in period 3 than in period 2. This arises because

equilibrium contracts impose a common cap that is higher in period 2 than in period 3. Since sickness risk increases over time, spot premia are higher in period 3, making the subsidized premium more attractive for those types. As a result, more states receive insurance against reclassification risk in period 3 than in period 2. Intuitively, as sickness risk increases over time, it is easier to provide reclassification risk insurance later in life.

Second, we extend the three-period model by allowing reclassification probabilities from period 2 to period 3 to depend on the risk class realized in period 2. In this setting, individuals with a history of sickness are more likely to be reclassified into a higher-risk group in the final period than those with a healthy history. This assumption provides a formal rationale for allowing firms to condition premia on past health status, which is the key feature of this second model. As in the previous extension, longer horizons increase the degree of front loading required in early periods to finance higher sickness risks later in life. With persistence, however, the reclassification probabilities in period 3 depend on the class realized in period 2. Being assigned to a high risk class in period 2 increases the likelihood of being reclassified into a higher-risk group in period 3. Buyers with a history of being in a high-risk class thus enter a worse risk pool in the final period, so the capped premium they face is higher and fewer states are subsidized. This reflects the fact that these buyers enter a worse risk pool in the final period. Moreover, because they already belong to the higher-risk group in period 2, they must devote more resources to covering sickness events at that stage, leaving less scope for front loading.

3. THE MODEL

3.1 *Environment and Contracts*

We retain several elements from the model adopted by [Hendel and Lizzeri \[2003\]](#). The information structure is the same: learning is symmetric and both consumers and insurers observe the realized risk class at the start of each period. The market environment is also the same: insurers are perfectly competitive and commit to future premia and benefits, while consumers may lapse to spot contracts after health states are revealed. The timing of contracting within each period is also identical. Risk classes are assigned in the same way, through a discrete probability distribution independent of health status.

Our model departs in a few respects. First, the model is extended from two to three periods. Second, in the event of sickness buyers do not die, as in their model, but instead incur a monetary loss $m > 0$. Third, utility is given by a single concave, twice-differentiable function $u(c)$ that is the same in sickness and in health.

Consider a three period model of the market for long-term care (LTC) insurance with insurance sellers and buyers who wish to insure a loss from sickness $m > 0$ in each period. Buyers' utility is given by $u(c)$ over consumption c both in the event of sickness or not. $u(c)$ is assumed to be both concave and twice differentiable.

The timing of the model is as follows:

1. *Period* $t = 1$.
 - 1.1. Insurers offer contracts
 - 1.2. Consumers choose a contract

- 1.3. Consumers either get sick or stay healthy;
2. *Period* $t = 2$.
 - 2.1. Uncertainty about health state is realized (and observed by all)
 - 2.2. Insurers offer new two period contracts
 - 2.3. Consumers choose whether to lapse or stick with the contract
 - 2.4. Consumers either get sick or stay healthy;
3. *Period* $t = 3$ is analogous to period 2.

In the first period, the sickness event occurs with probability q_1 , identical for all buyers. At the start of the second period, each individual is assigned to a health state i , where the probability of the event is q_2^i . In the third period, a new risk state j is realized, and the event occurs with probability q_3^j . Lower states have lower sickness probability: $q_t^1 < q_t^2 < \dots < q_t^N$. Sickness risk also increases over time, so that $q_2^1 > q_1$ and $q_3^1 > q_2^N$. Let $S = \{1, \dots, N\}$ be the set of risk states s and let $\pi = (\pi_1, \dots, \pi_N)$ be a probability vector with $\pi_i \geq 0$ and $\sum_{i=1}^N \pi_i = 1$. Here, π_i represents the probability of being reclassified in risk class i in both period 2 and 3. This uncertainty gives rise to reclassification risk. Learning about risk is symmetric: both consumers and insurers observe the realized risk state at the start of each period.

We assume there is perfect competition between insurance companies. We further assume one sided commitment, insurers can commit to future premia and benefits but consumers cannot. After health state is realized in period 2 and 3 consumers choose whether to stay in the long term contract or lapse to a spot contract. The set of first period contracts is the set of contracts to which the firm commits that end when the consumer stops paying premia, without penalty in case of termination.

A first period contract consists of premium Q_1 and benefit F_1 in the first period, and a vector $\{(Q_2^i, F_2^i)\}_{i=1}^N$ in the second period and likewise a vector $\{(Q_3^j, F_3^j)\}_{j=1}^N$ in the third period. These contracts specify the premium Q to be paid if the LTC event does not occur and the benefit F to be received in the event of sickness m . A second period contract for risk class i offers (Q_2^i, F_2^i) in period 2 and a vector $\{(Q_3^j, F_3^j)\}_{j=1}^N$ for period 3. Lastly, a third period contract consists of premium and benefit (Q_3^j, F_3^j) indexed by health status. We assumed symmetric learning, therefore insurers can offer different contracts to buyers in different classes at period 2 and 3. These later contracts are referred to as spot contracts.

Full insurance against reclassification risk is, in theory, possible. In an ideal world, buyers would front load enough to equalize premia across all risk classes within each period, which corresponds to equalizing marginal utilities of consumption across health states within periods. Under such contracts, consumers would have no incentive to lapse in the future and would be fully insured against reclassification risk, and all buyers would select the same contract.. The equalized premium would necessarily lie below the actuarially fair premium in each state to prevent lapsation.

To explain the observed variety of contracts in the LTC market, it is necessary to explain why consumers cannot front load sufficiently. Following [Hendel and Lizzeri \[2003\]](#), we assume incomplete capital markets: consumers cannot freely borrow against future income. Endowments are given by

$$(y_1, y_2, y_3) = (y - a, y + a - b, y + b),$$

where the parameters $a, b \geq 0$ capture heterogeneity in liquidity constraints. A larger a, b corresponds to tighter budget in period 1 and 2 respectively and thus a higher private cost of shifting resources forward.

A potential loss of size m occurs in the event of sickness, representing the costs of care for LTC. The sickness cost m is independent the period in which the expense incurred.

Given endowment y_t and period loss m , consumption is

$$c_t = \begin{cases} y_t - Q, & \text{if no sickness occurs at } t, \\ F - m, & \text{if sickness occurs at } t. \end{cases}$$

3.2 Solving the Model

To characterize competitive equilibrium contracts, we first formulate the optimization problem as a constrained utility maximization problem. In a competitive equilibrium, allocations must maximize expected lifetime utility subject to a zero profit condition and no lapsation constraints constraints at period two and three.

The equilibrium contract is therefore obtained as the solution to a constrained maximization problem in which the planner chooses premiums and benefits $\{(Q_1, F_1); (Q_t^i, F_t^i)\}$ to maximize consumers' expected utility:

$$\begin{aligned} EU &= (1 - q_1) u((y - a) - Q_1) + q_1 u(-m + F_1) \\ &+ \sum_{i=1}^N \pi_i \left[(1 - q_2^i) u((y + a - b) - Q_2^i) + q_2^i u(-m + F_2^i) \right] \\ &+ \sum_{j=1}^N \pi_j \left[(1 - q_3^j) u((y + b) - Q_3^j) + q_3^j u(-m + F_3^j) \right] \end{aligned} \quad (1)$$

subject to the Zero Profit condition:

$$(1 - q_1)Q_1 - q_1 F_1 + \sum_{i=1}^N \pi_i \left[(1 - q_2^i)Q_2^i - q_2^i F_2^i \right] + \sum_{j=1}^N \pi_j \left[(1 - q_3^j)Q_3^j - q_3^j F_3^j \right] = 0 \quad (2)$$

and the No-Lapsation constraints such that for all $\tilde{Q}_2^i, \tilde{F}_2^i$, and $\{(\tilde{Q}_3^j, \tilde{F}_3^j)\}_{j=1}^N$ with

$$(1 - q_2^i)\tilde{Q}_2^i - q_2^i\tilde{F}_2^i + \sum_{j=1}^N \pi_j \left[(1 - q_3^j)\tilde{Q}_3^j - q_3^j\tilde{F}_3^j \right] > 0, \quad \forall i \in \{1, \dots, N\}, \quad (3)$$

$$(1 - q_3^j)\tilde{Q}_3^j - q_3^j\tilde{F}_3^j > 0, \quad \forall j \in \{1, \dots, N\} \quad (4)$$

in any equilibrium contract:

$$\text{For period 2: } (1 - q_2^i) u((y + a - b) - Q_2^i) + q_2^i u(-m + F_2^i)$$

$$\begin{aligned}
& + \sum_{j=1}^N \pi_j \left[(1 - q_3^j) u((y + b) - Q_3^j) + q_3^j u(-m + F_3^j) \right] \\
& \geq (1 - q_2^i) u((y + a - b) - \tilde{Q}_2^i) + q_2^i u(-m + \tilde{F}_2^i) \\
& + \sum_{j=1}^N \pi_j \left[(1 - q_3^j) u((y + b) - \tilde{Q}_3^j) + q_3^j u(-m + \tilde{F}_3^j) \right] \tag{5}
\end{aligned}$$

$$\begin{aligned}
\text{For period 3: } & (1 - q_3^j) u((y + b) - Q_3^j) + q_3^j u(-m + F_3^j) \\
& \geq (1 - q_3^j) u((y + b) - \tilde{Q}_3^j) + q_3^j u(-m + \tilde{F}_3^j) \tag{6}
\end{aligned}$$

Equation (2) ensures that, in equilibrium, insurers make zero expected profits, as implied by the assumption of perfect competition. The no-lapse constraints (5) and (6) capture the absence of consumer commitment. After observing their health state, consumers have the right to lapse and purchase a competitive spot contract. As a result, the long-term contract must be designed so that it is weakly preferred to any profitable spot contract in every realized state at periods two and three.

3.3 Characterization of Equilibrium Contracts

We now characterize the structure of competitive equilibrium contracts. For each state i , let $Q_t^i(\text{FI})$ denote the actuarially fair premium that covers sickness event insurance within period t . Let $Q_2^i(\text{Spot})$ denote the competitive spot premium at period 2. Note that $Q_2^i(\text{Spot}) \geq Q_2^i(\text{FI})$, $\forall i \in \{1, \dots, N\}$, since any competitive spot contract in period 2 also provides some level of risk reclassification insurance losses in period three through front loading.

The following proposition is proved in the Appendix.

PROPOSITION 1. In the set of competitive equilibrium contracts:

- (i) In every state and period, consumers receive full event insurance, so that marginal utilities are equalized across health outcomes:

$$u'(y_t - Q_t^i) = u'(-m + F_t^i).$$

- (ii) For each a, b , there exists s_2 such that:

$$Q_2^i = \begin{cases} Q_2^i(\text{Spot}), & i < s_2, \\ Q_2^{s_2}, & i \geq s_2, \end{cases} \quad Q_2^{s_2} < Q_2^i(\text{Spot}) \text{ for } i > s_2,$$

and there exists s_3 such that:

$$Q_3^i = \begin{cases} Q_3^i(\text{FI}), & i < s_3, \\ Q_3^{s_3}, & i \geq s_3, \end{cases} \quad Q_3^{s_3} < Q_3^i(\text{FI}) \text{ for } i > s_3$$

(iii) For sufficiently low a, b , $a < \hat{a}$, $b < \hat{b}$ contracts are front loaded:

$$Q_1 > Q_1(\text{FI}), \quad Q_2^i > Q_2^i(\text{FI})$$

- (iv) Greater front loading provides better insurance against reclassification risk. Namely a higher Q_1 and Q_2^i reduces s_2 and s_3 . Consumers with less early period income (higher a, b) choose to front load less (lower Q_1 and Q_2^i) at the expense of risk reclassification insurance.
- (v) More states pay a common capped premium in period 3 than in period 2:

$$s_2 > s_3$$

Parts (i), (iii) and (iv) are analogous features of the equilibrium contracts in [Hendel and Lizzeri \[2003\]](#), while part (iii) and (v) have are some novel features of our model.

Point (i) shows that, under perfect competition, event insurance is provided at actuarially fair rates. As a result, buyers fully insure against sickness risk, ensuring that within-period medical expenditures are always covered.

Parts (i)–(iv) correspond to features of the framework in [Hendel and Lizzeri \[2003\]](#), while part (v) highlights the novel aspects of our model. Point (i) shows that, under perfect competition, event insurance is provided at actuarially fair rates. As a result, buyers fully insure against sickness risk, ensuring that all within-period medical expenditures are covered.

Point (i) shows that under perfect competition event insurance is offered at a fair rate. As a result, buyers will fully insure the risk of sickness, so that health expenditures within periods are always covered.

Point (iii) shows that premia must be front loaded to finance later subsidies as long as liquidity not too tight.

Point (iv) indicates that companies offer a set of contracts with different degrees of front loading based on the buyers first and second period income. Buyers with more income in period 1 and 2 (lower a, b respectively) receive more front loaded contracts that better insure against reclassification risk.

Point (ii) first establishes that premia in the riskiest health states are pooled at a common cap strictly below the spot value. In optimal contracts, resources are shifted from earlier to later periods to smooth variability in premia. Healthier risk types continue to pay actuarially fair spot premia, since in these states the no-lapse constraints bind. The novelty relative to [Hendel and Lizzeri \[2003\]](#) is that front-loading occurs in both periods 1 and 2.

Finally, point (v) shows that contracts offer better risk reclassification insurance later in life, so that more states pay a common capped premium in period 3 than in period 2. This follows from the assumption of increasing sickness probabilities over periods. Buyers with greater liquidity in periods 1 and 2 obtain a more even premium profile across periods.

3.4 Numerical Illustration

To illustrate the properties of the equilibrium characterized in Section 3.3, we work through a numerical example. The goal is to show the implications of Proposition 1 numerically under general, exemplifying, conditions and compare our numerical results with those of a two period model similar to that of [Hendel and Lizzeri \[2003\]](#). Note that the data is not drawn from any real world source as the exercise serves purely to complement the theoretical analysis of the model.

We first fix the parameters of the model, then determine all possible combinations of health states where the no lapsation constraints bind in period $t = 2$ and $t = 3$, and then select among these the allocation that maximizes expected utility subject to the zero profit and no lapse constraints.

Agents have utility function $u(c) = \ln c$ over consumption. The economy consists of three possible health (hence risk) categories at each of periods 2 and 3. The probability distribution over the three states for period $t = 2$ and $t = 3$ is given by

$$\pi = (0.40, 0.40, 0.20)$$

The monetary loss from a sickness event is set at $m = 40,000$. The probability of sickness in the first period is $q_1 = 0.02$, while in the second period the probabilities are $q_2 = (0.05, 0.10, 0.15)$ and in the third period $q_3 = (0.26, 0.28, 0.32)$. Endowments are chosen as

$$(y - a, y + a - b, y + b) = (20,000, 33,000, 36,250),$$

where $y = 29,750$ and $a = 9,750$, $b = 6,500$.

It is useful to compute the actuarially fair premia under full event insurance for each period and state. The fair premium in each state is given by $Q_t^i(FI) = q_t^i(y_t + m)$ (combine equation (24) and (17) in the appendix and set the result to 0). This yields

$$Q_1(FI) = 1,200, \quad Q_2(FI) = (3,650, 7,300, 10,950), \quad Q_3(FI) = (15,250, 19,062, 21,350)$$

These numbers provide a benchmark against which the equilibrium allocations can be compared. If there were no commitment on either the insurers' or the buyers' side, each individual would pay a premium that exactly reflects their expected loss in each state. Uncertainty in loss m would be fully insured, while uncertainty in π would be left uninsured.

With one-sided insurer commitment, no-lapse constraints (5) and (6) ensure that the healthiest types do not lapse to the spot market. As shown in Proposition 1, states where the no-lapse constraint binds pay the spot premium, while all other states share a common premium set below spot rates. In equilibrium contracts, the healthiest states are the first to bind. We denote s_t where $t = 2, 3$ the last binding state in each period. All states $i \leq s_t$ must be binding, since they are healthier than s_t . To each contract we associate two integers (s_2, s_3) that denote the last state (inclusive) where the no lapsation constraint binds in periods 2 and 3. When all states are nonbinding in the no lapsation constraints $(s_2, s_3) = (0, 0)$. There are 16 possible combinations of (s_2, s_3) . For each combination, we input a possible value for the nonbinding premium, we compute the remaining premia

and benefits consistent with the zero profit condition and no lapse constraints, then we check the feasibility of the contract and compute the expected utility. We iterate this process over all possible values of the nonbinding premium. The allocation with the highest expected utility among the feasible candidates is selected as the equilibrium.

The best three period equilibrium contract occurs at $(s_2, s_3) = (2, 0)$, meaning that two states bind at period 2 (healthiest states are offered spot premium) while all three states at period 3 are pooled at a common cap. The resulting premia are displayed in Table 1.

TABLE 1. Three-period equilibrium vs. full-insurance benchmarks

	Q_1	$Q_{2,1}$	$Q_{2,2}$	$Q_{2,3}$	$Q_{3,1}$	$Q_{3,2}$	$Q_{3,3}$	(s_2, s_3)
Equilibrium	1,289	7,645	11,295	14,500	14,000	14,000	14,000	(2,0)
Spot market premia	1,200	7,645	11,295	14,945	15,250	19,062	21,350	—
Fair insurance	"	3,650	7,300	10,950	"	"	"	—

For comparison, I also compute the two-period benchmark, in the spirit of [Hendel and Lizzeri \[2003\]](#). The parameters are the same as in the previous simulation, except now the third period is removed. The results are summarized in Table 2.

TABLE 2. Two-period equilibrium vs. full-insurance benchmarks

	Q_1	$Q_{2,1}$	$Q_{2,2}$	$Q_{2,3}$	State s_2
Equilibrium	1,200	3,650	7,300	10,950	3
Fair insurance	1,200	3,650	7,300	10,950	—

The optimal allocation in this case has $s_2 = 3$, meaning that all no lapsation constraints bind in equilibrium. In other words, at every health state in period 2 buyers are offered exactly the spot rate, hence there is no insurance against reclassification risk and, consequently, no front loading.

The numerical results illustrate the two testable implications of the model. First, premia are front loaded all periods but the last. Second, longer contracts can provide better protection against reclassification risk. This pattern is evident in the three period case, where front-loading finances the pooling of all states at $t = 3$ at a common capped premium. Instead, pooling is absent in the two-period benchmark: with $s_2 = 3$, all health states pay their spot rate, so there is neither insurance against reclassification risk nor any front-loading. Thus, comparing the two and three period settings illustrates that extending the horizon of the model increases the ability of firms to front load and therefore their ability to insure reclassification risk when health risks are more severe and the value of reclassification insurance is highest.

4. PERSISTENT HEALTH RISK

In this section we examine how persistence in health risk affects insurance against reclassification risk. Specifically, we study a setting where the risk class assigned at period 2 determines the distribution of future classes at period 3. In other words, the probabilities of being reclassified at 3 depend on the health class realized at 2. Premia at 3 can therefore be adjusted on the basis of the class realized in period 2. Most interestingly, we now allow insurers to condition period 3 premia on observed health histories in period 2, making past sickness realizations a key determinant of future contract terms. Additionally, we consider the extreme case where the risk class i , may persist over time. Naturally this can occur only after buyers have been assigned to a risk class in period 2, since in period 1 all individuals belong to a common health class. This type of health persistence effectively removes reclassification risk after the first reclassification occurs.

4.1 Persistence in Risk Class i

Once again we retain several elements from the model in [Hendel and Lizzeri \[2003\]](#). The information structure remains identical: both consumers and insurers observe the realized risk class at the beginning of each period. The market environment is unchanged, competitive insurers can commit to future premia and benefits, while consumers can lapse to spot contracts after finding out about their risk class. The timing of contracting also coincides with theirs.

Our model departs in several key respects. First, we extend the horizon from two to three periods, introducing a second reclassification date. Second, rather than exiting the market upon sickness, buyers suffer a monetary loss $m > 0$. Third, preferences are represented by a single concave, twice-differentiable utility function $u(c)$ that applies in health and a different function $v(c)$ with the same properties in case of sickness. Finally, and most importantly, reclassification now affects future reclassification probabilities: buyers in higher-risk classes are more likely to remain high risk. This allows premia at to depend on the previously realized class, giving insurers more power to adjust contracts based on observed histories. These two new features generate the main differences in our results relative to [Hendel and Lizzeri \[2003\]](#).

Consider a three period model identical to that in Section 3, except in this setting, the distribution of risk classes in period 3 depends on the class realized in period 2. For example, individuals in a high-risk class in period 2 are more likely to remain in high-risk classes in period 3. Persistence therefore conditions the probability distribution of future classes on the observed class in period 2. As a consequence firms can condition spot offers on the revealed distribution of future risks. As a result, each class i in period 2 is offered a distinct spot contract. Long term contracts distinguish between healthy and risky buyers in period 2 and provide terms that prevent lapsing based on their conditional reclassification risk probabilities.

Formally, consider the setting in Section 3. Recall that in the first period, sickness occurs with common probability q_1 in period 1, with q_2^i in state i in period 2, and with q_3^j in state j in period 3, where risks are ordered within periods ($q_t^1 < \dots < q_t^N$) and increase over time ($q_2^1 > q_1$, $q_3^1 > q_2^N$). At $t = 2$ individuals are allocated to a class $i \in \{1, \dots, N\}$,

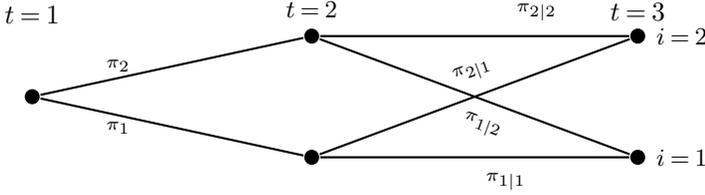


FIGURE 1. Simplified game tree, π_i is the independent probability distribution whereas $\pi_{j|i}$ is dependent on previous health states

according to a common distribution $\pi = (\pi_1, \dots, \pi_N)$. At $t = 3$, the distribution of classes depends on the class realized at $t = 2$. Formally, conditional on i at $t = 2$, the probability of being in class $j \in \{1, \dots, 3\}$ at $t = 3$ is $\pi_{j|i}$, with $\sum_j \pi_{j|i} = 1$. We assume that for $m > n$ the distribution of $\pi_{j|n}$ first order stochastically dominates (FOSD) $\pi_{j|m}$, so that $F(\pi_{i|m}) \leq F(\pi_{i|n})$ at all $i \in \{1, \dots, N\}$. See Figure 1 for a graphic interpretation of the model. A first period long term contract specifies premia and benefits conditional on the current class and, at $t = 3$, on the realized $t = 2$ class:

$$(Q_1, F_1), \quad \{(Q_2^i, F_2^i)\}_{i=1}^N, \quad \{(Q_3^{j|i}, F_3^{j|i})\}_{i,j=1}^N$$

As in Section 3, markets are assumed to be competitive, and while full insurance is theoretically attainable, it is precluded by liquidity constraints, modeled in the same way as earlier.

4.2 Solving the Model

To characterize competitive equilibrium contracts, we follow the same approach as in Section 3, formulating the problem as a constrained utility maximization. In a competitive equilibrium, allocations must maximize expected lifetime utility subject to a zero profit condition and no lapse constraints in periods 2 and 3. Accordingly, the equilibrium contract is obtained as the solution to a maximization problem in which the planner chooses premiums and benefits to maximize consumers' expected utility:

$$\begin{aligned} EU &= (1 - q_1) u((y - a) - Q_1) + q_1 u(F_1 - m) \\ &+ \sum_{i=1}^N \pi_i \left[(1 - q_2^i) u((y + a - b) - Q_2^i) + q_2^i u(F_2^i - m) \right] \\ &+ \sum_{i=1}^N \pi_i \sum_{j=1}^N \pi_{j|i} \left[(1 - q_3^j) u((y + b) - Q_3^{j|i}) + q_3^j u(F_3^{j|i} - m) \right] \end{aligned} \quad (7)$$

subject to the Zero Profit condition:

$$(1 - q_1)Q_1 - q_1F_1 + \sum_{i=1}^N \pi_i [(1 - q_2^i)Q_2^i - q_2^iF_2^i] + \sum_{i=1}^N \pi_i \sum_{j=1}^N \pi_{j|i} [(1 - q_3^j)Q_3^{j|i} - q_3^jF_3^{j|i}] = 0 \quad (8)$$

and the no-lapse constraints at $t = 2$ and $t = 3$. Formally, for all $\tilde{Q}_2^i, \tilde{F}_2^i$, and $\{(\tilde{Q}_3^{j|i}, \tilde{F}_3^{j|i})\}_{j=1}^N$ with

$$(1 - q_2^i)\tilde{Q}_2^i - q_2^i\tilde{F}_2^i + \sum_{j=1}^N \pi_{j|i} \left[(1 - q_3^j)\tilde{Q}_3^{j|i} - q_3^j\tilde{Q}_3^{j|i} \right] > 0, \quad \forall i \in \{1, \dots, N\}, \quad (9)$$

$$(1 - q_3^j)\tilde{Q}_3^{j|i} - q_3^j\tilde{Q}_3^{j|i} > 0, \quad \forall j \in \{1, \dots, N\}, \quad (10)$$

any equilibrium contract must satisfy:

$$\begin{aligned} \text{For period 2: } & (1 - q_2^i)u((y + a - b) - Q_2^i) + q_2^i v(F_2^i - m) \\ & + \sum_{j=1}^N \pi_{j|i} \left[(1 - q_3^j)u((y + b) - Q_3^{j|i}) + q_3^j v(F_3^{j|i} - m) \right] \\ & \geq (1 - q_2^i)u((y + a - b) - \tilde{Q}_2^i) + q_2^i v(\tilde{F}_2^i - m) \\ & + \sum_{j=1}^N \pi_{j|i} \left[(1 - q_3^j)u((y + b) - \tilde{Q}_3^{j|i}) + q_3^j v(\tilde{Q}_3^{j|i} - m) \right], \quad (11) \end{aligned}$$

$$\begin{aligned} \text{For period 3: } & (1 - q_3^j)u((y + b) - Q_3^{j|i}) + q_3^j v(F_3^{j|i} - m) \\ & \geq (1 - q_3^j)u((y + b) - \tilde{Q}_3^{j|i}) + q_3^j v(\tilde{Q}_3^{j|i} - m), \quad (12) \end{aligned}$$

Equation (2) ensures that, in equilibrium, insurers make zero expected profits, as implied by the assumption of perfect competition. The no lapsation constraints (11) and (12) capture the absence of consumer commitment. After observing their health state, consumers have the right to lapse and purchase a competitive spot contract. Buyers must therefore be offered a weakly better contract given their realized health class. As a result, the long-term contract must be designed so that it is weakly preferred to any profitable spot contract in every realized state at periods two and three.

4.3 Characterization of Equilibrium Contracts

We now characterize the structure of competitive equilibrium contracts. For each state i , let $Q_t^i(\text{FI})$ denote the actuarially fair premium that covers sickness event insurance within period t . Let $Q_2^i(\text{Spot})$ denote the competitive spot premium at period 2. Recall that $Q_2^i(\text{Spot}) \geq Q_2^i(\text{FI}), \forall i \in \{1, \dots, N\}$, since any competitive spot contract in period 2 also provides some level of risk reclassification insurance losses in period three through front loading.

The following proposition is proved in the Appendix.

PROPOSITION 2. In the set of competitive equilibrium contracts:

- (i) Full event insurance is obtained in every state and period.

(ii) For each a, b , there exists s_2 such that:

$$Q_2^i = \begin{cases} Q_2^i(\text{Spot}), & i < s_2, \\ Q_2^{s_2}, & i \geq s_2, \end{cases} \quad Q_2^{s_2} < Q_2^i(\text{Spot}) \text{ for } i > s_2. \quad (13)$$

Moreover, for each $i = 1, \dots, N$, there exists $s_{3|i}$ such that:

$$Q_3^{j|i} = \begin{cases} Q_3^j(\text{FI}), & j < s_{3|i}, \\ Q_3^{s_{3|i}}, & j \geq s_{3|i}, \end{cases} \quad Q_3^{s_{3|i}} < Q_3^j(\text{FI}) \text{ for } j > s_{3|i}. \quad (14)$$

Finally, given states m, n paying spot premia in period two with $m > n$ we have

$$Q_3^{s_{3|m}} > Q_3^{s_{3|n}}. \quad (15)$$

Namely, higher risk classes in period 2 receive higher capped premia in period 3.

(iii) For sufficiently low a, b , $a < \hat{a}$, $b < \hat{b}$ contracts are front loaded:

$$Q_1 > Q_1(\text{FI}), \quad Q_2^i > Q_2^i(\text{FI}). \quad (16)$$

(iv) Greater front loading provides better insurance against reclassification risk. Namely a higher Q_1 and Q_2^i reduces s_2 and s_3 . Consumers with less early period income (higher a, b) choose to front load less (lower Q_1 and Q_2^i) at the expense of risk reclassification insurance.

Equilibrium contracts in this environment closely mirror those in Section 3. In particular, items (i), (ii), and (iv) are analogous to Proposition 1: contracts feature full event insurance in every period, front loading of premia in period 1 and 2, and premium caps in period 2 and 3. Moreover, insurers offer a set of contracts so that consumers with higher a, b choose to front load less. Contracts with greater front loading provide better insurance against reclassification risk.

The striking difference arises in point (ii), specifically in period 3. As before, the absence of commitment imposes no lapsation constraints on equilibrium contracts.

In period 2, when these constraints are binding, premia are set just high enough to fully insure event risk and provide some risk reclassification insurance for period 3: $Q_2^i = Q_2^i(\text{Spot})$ for all $i < s_2$. When the constraints are slack, premia are set below the spot level, which may or may not lie below the actuarially fair premium, depending on the set of binding no lapsation constraints in period 3.

In period 3 when no lapsation constraints are binding premia are actuarially fair and do not depend on the period 2 history: $Q_3^{j|i} = Q_3^j(\text{FI})$ for all $j < s_{3|i}$. When no lapsation constraints do not bind, buyers pay a common capped premia across risk classes which is below actuarially fair levels but depends on the period 2 history, namely $Q_3^{s_{3|i}}$ for each $i = 1, \dots, N$. Riskier period 2 classes have a higher period 3 capped premium, formally $Q_3^{s_{3|m}} > Q_3^{s_{3|n}}$ for $m > n$. This occurs because, with identical income at $t = 2$, higher-risk classes bear larger event risk at $t = 2$ (higher q_2^i) and face greater reclassification risk for $t = 3$, since $\pi_{j|n}$ FOSD $\pi_{j|m}$ implies that higher risk classes are assigned to worse risk

pools in the future. As a result, there is less scope to insure against reclassification risk in period 3.

Taken together, lack of commitment and history-dependent reclassification reduce the extent of reclassification-risk insurance in later periods for those who enter riskier classes at $t = 2$ compared to a scenario where there is full commitment.

4.4 Complete Persistence in Risk Class i

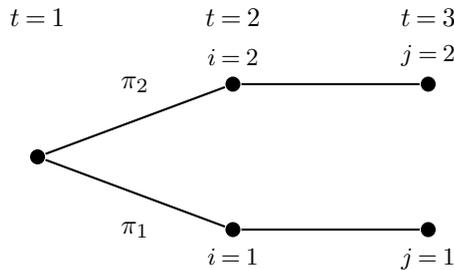


FIGURE 2. Simplified game tree, π_i is the independent probability distribution whereas j is the fully persistent class after $t = 2$

Consider a three period model identical to that in Section 3, except we now consider the extreme case in which the health class realized in period two persists in the future. Formally, for any class $k \in \{1, \dots, N\}$ in period 2, $\pi_{j|k} = 1$. In other words, once an individual is classified as type i in period 2, the same risk level applies in period 3 and there is no further reclassification. In this environment there is no uncertainty about future reclassification once the state in period 2 is revealed. This environment is illustrated in Figure 2.

The absence of reclassification risk after period 2 eliminates any need for reclassification risk insurance through capped premia beyond that date. Long term contracts therefore reduce to spot contracts in period 3. Equilibrium first period contracts still provide full insurance against the sickness event within each period, but in period 3 the premia coincide with the competitive, actuarially fair, spot premia. The only potential role for front loading is to insure against reclassification risk in period 2 through front loading in period 1.

5. CONCLUSION

This paper set out to understand how dynamic contracting problems play out in the private long-term care (LTC) insurance market, specifically to what extent they limit consumers' ability to obtain coverage and how they shape LTC insurance contracts. The literature has shown that LTC insurance contracts are front loaded to insure against the possibility of being placed in higher risk classes in the future [Finkelstein et al., 2005], yet the role of this mechanism in explaining the persistently small size of the LTC insurance

market remains unclear. The contribution of this study is to complement previous findings with two theoretical models, both inspired by [Hendel and Lizzeri \[2003\]](#), to better understand the structure of LTC insurance contracts and their implications for the insurance market. We expected these frictions to limit insurance coverage by making long term contracts either unattractive for many consumers or non feasible for insurers.

The first model extends the two period framework of [Hendel and Lizzeri \[2003\]](#) to a three period model. We retain the assumptions of one-sided commitment by insurers and symmetric learning, in line with LTC insurance markets. In addition, probabilities of being reclassified as high risk are assumed to be independent of previously assigned risk class, an assumption we later relax. The longer horizon allows insurers to offer more protection against reclassification risk, but only by front-loading premia in both the first and second periods. Since event risk is sharply higher in the final period, the value of reclassification insurance is also greater at that period, which leads to stronger coverage. The main novel testable implications of our adaptations are: first, premia are be front-loaded in all but the final period; and second, contrary to our initial expectation, contracts provide stronger reclassification insurance in later periods.

The second model relaxes the assumption that reclassification probabilities are independent of previously assigned risk class. In this setting, premia are allowed to depend on the realized class at period 2, so that insurers are allowed to offer equilibrium contracts where third period premia are contingent on health status in period two. The result is that individuals in worse classes in period two are placed into worse pools in period three and receive less insurance against reclassification risk. This occurs because, with identical income in period two, higher-risk classes face both larger event risk and more reclassification risk, reducing the extent of reclassification risk insurance in period three. The key implication is that when premia are allowed to be contingent on realized health states riskier buyers receive less insurance against reclassification risk. This outcome is consistent with our expectation that long term contracts may present new features limiting insurance coverage in LTC markets.

These results suggest that expanding access to LTC insurance may require interventions that reduce the burden of front-loading (for instance, through subsidies or tax incentives) and mechanisms that mitigate unequal treatment of higher-risk buyers (such as limitations on health based pricing).

Future research should explore how these dynamic contracting frictions interact with asymmetric information as was found by [Finkelstein and McGarry \[2006\]](#) and with the design of public insurance programs, as well as investigate empirically the extent to which front-loading and persistence in risk classes affect observed take-up. By clarifying the limits of private contracts, such extensions could help show the limits of private contracts and inform policies that make LTC coverage broader and more sustainable.

APPENDIX

Proof of Proposition 1 in Section 3

Define the profit in each period and state:

$$\Pi_t^i = (1 - q_t^i)Q_t^i - q_2^i F_t^i \quad (17)$$

Substitute the constraints in (3) and (4) with these equivalent, simpler constraints:

$$\Pi_2^i + \sum_{j=1}^N \pi_j \Pi_3^j \leq 0, \quad \forall i \quad \text{for spot contracts offered in period 2,} \quad (18)$$

$$\Pi_3^j \leq 0, \quad \forall j \quad \text{for spot contracts offered in period 3.} \quad (19)$$

To see this, note that if (18) and (19) hold, then there is no state i in $t = 2$ with $\{(\tilde{Q}_2^i, \tilde{F}_2^i), (\tilde{Q}_3^j, \tilde{F}_3^j)\}_i$ that makes positive profits and gives buyers higher utility in that state. Likewise in state j in $t = 3$ with $(\tilde{Q}_3^j, \tilde{F}_3^j)$ that makes positive profits and gives buyers higher utility in that state. Thus, (3) and (4) are satisfied. Conversely, suppose that (19) is violated so that $\Pi_3^j > 0$ for some j . A competitor at $t = 3$ can offer a slightly better pair $(\tilde{Q}_3^j, \tilde{F}_3^j)$ that yields strictly higher utility to the buyer and still makes positive profit, so the period-3 no-lapse condition in (4) would fail. Likewise, if (18) is violated, $\Pi_2^i + \sum_j \pi_j \Pi_3^j > 0$ for some i and a competitor at $t = 2$ can replicate the incumbent's continuation schedule $\{(Q_3^j, F_3^j)\}_j$ and improve the period-2 terms by an arbitrarily small amount; the total profit along the i -path, remains strictly positive, so the deviation is both profitable and strictly preferred by the buyer, contradicting (3).

The planner chooses $\{(Q_1, F_1), (Q_2^i, F_2^i), (Q_3^j, F_3^j)\}$ to maximize expected utility subject to the zero profit (ZP) and no lapsation (NL) constraints:

$$\max_{\{Q, F\}} \text{EU} \quad \text{s.t. } Z = 0, \quad \Pi_2^i + \sum_{j=1}^N \pi_j \Pi_3^j \leq 0, \quad \Pi_3^j \leq 0. \quad (20)$$

The Lagrangian of the problem is:

$$\begin{aligned} \mathcal{L} = & \text{EU} + \phi \left(\Pi_1 + \sum_{i=1}^N \pi_i \Pi_2^i + \sum_{j=1}^N \pi_j \Pi_3^j \right) \\ & - \sum_{i=1}^N \beta_2^i (\Pi_2^i + \sum_{j=1}^N \pi_j \Pi_3^j) - \sum_{j=1}^N \beta_3^j \Pi_3^j \end{aligned}$$

with multipliers $\phi \in \mathbb{R}$ (ZP), $\beta_2^i \geq 0$, $\beta_3^j \geq 0$ (NL).

Differentiating with respect to each (Q_t, F_t) and (as usual) dividing by the corresponding probability weights in expected utility yields:

$$u'((y - a) - Q_1) = \phi, \quad u'(-m + F_1) = \phi, \quad (21)$$

$$u'((y + a - b) - Q_2^i) = \phi - \frac{\beta_2^i}{\pi_i}, \quad u'(-m + F_2^i) = \phi - \frac{\beta_2^i}{\pi_i}, \quad \forall i, \quad (22)$$

$$u'((y + b) - Q_3^j) = \phi - \Lambda_2 - \frac{\beta_3^j}{\pi_j}, \quad u'(-m + F_3^j) = \phi - \Lambda_2 - \frac{\beta_3^j}{\pi_j}, \quad \forall j, \quad (23)$$

where $\Lambda_2 \equiv \sum_{i=1}^N \beta_2^i$. The complementary slackness conditions are

$$\beta_2^i \geq 0, \quad \Pi_2^i \leq 0, \quad \beta_2^i \Pi_2^i = 0, \quad \beta_3^j \geq 0, \quad \Pi_3^j \leq 0, \quad \beta_3^j \Pi_3^j = 0.$$

To prove point (i) combine together (21), (22) and (23) to obtain full insurance in every period and state:

$$(y-a) - Q_1 = -m + F_1, \quad (y+a-b) - Q_2^i = -m + F_2^i \quad \forall i, \quad (y+b) - Q_3^j = -m + F_3^j \quad \forall j \quad (24)$$

To prove point (ii) first combine (24) with (17) and simplify the expression for the per-state profits $\Pi_t^i = (1 - q_t^i)Q_t^i - q_t^i F_t^i = Q_t^i - q_t^i(y_t + m)$. Define the expected profits at period 3 as $R \equiv \sum_{j=1}^N \pi_j \Pi_3^j \leq 0$.

If constraint i in (18) binds at $t = 2$ then $\Pi_2^i + R = 0$, so

$$Q_2^i = q_2^i((y+a-b) + m) - R \quad (25)$$

Hence for states i and j binding in (18) with $q_2^i > q_2^j$ we have $Q_2^i > Q_2^j$.

If constraint k is nonbinding in (18), (23) gives:

$$u'((y+a-b) - Q_2^k) = \phi \quad (26)$$

Thus, if constraints k and l are nonbinding, $Q_3^k = Q_3^l$.

If constraint i is binding in (18), (23) gives:

$$u'((y+a-b) - Q_2^i) = \phi - \frac{\beta_2^i}{\pi_i} < \phi \quad (27)$$

where the inequality holds because if constraint i is binding, $\beta_2^i > 0$. Thus if i is binding and k is not, $Q_2^i < Q_2^k$. Now we have to show that this implies $i < k$.

For a non-binding class k , (18) is slack: $\Pi_2^k + R < 0$, so

$$Q_2^k < q_2^k((y+a-b) + m) - R \quad (28)$$

Combining (25) and (28) and the fact that $Q_2^i < Q_2^k$, we obtain

$$q_2^i((y+a-b) + m) - R < q_2^k((y+a-b) + m) - R \quad (29)$$

Hence $q_2^i < q_2^k$, the binding classes are the lower-risk classes.

To prove point (iii) we must show that $Q_1 > Q_1(FI)$ and $Q_2^i > Q_2^i(FI)$. If any of the no-lapsation constraints (18) and (19) are non binding then $Q_1 > Q_1(FI)$ and $Q_2^i > Q_2^i(FI)$ is immediate from (18) and the zero profit condition (2). When instead all no-lapsation constraints are binding at $t = 2$, substituting from (21) into (23) we obtain:

$$u'((y+a-b) - Q_2^i) = u'((y-a) - Q_1) - \frac{\beta_2^i}{\pi_i} \quad (30)$$

Hence $Q_1 > Q_2^N - (2a-b)$, which requires $Q_1 > Q(FI)$ if $2a-b$ is sufficiently small.

When instead all no-lapsation constraints are binding at $t = 3$, substituting from (23) into (22) we obtain:

$$u'((y+b) - Q_3^j) = u'((y+a-b) - Q_2^i) - \frac{\beta_2^i}{\pi_i} - \Lambda_2 - \frac{\beta_3^j}{\pi_j} \leq u'((y+a-b) - Q_2^i) \quad (31)$$

Therefore $Q_2^i > Q_3^N - (2b - a)$, which requires $Q_2^i > Q_2^i(FI)$ if $2b - a$ is sufficiently small.

Point (iv) is intuitive. As the income difference increases (a, b), income is transferred in the future, more of the no-lapsation constraints become binding, this leads to a higher s_t , which in turn lower front loading.

To prove point (v) first recall the equilibrium long term contract structure. In Proposition 1, point (ii), we determined that once a states' spot premium surpasses the cap, the state pays the capped premia. Moreover, once a binding state i pays the capped premium, every state higher than i also pays the capped premium. Hence it is sufficient to demonstrate that the binding premia at period 3 cross the cap before binding premia at 2. In other words it is sufficient to determine that $Q_2^{\text{binding}} < Q_3^{\text{binding}}$ and $Q_2^{\text{Non-Binding}} > Q_3^{\text{Non-Binding}}$ to ensure that the last binding state $s_2 > s_3$.

First compute binding premia at $t = 2$ and $t = 3$. For any binding period 2 state $i \leq s_2$ in (18) (i.e., $\Pi_2^i + R = 0$),

$$Q_2^i = q_2^i((y + a - b) + m) - R \quad (32)$$

For any binding period 3 class $j \leq s_3$ in (19) (i.e., $\Pi_3^j = 0$):

$$Q_3^j = q_3^j((y + b) + m) \quad (33)$$

Now compare the binding premia at $t = 2$ and $t = 3$ found above. Divide (32) and (33) by q_2^i and q_3^j respectively:

$$\frac{Q_2^i}{q_2^i} = ((y + a - b) + m) - \frac{R}{q_2^i}, \quad \frac{Q_3^j}{q_3^j} = ((y + b) + m) \quad (34)$$

Subtract and rearrange:

$$\frac{Q_2^i}{q_2^i} - \frac{Q_3^j}{q_3^j} = (a - 2b) - \frac{R}{q_2^i}, \quad (35)$$

$$Q_3^j = \frac{q_3^j}{q_2^i} \left(Q_2^i + R + q_2^i(2b - a) \right). \quad (36)$$

Since risks are increasing across periods ($q_3^j > q_2^i$), the factor $\frac{q_3^j}{q_2^i} > 1$. Therefore:

$$Q_3^j > Q_2^i + R + q_2^i(2b - a). \quad (37)$$

Hence $Q_3^j > Q_2^i$ whenever $R + q_2^i(2b - a) > \bar{g}$. Since $R \equiv \sum_{j=1}^N \pi_j \Pi_3^j$ this condition requires that the expected losses from pooling at period 3 are not too large. This is likely because event risk at $t = 2$ is high, limiting excessive front loading, and because $(2b - a)$ is sufficiently large, so that buyers' liquidity constraints are strong enough. Under these conditions, the inequality reduces to $Q_3^{\text{Binding}} > Q_2^{\text{Binding}}$. Now compare non binding premia: if the no-lapse is not binding in state k in period 2 and m in period 3 ($\beta_2^k, \beta_3^m = 0$), the FOCs (22) and (23) determine:

$$u'((y + a - b) - Q_2^k) = \phi, \quad u'((y + b) - Q_3^m) = \phi - \Lambda_2 \quad (38)$$

with $\Lambda_2 > 0$ when any period 2 no lapsation constraints bind. By monotonicity of u' , this yields $Q_2^k \geq Q_3^m - (2b - a)$. If the level of $(2b - a)$ is small, due to borrowing limitations or slow income growth, then $Q_2^{\text{Non-Binding}} \geq Q_3^{\text{Non-Binding}}$.

Lastly put all the above together:

$$Q_2^{\text{Non-Binding}} > Q_3^{\text{Non-Binding}}, \quad Q_2^{\text{Binding}} < Q_3^{\text{Binding}} \quad (39)$$

more risk classes hit their cap at $t = 3$, so fewer bind at $t = 3$ than at $t = 2$, namely $s_2 > s_3$.

Proof of Proposition 2 (Section 4)

With class dependent reclassification probabilities at $t = 3$, substitute the period 2 and period 3 no lapse (NL) constraints with the following constraints. The argument for constraint equivalence is analogous to the one in the proof of Proposition 1 above.

$$(1 - q_2^i)Q_2^i - q_2^i F_2^i + \sum_j \pi_{j|i} [(1 - q_3^j)Q_3^{j|i} - q_3^j F_3^{j|i}] \leq 0, \quad \forall i, \quad (40)$$

$$(1 - q_3^j)Q_3^{j|i} - q_3^j F_3^{j|i} \leq 0, \quad \forall i, \forall j. \quad (41)$$

Let $\phi \in \mathbb{R}$ be the multiplier for the Zero Profit constraint, $\beta_2^i \geq 0$ the multipliers on (40), and $\beta_3^{j|i} \geq 0$ on (41). Differentiating w.r.t. each (Q_t, F_t) gives:

$$u'((y - a) - Q_1) = \phi, \quad v'(-m + F_1) = \phi, \quad (42)$$

$$u'((y + a - b) - Q_2^i) = \phi - \frac{\beta_2^i}{\pi_i}, \quad v'(-m + F_2^i) = \phi - \frac{\beta_2^i}{\pi_i}, \quad \forall i, \quad (43)$$

$$u'((y + b) - Q_3^{j|i}) = \phi - \frac{\beta_2^i}{\pi_i} - \frac{\beta_3^{j|i}}{\pi_i \pi_{j|i}}, \quad v'(-m + F_3^{j|i}) = \phi - \frac{\beta_2^i}{\pi_i} - \frac{\beta_3^{j|i}}{\pi_i \pi_{j|i}}, \quad \forall i, j. \quad (44)$$

Point (i) can be verified as in Proposition 1: marginal utilities are equalized within each period and state, implying full event insurance:

$$u'(y_t - Q_t^i) = v'(-m + F_t^i).$$

The proof of points (iii) and (iv) is analogous to that for the respective points in Proposition 1 and is therefore omitted.

The proof of premium caps in point (ii) is analogous to the proof of point (ii) from Proposition 1. We therefore restrict the proof to the novel findings regarding premia in period 3. In particular, we prove that $Q_3^{j|i} = Q_3^j$ for all binding states j and $Q_3^{s_3|m} > Q_3^{s_3|n}$ for $m > n$.

First note that premia at $t = 3$ in states where no lapsation constraints (41) bind are actuarially fair and independent of realized states i in period $t = 2$. If constraint (41) is binding, then

$$q_3^j F_3^{j|i} = (1 - q_3^j)Q_3^{j|i}.$$

Plugging this into the FOC (44), we obtain

$$(1 - q_3^j)Q_3^{j|i} = q_3^j v'^{-1} \left(u'((y + a - b) - Q_3^{j|i}) \right) + m.$$

Clearly $Q_3^{j|i} = Q_3^j$ for all binding states j , independent of realized state in period 2, i .

We now show that for any non binding state k in period $t = 3$, $Q_3^{k|i} = Q_3^{s_3|i}$ and, assuming $m > n$, we have $Q_3^{s_3|m} > Q_3^{s_3|n}$. Take any non binding constraint k , the multiplier for (41) is zero ($\beta_3^{k|i} = 0$), so the FOC (44) reduces to:

$$u'((y + b) - Q_3^{k|i}) = \phi - \frac{\beta_2^i}{\pi_i}.$$

Thus, if constraints k and z are non binding, clearly: $Q_3^{k|i} = Q_3^{z|i}$.

If any constraint i in period 2 is binding, then rearranging the no-lapse constraint (40) we obtain

$$q_2^i F_2^i = (1 - q_2^i)Q_2^i + \sum_j \pi_{j|i} [(1 - q_3^j)Q_3^{j|i} - q_3^j F_3^{j|i}].$$

Moreover, substituting this into (43) we obtain:

$$(1 - q_2^i)Q_2^i + \sum_j \pi_{j|i} [(1 - q_3^j)Q_3^{j|i} - q_3^j F_3^{j|i}] = q_2^i v'^{-1} (u'((y + a - b) - Q_2^i)) + m. \quad (45)$$

Take two binding constraints $m > n$ in period 2 (so that $q_m > q_n$) and assume that $Q_3^{k|m} \leq Q_3^{k|n}$. First note that buyers with higher previous risk class i in period 2 have higher expected losses (i.e. $\sum_j \pi_{j|i} [(1 - q_3^j)Q_3^{j|i} - q_3^j F_3^{j|i}]$ is increasing in i). To see this, note that risk of sickness q_3^j is independent of i , yet third period premia decrease with i by assumption.

Given this fact and $q_m > q_n$, for equation (45) to simultaneously hold with equality for any two binding states m and n we must have $Q_2^m > Q_2^n$.

Yet combining our assumption $Q_3^{k|m} < Q_3^{k|n}$ with equation (44) we obtain:

$$\phi - \frac{\beta_2^m}{\pi_m} < \phi - \frac{\beta_2^n}{\pi_n}$$

Namely marginal utility in state m is lower than in state n . Substituting into the marginal utilities at optimum for Q_2^i from (43), we obtain $Q_2^m \leq Q_2^n$. This contradicts our previous finding, so for a non binding state k in period 3 and two binding states m, n in period 2 it must be that $Q_3^{k|m} > Q_3^{k|n}$.

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