



Ca' Foscari
University
of Venice

Master's Degree programme

in Economics and Finance
"Second Cycle (D.M. 270/2004)"

Final Thesis

The Portfolio Insurance in the ETFs Market

Analysis and application of asset allocation strategies

Supervisor

Ch. Prof. Marco Corazza

Assistant supervisor

Ch. Prof. Lorian Pelizzon

Graduand

Giulio Franceschetto

Matriculation Number 860931

Academic Year

2017 / 2018

Table of contents

INTRODUCTION	4
CHAPTER 1: THE PORTFOLIO INSURANCE	6
1.1 HEDGING STRATEGIES	7
1.1.1 <i>Buy and Hold (BH)</i>	8
1.1.2 <i>Stop-Loss Portfolio Insurance (SLPI)</i>	11
1.1.3 <i>Constant Mix (CM)</i>	13
1.1.4 <i>Constant Proportion Portfolio Insurance (CPPI)</i>	16
1.1.5 <i>Time Invariant Portfolio Protection (TIPP)</i>	19
1.1.6 <i>Variable Proportion Portfolio Insurance (VPPI)</i>	21
1.1.7 <i>Option Based Portfolio Insurance (OBPI)</i>	23
1.2 COMPARISON	29
1.3 REBALANCING TECHNIQUES	32
CHAPTER 2: THE EXCHANGE-TRADED FUNDS.....	34
2.1 HISTORY OF ETFs	35
2.1.1 <i>Origin and development</i>	35
2.1.2 <i>Current scenario</i>	37
2.2 STRUCTURE AND FUNCTIONING	39
2.2.1 <i>ETF characteristics</i>	40
2.2.2 <i>Creation/Redemption Process</i>	42
2.3 REGULATION	44
2.3.1 <i>In the United States</i>	44
2.3.2 <i>In Europe</i>	45
2.4 ETF MANAGEMENT	46
2.5 INDEX REPLICATION.....	48
CHAPTER 3: EMPIRICAL ANALYSIS	51
3.1 ETFs SELECTION	52
3.2 PERFORMANCE RATIOS	53

3.2.1	<i>Sharpe ratio</i>	53
3.2.2	<i>Sortino ratio</i>	54
3.2.3	<i>Calmar ratio</i>	54
3.2.4	<i>Information ratio</i>	55
3.3	APPLICATIONS	55
3.3.1	<i>LYXOR EURO STOXX 50 UCITS ETF (MSE)</i>	58
3.3.2	<i>SPDR MSCI Small Cap Europe UCITS ETF (SMCX)</i>	61
3.3.3	<i>Lyxor MSCI Europe UCITS ETF (MEU)</i>	65
	CONCLUSION	69
	REFERENCES	72
	APPENDIX A	75
	APPENDIX B	80
	APPENDIX C	81
	MATLAB CODE	89

Introduction

An investment is subject to different types of risk. The two main categories are the so-called systematic risk and specific risk. While the former concerns the chance of an investor experiencing losses due to circumstances affecting the overall performance of the reference market, the latter refers to specific factors influencing only individual securities. The specific (unsystematic) risk may be reduced through the diversification of the investment. In his studies Markowitz (1952) recognized that the unsystematic risk decreases significantly when 30 or more assets are held in the portfolio. The Exchange-traded funds, firstly developed in the 1990s, are financial instruments that, allocating capital to a variety of securities, allow the investors to diversify their investment, thus reducing the sources of risk specific to individual assets. Conversely, the systematic (market) risk can't be reduced through diversification, but it can be hedged against. In this regard, the portfolio insurance represents a variety of investment and allocation strategies which permits to hedge a portfolio of security against market risk.

Hence, the aim of this thesis is to explore the portfolio insurance theory in relation to the exchange-traded funds market. Therefore, in the first chapter the most popular asset allocation schemes, employed for portfolio insurance purposes, will be defined and compared as theorized by the scholars. In particular, the emphasis will be placed on the four main families of strategies: the Buy and Hold, the Constant Mix, the Constant Proportion, the Option Based with all their variations. The second chapter will provide a comprehensive overview of the exchange-traded funds sector. In particular, the origin and development of the industry will be mentioned and the structure as well as the functioning of this type of financial instruments will be explained. Finally, the third chapter will report an empirical analysis of the diverse portfolio insurance strategies. These have been applied to three exchange-traded funds descriptive of different market scenarios through simulations based on historical market data. Eventually, the strategies have been evaluated by the calculation of some performance measures aimed at stressing the advantages and

the drawbacks of each one and at identifying whether and in which cases one would outperform another.

Chapter 1: The Portfolio Insurance

The first chapter of this thesis will define the concept of portfolio insurance and will analyze some of the strategies portfolio managers employ to enable a form of protection against the depreciation risk of the securities they handle. The portfolio insurance (henceforth PI), in fact, refers to a particular investment approach which entails using financial products individually or in combination to reduce the market risk a specific portfolio of securities is exposed to. H. Leland and M. Rubinstein implemented in 1976 a strategy to insure the financial portfolios according to a hedging technique based on options starting from the Black, Scholes and Merton model (1973). The approach may involve various products such as equities and bonds as well as, in some derivations, derivatives like futures and options. In conditions of uncertainty, a hypothetical investor may want to take advantage of a PI strategy to deal with a risky scenario and limit the volatility of his exposure. However, this entails both the protection against the losses and the erosion of potential profits. Therefore, the goal of the most advanced dynamic strategies is to capture and remove only the risk of downside movements in the asset price while exploiting the favourable situation, in case of upward variations. The 'dynamic' aspect concerns the continuous effort in rebalancing the parameters, ratios and weights affecting the composition of the insured portfolio.

After the launch in 1976, the PI met such a sudden spread that it is argued whether it influenced the 1987 world markets crash. Leland and Rubinstein, together with O'Brien, established in 1980 the LOR¹ and started the industrial development of these new allocation strategies. In the 1987, right before the crisis, the portfolio insurance related products amounted to around 100 billion of dollars of the global asset under management². These numbers lead some critics, at odds with Rubinstein, to consider the PI as partially responsible for the increased market volatility during the crisis. In the meantime, other

¹ Leland O'Brien Rubinstein Associates Incorporated.

² See Eric Bouyé (2009), *Portfolio Insurance: A Short Introduction*. Available at SSRN: <https://ssrn.com/abstract=1416790>.

variations of the initial option based approach were introduced: in 1986 Perold studied the so-called Constant Proportion Portfolio Insurance, while Rubinstein analyzed a stop-loss strategy (1985). During the last few decades, as its principles proved to be consistent, the PI has kept growing. In this regard, a fundamental role was played by its peculiar property, which allows a market participant to limit the riskiness of an investment while preserving the potential rewards. The variety of applications, guaranteed by the PI, gained popularity in particular among the institutional investor such as mutual funds, pension funds and exchange-traded funds which will be further analyzed in the next chapter. The latter, indeed, could take advantage of these investment opportunities to face steep market crash as well as general and periodic downturns.

1.1 Hedging strategies

A portfolio insurance strategy may be defined as an allocation technique, that allows the investor to guarantee at least a percentage of the initial capital at a given maturity date.

Therefore, basic elements of any PI strategy are:

- the underlying portfolio the holder may want to insure;
- the time horizon of the guarantee;
- the level of protection, that is the minimum fraction of the initial portfolio value the investor wants to receive at maturity.

In this respect, three different scenarios appear:

- the portfolio value at maturity has to be at least equal to the initial value;
- the portfolio value at maturity is smaller than the initial capital, but at least equal to a given percentage (for example in case of a 95% level of protection);
- the portfolio value at maturity has to be higher than the original investment, such as to get a minimum yield.

Reasonably the last is the most likely situation since a common investor may want to receive at least the same profit as if he bought a risk-free government bond typically giving around a 3%³ annual return.

Before engaging in a deeper analysis of each particular strategy, it is important to mention some common characteristics. A crucial feature that distinguishes the insurance strategies is the difference between a static and a dynamic approach. A static strategy consists on properly choosing the desired asset allocation, taking all the positions on debts, equities or derivatives and holding them till the maturity date holding the quantities constant without. This may be the case of the buy-and-hold strategy. On the other hand, the dynamic view requires a continuous periodic effort of rebalancing the weights of the asset classes the initial capital was allocated to. In this regard, some different possibilities concerning the rebalancing issue will be examined in the next chapters. An example of a dynamic strategy is the Constant Proportion Portfolio Insurance, while others like the Option Based Portfolio Insurance and the Stop-Loss present different versions that can adopt both a static and a dynamic view. The next subsections examine every strategy in the detail. Specifically, seven different approaches will be described: Buy and Hold (BH), Stop-Loss (SL), Option Based Portfolio Insurance (OBPI), Constant Mix (CM), Constant Proportion Portfolio Insurance (CPPI), Variable Proportion Portfolio Insurance (VPPI) and their derivations.

1.1.1 Buy and Hold (BH)

The Buy and Hold consists on a straightforward static allocation method where a portion of the initial capital is invested in a risky asset (usually some equities), while the remaining amount is employed to buy a risk-free bond. The starting allocation, then, is fixed until maturity and no more negotiations take place, no matter what happens to the relative values. As a static strategy, the Buy and Hold is quite inexpensive since it has no

³ 2,77% is the annual return given by a US 10-yr Treasury bond as of 02/04/2018.

management fee or transaction cost, except for those concerning the opening trades. However, its drawback consist in the inability of capturing all the potential profits deriving from the stocks held in the portfolio. In particular, once the initial mix of equity and debt has been established, the investor buys:

- risk free-bonds with face value equal to minimum amount desired, as per level of protection, and maturity date corresponding to the time horizon of the insurance strategy,
- stocks with the remaining capital.

Assuming:

- 100€ initial capital,
- 70% level of protection,
- 3% annual risk-free rate (r_f) corresponding to the yield of an ideal treasury bond,
- 1-year horizon (T)⁴,

the investor should buy risk-free debt for around 68€⁵ which returns in one year the face value of 70€ and invest the 32€ left on the stock market, such that the allocation mix is 32/68 of stocks and bonds. At maturity he gets at least the insured value of 70€ plus a variable amount depending on the evolution of the stock price. In case of remarkable returns in the stock market (let's say $r_s=15\%$), the gain captured from the insurance strategy will be limited and equal to the weighted average of the returns realized by the two asset classes

$$r_{BH} = r_f w_f + r_s w_s \quad (1)$$

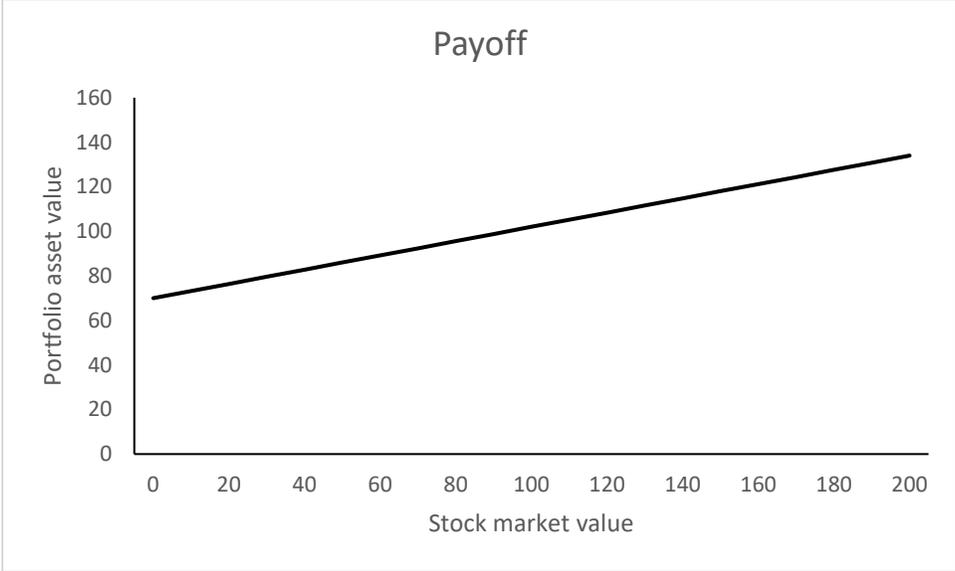
in this case $r_{BH} = 6,84\%$ ($0,03*0,68+0,15*0,32$).

⁴ The same amounts of initial capital, risk-free rate, and time horizon will be used for all the further examples.

⁵ The cost of the bond corresponds to its face value discounted for the time horizon at the risk free rate: $B = \frac{FV}{(1+r_f)^T} = \frac{70}{1,03} = 67,96$.

Figure 1.1 below illustrates the payoff of the BH strategy depending on the path of the stock price, while Figure 1.2 represents the gain captured⁶, given the allocation mix as in the example above.⁷

Figure 1.1 Payoff graph at maturity for a 32/68 BH strategy

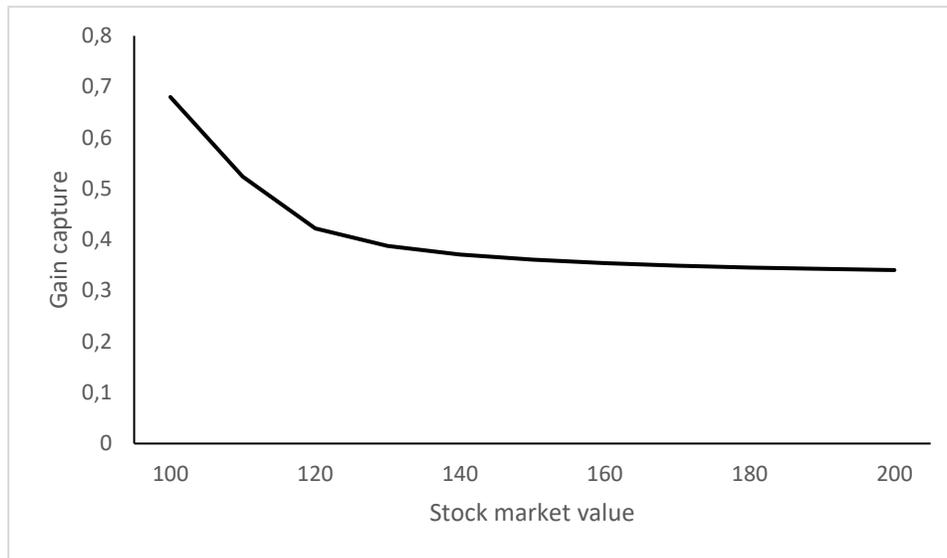


In Figure 1.1 the slope equals the percentage destined to the equity market, the minimum guaranteed amount is 70€ and the return is potentially unlimited increasing as the stock price at maturity rises. In addition, the greater is the initial weight allocated to the stock the higher the gain realized by the BH when the stocks outperform the bonds and the lower the proceeds when the risk-free rate overcome the return on the equity market. But, on the contrary, while the return increases as the stock price at maturity rises, the fraction of the total potential profit captured by the strategy is decreasing.

⁶ The gain captured is the portion of the total potential gain the strategy is able to realize. The total potential gain is the best achievable profit given by a full investment in the equity (as asset which commonly returns the highest yield).

⁷ See Appendix A.1 for the construction of the graph.

Figure 1.2 Gain capture for a 32/68 BH strategy



1.1.2 Stop-Loss Portfolio Insurance (SLPI)

The second alternative methodology of PI is a Stop-loss strategy. It is the simplest dynamic technique, which lets the investor to hold a fully risky portfolio as long as its value remains above a preset minimum level (the floor F). Only if it drops under the floor, the investor would liquidate her position and reinvest the proceeds in a risk-free bond. The floor value (F_t) is, indeed, variable and corresponds to the present value of the capital the investor may want to guarantee at maturity (F_T). In other words, the floor value may be considered as the value of a risk-free bond, whose maturity and face value correspond to the time horizon of the protection and the guaranteed capital, respectively. So the underlying portfolio is not an allocation mix of equities and bonds, but it is rather composed of either risky or risk-free assets depending on the moment of observation. The SLPI may leads to the following three different scenarios:

- The value of the risky position on the underlying (U_t) never crosses the threshold, represented by the net present value of the floor ($NPV(F_T)$): $U_t > F_t$ in any moment, then the investor can benefit from all the profits realized on the stock market (in this

scenario the strategy is static since no trades are necessary to rebalance the portfolio).

- U_t falls below F_t : the stop-loss order is immediately triggered, so the position on the underlying is liquidated and reinvested in risk-free assets. The investment will, then, grow constantly at the risk-free rate for the remaining time horizon. If risky portfolio value at maturity remains under the floor, then the investor gets the guaranteed capital (F_T) without any cost, besides the transaction costs⁸.
- Same situation as in the previous point except that the return on the risky asset at maturity overcomes the return from the risk-free position: in this case the investor gets the guaranteed capital anyway, but faces an opportunity-cost as loss of the potential profits given by the risky portfolio.

This last aspect of the strategy represents, in fact, also its main limit since the investor loses the chance of participating from any upward movement in the stock market, once the portfolio has been switched to the risk-free configuration and the strategy is activated.

Imagine again: $U_0 = 100\text{€}$, $F_T = 100\text{€}$, $T = 1$ Year, $r_f = 3\%$

- If $U_t > 100\text{€} \forall t \in [0,1]$ then the return of the strategy is $r_{SLPI} = \frac{U_T}{100} - 1$ (2)
- If $U_t \leq 100\text{€}$ for some $t \in [0,1]$ (for example $t=0,5$), then the investor sells its position on the risky asset and buy a risk-free bond for $NPV(F_{0,5}) = \frac{100}{(1,03)^{0,5}} = 98,53\text{€}$ getting 100€ at maturity.

However, in this last case, whether $U_T > 100$ he would lose the potential return given from the equation (2) above.

⁸ Ideally, the investor reacts as soon as the stop-loss order is triggered. Therefore, she would be able to liquidate and reinvest the position on the underlying when its value is below but still very close to F_t . In this way it is possible to get a maturity a guaranteed capital which is approximately very close to F_T .

1.1.3 Constant Mix (CM)

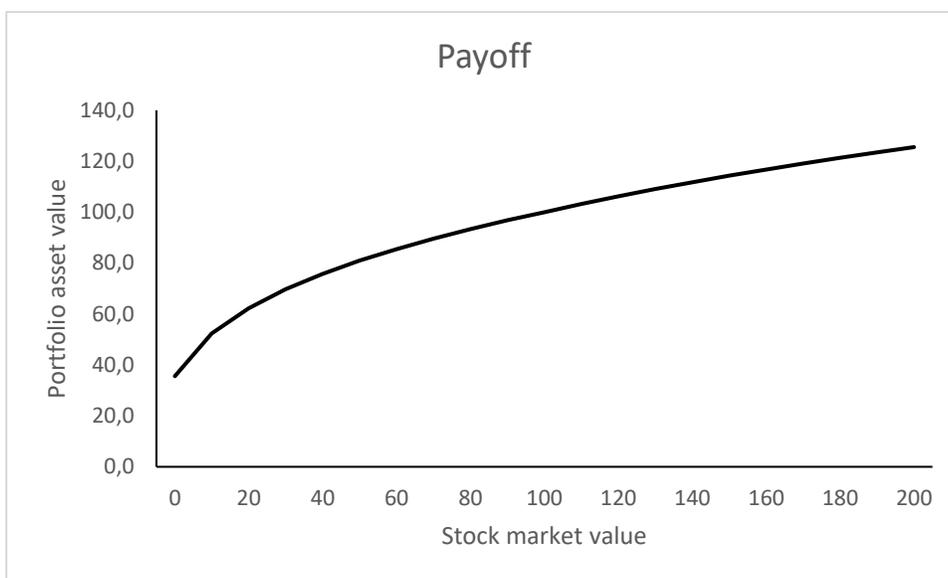
Another dynamic approach to investment decision-making can be found in the family of Constant Mix strategies. They imply keeping an exposure to stocks and risk-free assets which is a constant proportion of initial wealth. While in the BH the initial quantities are the constant item, here the initial portfolio proportions are so. Therefore, whenever the relative values of the assets change, new negotiations are required to readjust the desired allocation mix. In practice, if the equity market grows (i.e. prices go up) the investor needs to sell some stock and purchase risk-free bonds to rebalance the portfolio. Conversely, when prices drop, more stock are necessary to preserve the constant target mix, hence the investor applies a “buy low and sell high” strategy. Intuitively, CM seems to perform better than BH when the market is unstable and frequent oscillations occur. In fact, every reduction in stocks price, implying a purchase, will be reasonably followed by a subsequent price increase allowing the investor to realize a capital gain. Consequently, frequent rebalancing trades lead to higher transaction costs. Let us consider an initial mix 50/50 of equity and risk-free asset, for instance 50€ allocated to stocks and 50€ invested in risk-free bonds, with an initial total wealth of 100€. If equity market experiences a 20% rise, bringing the stocks value to 60€, the total portfolio would be worth 110€. Hence, the 54,5% of the total portfolio (60€ of 110€) is allocated to stocks and the 45,5% (50€ of 110€) to bonds. Meanwhile, the original mix would require the portfolio to be equally composed of 55€ (50% of 110€) in risky and 55€ in risk-free assets. To restore the initial proportion, then the investor needs to sell 5€ of stocks and reinvest the same amount in treasury bills. In this example the equity market is moving up while bonds price does not change, if they both realized identical returns clearly there would be no need of rebalancing⁹.

Being dynamic, the frequency of rebalance is a fundamental issue for the Constant-mix strategies. Ideally the allocation mix should be continuously adjusted, but this is of course impossible. More realistically a daily rebalancing technique, which captures the prices at

⁹ If both stocks and bills give the same 5% annual return in one year the total portfolio would be worth 105€ ($50 \cdot 1,05 + 50 \cdot 1,05 = 100 \cdot 1,05 = 105$), where 52,5€ of stocks and 52,5€ of bills, hence the original desired 50/50 mix is confirmed.

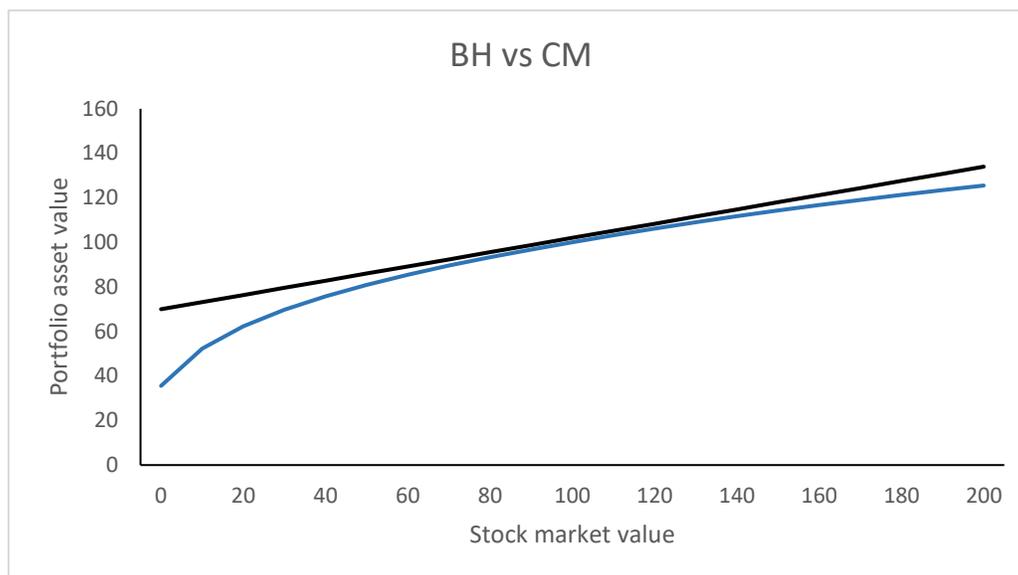
every market closure, would be achievable, but it still shows some drawback since it would be extremely costly due to the high frequency of transactions. Therefore, in order to avoid too many unnecessary transactions, usually different approaches can be adopted which imply the activation of the rebalancing over a certain threshold (for example when the equities grow more than 10%) or after a longer period of time (monthly or quarterly). However, the problem concerning the rebalancing method of the portfolio in case of dynamic insurance strategy will be better analyzed going forward. See Figures 1.3 and 1.4 for the graph of the payoff of a 32/68 CM strategy, also compared to the BH with the same starting allocation.¹⁰

Figure 1.3 Payoff graph at maturity for a 32/68 CM strategy



¹⁰ See Appendix A.2 for the detailed construction of the graphs.

Figure 1.4 Comparison between the BH and CM payoff at maturity with the same initial allocation mix



Looking at the figures, the BH seems to dominate the CM since for every level of the stock market a prospective investor would prefer to undertake the BH, whose payoff line always lies above the CM one. Nonetheless, the CM proves to be convenient when the market fluctuations let the investor profit from the capital gains related to the “buy low and sell high” rule (the example above just considered linear market movements: the stock market moves just in one direction, upward or downward). The CM is, in fact, a path-dependent strategy.

Assume first a 10% drop in the stock market value, decreasing from 100€ to 90€, which implies the risky portion of the portfolio to go to 45€ and the total wealth to 95€. After rebalancing, the portfolio would be composed of 47,5€ in risky asset and the same amount in non-risky products. If then the equity market regains the value it has previously lost, increasing from 90€ to 100€, the allocation in stock would be worth 52,78€ ($47,5 \times 1,11$)¹¹. Finally, the total portfolio would be equal to 100,28€ (52,78€ of stocks plus 47,5€ of bills) where the additional 0,28€ represents the realized capital gain from the stocks trading. On the contrary, the BH is path independent, meaning that in case of market fluctuation leading

¹¹ Going from 100 to 90 represents a 10% drop, while going back from 90 to 100 is a 11,11% increase.

to the same starting and final point there are no capital gains. In a nutshell, CM's performance depends both on the final assets value and on the path it follows from the starting point for all the subsequent periods, while the payoff of the BH relies just on the final level of the stock market. In general CM should outperform BH when the market is flat but oscillating (high volatility, frequent or large fluctuation, but final portfolio value not too far from the initial one, while scenarios where fluctuations are small or infrequent and the stock path ends up quite far from its starting point are more likely to foster BH strategies¹².

1.1.4 Constant Proportion Portfolio Insurance (CPPI)

The Constant Proportion strategy, introduced by Black and Perold in 1987, is a more structured form of dynamic portfolio insurance. In particular, unlike the previously analyzed approaches, it does not require the specification of the time horizon of the coverage as basic parameter. In fact, in theory it may allow an eternal protection. The CPPI lets the investor allocate his initial wealth in both risky and non-risky assets, where the respective proportions depend on the definition of two fundamental parameters: the floor (F) and the multiplier (m). The floor, as explained above, is the minimum desired level of the portfolio value at a given date, in other words the guaranteed value, moreover it behaves as a risk-free bond growing over time at the risk-free rate. The equation defining the strategy is:

$$E_t = mC_t, \quad (3)^{13}$$

i.e. the exposure in stocks (E) has to be equal to the cushion (C) multiplied by a given multiplier (m). The cushion is, in fact, the difference existing between the total underlying

¹² See A. Perold, W. Sharpe (1988), "Dynamic Strategies for Asset Allocation", *Financial Analysts Journal*, 44, pp 16-27.

¹³ According to this formulation the CPPI strategy, there is no restriction on the exposure on stocks and on the amount to be allocated to risk-free bills, that is to say borrowing and short selling are allowed. In particular, the investor tends to go short on risk-free asset when the price of the risky one is high and, on the other hand, he would prefer to get a short position on stocks when their price is low. But many applications rule out the short selling such that the equation (3) is reformulated as follow: $E_t = \min(mC_t; U_t)$.

portfolio (U_t) and the net present value of the floor (F_t), $C_t = U_t - F_t$. The remainder of the investor's wealth is invested in a risk-free asset. The net present value concept implies the definition of the floor in relation to a given maturity date, in contrast to the description of the CPPI as a potentially eternal protection strategy. Nonetheless, assuming that the floor (F_T) keeps growing at the risk-free rate once the maturity date (T) has been reached, the scheme still works without any further modification.

A crucial issue in the outlining of a CPPI strategy is the selection of an appropriate multiplier. Theoretically, there is no restriction on its value but it is very important as it gives the economic meaning to the scheme itself. The reciprocal of the multiplier ($1/m$) is, indeed, the maximum sudden drawdown in the stocks that may occur such that the floor is not violated. For instance, given a multiplier of $m = 3$, the stocks may experience at most a drop of 33% ($1/3$) in one period before rebalancing not to make the portfolio falling below the floor's present value. This is why in the CPPI, and in all the dynamic strategies in general, it is fundamental to constantly monitor the maintenance of the floor and the exposure to the risky asset to rebalance the portfolio appropriately. Besides, the case where $m = 1$ represents a Buy-and-Hold strategy, while for $0 < m < 1$ the scheme falls in the family of the Constant Mix strategy. Thus, we consider the insurance scheme as belonging to the CPPI only for values of the multiplier greater than 1.

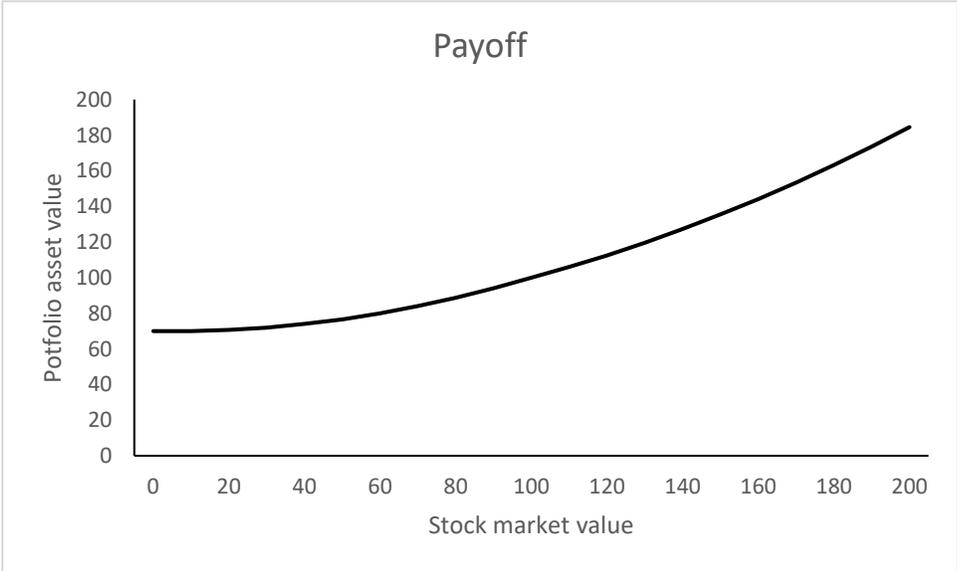
Example

For instance having 100€ of initial capital, a floor of $F = 70€$ and a multiplier $m = 2$, the cushion would be $C = 30$, the exposure in risky assets $E = 60€$ and the allocation mix of stocks and risk-free bonds would be 60/40. Then, assuming that the stock market falls by 10% from 100 to 90, the risky portion of the portfolio drops to 54€ ($60 * 0,9$) bringing the total asset value to 94€. Now the new cushion would be equal to 24€ ($94 - 70$) and the related investment on stocks equal to 48€ ($24 * 2$). Hence to re-equilibrate the portfolio the investor should sell stocks for 27€ ($90 - 63$) and reinvest the proceeds in risk-free bond. This proves how the CPPI follows a "buy high sell low" rule implying the purchase of stocks when their

price rises and the sell when the value falls, opposed to the CM approach, such that we can expect the payoff diagram to be convex. Therefore, a portfolio insured according to a CPPI strategy will at least return the floor, as guaranteed capital, when the stock market declines, since the portfolio tends to switch stocks for bills as the value of the former approaches zero. In bull markets, instead, the strategy performs very well since it calls for buying stocks as their price rises, returning increasing marginal profits. As opposed to CM, then, the CPPI is not optimal in a flat market scenario, as it is harmed from frequent price oscillations. Finally, as aforementioned, the multiplier, the only case in which the insurance may not return at least the floor is when the stock market experiences an abrupt fall before having the chance to rebalance the portfolio.

Below is the payoff diagram of the CPPI¹⁴

Figure 1.5 Payoff diagram for a CPPI strategy, with 70€ as floor and multiplier $m=2$.



¹⁴ See Appendix A.3.

1.1.5 Time Invariant Portfolio Protection (TIPP)

The Time Invariant Portfolio Protection (also constant proportion with lock-in of profit¹⁵), developed by Estep and Kritzman (1988)¹⁶, originates from a modification of the CPPI. While the CPPI, as all the strategies analyzed up to this point, aims at protecting the original capital up to a pre-defined floor, the TIPP also allows the investor to insure any interim capital gain. In this regard, the extension provided by the TIPP concerns the definition of the floor. It is not a given fixed parameter anymore, but it can rather be ratchet up when the portfolio value grows. In practice, as before, first are determined the multiplier and the initial floor (F) as percentage (f) of the initial wealth (U_0). Then in any period the new portfolio value is computed and multiplied by the level of protection (floor percentage). If the result is greater than the previous floor, this new level becomes the floor otherwise the old value holds. While enabling the capture of the capital gains in the stock market, the TIPP presents a fundamental drawback. As in the CPPI, once the floor is reached, the investor transfers all its wealth on the non-risky assets irreversibly, such that she can't benefit anymore from potential subsequent upwards movements in the equity market.

Example

Recalling the example above, when the equity market rises from 100€ to 110€ (10% increase) then the stocks value grows to 66€, the floor is readjusted to 74,2€¹⁷ ($106 \times 0,7$). Thus the new allocation mix on stocks and risk-free assets would be computed as follows:

$$C = U - F = 106 - 74,2 = 31,8€;$$

$$E = m \times C = 2 \times 31,8 = 63,6€;$$

¹⁵ See R. Cesari, D. Cremonini (2003), "Benchmarking, portfolio insurance and technical analysis: a Monte Carlo comparison of dynamic strategies of asset allocation", *Journal of Economic Dynamics & Control*, 27, pp. 987-1011.

¹⁶H. Dichtl, W. Drobetz (2011), "Portfolio insurance and prospect theory investors: Popularity and optimal design of capital protected financial products", *Journal of Banking & Finance*, 35, pp. 1683-1697.

¹⁷ The initial floor in the CPPI example was equal to 70€, that is the 70% of the initial capital of 100€.

$$B = U - E = 106 - 63,6 = 42,4\text{€}.$$

As it has been done before, it is possible to simulate a stock path to draw the payoff graph of the TIPP strategy (Figure 1.6) and compare it to the CPPI one (figure 1.7).¹⁸

Figure 1.6 Payoff diagram for a TIPP strategy with 70€ as initial floor and multiplier $m=2$.

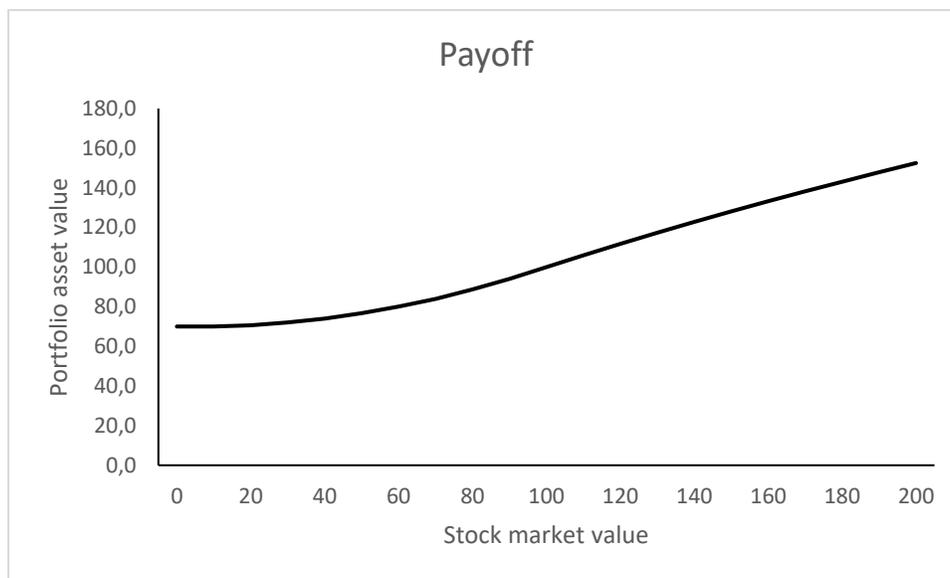
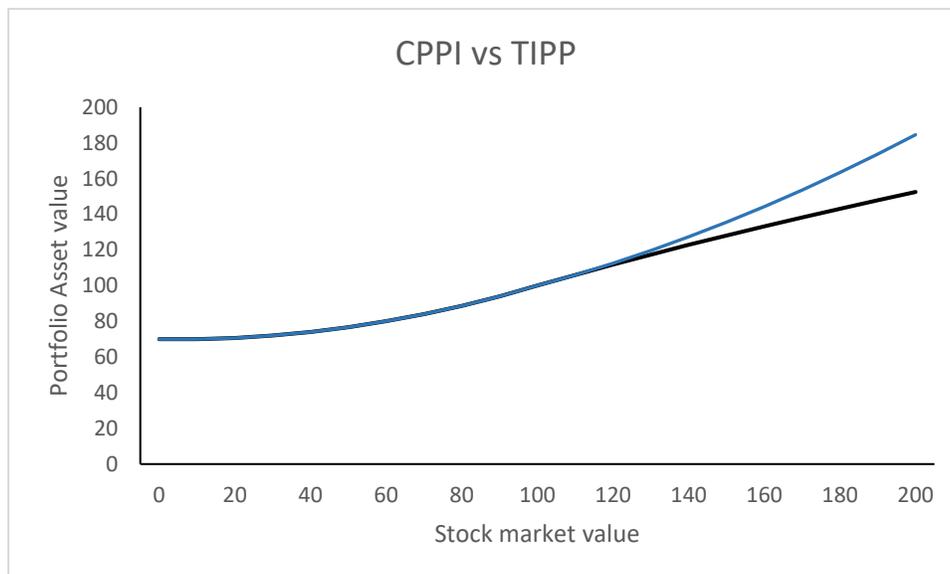


Figure 1.7 comparison between the TIPP (black) and CPPI (blue) payoffs with the same initial floor and multiplier.



¹⁸ See Appendix A.4.

As shown in the graphs, the CPPI strategy seems to dominate the TIPP payoff which is flatter when the stock market rises. In fact, the TIPP allocates an increasing share of the total portfolio to the risk-free asset when stock price grows but this allows the investor to lock-in the profit from the capital gain and turns to be convenient in case of reversals on the stock market.

1.1.6 Variable Proportion Portfolio Insurance (VPPI)¹⁹

Another derivation from CPPI is the Variable Proportion Portfolio Insurance. As opposed to the simplicity of a constant multiple characterizing the CPPI, the VPPI employs a dynamic multiple in order to improve the effectiveness of the insurance strategy. Specifically, this strategy provides a parameter which grows as the equity market increases and gets smaller as the stock price falls. Indeed, the function of the dynamic multiple is to improve the upside capture when the market grows as well as to allow for more downside protection when the stock price decreases. Now let us analyze as example a particular type of VPPI: the Exponential Proportion Portfolio Insurance.

Exponential Proportion Portfolio Insurance (EPPI)

According to this model the multiplier (m) changes following an exponential dynamic as described by the equation below:

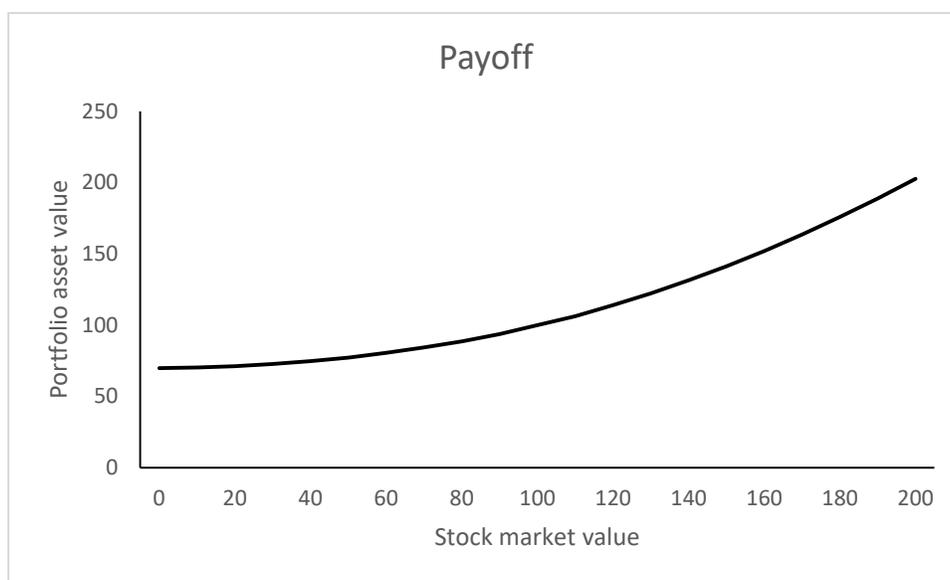
$$m = \eta + e^{aln(S_1/S_0)} \quad (4)$$

where $\eta > 1$ is an arbitrary constant, $e^{aln(S_1/S_0)}$ is the dynamic multiple adjustment factor (DMAF) with $a > 1$ as parameter, S_1 is the stock value after rebalance while S_0 is the value before rebalancing. Both parameter are greater than 1, thus improving the upside capture

¹⁹ See Lee, Chiang, Hsu (2011), "A new choice of dynamic asset management: the variable proportion portfolio insurance", *Applied Economics*, 40:16, p. 2135-2146.

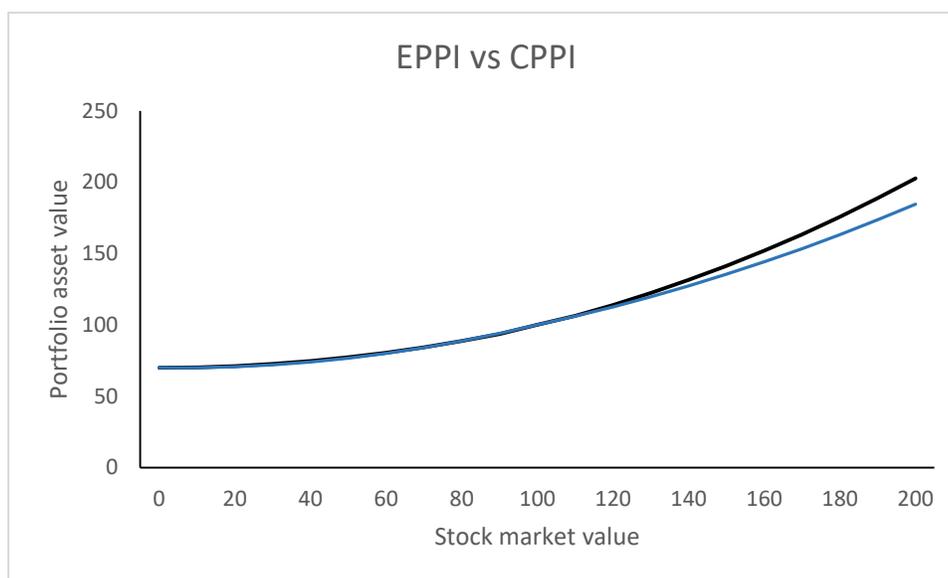
in accordance to the “buy high sell low” rule. In fact, the position on stock has to increase as the price goes up. Then we can adopt the same approach as with the CPPI to plot and compare the payoff diagram of a EPPI strategy with $\eta = 1,1$ and $a = 2$ (Figures 1.8 and 1.9)²⁰. Such parameters have been chosen to maintain a configuration similar to that of the strategies previously described. In Figure 1.9 is visible how the graph of the EPPI clearly overcomes the graph of the CPPI for high values of the stock market, due to exponential effect of the multiplier.

Figure 1.8 Payoff graph for a EPPI strategy with 70€ as floor, $\eta = 1,1$ and $a = 2$.



²⁰ See Apeendix A.5

Figure 1.9 comparison between the payoffs of the EPPI (black) and CPPI (blue) strategies.



1.1.7 Option Based Portfolio Insurance (OBPI)

The Option Based Portfolio Insurance is, in principle, a static protection scheme, suggested by Leland and Rubinstein (1981), which relies on the studies made by Black and Scholes resulted in the famous model on option pricing (1973). The option pricing methodology can be exploited to implement few different alternative strategies. In its basic definition the OBPI employs put options, which are a derivative financial instrument (written on a specific underlying asset) which gives to the buyer the right to sell the underlying asset for a pre-defined price (called strike price). If the option is European style, then the right can be exercised only at the given maturity date (expiration date), if on the contrary it is American it can be exercised also in any moment before expiration. In this thesis the options take into consideration will be only European. This is indeed a static configuration as the investor's protected portfolio, composed of the risky asset and the put option written on it, will remain fixed for the whole horizon of the insurance scheme.

Imagine a portfolio holder who wants to buy an insurance to secure her assets up to a desired floor (K). If there exists on the market any exchange traded options written exactly on his portfolio, the investor could buy a put option with strike price (K) corresponding to the desired protection level. Thus the new portfolio $S + p(S, K)$, composed of the underlying asset worth (S) and the put option, would guarantee at maturity (T):

- The floor (K) when $S_T < K$, given by the exercise of the option, that is the right to sell the underlying for the pre-determined strike price K ;
- Any capital gain when $S_T > K$, due to upward movements of the value of the stocks held in the portfolio.

Example

In $t = 0$ an investor holds a portfolio worth 100€ and he would like to insure its value for a 1-year time horizon. Then he can buy in the market a put option for (when exists) written on his portfolio with expiration date in 1 year and strike price $K = 100€$ (full coverage).

In $t = 1$:

- If the portfolio value has increased (assume by 10%), then the investor takes no action and he would gain 10€ simply as capital gain realized on the equity market
- If the portfolio value has dropped, then he would exercise the option selling the underlying and getting the insured amount of 100€.

Here the cost of purchasing the option has been disregarded, as the insured portfolio $S + p(S, K)$ does not match the investor's initial wealth. To create a self-financing portfolio, the investor should liquidate first a portion (p) of his initial capital, corresponding to the cost of the option, to buy the protection and just reinvest the remainder in the stock market.

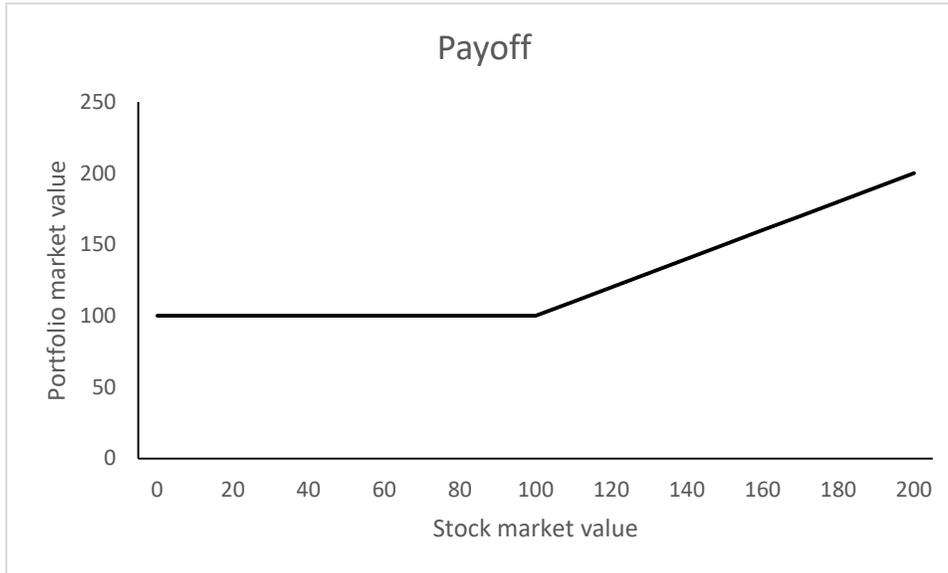
The payoff from this strategy is given by:

$$\begin{cases} S_T, & S_T > K \\ K, & S_T < K \end{cases}$$

And considering also the cost (p) of purchasing the option:

$$\begin{cases} S_T - p, & S_T > K \\ K - p, & S_T < K \end{cases}$$

Figure 1.10 Payoff graph for a static OBPI strategy, where the cost of the protective put with $K=100$ is not considered



To move forward through the examination of the alternative option based approaches, first the option pricing problem needs to be formulated according to the Black-Scholes model. It allows to price both a put and a call option (respectively p and c) at time t , with strike price K and time to maturity T as follows:

$$c(S_t, t) = S_t N(d_1) - K e^{-r(T-t)} N(d_2) \quad (5)$$

$$p(S_t, t) = K e^{-r(T-t)} N(-d_2) - S_t N(-d_1) \quad (6)$$

Where:

- $d_1 = \frac{\ln \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}$;
- $d_2 = d_1 - \sigma\sqrt{(T-t)}$;
- r is the risk-free rate;
- σ is the standard deviation of the stock's return;²¹
- $N()$ is the standard normal cumulative distribution function.

²¹ For operational purpose the volatility and the return need to be estimated based on historical data.

Constant Value Capture and Constant Gain Capture

Two possible alternative static option based portfolio insurance approaches are the CVC (Constant Value Capture) and the CGC (Constant Gain Capture)²². In particular, they are considered as self-financing methodologies, thus the cost of the protection is included in the insurance scheme. Practically, the underlying portfolio value is reduced of an amount Z , necessary to buy the protection (the cost of the option). The differences arise on how the remaining fraction $(1 - Z)$ of the capital is invested. While, in the CVC scheme the whole amount $(1 - Z)$ stays in the underlying portfolio²³, according to the CGC a part of that amount can be reinvested in a risk-free bond. Therefore, in the CVC, Z reflects effectively the cost of the protection and the amount $(1 - Z)$ is the capture ratio (VC); then, given Z and the return of the underlying portfolio r_p , the return from the insurance strategy r_{CVC} is defined as:

$$r_{CVC} = VC(1 + r_p) - 1 \quad (7)$$

when the portfolio value at maturity overcomes the minimum level of protection. Otherwise the investor will get the amount insured. Let us consider the following situation where the self-financing portfolio $S + p(S, K)$ has been constructed according to the Black-Scholes assumptions²⁴: 100€ as initial wealth, 5,6€ liquidated to buy the protection ($Z = 0,056$ and $VC = 0,944$), a full protection ($F_T = 100$) and 1-year maturity. If, for example, the portfolio value at maturity gets to 108€ ($r_p = 8\%$) then, as per Equation 7, the return from the insurance strategy would be $r_{CVC} = 1,95\%$, corresponding to a value of the scheme of 101,95€ and to a realized gain of 1,95€. Therefore, the realized gain represents only the 24,4% of the total potential gain of the portfolio (1,95 divided by 8). Thus, this ratio represents the percentage of gain captured. If, instead, the portfolio value rises to 110€ the realized return would be 3,84€, that corresponds to the 38,4% of gain capture (3,84€ realized versus a potential of 10€). Obviously, when r_{CVC} is smaller than 0, that is r_p lower

²² H. Leland (1999), *L'Assicurazione di Portafoglio: Elementi Teorici e Applicativi*, Il Mulino, Bologna.

²³ It is the same "put plus stock" scheme, as seen before, just allowing for the self-financing.

²⁴ See Appendix B.1 for the detailed calculations.

than 5,9%, the investor gets the guarantee amount, in this case 100€. So it can be easily noticed how the CVC strategy captures a constant ratio of the portfolio value at maturity (namely the VC), while the share of the realized gain is variable.

This fraction of the profit realized by the underlying portfolio at maturity, when it reaches the minimum level of protection, is called the gain capture (GC) and it becomes a constant parameter in the formulation of the CGC model. It lies on the principle that the capital $(1 - Z)$, left over as a result of the protection purchase, is not fully reinvested in the underlying. Thus, we can say that a share α of the portfolio is invested in the $S + p(S, K)$ while the remainder $(1 - \alpha)$ is employed to buy a risk-free bond with face value $(1 - \alpha)K$. Therefore, a CGC strategy lies on the following constraint:

$$\alpha(S + p) + (1 - \alpha) \frac{K}{(1 + r)^T} = \text{Initial wealth} \quad (8)$$

where the strike price (K) matches the floor (F) and the constant parameter (α) corresponds to the GC. So, given an initial capital of 100€ and a full level of protection, an investor may adopt a CGC approach, with GC = 0,744 such that the return from the scheme in case the portfolio value a maturity goes to 110€, is $r_{CGC} = 7,44\%$ ($10 * 0,744$).²⁵

OBPI using call options

The classic OBPI scheme based on put options may be restructured using call options²⁶ instead of puts. It lies on a fundamental property deriving from the Black-Scholes model: the put-call parity formulated as follows

$$c - p = S_t - Ke^{-rT}, \quad (9)$$

hence

²⁵ See Appendix B.2 for the detailed calculations.

²⁶ A call grant to the buyer the right to purchase the underlying asset the option is written on for a pre-specified strike price K, at a given maturity date T.

$$p + S_t = c + Ke^{-rT}. \quad (10)$$

Then it can be easily noticed how the insured portfolio $S + Put(S, K)$ is equivalent to a portfolio composed of a call option written on the underlying asset S and a risk-free zero-coupon bond with face value equal to the strike price K of the option. Holding this particular allocation mix, the investor at maturity would either:

- get the face value of the bond equal to K , when $S_T < K$
- or exercise the call option purchasing the underlying stock worth S_T , when $S_T > K$.

Hence the payoff function of the strategy is equal to:

$$\begin{cases} S_T, & S_T > K \\ K, & S_T < K \end{cases}$$

exactly identical to the scheme described before combining stocks and put option.

OPBI with option replication (synthetic put strategy)

As aforementioned, the original OBPI (also called protective put strategy) is based on a static allocation mix between stocks and derivatives. Now a particular OBPI strategy, which allows for the dynamic rebalance of the portfolio, will be examined. This dynamic approach aims at artificially creating a synthetic put option, that would serve as a protection for the risky portion of the portfolio, making the insurance scheme more flexible and adaptable. In this way there is no need anymore for the availability on the market of an adequate exchange traded option written exactly on the portfolio held by the investor, with the appropriate strike price and the desired expiration date since the same payoff from a protective strategy may be replicated as follows. In fact, relying on the Black-Scholes equations the portfolio $S + p(S, K)$ turns to be equivalent to a continuously adjusted portfolio combining an investment in risky and risk-free assets. Combining formulas (5) and (10) the equation for the portfolio $S + p(S, K)$ can be derived as follows:

$$p + S_t = S_t N(d_1) + K e^{-r(T-t)} N(-d_2) \quad (11)$$

where

$$N(-d_2) = 1 - N(d_2)$$

Then it is defined:

$$\Delta = \frac{\partial(p + S_t)}{\partial S_t} = N(d_1) \quad (12)$$

as the first derivative of the portfolio equation with respect to the stock price.

This delta (Δ) represents the portion of the replicating portfolio to be invested in the risky asset and this is why the method is also called “delta hedging”.

Therefore, an investor willing to follow a synthetic insurance strategy replicating the same payoff of a put option, would allocate its initial wealth in Δ -shares of the risky asset and the remainder in a risk-free bond. To perfectly match the protective and the synthetic put strategies, ideally the replicating portfolio should be continuously rebalanced²⁷ any time the stock price changes, which is not obviously possible in reality. Hence, depending on the rebalancing period, the replicating portfolio performs a tracking error with respect to its benchmark and the higher the adjustment frequency the higher the transactions costs. Moreover, the strategy implies an increasing proportion of the risky assets in the portfolio as the stock price rises and vice versa.

1.2 Comparison

Let us try to summarize in the following table all the characteristics of the hedging strategies analyzed.

²⁷ Accordingly to the Black-Scholes model’s assumption of absence of any transaction costs.

Table 1.1 Recapitulatory table of the main features of the portfolio insurance strategies analyzed.

	Portfolio Value	Definitions	Exposition in risky asset	Exposition in risk-free asset	Static/Dynamic	Payoff diagram
BH	$P_t = E_t + B_t$	Floor: $F_t = Fe^{-rT}$	$E_t = S_t$	$B_t = F_t$	Static	Linear
SLPI	$P_t = E_t + B_t$	Floor: $F_t = Fe^{-rT}$	<ul style="list-style-type: none"> • if $P_t > F_t$ $E_t = P_t$ • if $P_t < F_t$ $E_t = 0$ 	<ul style="list-style-type: none"> • if $P_t > F_t$ $B_t = 0$ • if $P_t < F_t$ $B_t = P_t$ 	Dynamic	Generally non linear
CM	$P_t = E_t + B_t$	Floor: $F_t = Fe^{-rT}$ Weights: $w_s = k$ $0 < w_s < 1$	$E_t = w_s P_t$	$B_t = (1 - w_s)P_t$	Dynamic	Concave
CPPI	$P_t = E_t + B_t$	Floor: $F_t = Fe^{-rT}$ Cushion: $C_t = P_t - F_t$ Multiplier: m	$E_t = m \times C_t$	$B_t = P_t - E_t$	Dynamic	Convex
TIPP	$P_t = E_t + B_t$	Floor: if $P_t > P_0$, $F_t = fP_t$ if $P_t < P_0$, $F_t = Fe^{-rT}$ Cushion: $C_t = P_t - F_t$ Multiplier: m	$E_t = m \times C_t$	$B_t = P_t - E_t$	Dynamic	Convex
EPPI	$P_t = E_t + B_t$	Floor: $F_t = Fe^{-rT}$ Cushion: $C_t = P_t - F_t$ Multiplier: $m = \eta + e^{a \ln(S_1/S_0)}$	$E_t = m \times C_t$	$B_t = P_t - E_t$	Dynamic	Convex
OBPI	$P_t = E_t + Put$	$p + S_t = S_t N(d_1) + Ke^{-rT} N(-d_2)$ $\Delta_t = \frac{\partial(p + S_t)}{\partial S_t} = N(d_1)$	$E_t = \Delta S_t$		Dynamic	Convex

Now let us focus on the most relevant differences existing between them, with respect to the type of payoff graph and the scheme dynamics.

Concave vs Convex

A first element, instantly clear, by looking at the payoff diagrams is the contrast between concave and convex strategies. Except for the BH graph which is just a straight line, all the other schemes' payoff plot concave or convex lines. Taking CM (fig. 2) and CPPI (fig. 3) as

the simplest examples of concave and convex strategies, it is quite intuitive as the concave curves increase at a decreasing rate as stock market value rises and decrease at an increase rate. Conversely, convex lines increase with rising slope, while decline with a decreasing slope. This is meaningful of how the concave ones perform quite poorly in upside market and at the same time are unable to give an effective protection in case of downward movements, but prove to be adequate in flat but oscillating markets taking advantage from the frequent reversals. On the contrary, convex strategies grant more upside capture in case of growing markets as well as more downside protection when performing worse. In practice we can relate the three kinds of diagram to three different investment rules:

- “do nothing” (straight lines);
- “sell high, buy low” (concave curves),
- “buy high, sell low” (convex curves).

In the first category the BH and the OBPI with the protective put approach are included, in the second the CM is identified, while in the third the CPPI and its derivation as TIPP and EPPI are recognized. We can also highlight how the EPPI strategy is more convex than the CPPI (due to the exponential factor influencing the multiplier) which is in turn more convex than the TIPP.

Static vs Dynamic

In general, the opposition between “do nothing” and “do something” strategies characterize also the distinction between static and dynamic approaches. The static ones are usually the simplest and the cheapest to implement since the investor makes his investment decision at the beginning of the time horizon without producing any further change in the portfolio allocation mix or undertaking any other negotiation. This is the case of BH and OPBI (with the protective put) schemes. On the contrary, dynamic strategies may be more complex but they allow, at the same time, for the creation of more flexible

insurance schemes, suitable to the specific investor's needs. In fact, the operative application of dynamic schemes reveals few complications and potential issues.

The first issue concerns the transactions cost. Indeed in all the dynamic strategies, the periodic rebalance (which should be ideally continuous to preserve the theoretical functionality of the scheme) of the portfolio implies new negotiations and trades that involve transaction costs, thus eroding the potential profit. Therefore, in this regard the challenge is to smartly select the appropriate rebalancing frequency, such as to limit the transaction costs while guaranteeing the allocation adjustments to be effective. The second issue regards the chance the stock market exhibits jumps in prices. In case of sudden and significant variations in the market prices the investor may not be able to promptly rebalance his portfolio (for instance when the discontinuity in the stock path occurs during market closures), such as the protective strategies can be heavily undermined. In particular, the CPPI may suffer from this kind of events. The last problem concerns specially the OBPI with options replication as it comes from a fundamental assumption of the Black-Scholes model, namely the estimation of the volatility of the underlying portfolio as a constant parameter. First, the parameter is indeed an estimate and may not properly reflect the actual evolution of the stock path. Secondly, it is assumed to be constant which has not always proved to be true in history. For instance, in the 90's the volatility of the S&P500's daily returns fluctuated from a minimum 11% in the 1993 to a maximum 21% in the 1998.²⁸

1.3 Rebalancing techniques

As introduced in the previous subchapters, the rebalancing issue has a fundamental role in outlining a dynamic portfolio insurance strategy. In fact, the same scheme may lead to different results depending on the specific technique employed to readjust the portfolio allocations. In particular, three different rebalancing methodology may be distinguished:

²⁸H.Leland (1999), *L'assicurazione di portafoglio: elementi pratici e applicativi*, Il Mulino, Bologna.

- Periodic (daily, monthly, quarterly, exc...);
- By threshold;
- By range.

The periodic rebalancing strategy simply implies the regular reallocation of the predefined target portfolio weights at the end of each period. Typically, it may be implemented daily, weekly, monthly or quarterly. Obviously, the more frequent is the adjustment the more consistent is the insurance strategy, but the higher the transaction costs. The threshold rebalancing suggests that the allocation mix needs to be rearranged to the target proportions only when the original portfolio weights move over a given threshold, for example plus or minus 5%. Finally, the range rebalancing may be seen as a modification of the threshold one and it works as follows. First of all, a threshold (let us assume $\pm 5\%$) and a further range of tolerance over the threshold (assume $\pm 3\%$) are imposed. Then, the reallocation is not triggered until not only the initial threshold, but also the range are breached. Moreover, the rebalance does not imply the portfolio readjustment to the original target weights, but to the nearest edge of the predefined interval given by the threshold. In practice, imagine an original allocation mix 50/50 of stocks and bonds. The reallocation occurs when the original mix reaches at least a 8% variation. Assume, for instance, that the equity value grows to 60€ and, consequently, the total portfolio reaches a value of 110€. Thus the relative percentage of the portfolio allocated to stocks is then equal to 54,5%, which represents a 8% variation as compared to the target proportion of 50%. In this case, therefore, the allocation is not readjusted to the original 50/50 mix but to 52,5/47,5 as per the 5% threshold.

Chapter 2: The Exchange-Traded Funds

An exchange-traded fund (ETF) is an investment vehicle, traded on a stock exchange, which allows the investors to track an index, a commodity, bonds or a basket of assets through a single liquid instrument. The ownership of an ETF is divided into shares, in such a way that the shareholders directly own a fraction of the fund which reflects a corresponding portion of the underlying assets, thus indirectly held. ETFs are a type belonging to a wider category of financial instrument referred to as exchange-traded products (ETPs). The ETPs are typically open-ended²⁹ investments listed on exchange and traded like shares, whose aim is usually to passively replicate the same return as a specified benchmark. As opposed to mutual funds that are priced once at the end of the day, the ETPs enable also the intraday trading, such as stocks, and they usually have low managing costs and expense ratios. The three typical forms ETPs split into are in fact:

- Exchange Traded Funds (ETFs);
- Exchange Traded Currencies and Commodities (ETCs);
- Exchange Traded Notes (ETNs).

As aforementioned, despite these different categories, all ETPs denote some basic common characteristics:

- Listed on stock exchange;
- Traded like shares;
- Liquid products;
- They track a benchmark;
- Passive vehicles.

ETFs in particular give access to equity, commodity and fund of funds indexes, fixed income and money markets. While ETCs allow investments in currencies, individual and baskets of

²⁹ An open-ended fund is a collective investment vehicle with no restriction on the number of shares that can be issued. Usually, investors buy and sell shares directly to the fund. Therefore, the purchase creates new shares while the redemption occurs when there is a sale.

commodities and ETNs provide access to assets or benchmark employing an uncollateralized debt security. The structure and the operation of the ETFs will be further analyzed in the next subsections.

2.1 History of ETFs

2.1.1 Origin and development

The structuring of Exchange-traded funds as financial products is a consequence of the development of the so-called program trading, a new technologic approach to financial markets born between the late 70's and the early 80's. The technology improvement supported this innovative methodology, allowing for the trade of an entire portfolio with a single market order placed at a brokerage firm. This new method, mainly employed to trade S&P 500 stocks, was accompanied by the introduction in the market of S&P 500 index futures contracts. Both instruments together brought a simplification of the trading process and a broadening of combinations attracting different kinds of investors. Specifically, beyond the main institutional investors, the interest for easily and readily tradable baskets of securities also appealed to private and smaller institutional investors.

The first prototype of ETF is considered to be the Toronto Index Participation Shares (TIPs). Designed to track the TSE-35 Index, it was launched in 1990 on the Toronto Stock Exchange providing access to the 35 largest Canadian companies. In fact, it was born as an evolution of the Index Participation Shares (IPS), a first-in-kind product, which started trading in 1989 on the American Stock Exchange, soon liquidated since a lawsuit declared it as illegal. Extremely low cost, the TIPs proved to be quite successful at first, but precisely because of its low expense ratio, it turned to be too costly for the Exchange and was shut down at the beginning of 2000. Actually, the Standard and Poor's Depository Receipts (SPDRS), launched in 1993 on the American Stock Exchange, is recognized as the first true ETF (traded in the US market). It represents the shares of a single unit trust holding an S&P

500 portfolio susceptible to adjustments as the value of the index changes. Thus the SPDRs provided access to a major market index and relied on a creation-redemption mechanism allowing the market value and the fair value to move closely. These characteristics along with the unit trust structure³⁰, motivated by an initial caution in keeping the costs low, were the fundamentals for the success of the ETF, such that other ETFs later traded have been developed with the same structure. The SPDRs, in fact, at the end of its first year held \$475 million of asset under management and it currently is the largest ETF in the world³¹. However the ETFs sector grew slowly during the early 90's, in fact the second exchange-traded fund was introduced in the market only in 1995.

A turning point in the history of the ETFs industry is represented by the issue of the World Equity Benchmark Shares (WEBS)³², by Morgan Stanley in 1996, which can be considered innovative products for different reasons. First of all, despite US based, these funds also hold foreign stocks (belonging to non-US based companies) thus providing the investors with exposure to additional country indexes and giving them access to a variety of different equity markets. Secondly, they employed a structure similar to mutual funds instead of the unit trust, allowing further flexibility and is widely adopted nowadays, as well. In 1998 therefore, the total asset under management in the whole ETFs industry amounted to \$15,6 billion. Afterwards, as the interest for ETFs was growing, new major players started entering in the market. Precisely, in 1999 the Bank of New York launched the Qs, focused on technology stocks and based on the NASDAQ 100 Index, which alone raised investments for \$18,6 billion in the first year, while BGI structured 50 new ETFs composing the iShares series (including also the former WEBS). In the 2000's another main firm entered the industry with its Vanguard Index Participation Equity Receipts (VIPERs) whose innovation was to be issued as particular share classes of already existing mutual funds. As it has been discussed so far, the first generation of ETFs concerned solely the equity market, so the natural evolution for the ETFs providers was aimed at looking for new opportunities in the commodity and

³⁰ An investment trust is a close-end fund constituted as public limited company.

³¹ J.M. Hill, D. Nadig, M. Hougan (2015), *A Comprehensive Guide to Exchange-Traded Funds (ETFs)*, The CFA Institute Research Foundation.

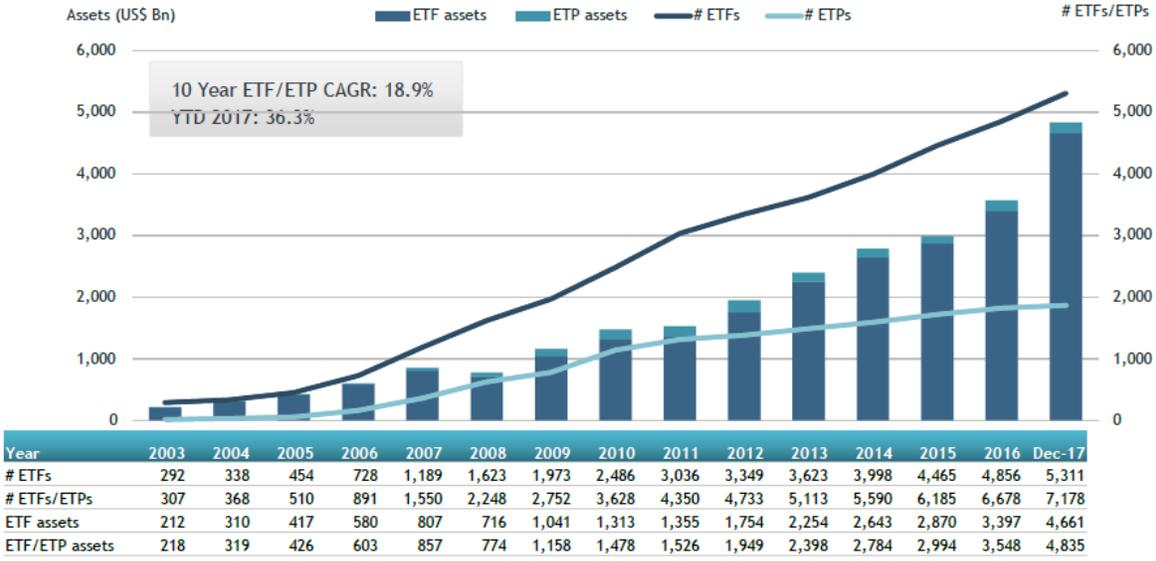
³² Now rebranded as iShares MSCI Series, since the intervention of Barclays Global Investors (BGI).

fixed-income markets. In this regard, the launch of SPDRS Gold Shares in 2006 had a significant success. Moreover in the same period, the permission accorded to ProShares by the US regulatory agency to introduce leveraged and inverse ETFs, involving short selling and derivatives trading, contributed to boost the industry development

2.1.2 Current scenario

25 years after the inception of the first ETF, today the this industry is still considered as quite a new sector, continuously and rapidly evolving. In the last ten years the ETFs market kept developing and spreading considering both the rising number of ETFs registered and available and the amount of assets under management. In particular, in the very last few years it recorded an impressive and sharp growth. In fact the global ETF assets under management rose, only in 2017, by \$1,264 trillion reaching a total of \$4,661 trillion (of whom \$3,331 trillion concerns US stocks), while the number of ETFs available in the market got to 5311 (of whom 1834 are US based) as of December 2017. The detailed growth of the ETF industry starting from 2003 up to now can be observed in the Figure 2.1.³³

Figure 2.1 Global ETF and ETP growth by number of registered products and by asset under management



³³ Source: ETFGI’s December 2017 Global ETF and ETP industry insights report.

Some further interesting data, related to the ETF industry's wealth, concern the types of asset class which best suit the ETF business. As of March 2014 ETF managers mainly invested in US equity (54% of the total global assets under management), then in non-US equity (24% of the total AUM) while they invested only smaller fractions in US fixed income (about 14%) and commodities (around 4%). Summing up, the stock market, all the ETFs were born for originally, still largely represents the main asset class in ETF portfolios accounting for almost the 80% of the total global AUM invested in ETF. Since the industry started looking at different asset classes only in 2006, these still play a more marginal role. Specifically, the investments in fixed income amount to 15% of AUM and to 4% in commodity. As far as the remainders are concerned (currency, leveraged, inverse), they all oscillates between zero and 1%. In addition, from the data it looks clear that the US represent the main market for ETFs.³⁴

In fact, the two largest ETFs by AUM are currently the SPDR S&P 500 (SPY) with \$255 billion and the iShares Core S&P 500 (IVV) with \$143 billion both tracking the S&P 500 index and the third, the Vanguard Total Stock Market, based on US equity as well, accounts for \$94 billions of AUM³⁵. Furthermore nowadays, the major ETFs providers are: BlackRock (which purchased Barclays Global Investors in 2009) issuing the iShares ETFs, State Street Global Advisors (SSgA) controlling the SPDR S&P 500 and Vanguard. These three together dominate the industry summing together the 70% of the assets under management by ETFs worldwide.

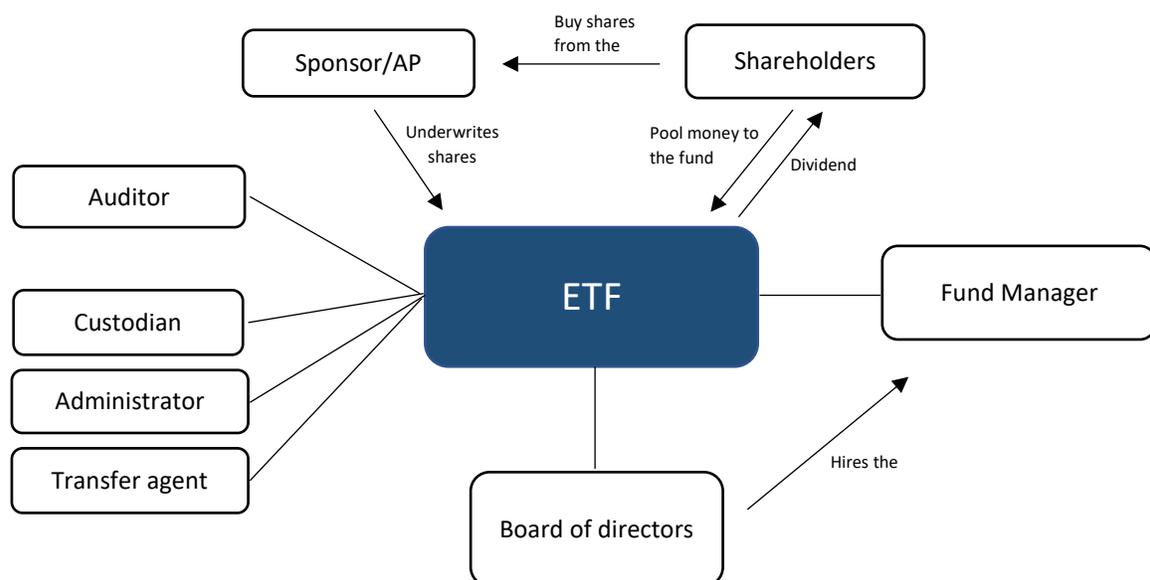
³⁴ Data source: J.M. Hill, D. Nadig, M. Hougan (2015), *A Comprehensive Guide to Exchange-Traded Funds (ETFs)*, The CFA Institute Research Foundation.

³⁵ Market data as of 30/04/2018.

2.2 Structure and Functioning

The players involved in the organizational structure of an Exchange-traded fund are numerous. First of all, each fund is bound to a board of directors primarily acting in the shareholders' interest and overseeing the fund's activities. The shareholders, whose capitals are pooled into the fund, own a fraction of the fund proportional to their investment (i.e. a share) and, in some cases, they may be entitled of dividend paybacks as it happens for normal stocks. Secondly, a management company (also investment advisor), nominated by the board, is in charge of handling the portfolio as defined by the prospectus (the document providing information about the characteristics of the fund, such as the types of targeted asset class and the entailed riskiness). Thirdly, a fundamental role is also played by the sponsor which, as main underwriter, has the right by contract to buy the fund's shares and to resell them to the public (usually through the so-called "authorized participants"). Finally, the other participants involved in the organization of an ETF are the administrator, the custodian and the transfer agency that are respectively in charge of performing the accounting functions, executing the trades between the shareholders and

Figure 2.2 ETF structure



the fund, physically holding the fund's asset. Sometimes these agents may even coincide or some of the functions, such as the administrative ones, may be handled internally to the management company. However, every fund needs to have an independent external auditor. Typically the organization of an ETF may be based following the below scheme.

2.2.1 ETF characteristics

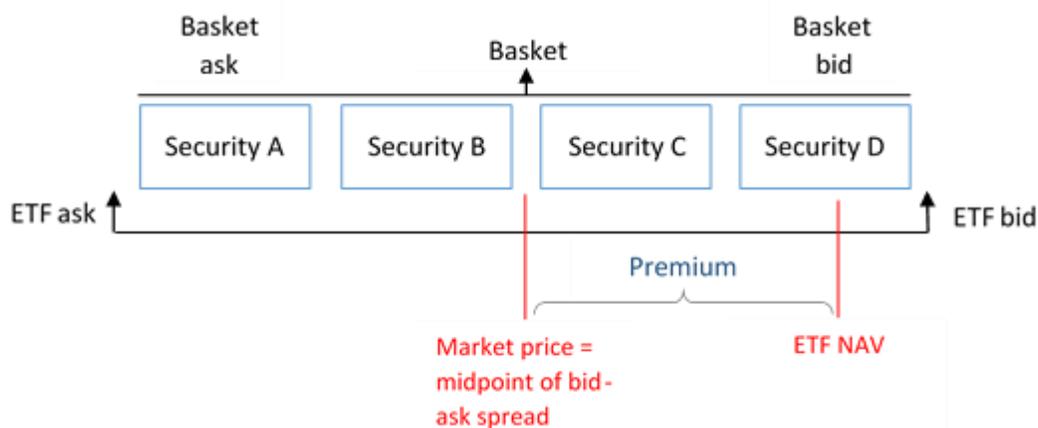
The structure described in the previous section is generally common to a wider category of funds, such as mutual funds, thus it is not specifically linked to ETFs. But there are more features that characterize the ETF mode of operation distinguishing it from other kinds of funds. In principle, the 'exchange-tradability' is the source of a variety of benefits. The first one could be said to be the easy accessibility to a broad range of investment opportunities, markets, asset classes and levels of risk that ETFs can offer to investors, regardless the asset size. As mentioned earlier, in fact, an individual may get exposure to foreign equities or fixed-income as well as commodities and currencies, through a single investment in an ETF. Secondly, the ETF market guarantees a high level of transparency. Most ETF issuers publish their portfolios composition and report the performances on a daily basis and their products are continuously priced as common stocks; on the other hand, the prices of mutual funds are released only at the day end or in some cases even weekly or monthly. Generally, an ETF shareholder can always control how the fund managers are investing the capital and which are the positions on the underlying, while mutual funds have longer reporting periods. Thirdly, the ETFs are highly liquid instruments, since they can be sold and purchased intraday on the secondary market.

Controversial is, instead, the cheapness of ETFs as compared to similar mutual funds³⁶. In fact, on the one side the average expense ratio for ETFs is indisputably lower (about 0,57%), because most ETFs tracking an index are passively managed while mutual funds, as

³⁶The Vanguard Group, *ETF Knowledge Center*, available at: <https://advisors.vanguard.com/VGApp/iip/site/advisor/etfcenter>

mostly active, charge higher expenses (1,24%)³⁷. Meanwhile, on the other side ETFs are subject to additional expenses due to the transactions taking place in the secondary market such as: commissions, bid-ask spreads and premiums or discounts deriving from the potential difference between the fund market price and the market value of the underlying assets. The market value of the underlying assets is represented by the NAV (Net Asset Value), that is a per share quantity given by the total portfolio value divided by the number of shares composing the fund, commonly released at the end of each trading day. These premiums or discounts in the ETFs prices may depend on several factors. In case of international ETFs, the difference is usually due to the lag in the trading hours (time zones or different business days) of the ETF local market with respect to foreign markets where the underlying securities are exchanged. On the other hand, in case of domestic ETFs, the premium may be related to the demand, thus depending on the bid-ask spread. For instance, while the ETF market price is computed as the midpoint of the bid-ask at a given time, the underlying securities may be priced according to the bid value. Then, of course, variations on the demand for ETF drive the fluctuation of the bid-ask quote for a better understanding see the Figure 2.3 below.³⁸

Figure 2.3 Premium and bid ask-spread relation



³⁷ Source: Morningstar 2015.

³⁸ Source: The Vanguard Group, Inc [US], *ETF Knowledge Center*.

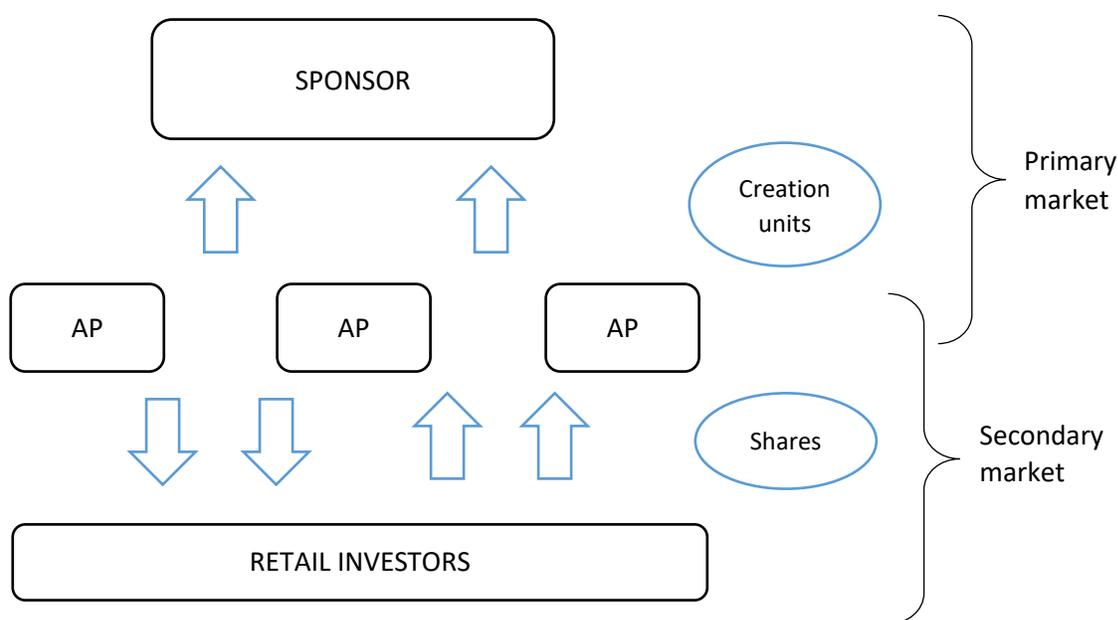
2.2.2 Creation/Redemption Process

A very unique feature characterizing the functioning of ETFs is the shares creation/redemption mechanism. Although, ETFs trade on exchanges similarly to normal stocks, unlike these, they are not issued through an IPO (Initial Public Offering) and they are mostly open-ended. In fact, the number of shares in an ETF is not fixed, but they may be continuously created and redeemed depending on demand, as opposed to stocks whose trading process simply requires each buyer to meet a seller in the secondary market (with the intermediation of a broker). Also, while usually any investor can purchase new shares directly from the mutual fund company, only some large institutional authorized participants (APs) take part in the creation/redemption process of ETFs shares. The interaction between the APs and the ETF issuer take place in the primary market typically through transactions in-kind involving large block of units (a “creation unit” usually corresponds to 50000 ETF shares), while cash transfers are employed only rarely. Thus the APs willing to purchase ETF shares firstly needs to buy in the market a certain basket of securities the ETF manager wants to hold as underlying (the “creation basket”); then they can deliver it to the fund management in exchange for a corresponding value of ETF shares. In this way new shares have been created. Conversely, the APs, willing to liquidate their shares in the ETF, can redeem them getting the corresponding value of underlying assets in return. When new shares are created, the APs can resell them on the secondary market; when they are redeemed, the APs can place the underlying basket of securities on the exchange.

The creation basket is disclosed by the ETF manager in the morning and the creation and redemption orders are executed only at the end of the day, while the APs can trade the underlying basket throughout the day. This timing allows for arbitrage opportunities, but it is fundamental to keep the ETF market price in a narrow range to the NAV. Imagine that the investors’ demand for ETF shares grow consistently, such that their price would increase (as it happens for stocks) to the extent that it would not reflect anymore the true value of the underlying assets held in the portfolio. At this point the AP, informed of the daily creation

basket, may want to sell the overrated ETF shares in the secondary market and buy the cheaper underlying securities which can be exchanged, then at the end of the day, to create new shares at a fair price. Thus, the APs selling the ETF shares would push their price down and, conversely, significant purchase orders on the underlying basket would make the price of the securities goes up. This continuous process reiterates until the relative market price of the ETF and the value of the underlying (the NAV) readjust to an equilibrium level. Clearly, APs bear the transaction costs, thus the equilibrium level is realistically a range, in which premiums and discounts may occur. The scheme below illustrates the functioning of the creation/redemption mechanism.³⁹

Figure 2.4 Creation/redemption process



³⁹ M. Liera (2006), *Tutti gli strumenti del risparmio, dai titoli di stato agli hedge fund, dalle azioni agli ETF*; Schroders, Milano.

2.3 Regulation

Since the ETF industry was born in the US, it is necessary to provide first a brief description of the American regulatory framework as well as of the European context, where ETFs were introduced later.

2.3.1 In the United States

The two pillars of the US regulation on ETFs are the Securities Act of 1933 and the Investment Company Act of 1940 issued by the US Congress. The former, in particular, rules any kind of exchange-traded product available for public investments behaving as common equities, while the latter governs the organizations mainly involved in trading and investing in securities. In particular, introducing the figure of Registered Investment Company (RIC), the Investment Company Act of 1940 defines the structure of mutual funds, while it is not fully suitable to ETF as exchange-traded product. Hence, another Investment Company Act rectifying those exemptions applicable to ETF was released in 1990. The peculiar characteristics of the RIC, as defined by the act of 1940, are the independence of the board and the ability of continuously create and redeem shares. The main extensions provided by the act of 1990 applicable to ETFs are, instead, the possibility of trading ETF shares at a market price not reflecting exactly the NAV and the concept of creation unit as opposed to the creation and redemption of individual shares. Today, the vast majority of ETFs are registered as RICs under the 1940 Act, but not all of them. In fact, the first ETF ever to be issued, the SPDR S&P 500, although regulated by the 1940 Act, is registered as Unit Investment Trust (UIT). This is a simpler and stricter structure, only allowing for fully passive management.

2.3.2 In Europe

In the European context, the ETFs are registered as Undertaking for Collective Investments in Transferable Securities (UCITS), which identifies a wider category of funds, and regulated by the European Securities and Markets Authority (ESMA) under the Directive 2009/65/EC of the European Parliament (“UCITS Directive”). “A UCITS exchange-traded fund is a UCITS at least one unit or share class of which is traded throughout the day on at least one regulated market with at least one market maker which takes action to ensure that the stock exchange value of its unite or share does not significantly vary for their net asset value”⁴⁰. Moreover, most UCITS ETFs are considered Index-tracking UCITS and aim to replicate the performance of an index. As such, they are subjected to specific provisions regarding the prospectus disclosure. Thus, the prospectus of an index-tracking UCITS should include in particular: a basic presentation of the fund (creation date, domiciliation, management company, sponsors), the risk profile, commissions and expenses, the description of the reference index including details on the underlying securities, information on the replication method (physical or synthetic) and the description of the factors that are likely to influence an accurate tracking of the index.⁴¹ In addition, in the annual reports the size of the tracking error⁴² should be stated and justified⁴³. However, among the UCITS ETF there also exist some actively managed UCITS. These should disclose additional information in the prospectus: how it will meet the stated investment policy (in particular the intention of outperforming), its policy about portfolio transparency and how the iNAV⁴⁴ is calculated.⁴⁵

⁴⁰ ESMA (2014), Guidelines on ETFs and other UCITS issues, Guidelines for competent authorities and UCITS management companies, ESMA/2014/937.

⁴¹ Directive 2009/65/EC of the European Parliament and of the Council of 13 July 2009, *on the coordination of laws, regulations and administrative provisions relating to undertakings for collective investment in transferable securities (UCITS)*.

⁴² The tracking error represents the discrepancy between the performance of the ETF and the performance of the reference benchmark.

⁴³ ESMA (2014), *Guidelines on ETFs and other UCITS issues*, Guidelines for competent authorities and UCITS management companies, ESMA/2014/937.

⁴⁴ Indicative Net Asset Value, that is a measure of the intraday value of the NAV.

⁴⁵ ESMA (2012), *ESMA’s guidelines on ETFs and other UCITS issues*, Consultation paper, ESMA/2012/44.

Finally, UCITS are also subjected to some constraints concerning the portfolio composition that may slightly vary, depending on the country of issue. For instance, as reported in the figure below, a Luxembourg based UCITS may invest up to:

- 20% of its net asset in shares or bonds issued by the same body except for specific cases where the limit is raised to 35%;
- 10% in transferable or money market instruments, where the limit is raised to 20% in case of Index Funds;
- 20% in bank deposits;
- from 5% to 10% in OTC derivatives.⁴⁶

Table 2.1 Individual and combined investment limits per type of security as percentage of net assets

Type of instruments	Limits	Exceptions
Transferable securities	10%	up to 20%
Money market instruments	10%	
Bank deposits	20%	
OTC financial derivatives	5% / 10%	
Combined total exposure per issuer	20%	up to 35%

2.4 ETF Management

Theoretically, ETFs investment strategies, as general portfolio management strategies, are tagged as passive or active. In origin, ETFs were born exclusively as passive instruments and only later, in the mid 2000s, the first active exchange-traded funds started to trade. Today, the passively managed ETFs represent the great majority of the industry; in 2015 the

⁴⁶ Source: PricewaterhouseCoopers Société coopérative (Luxembourg), 2014, *Eligible investment and investment restrictions*.

number of active ETFs available in the US market was 132 versus 1411 passively managed ETFs⁴⁷.

However, along with the ETF industry growth and the development of alternative strategies, the distinction between active and passive management has become not always so clear. In theory, passive ETFs simply aim at replicating the performance of the market by tracking a reference index, employed as benchmark. In this regard, the ETF portfolio is built through the replication of its benchmark's composition, whether exact or selecting some significant underlying securities as sample. A passive approach allows to minimize the amount of transactions and the portfolio turnover, implying respectively costs, capital gains and the related tax. Nonetheless, it relies on rule-based schemes such as to eliminate the manager's discretion and to reduce the fees. The passive management is based on the classic assumption of market efficiency (E. Fama 1970), stating that the market prices already reflect all the available information, such that an investor is not able to outperform the market in the long run. Furthermore, typically the true passive ETFs are only those replicating indexes built on market capitalization-weighting schemes. Namely, portfolio's assets are weighted according to their market capitalization.

In general, ETFs are defined as active when their objective, as per prospectus, is to outperform the reference index. An actively-managed ETF is, by definition, non-rule-based, and it relies on the ability of the manager who, smartly selecting an appropriate portfolio composition, seeks to beat the performance of the market. However, an active ETF refers to a benchmark, but the manager can count on some degrees of freedom using his discretionary power to deviate from the index. With this approach, the security selection and the market timing are critical. The manager may change the portfolio allocations, according to the market conditions, for instance, liquidating underperforming assets (even if included in the benchmark) and adding securities referring to sectors or asset classes in good wealth. Moreover, choosing the right moment to liquidate or take a new position, is fundamental for the overall performance of the fund. As known, the beta is the indicator of the

⁴⁷ J. Dickinson, T. Known, J. Rowley (2015), *Choosing between ETFs and mutual funds: Strategy, then structure*; Vanguard Research.

systematic risk related to the performance of the market and alpha represents the measure of the additional return realized through the manager's active strategy. As of today, it is argued whether it is convenient to invest in active ETFs or not, since only few of them proved to really outperform their benchmarks. The idea of active ETF industry as a "zero-sum game" is commonly spread, that is to say for any manager who succeeds in beating the market there is one who performs worse, leading the average alpha of the industry to zero.

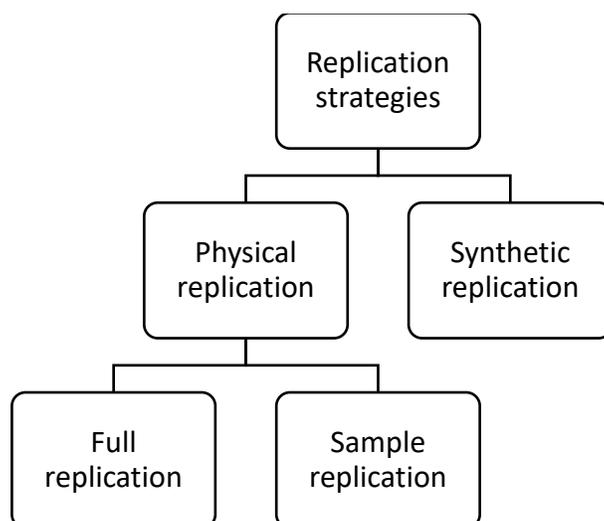
As aforementioned, the difference between active and passive approaches is not always so clear. In fact, in a wider definition of active ETFs, may be also included those which, although passively replicating an index, employ weighting schemes alternative to the usual capitalization-based. This is the case of a recently developing variety of ETFs relying on "smart-beta" strategies. The main types of smart-beta ETFs are:

- fundamentally-weighted: they track an index whose securities are allocated according to fundamental factors such as earnings, dividend, revenues, book value, cash flows;
- equally-weighted: the assets in the portfolio have the same weight
- low volatility: securities allocation depends on their historical volatility (low volatile assets are preferred)

2.5 Index Replication

The index replication represents a fundamental issue in management of an ETF, since it may significantly affect the performance of the fund. Index-tracking exchange-traded funds may adopt different methods to replicate their benchmark portfolios. Figure 2.5 shows a classification of the different replication strategies. In principle, the replication may be either physical or synthetic. The physical approach implies that the ETF actually owns the same securities composing the reference index, fully or just in part. The replication is full

Figure 2.5 Replication strategies



when the ETF's basket comprises all the assets underlying the benchmark in the same proportions. It can guarantee a faithful match of the index characteristics, but it is conveniently applied only when the number of the assets is relatively small and there is a low level of portfolio turnover, otherwise it would require many resources and transaction costs. Instead an ETF, based on a sample replication, does not hold all the components of the index, but only a significant sample representative of the key features of the index. The sample replication is useful when the benchmark is composed of a large number of securities which are frequently changing, but it is more likely to exhibit a tracking error. A variation of the sample approach, is represented by the replication by optimization. This still relies on a sampling technique, but it employs quantitative models which, analyzing historical asset prices and correlation, seek the optimal basket of securities that best represents the index.

On the other hand, the synthetic replication involves a more complex process. A synthetic ETF, instead of directly holding the assets representative of the index, enters in a swap agreement with a counterparty to receive the benchmark's return as cash flows in exchange for a collateral. Specifically, the ETF uses the capital, deriving by the shareholders' subscriptions, to enter in the swap contract. Therefore, it agrees to give this capital to the counterparty, in cash or by buying a substitutive basket of securities (deposited as

collateral), which in turn promises to provide to the ETF the gains realized by the index. Clearly, the ETFs based on synthetic replication are subjected to the counterparty risk. In fact, in case of default of the counterparty the ETF takes back the collateral deposited as guarantee, however it may not be able to repay its investors.

Chapter 3: Empirical Analysis

In this chapter the portfolio insurance strategies previously described will be further explored and applied to portfolios composed by ETFs. In particular, the analysis of the strategies and the evaluation of their performances have been conducted, through the software Matlab, considering the historical data over a 10-years time span stretching from 2008 to 2017. To provide an effective estimate of the performance of the various strategies the period has been chosen as to include diverse market phases, from the downturn in the equity market started in 2008 to the solid growth occurred in the most recent years.

As described from a theoretical point of view in chapter 1, the asset allocation schemes are developed as a combination of two investments, one in an ETF and the other in the money market (risk-free). Then, the results from the different strategies are reciprocally compared to the performance of a full investment in the fund or in the risk-free asset, through some classical risk adjusted performance ratios: the Sharpe ratio, the Information ratio, the Calmar ratio and the Sortino ratio. Furthermore, for the purposes of a wider analysis the same insurance strategies have been replicated on four ETFs, all referring to the European corporate framework, but involving investments in different equity classes based on their market capitalization, for diversification purposes. Equity ETFs have been preferred to those investing in sovereign bonds or alternative assets for several reasons. Firstly, the ETFs have historically been developed for the stock market and, today, the vast majority of them still invests only on equities, insofar as more data are available as compared to lots of bond and alternative ETFs recently launched having less than ten years of historical prices. Secondly, since the aim of this thesis is to verify the effectiveness of portfolio insurance strategies balancing a risky investment with a risk-free one, it is important to consider equities instead of bonds which commonly already entail less riskiness by nature.

3.1 ETFs Selection

To develop the portfolio insurance strategies presented in chapter 1, the average 12-months Euribor over 10 years (2008-2017) has been considered as risk-free rate. The Euribor has been preferred to the interest rate of short term government bonds of reliable countries, typically used as proxy for the risk-free rate, because in the period analyzed these exhibited negative values for a long time range. Moreover the following four exchange traded funds have been selected as alternatives for the risky portion of the portfolio:

- The Lyxor EURO STOXX 50 UCITS ETF, which provides exposure to the 50 European blue-chip companies, with the largest market capitalization⁴⁸, from twelve different countries of the euro zone. Launched in 2001, it currently holds \$6,9 billion⁴⁹ in asset under management and relies on a full physic replication method to track its benchmark, the EURO STOXX 50.
- The SPDR MSCI Europe Small Cap UCITS ETF, which is designed to replicate the performance of the related index, the MSCI Europe Small Cap Index. It captures the performance of the small-cap⁵⁰ equities in the European market. Launched in 2005, the fund is based on a physical replication of the benchmark by sampling and counts around €95 million⁵¹ of AUM.
- The Lyxor MSCI Europe UCITS ETF, which tracks the corresponding cap-weighted index, the MSCI Europe developed to measure the performance of the broad European equity market. Launched in 2006, the fund employs a synthetic replication technique and it holds €1,9 billion⁵² in AUM.

⁴⁸ Over \$10 billion of market capitalization.

⁴⁹ As of 30/05/2018.

⁵⁰ Small cap commonly refers to the companies with a market capitalization value included between \$300 million and \$2 billion.

⁵¹ As of 30/05/2018.

⁵² As of 30/05/2018.

3.2 Performance Ratios

To evaluate and compare the performance of the portfolio insurance strategies some performance indicators will be employed alongside the simple return, volatility and maximum drawdown. They are defined as risk-adjusted performance ratios since they measure the results not in absolute value but in relation to the level of risk faced. Specifically four alternative performance ratios will serve the analysis execution:

- the Sharpe ratio,
- the Information ratio,
- the Calmar ratio,
- the Sortino ratio.

3.2.1 Sharpe ratio

The Sharpe ratio is defined as the average excess return of the portfolio over the risk-free rate per unit of volatility (the total return and the total risk in the period may be considered as well, instead of the average values). Hence it is computed according to the following formula, where r_p is the portfolio return and σ_p is the unit risk measure given by the standard deviation of the portfolio returns.

$$\text{Sharpe ratio} = \frac{r_p - r_f}{\sigma_p} \quad (13)$$

Even though the Sharpe ratio is the most famous and widely used among the various performance index, it is not always very accurate. First of all, it relies on the hypothesis of normality of the distribution of returns⁵³, which is in reality not always true. Secondly, the risk component takes into account both the upward and downward deviations from the

⁵³ Going forward all the analysis is based on the assumption that the prices are log-normally distributed such that the log-returns are normal.

mean, while for a typical investor the risk to avoid only concerns the downside movements in the price.

3.2.2 Sortino ratio

The Sortino ratio derives from an evolution of the Sharpe index. In fact, as the Sharpe does, it measures the average excess portfolio return per unit of risk, but the risk coefficient only captures the downside deviation, instead of the classic standard deviation. The formula for the Sortino ratio is:

$$\textit{Sortino ratio} = \frac{r_p - r_f}{DR} \quad (14)$$

where the downside risk (DR) is the standard deviation of the negative asset returns. An alternative stricter definition of the Sortino index considers the DR not as related to the negative returns, but as the standard deviation of the returns falling behind a minimum acceptance level, which may be represented by the risk-free rate.

3.2.3 Calmar ratio

The Calmar ratio analyses the performance of the investment in relation to its drawdown risk. That is to say, the excess return on the portfolio is divided by the maximum drawdown in the given period:

$$\textit{Calmar ratio} = \frac{r_p - r_f}{\textit{Max Drawdown}} \quad (15)$$

The maximum drawdown represents the highest potential loss for an investor who may purchase a security at its maximum level and sell it at the lowest.

3.2.4 Information ratio

The information ratio follows the same structure as the indexes described, in other words it measures the excess return on the portfolio adjusted per unit of volatility. However, in this case the excess return is computed as the difference between the portfolio return and the return of a given benchmark, while the risk unit is represented by the tracking error, which is the standard deviation of the aforementioned difference. In this sense, the information ratio is a measure of the 'active return' on the investment.

$$\text{Information ratio} = \frac{r_p - r_b}{\sigma(r_p - r_b)} \quad (16)$$

3.3 Applications

In this subsection the portfolio insurance strategies will be explored in depth and they will be effectively applied to daily historical data. However, before starting to implement the simulations, some general assumptions need to be made. The same hypotheses, with some slight variations, are valid for all the cases and they have been chosen in a way that allows the strategies to be reasonably comparable, despite their diverse functioning and structure with different parameters.

First of all, the investment strategies use a starting capital of €1000 and the risk-free rate is represented by the average 12-month Euribor over the considered period, that is $r_f = 1,182\%$. Then, each strategy (except for the option based) has been set up with an initial floor equal to the 70% of the starting investment, corresponding to a guaranteed capital at maturity of about €800⁵⁴. This level has been chosen to let the insurance as such protect a high percentage of the initial capital. However, values of the floor higher than 70% would

⁵⁴ The guaranteed capital at maturity corresponds to the future value of the initial floor. Therefore:
 $F_T = F_0 * e^{r_f * T} = 700 * e^{0,01182 * 10} = 787,83\text{€}$.

have brought to an imbalance in the initial allocations for some strategies. As far as the Buy and Hold and the Constant Mix are concerned, this floor value implies an initial 70/30 allocation mix of bonds and ETF shares then holding for the entire time horizon, while in the CPPI, EPPI and TIPP it varies depending on the cushion and the multiplier. As regards the CPPI and the TIPP, the constant multiplier m has been set to 2 while for the EPPI the parameters a and η have been set to 2 and 1,1 respectively. These values have been chosen to guarantee an initial allocation mix balanced between the risky and the risk-free assets and similar to the setup of the other strategies. In this way the comparison between the different approaches can be more effective. In fact, given these parameters all the three strategies initially allocate about the 60% of the capital in the ETF and the 40% in bonds. Moreover, for a more complete analysis, three alternative rebalancing options have been simulated, in fact the portfolio allocations have been readjusted both periodically (daily and monthly⁵⁵), and by threshold (8%). In particular, the threshold has been considered as variation edge of the given mix for the Constant Mix and of the target multiplier for the CPPI, TIPP and EPPI. In other words, applying the CM, the portfolio has been rebalanced anytime the fraction invested in the ETF has changed by at least 8%, while for the other strategies the threshold has been imposed on the value of the multiplier. The reference multiplier is a constant parameter in the CPPI and TIPP, while in the EPPI the rebalance is triggered when the effective multiplier exceeds the theoretical one by the given threshold. In addition, whenever there is a reallocation of the assets which thus involves the purchase of ETF shares and the sale of bonds, or vice versa, a transaction cost of 10 basis points is applied.

As far as the Option Based Portfolio Insurance is concerned, the setting of the strategy requires few different preconditions. First of all, the simulation has been implemented in two alternative ways. One implies a static allocation scheme involving the purchase at time zero of ETF shares and a corresponding amount of put options to protect the investment, which does not change over the entire time horizon. Given the absence of put options

⁵⁵A month is considered to be composed of 22 business days, such that the portfolio readjustments occur every 22 days.

written on the fund and of historical market prices of options written on the benchmark, the put price at time zero has been calculated through the Black-Scholes formula (equation 6). In this regard, the option is calculated as it is written on the index with: expiration in 10 years and strike price equal to 0,9 times the benchmark value at time zero (such to keep the guaranteed capital close to the 80% of the initial investment, as for the other strategies)⁵⁶. Therefore, in this situation the portfolio would be composed of n put options and αn ETF shares where α is the ratio between the index value and the ETF price at time zero. Furthermore, the initial portfolio composition holds till maturity with no rebalances, since the options are European-style, thus not early exercisable. The other alternative option-based strategy is based on a dynamic scheme, which allows the investor to synthetically replicate an option written on the fund through a position in the ETF and an investment in the money market⁵⁷. In this particular scenario the strategy does not replicate only the put, but the whole portfolio of options and shares returning a guaranteed capital computed with the same principle as in the static scheme⁵⁸. Because of the complexity of the model, in this case the transaction costs are disregarded, so the frequency of rebalances is limited to three months, which ties to the usual expiration cycles of option contracts. For both the option-based strategies the portfolio is considered to be self-financing, such that the cost of purchasing the option is subtracted from the initial capital.

Finally, for all the strategies the constraint of short selling has been imposed, such that in any moment the fraction of the capital allocated to the different asset classes has to be included between 0 and 1. As far as the Stop Loss Portfolio Insurance is concerned, it has not been included in the analysis, since it would not have led to interesting results. In fact, due to the heavy downturn in the world economy at the beginning of the time span

⁵⁶Given the initial capital (I) the strategy is assumed to be self-financing, in other words a fraction (z) of the capital is liquidated to buy the protection (p). Therefore, whether the strike price (K) is set to the 80% of the Index value, the resulting put option (p) would instead protect only the 80% of $(I - z)$. Since the aim is to insure the 80% of I , it is necessary to raise the level of K . At this point, the value of 0,9 has been chosen arbitrarily. In fact, the exact value of K to reach the target level of protection changes for each fund, depending on the index value.

⁵⁷ In this particular case, the synthetic replication involves buying Δ stocks and taking a long position in the risk-free asset.

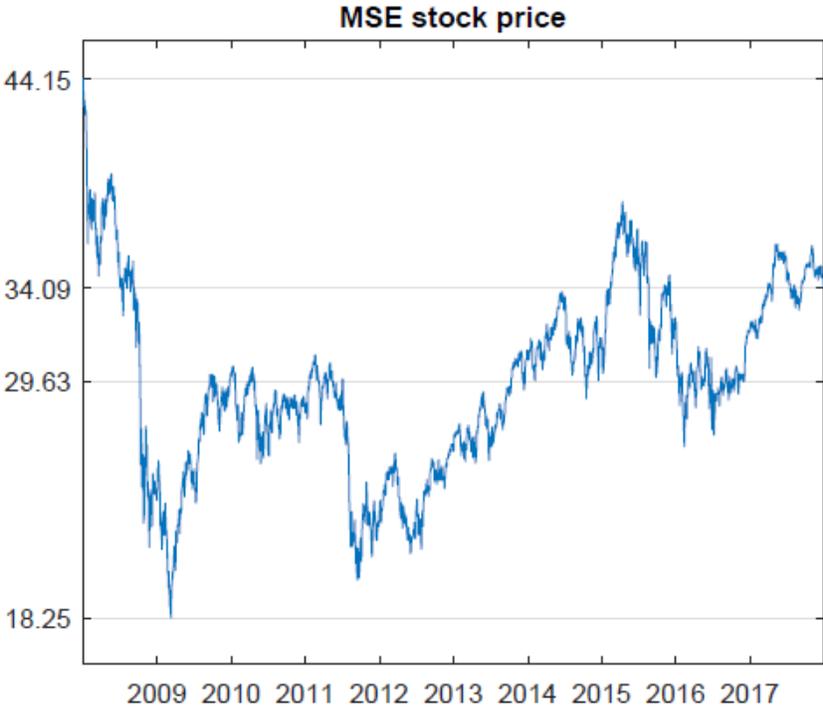
⁵⁸ In this case, the K of the synthetic portfolio has been fixed again to the 90% of the capital $(I-z)$, where $(I-z)$ has been set through a target seek function, such that $I = \text{€}1000$.

considered, the strategy would have required to liquidate the whole position in the ETF in favor of the risk-free asset very soon, thus impeding to get a satisfying return at maturity.

3.3.1 LYXOR EURO STOXX 50 UCITS ETF (MSE)

The LYXOR EURO STOXX 50 UCITS ETF is the first fund employed to test the various insurance strategies. See in figure 3.1 the graph of the daily price of the fund over the period 2008-2017, where €44,15 is the maximum value; €18,25 the lowest; €29,63 the mean price over the period and €34,09 is the final value.

Figure 3.1 MSE price trend Jan 1st 2008 - Dec 29th 2017



Besides, the fund exhibits the statistics summed up in the table 3.1⁵⁹. The most relevant aspect, disclosed by the graph and data, is the negative overall performance (-25,87%) realized by the ETF. In particular, it suffered a heavy decrease in value during 2008 and 2009, due to the general economic downturn, followed also from another negative peak in mid-2011, such that it was not able to generate sufficiently high returns in the subsequent years to reach the pre-crisis level. The maximum drawdown, happening between day 1 and day 310 of the time series, also reveals to be very consistent.

Table 3.1 risk-return statistics for MSE ETF

st. deviation	annual return	total return	max drawdown
23,373%	-2,589%	-2,587%	58,666%

Now let us apply the dynamic allocation strategies to the fund and see how an insured portfolio could have performed in the same period. First of all, each singular strategy (CM, CPPI, TIPP and EPPI) has been ran in three different scenarios corresponding to the alternative frequencies of rebalance (daily, monthly and by threshold). The performance of the resulting insured portfolios (see Appendix C.1) only show small discrepancies with respect to the three scenarios. In this regard, the four daily rebalanced portfolios seem to slightly underperform the corresponding ones based on monthly readjustments, while the portfolios rebalanced with an 8% threshold maintain approximately the same level. In fact, the daily adjustments entail a greater amount of transaction costs. For this reason, only the monthly rebalanced portfolio are considered for the following comparison. Similarly, the dynamic OBPI has been preferred to the static one, since the former outperforms the latter⁶⁰. Here the considerable gap between the two alternative scheme is due to a considerable tracking error of the ETF with respect to its benchmark.

⁵⁹ The annual return is given by the mean daily return annualized, while the standard deviation displayed in the table is the annualized standard deviation of the daily returns.

⁶⁰ See Appendix C.1.

As displayed in table 3.2, for the LYXOR EURO STOXX 50 UCITS ETF all the best performance indicators (highlighted in green) are realized through the static Buy and Hold portfolio insurance strategy, allocating a fixed percentage of the initial capital to equities and bonds in the 30/70 proportion. In this case, the BH is undoubtedly dominant with respect to the others, both in a mean-variance framework and according to all the risk adjusted performance measures. On the other hand, the dynamic OBPI appears to be the worst strategy applied to the large-cap ETF (in the chart the worst indicators are highlighted in red). In particular, the BH is the only one which allows to realize an absolute positive return over the 10-year period, while the remainders only exhibit negative performance, even if above the floor level anyway.

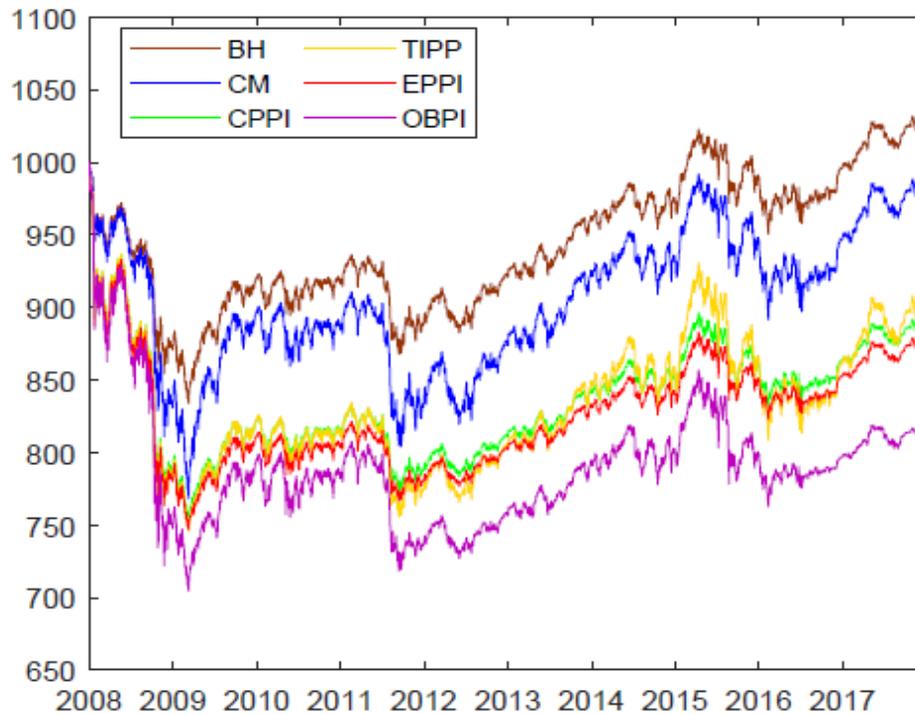
Table 3.2 Performance of the insurance strategies applied to the MSE ETF

	BH	CM	CPPI	TIPP	EPPI	OBPI
Annual standard deviation	4,81%	6,85%	5,71%	7,26%	5,63%	7,55%
Annual return	0,19%	-0,28%	-1,26%	-1,21%	-1,40%	-2,04%
Total return	1,92%	-2,78%	-12,61%	-12,09%	-13,96%	-20,39%
Min daily return	-2,27%	-2,74%	-4,72%	-4,72%	-4,98%	-5,09%
Max drawdown	16,61%	23,18%	24,79%	25,41%	25,30%	29,52%
Sharpe ratio	-0,21	-0,21	-0,43	-0,33	-0,46	-0,43
Sortino ratio	-0,32	-0,33	-0,62	-0,50	-0,66	-0,63
Calmar ratio	-0,06	-0,06	-0,10	-0,09	-0,10	-0,13
Information ratio	0,15	0,14	0,07	0,08	0,06	0,03

The portfolio trends are also reported in figure 3.2, where the performances are easily perceivable. The peculiar dominance of the BH over the other strategy may be explained by the negative absolute return realized by the ETF (-25,89%). In fact, the BH starts from an initial allocation in risk-free assets equal to €700, which then grows constantly at the risk free-rate. In the CM, the starting allocation in bond and equity is the same as in the BH, but then the bond exposure, as fixed percentage of the total portfolio value, decreases if so does the overall portfolio. On the contrary, the CPPI, the TIPP, the EPPI and the OBPI all start with a lower exposure to risk-free asset, respectively the 40% (both CPPI and TIPP),

37% and the 36%. Its evolution is then subjected to the effect of the cushions, multipliers or the delta for the OBPI. Summing up, since the ETF displays negative returns while the risk-free asset grows perpetually, the BH works better because it invests more money to the healthy asset.

Figure 3.2 Trends of the portfolios constructed according to the different allocation strategies



3.3.2 SPDR MSCI Small Cap Europe UCITS ETF (SMCX)

The second fund the portfolio insurance strategies have been tested on is the SPDR MSCI Small Cap Europe UCITS ETF. The ETF invests in European equities belonging to the small-cap segment of the market, which are usually more volatile as compared to the large-cap companies. Thus, an investment in this fund, although riskier, may provide higher returns. Between 2008 and 2017 the fund encountered variable scenarios. Starting at €108,51, after the financial crisis (where it reached its minimum level of €45,78) the ETF exhibited a steep

and continuous growth, despite some natural oscillations. By the end of the period, in fact, the price reached the maximum value of €231,26, while the mean level over the time horizon stuck to €130,69 (see figure 3.3).

Figure 3.3 SMCX price trend Jan 1st 2008 – Dec 29th 2017

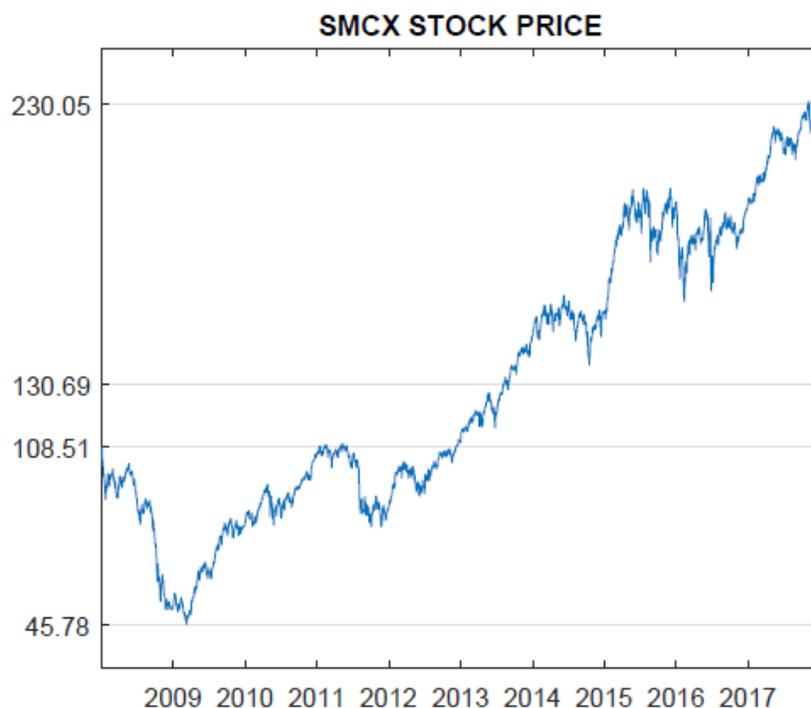


Table 3.3 also reports the risk-return statistics for the fund. The SMCX suffered the maximum drawdown precisely during the same daily range as the previous fund: that is easily explainable by the general downturn of the equity market. However, the data regarding the risk and return are somehow surprising. In fact, the return realized by the fund is impressive (7,52% annual), while its variability is anyway lower than that of the previous fund.

Table .3.3 risk-return statistics for the SMCX ETF

st. deviation	annual return	total return	max drawdown
19,06%	7,52%	75,15%	57,81%

As in the previous case, the dynamic insurance strategies have been implemented with the three diverse rebalancing techniques, except for the OBPI which has been developed both in a static and in a dynamic environment (see Appendix C.2). Again, the discrepancies among the three methods are irrelevant, since for all the strategies the three portfolios reach very similar levels, even though they are readjusted differently. In fact, in a context of sharply and monotonically rising or decreasing equity market, the negative effect of the transaction costs, eroding the portfolio value, may be offset by a mechanism inherent the strategy itself. In other words, when the exposure on the risky asset depends on a cushion (CPPI, TIPP and EPPI), the rebalance allows to allocate more capital to the asset which is instantly better performing. Therefore, if the ETF price drops, the wealth is transferred to the risk-free asset. On the contrary, when the risky asset is outperforming the bonds, the insurance works in the other way around. Take for example the equation (3) of the CPPI. As both the floor and the bond exposure grow constantly at the risk-free, the total portfolio value decreases only when the risky asset realizes negative returns. Then, if the portfolio value drops, the cushion gets smaller as well and so does the equity exposure as it is directly proportional to the cushion. This mechanism has lower effectiveness in case of flat market and it seems not to work at all when a CM strategy is applied. The graphs of the CM⁶¹, in fact, is the only one clearly showing how the portfolio daily rebalanced underperforms the alternative ones.

According to the performance indicators (see table 3.4), the BH and the CM still seem to be the best strategies in term of volatility and maximum drawdown. This result was highly predictable, since both the portfolios, due to the 70/30 initial allocation mix, are mainly invested in bonds which are obviously less risky than the ETF shares. For the same reason, on the other hand they also exhibit the worst return. However, as far as the returns and the performance ratios are regarded, the OBPI behaves much better as compared to the other strategies. The OBPI, in fact, at maturity has the power to guarantee the same return of a portfolio totally invested in the risky asset, whether the equity price overcomes the strike price. Therefore the synthetic $S + p(S, K)$ portfolio, replicated by a position in stocks and

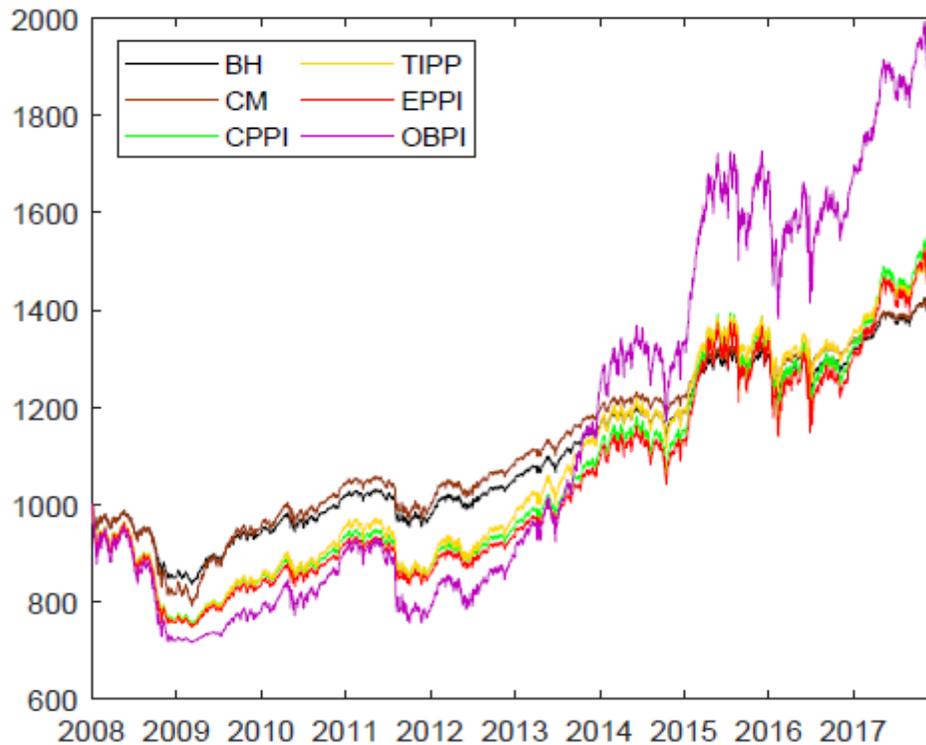
⁶¹ See Appendix C.2

risk-free assets, is composed in such a manner that the put price tends to zero as maturity approaches and the equity value increases with respect to the strike price. In particular, the Sharpe, the Calmar and the Information ratio are those which best qualify the OBPI as an effective insurance strategy, while the CPPI, the TIPP and the EPPI do not provide satisfying risk-adjusted returns. Furthermore, the Sortino ratio of the OBPI here may assume a significant meaning, even though it is not the best. In fact, measuring the return adjusted only for the downside risk, a high value of the Sortino ratio proves that a relevant portion of the strategy's volatility is influenced by beneficial upward deviations.

Table 3.4 Performance of the insurance strategies applied to the SMCX ETF.

	BH	CM	CPPI	TIPP	EPPI	OBPI
Annual standard deviation	5,53%	5,59%	9,82%	8,48%	9,97%	13,27%
Annual return	3,53%	3,50%	4,31%	4,04%	4,17%	6,84%
Total return	35,33%	35,02%	43,06%	40,37%	41,67%	68,42%
Min daily return	-3,38%	-2,50%	-7,04%	-4,65%	-7,25%	-8,60%
Max drawdown	16,36%	20,90%	24,62%	25,21%	25,16%	28,33%
Sharpe ratio	0,43	0,42	0,32	0,34	0,30	0,43
Sortino ratio	0,66	0,65	0,48	0,52	0,45	0,65
Calmar ratio	0,14	0,11	0,13	0,11	0,12	0,20
Information ratio	-0,29	-0,30	-0,29	-0,31	-0,30	-0,07

Figure 3.4 Trends of the portfolios constructed according to the different allocation strategies



3.3.3 Lyxor MSCI Europe UCITS ETF (MEU)

The last fund employed to test the portfolio insurance efficiency is the Lyxor MSCI Europe UCITS ETF, which replicates the performance of the broad European equity market. In figure 3.5 the path of ETF stock price is displayed. In the considered period the MEU ETF went from an initial price of €127,76 to a final value of €128,8. In the meantime the stock reached its minimum level at €55,79 in 2009 and its maximum at €141,86 in the mid-2015. Moreover, the average price over the period was equal to €103,79. The data listed, confirmed by the risk-return statistics displayed in table 3.5, are significant of the trend of

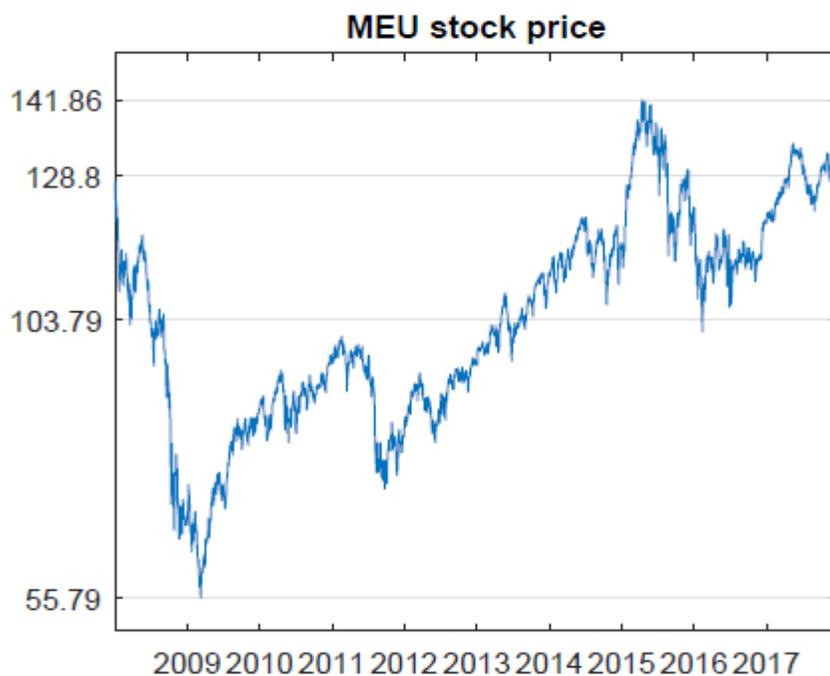
Table 3.5 risk-return statistics for the MEU ETF

st. deviation	annual return	total return	max drawdown
20,11%	0,08%	0,81%	56,33%

the stock price. Despite the total return in the 10-years span have been very low, the fund exhibited a high volatility due to steep growths alternated by abrupt declines. Due to well-known events, the hugest drop occurred between 2008 and 2009, as for the other fund analyzed, but it was followed also by two additional significant declines in 2011 and 2015. In the first time range, therefore, the maximum drawdown occurred.

For this ETF, the comparison of the rebalancing methods (see Appendix C.3) highlights a more accentuated difference among the alternative frequencies of portfolio readjustment. In particular, this discrepancy is quite evident in the CPPI, TIPP and EPPI, while it is negligible in the CM scheme. Even though the previous subchapter explained how the effect of the transaction costs was offset by the cushion mechanism, in this case the rebalances move differently and a reason for this behavior may be found in the diverse stock price trends.

Figure 3.5 MEU price trend Jan 1st 2008 - Dec 29th 2017



The ETF previously analyzed registered a very high yield, so the insurance, which frequently reallocates the wealth to equities, permits to continuously capture its additional outstanding returns. However, the MEU showed a much more indented evolution and it practically realized a zero annual return. This means that the frequent reversals, weakening the cushion effect, induce the transaction costs to be more influent on the portfolio value. Instead, the difference between the static and the dynamic OBPI strategy is due, again, to the non-perfect match among a protective put written on the index and a protection synthetically replicated on the fund.

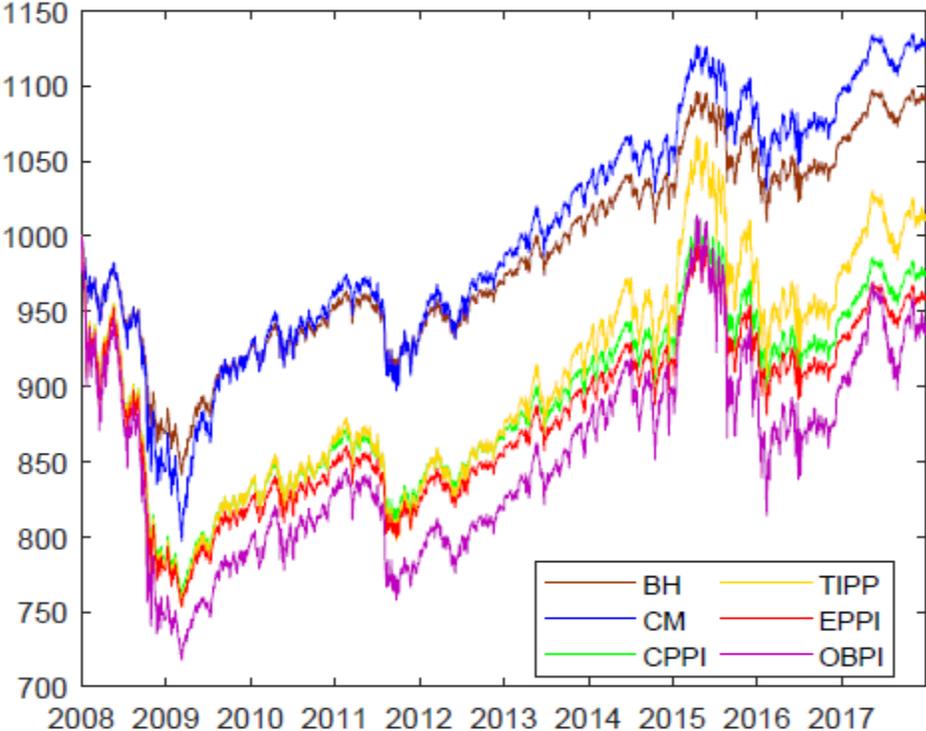
Looking at the performances of the insurance strategies (table 3.6), the BH always displays the lowest risk statistics as the standard deviation and the maximum drawdown because of the abovementioned reasons, while the CM appears to be the most suitable strategy to the specific ETF in term of absolute and risk-adjusted return. The fund, in fact, registered an annual mean return over the 10-year span period very close to zero (0,81%), so even lower than the 1,182% assumed as risk-free rate. It is quite clear, therefore, how the CM and BH are preferred to the others, since they initially invest more capital to the risk-free asset. Furthermore, in this situation it is also interesting to notice how the CPPI, the EPPI and the OBPI are returning a negative yield, even though an investment in the sole fund would have provided a low, but positive profit. Any insurance strategy, in fact, realistically bears the cost of buying the protection. This is usually the most evident and

Table 3.6 Performance of the insurance strategies applied to the SMCX ETF.

	BH	CM	CPPI	TIPP	EPPI	OBPI
Annual standard deviation	4,65%	5,86%	6,46%	7,68%	6,44%	9,21%
Annual return	0,86%	1,19%	-0,27%	0,10%	-0,43%	-0,67%
Total return	8,64%	11,87%	-2,69%	1,02%	-4,34%	-6,77%
Min daily return	-2,24%	-2,92%	-3,81%	-3,81%	-4,01%	-4,33%
Max drawdown	15,91%	20,31%	24,11%	24,71%	24,68%	28,19%
Sharpe ratio	-0,07	0,00	-0,22	-0,14	-0,25	-0,20
Sortino ratio	-0,10	0,00	-0,33	-0,21	-0,37	-0,30
Calmar ratio	-0,02	0,00	-0,06	-0,04	-0,07	-0,08
Information ratio	0,05	0,08	-0,02	0,00	-0,04	-0,06

influential in the OBPI. The net performance of the insured portfolio, in fact, would have been slightly positive if it were not for that amount z initially liquidated from the total capital to purchase the protection.

Figure 3.6 Trends of the portfolios constructed according to the different allocation strategies



Conclusion

The application of the portfolio insurance strategies to ETFs representative of various market scenarios has brought to light different results. None of the three cases analyzed returned a unique outcome, since no allocation scheme proved to be the best-performing in all the situations. Any strategy, in fact, behaved differently depending on the path of the fund stock price. In the first case, the ETF exhibited a considerable negative return over the given time span, but the application of the insurance strategies managed to raise the performance of the investment (even if still negative) while reducing its risk in term of volatility. Thus any of the insurance schemes would be preferred to a non-insured framework. Specifically, given the unprofitability of the ETF, the value of the strategy is due exclusively to the riskless portion of the portfolio and therefore the Buy and Hold results the most desirable approach. In the second case, instead, the ETF proved to be highly profitable. In this scenario (equity return much higher than the risk-free), an insurance strategy can only work in reducing the risk of the investment, but the return on the portfolio is necessarily undermined by the cost inherent the insurance purchase. In this situation an Option Based Portfolio Insurance is preferred, because it is the one which permits to exploit the most the upward potential of the risky asset, even though it also bears the highest costs. Finally in the third case the extremely low return on the ETF (lower than the risk-free rate) led few strategies (BH and CM) to outperform the fund, while others (TIPP with anyway positive return, CPPI, EPPI and OBPI with negative ones) to underperform it. In particular, the successful schemes are either the less costly (BH) or the less exposed to equities (BH and CM), while the higher equity exposure and costs implied a lower return (TIPP), sometimes even negative (CPPI, EPPI, OBPI).

Moreover, the insurance strategies are path dependent. In fact, as mentioned in chapter 1 and proved in chapter 3 the CM (based on a 'buy low-sell high' rule) is convenient when the equity market is oscillating (high volatility), but relatively flat (final price quite close to the initial value). This is the case, in particular, of the third scenario analyzed, represented by the MEU ETF. Instead, the CPPI, the TIPP and the EPPI (exploiting the 'buy high-sell low'

rule) are better performing in bull markets (as for the SMCX ETF), thus returning increasing marginal profits due to the cushion effect. Then, the more frequent are the reversals (price oscillations) the better is the performance of the TIPP as compared to the CPPI and EPPI (MSE and MEU) and vice versa (SMCX).

Any allocation scheme is strictly related to the floor value desired by the investor. Thus, even in case of negative overall returns each strategy achieves its aim, that is to guarantee a minimum level of capital to the investor in accordance with her tolerance to potential losses, while reducing the risk (volatility) of the investment. In principle, the analysis in this thesis has been driven by the decision to make the diverse approaches comparable starting from equal initial conditions as far as the floor value. At the same time the initial portfolios are meant to be kept to a minimum equilibrium in the allocation mix. Thus, in this perspective the initial floor has been set to €700, in order to maintain a guaranteed capital at maturity equal for all the strategies to the 80% of the initial investment. However, this construction was a bit restrictive. In fact, in reality the formulation of an insurance strategy should better consider a full protection of the investment or permit only a very small percentage of tolerated losses. However, a hypothetical initial floor of €900 would require, in the BH and in the CM, a 90/10 initial allocation mix of bonds and stocks which might imply an insured portfolio too tied to the return on the risk-free asset. Therefore, a deeper analysis may be developed by testing the same strategies on portfolios with different floor values and initial allocations. For instance, a CPPI allowing a multiplier equal to 3 would imply an allocation mix which may better exploit the upward potential of the equity allocation while maintaining the same guaranteed capital.

It has been demonstrated how the effectiveness of an insurance strategy varies depending on the underlying portfolio composition and performance, so the market conditions and trends are fundamental to define the best strategy to apply. However, given these results, is it somehow possible to identify a priori which portfolio insurance approach undertake? It might be possible, but the answer would be anyway subject to some parameters. Firstly, it would be necessary to get an accurate estimation of the risk-free rate as well as of the expected return and volatility of the reference market. In addition, crucial

is also the level of risk aversion of the investor. One may, in fact, tolerate higher potential losses (lower floor) to take advantage of higher potential returns on the equity market. Therefore, in anticipation of a highly volatile market and solid risk-free interest rate an investor may prefer a BH or a CM scheme with a consistent portion of the portfolio allocated to non-risky investments. Conversely, when the market is expected to show more stability, one of the remainder approaches may be desirable. Clearly, the shorter the time horizon of the protection scheme, the more likely is the estimation to be accurate.

References

- Abner D. (2015), "Understanding ETFs: Trading and Valuation", *The Journal of Trading*, 10 (3), pp. 24-30.
- Arthur K. (2013), "ETF Asset Managers", *The Journal of Index Investing*, 4 (2), pp. 95-97.
- Bertrand P., Pringent J. (2005), "Portfolio Insurance Strategies: OBPI vs CPPI", *Finance*, 26 (1), pp. 5-32.
- Bird R., Cunningham R., Dennis D., Tippet M. (1990), "Portfolio insurance: A simulation under different market conditions", *Insurance Mathematics and Economics*, 9 (1), pp. 1-19.
- Bouyé E. (2009), *Portfolio Insurance: A Short Introduction*, <https://ssrn.com/abstract=1416790>.
- Cesari R., Cremonini D. (2003), "Benchmarking portfolio insurance and technical analysis: a Monte Carlo comparison of dynamic strategies of asset allocation", *Journal of Economic Dynamics & Control*, 27, pp. 987 – 1011.
- Costa J., Gaspar R. (2014), "Portfolio insurance – a comparison of naïve versus popular strategies", *Insurance Markets and Companies: Analysis and Actuarial Computations*, 5 (1).
- Dichtl H., Drobetz W. (2011), Portfolio insurance and prospect theory investors: popularity and optimal design of capital protected financial products, *Journal of Banking & Finance*, 35, pp. 1683-1697.
- Dichtl H., Drobetz W., Wambach M. (2014), "Where is the value added of rebalancing? A systematic comparison of alternative rebalancing strategies", *Financial Markets and Portfolio Management*, 28, pp. 209-231.
- Dickson J., Kwon D., Rowley J. (2015), *Choosing between ETFs and mutual funds: Strategy, then structure*, Vanguard Research.

Directive 2009/65/EC of the European Parliament and of the Council of 13 July 2009, *on the coordination of laws, regulations and administrative provisions relating to undertakings for collective investment in transferable securities (UCITS)*.

ESMA (2012), *ESMA's guidelines on ETFs and other UCITS issues*, Consultation paper, ESMA/2012/44.

ESMA (2014), *Guidelines on ETFs and other UCITS issues, Guidelines for competent authorities and UCITS management companies*, ESMA/2014/937.

Gastineau G. (2001), "Exchange-Traded Funds: An Introduction", *The Journal of Portfolio Management*, 27 (3), pp. 88-96, Wiley.

Gastineau G. (2008), "Exchange-Traded Funds", *Handbook of Finance – Financial Markets and Instruments*, Vol.1, Ch. 61, pp. 633-642.

Hehn E. (2005), *Exchange-Traded Funds: Structure, Regulation and Application of a New Fund Class*, Springer.

Hill J., Nadig D., Hougan M. (2015), *A Comprehensive Guide to Exchange-traded Funds (ETFs)*, CFA Institute Research Foundation.

Kealy L., Daly K., Melville A., Kempeneer P., Forstenhausler M., Michel M., Kerr J. (2017), *Reshaping around the investor – Global ETF Research 2017*, EYGM limited.

Lee H., Chiang M., Hsu H. (2008), "A new choice of dynamic asset management: the variable proportion portfolio insurance", *Applied Economics*, 40:16, pp. 2135-2146.

Leland H., Rubinstein M. (1988), "The Evolution of Portfolio Insurance", in: Luskin D., ed., *Portfolio Insurance: A Guide to Dynamic Hedging*, Wiley.

Leland H. (1999), *L'assicurazione di portafoglio: elementi pratici e applicativi*, Il Mulino, Bologna.

Liera M. (2006), *Tutti gli strumenti del risparmio, dai titoli di stato agli hedge fund, dalle azioni agli ETF*, Schroders, Milano.

Madhavan H. (2014), "Exchange-Traded Funds: An Overview of Institutions, Trading, and Impacts", *The Annual Review of Financial Economics*, 6, pp. 311-341.

Perold A., Sharpe W (1988), "Dynamic Strategies for Asset Allocation", *Financial Analyst Journal*, 44, pp. 16-27, CFA Institute.

Zagst R., Kraus J. (2011), "Stochastic dominance of portfolio insurance strategies: OBPI versus CPPI", *Annals of Operations Research*, 185, pp. 75-103.

Bloomberg Terminal: <https://www.bloomberg.com/>

Investopedia: <https://www.investopedia.com/>

Ishares website: <https://www.ishares.com/it/investitore-privato/it/prodotti/>

Lyxor website: <https://www.lyxoretf.it/it/retail>

The Vanguard Group, ETF Knowledge Center:

<https://advisors.vanguard.com/VGApp/iip/site/advisor/etfcenter>

APPENDIX A

A.1 Buy and Hold

Table A.1 Buy and Hold (32/68 stocks/bonds)

Stock market value	Exposition in stocks	Exposition in bonds	Portfolio asset value
0	0	70	70
10	3.2	70	73.2
20	6.4	70	76.4
30	9.6	70	79.6
40	12.8	70	82.8
50	16	70	86
60	19.2	70	89.2
70	22.4	70	92.4
80	25.6	70	95.6
90	28.8	70	98.8
<u>100</u>	<u>32</u>	<u>70</u>	<u>102</u>
110	35.2	70	105.2
120	38.4	70	108.4
130	41.6	70	111.6
140	44.8	70	114.8
150	48	70	118
160	51.2	70	121.2
170	54.4	70	124.4
180	57.6	70	127.6
190	60.8	70	130.8
200	64	70	134

A.2 Constant Mix

Table A.2 Constant Mix (32/68 stocks/bonds)

Stock market value	Exposition in stocks before rebalancing	Exposition in stocks after rebalancing	Exposition in bonds	Portfolio asset value
0	0.0	11.4	24.2	35.6
10	10.0	16.7	35.6	52.3
20	14.9	19.9	42.4	62.3
30	18.2	22.3	47.4	69.7
40	20.7	24.3	51.5	75.8
50	22.8	25.9	55.1	81.0
60	24.6	27.4	58.2	85.5
70	26.1	28.7	60.9	89.6
80	27.5	29.9	63.5	93.4
90	28.8	31.0	65.8	96.8
100	32.0	32.0	68.0	100.0
110	35.2	33.0	70.2	103.2
120	36.0	34.0	72.2	106.2
130	36.8	34.9	74.1	109.0
140	37.6	35.7	76.0	111.7
150	38.3	36.6	77.7	114.3
160	39.0	37.3	79.4	116.7
170	39.7	38.1	80.9	119.0
180	40.3	38.8	82.5	121.3
190	41.0	39.5	83.9	123.4
200	41.6	40.2	85.4	125.5

A.3 CPPI

Table A.3 CPPI (floor = 70, m = 2), in this formulation the scheme allows for the short selling

Stock market value	Exposition in stocks before rebalancing	Exposition in stocks after rebalancing	Exposition in bonds	Floor	Cushion	Portfolio asset value
0	0.0	0.0	70.0	70.0	0.0	70.0
10	0.7	0.0	70.0	70.0	0.0	70.0
20	2.7	1.3	69.3	70.0	0.7	70.7
30	6.0	4.0	68.0	70.0	2.0	72.0
40	10.7	8.0	66.0	70.0	4.0	74.0
50	16.7	13.3	63.3	70.0	6.7	76.7
60	24.0	20.0	60.0	70.0	10.0	80.0
70	32.7	28.0	56.0	70.0	14.0	84.0
80	42.7	37.3	51.3	70.0	18.7	88.7
90	54.0	48.0	46.0	70.0	24.0	94.0
100	60.0	60	40.0	70.0	30.0	100.0
110	66.0	72.0	34.0	70.0	36.0	106.0
120	78.5	85.1	27.5	70.0	42.5	112.5
130	92.2	99.3	20.4	70.0	49.6	119.6
140	106.9	114.5	12.7	70.0	57.3	127.3
150	122.7	130.9	4.5	70.0	65.5	135.5
160	139.6	148.4	-4.2	70.0	74.2	144.2
170	157.6	166.9	-13.5	70.0	83.5	153.5
180	176.7	186.5	-23.3	70.0	93.3	163.3
190	196.9	207.3	-33.6	70.0	103.6	173.6
200	218.2	229.1	-44.5	70.0	114.5	184.5

A.4 TIPP

Table A.4 TIPP (floor = 70, m = 2)

Stock market value	Exposition in stocks before rebalancing	Exposition in stocks after rebalancing	Exposition in bonds	Floor	Cushion	Portfolio asset value
0	0.0	0.0	70.0	70.0	0.0	70.0
10	0.7	0.0	70.0	70.0	0.0	70.0
20	2.7	1.3	69.3	70.0	0.7	70.7
30	6.0	4.0	68.0	70.0	2.0	72.0
40	10.7	8.0	66.0	70.0	4.0	74.0
50	16.7	13.3	63.3	70.0	6.7	76.7
60	24.0	20.0	60.0	70.0	10.0	80.0
70	32.7	28.0	56.0	70.0	14.0	84.0
80	42.7	37.3	51.3	70.0	18.7	88.7
90	54.0	48.0	46.0	70.0	24.0	94.0
100	60.0	60	40.0	70.0	30.0	100.0
110	66.0	63.6	42.4	74.2	31.8	106.0
120	69.4	67.1	44.7	78.2	33.5	111.8
130	72.7	70.4	46.9	82.2	35.2	117.4
140	75.8	73.7	49.1	86.0	36.8	122.8
150	78.9	76.8	51.2	89.6	38.4	128.1
160	82.0	79.9	53.3	93.2	40.0	133.2
170	84.9	82.9	55.3	96.7	41.4	138.2
180	87.8	85.8	57.2	100.1	42.9	143.0
190	90.6	88.7	59.1	103.5	44.3	147.8
200	93.4	91.5	61.0	106.7	45.7	152.5

A.5 EPPI

Table A.5 EPPI (floor = 70), in this formulation the scheme allows for the short selling

Stock market value	Exposition in stocks before rebalancing	Exposition in stocks after rebalancing	Exposition in bonds	Floor	Cushion	Multiplier	Portfolio asset value
0	0.0	-0.1	70.0	70.0	-0.1	1.10	69.9
10	0.9	0.4	69.9	70.0	0.3	1.35	70.3
20	2.9	1.8	69.4	70.0	1.2	1.54	71.2
30	6.1	4.4	68.3	70.0	2.6	1.66	72.6
40	10.4	8.1	66.6	70.0	4.7	1.74	74.7
50	16.0	13.0	64.2	70.0	7.3	1.79	77.3
60	22.8	19.2	61.3	70.0	10.5	1.83	80.5
70	30.9	26.6	57.7	70.0	14.3	1.87	84.3
80	40.2	35.3	53.4	70.0	18.7	1.89	88.7
90	56.7	45.3	48.4	70.0	23.7	1.91	93.7
100	63.0	63	37.0	70.0	30.0	2.10	100.0
110	69.3	83.9	22.4	70.0	36.3	2.31	106.3
120	91.5	100.6	13.3	70.0	43.9	2.29	113.9
130	109.0	118.9	3.4	70.0	52.3	2.27	122.3
140	128.1	138.9	-7.4	70.0	61.5	2.26	131.5
150	148.8	160.4	-19.1	70.0	71.4	2.25	141.4
160	171.1	183.7	-31.6	70.0	82.1	2.24	152.1
170	195.1	208.5	-45.0	70.0	93.5	2.23	163.5
180	220.8	235.0	-59.2	70.0	105.8	2.22	175.8
190	248.1	263.2	-74.3	70.0	118.9	2.21	188.9
200	277.1	293.1	-90.3	70.0	132.7	2.21	202.7

Appendix B

B.1 Constant Value Capture

Table B.1 CVC portfolio, assumed a 10% standard deviation, where $K = F/(1 - Z)^{62}$

r =	3%	t	S	d1	d2	c	p
σ =	10%	0	100	-0.225	-0.325	2.80	5.59
K =	105.92						
F =	100						
Z =	0.056						

B.2 Constant Gain Capture

Table B.2 CGC portfolio, assumed a 10% standard deviation, where $K = F$

r =	3%	t	S	d1	d2	c	p
σ =	10%	0	100	0.350	0.250	5.58	2.63
K =	100						
F =	100						
Z =	2.63						
α = GC		αp	(1-α)F/(1+r)	αS	Initial wealth		
		0.744	1.955	24.814	74.442	100	

⁶² See H.Leland (1999), *L'assicurazione di portafoglio: elementi teorici e applicativi*, Il Mulino, Bologna.

Appendix C

C.1 LYXOR EURO STOXX 50 UCITS ETF (MSE)

Figure C.1 CM portfolio value with daily, monthly and 8% threshold rebalances.

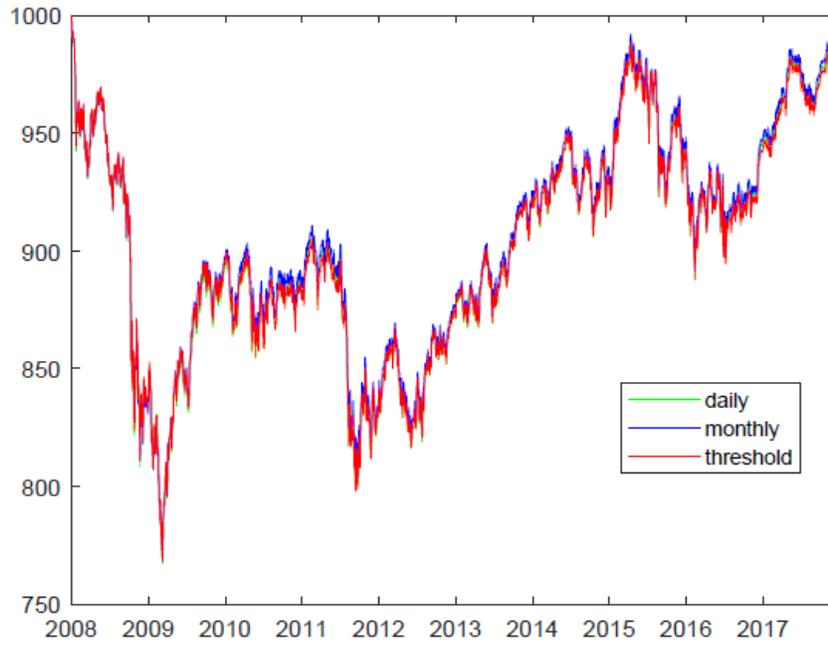


Figure C.2 CPPI with daily, monthly and 8% threshold rebalances.

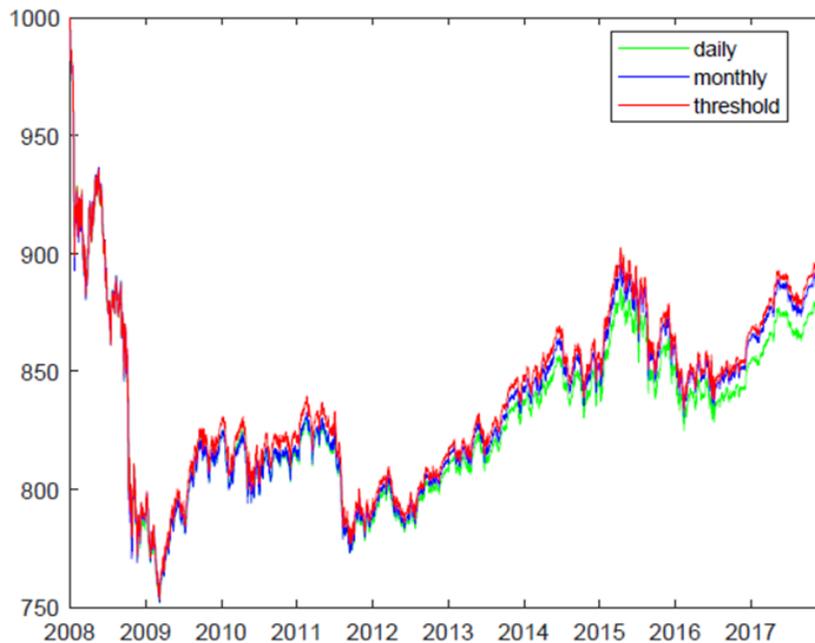


Figure C.3 TIPP with daily, monthly and 8% threshold rebalances.

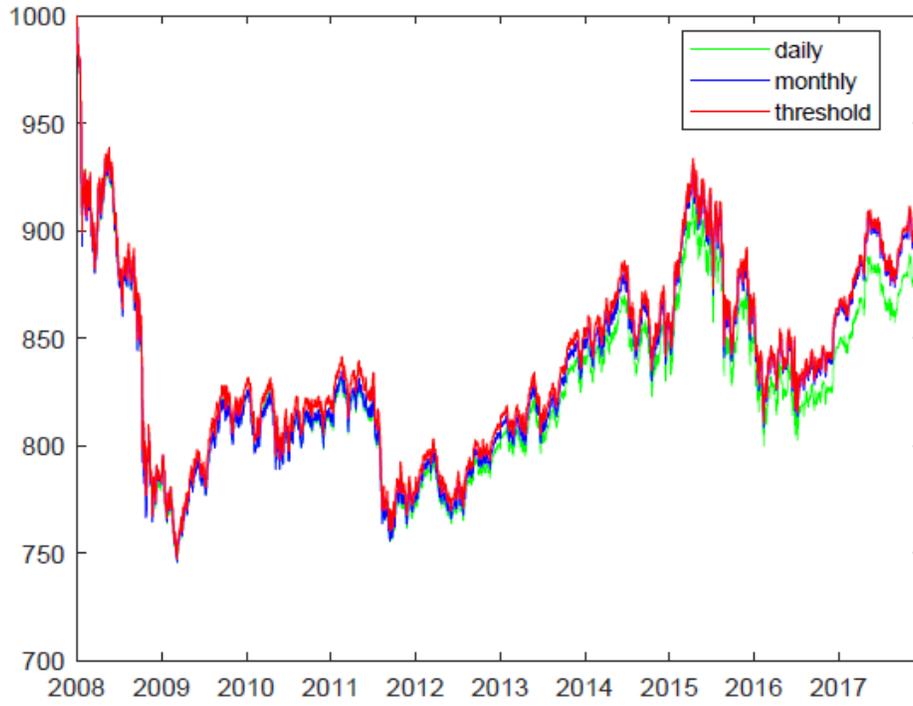


Figure C.4 EPPI with daily, monthly and 8% threshold rebalances.

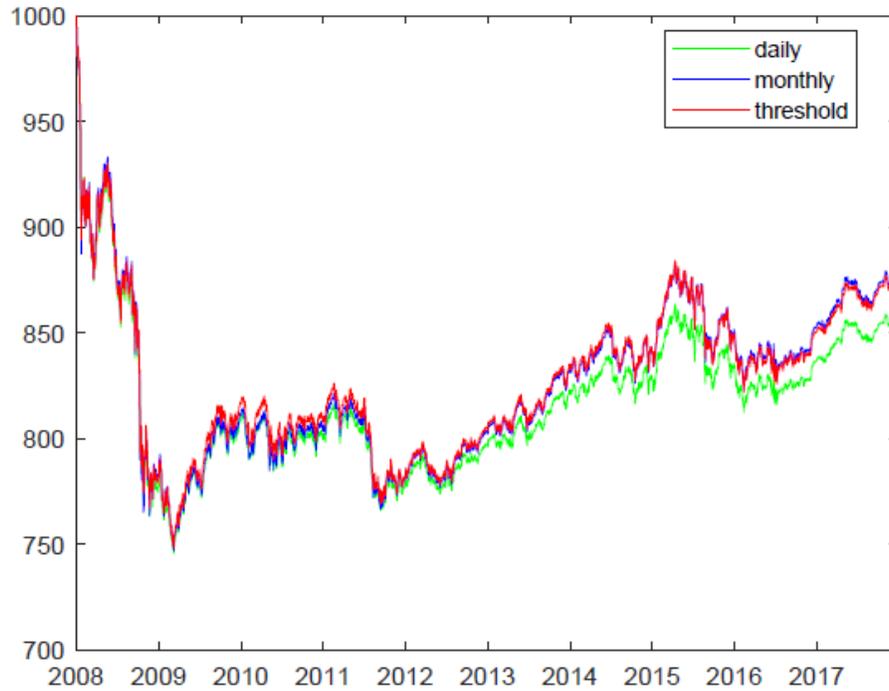


Figure C.5 OBPI with a protective put (static) and a synthetic put (dynamic) scheme.



C.2 MSCI SPDR Small Cap Europe UCITS ETF (SMCX)

Figure C.6 CM portfolio value with daily, monthly and 8% threshold rebalances.

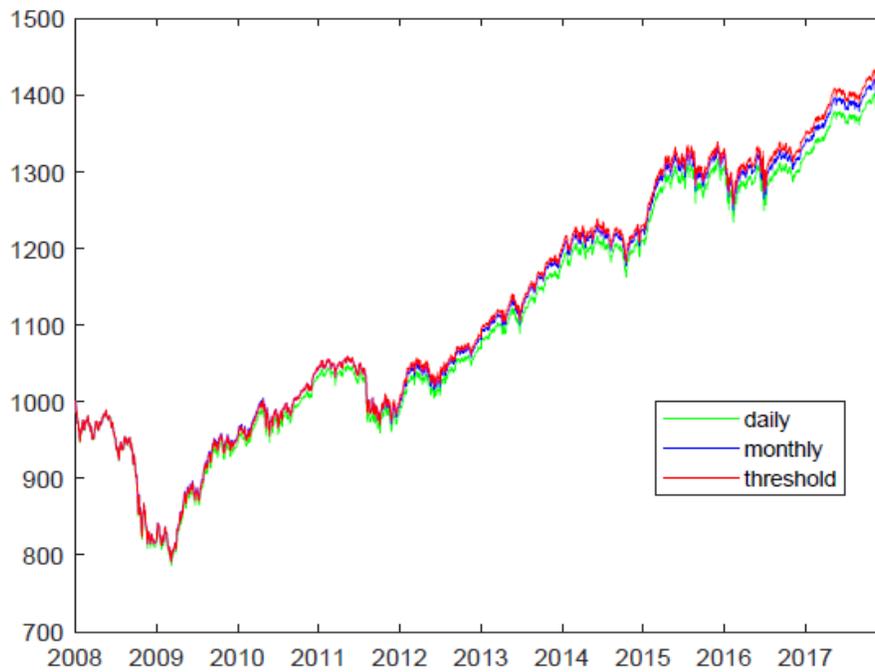


Figure C.7 CPPI with daily, monthly and 8% threshold rebalances.

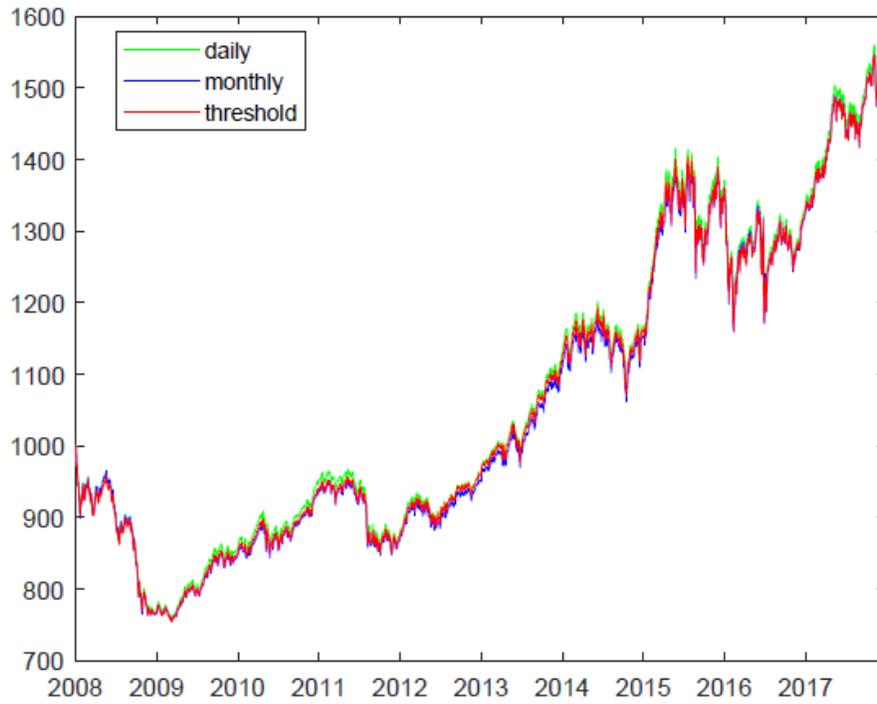


Figure C.8 TIPP with daily, monthly and 8% threshold rebalances.

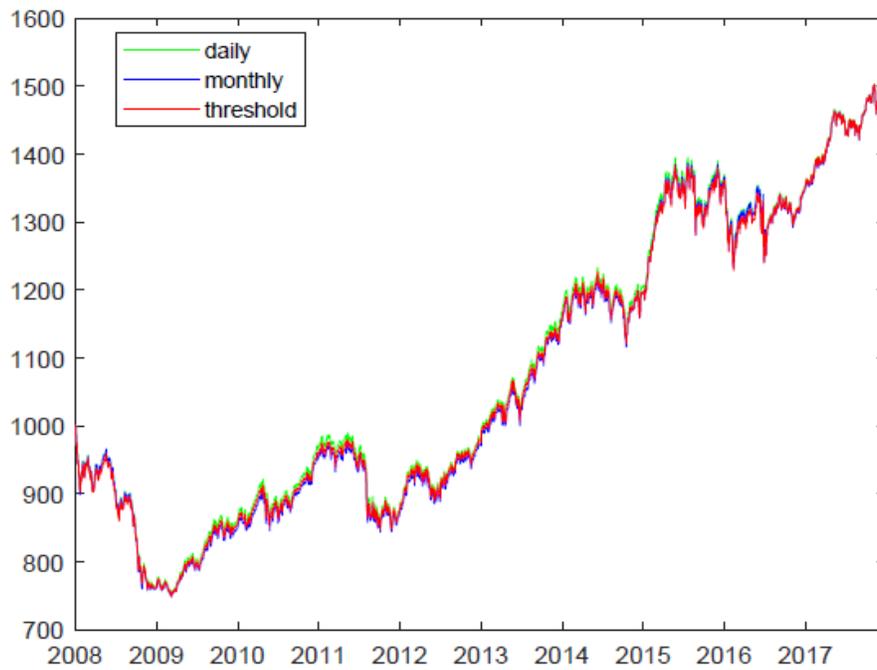


Figure C.9 EPI with daily, monthly and 8% threshold rebalances.

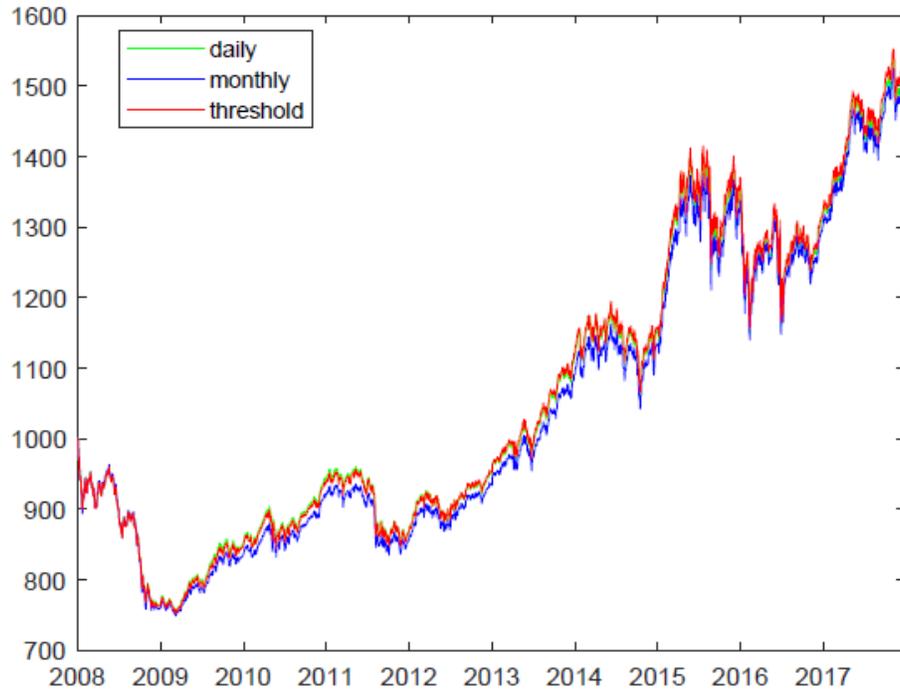
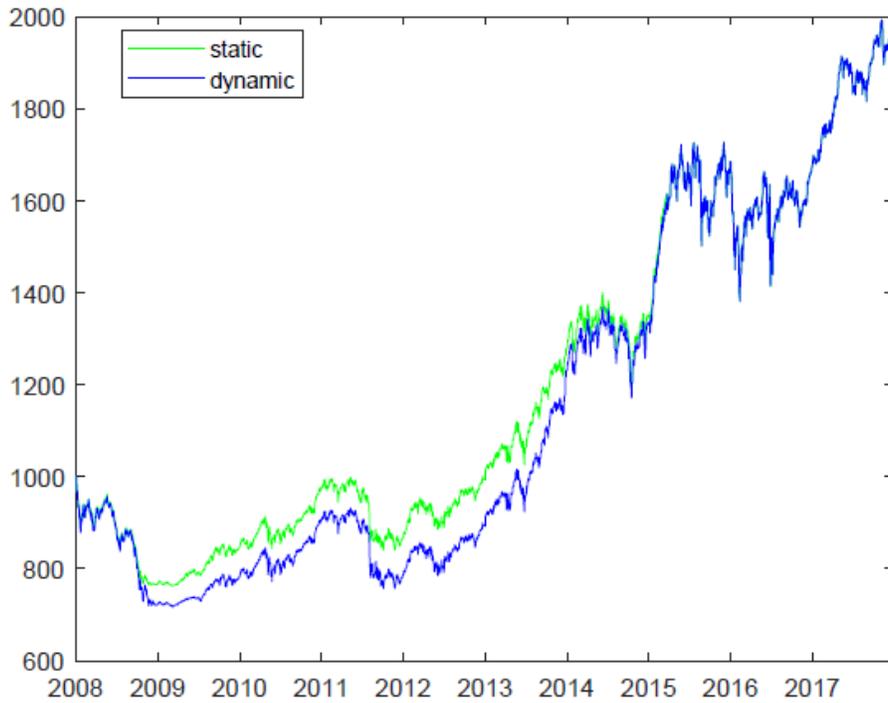


Figure C.10 OBPI with a protective put (static) and a synthetic put (dynamic) scheme.



C.3 Lyxor MSCI Europe UCITS ETF (MEU)

Figure C.11 CM portfolio value with daily, monthly and 8% threshold rebalances.

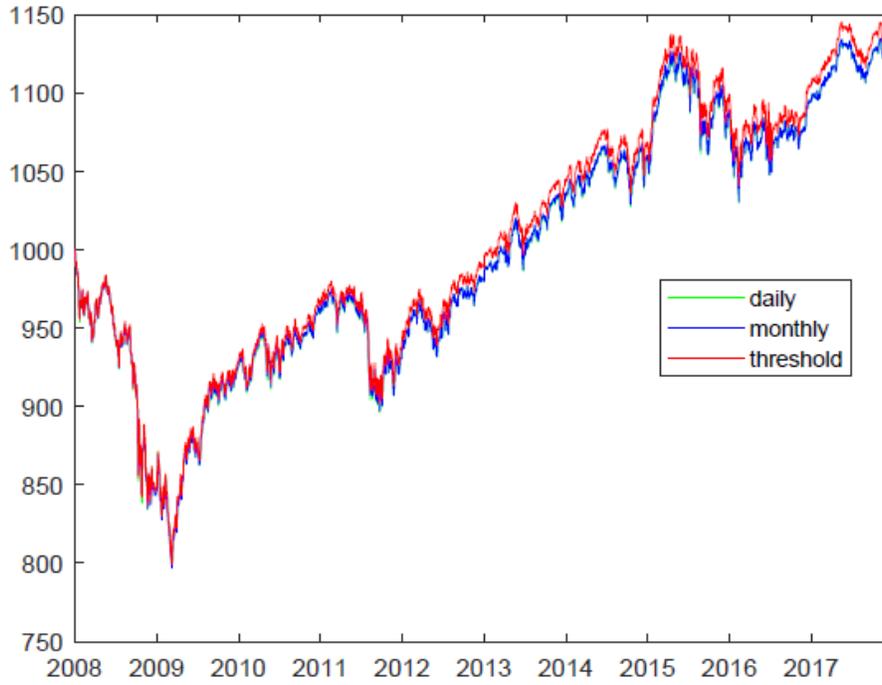


Figure C.12 CPPI with daily, monthly and 8% threshold rebalances.

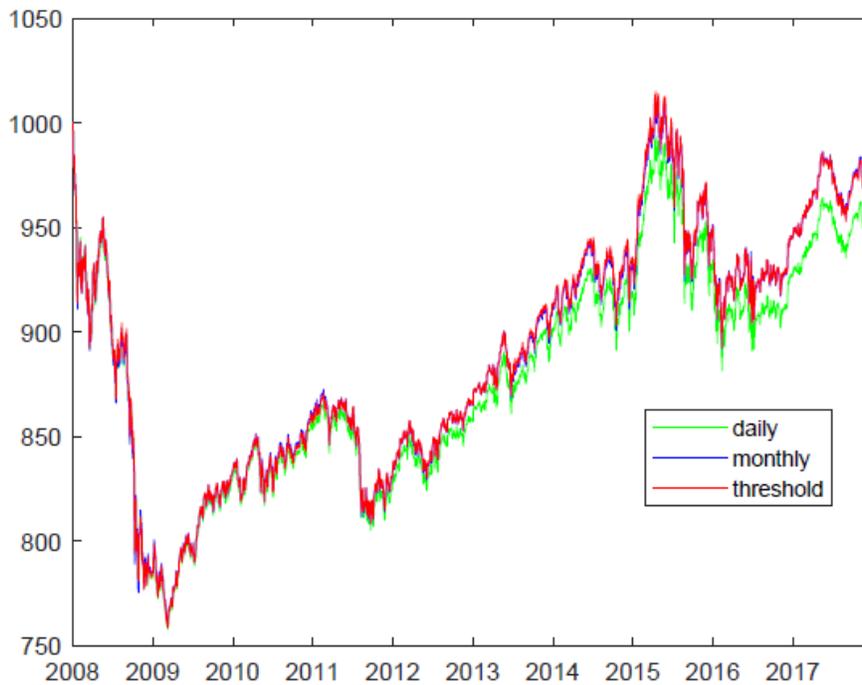


Figure C.13 TIPP with daily, monthly and 8% threshold rebalances.

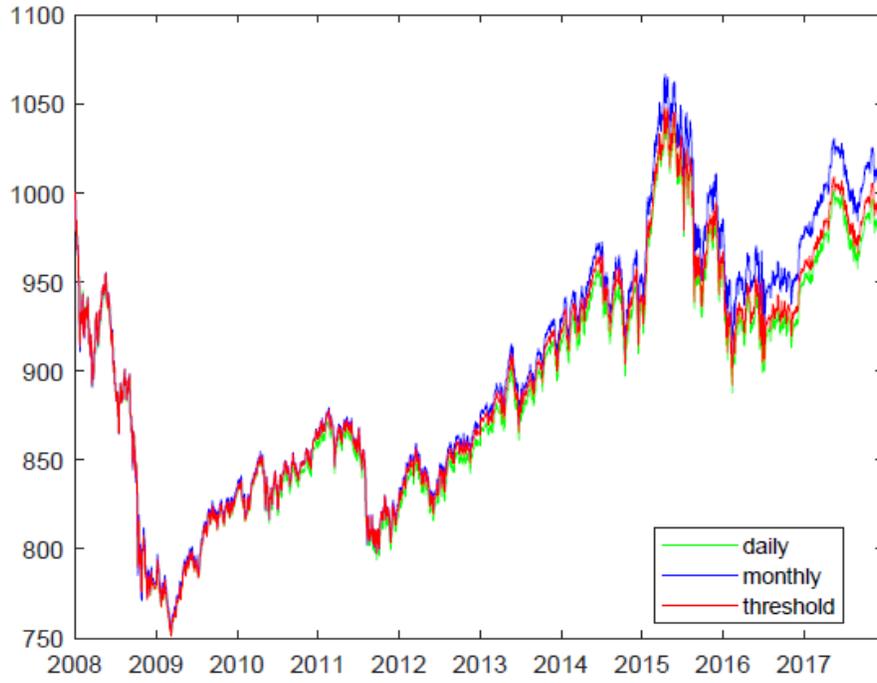


Figure C.14 EPPI with daily, monthly and 8% threshold rebalances.

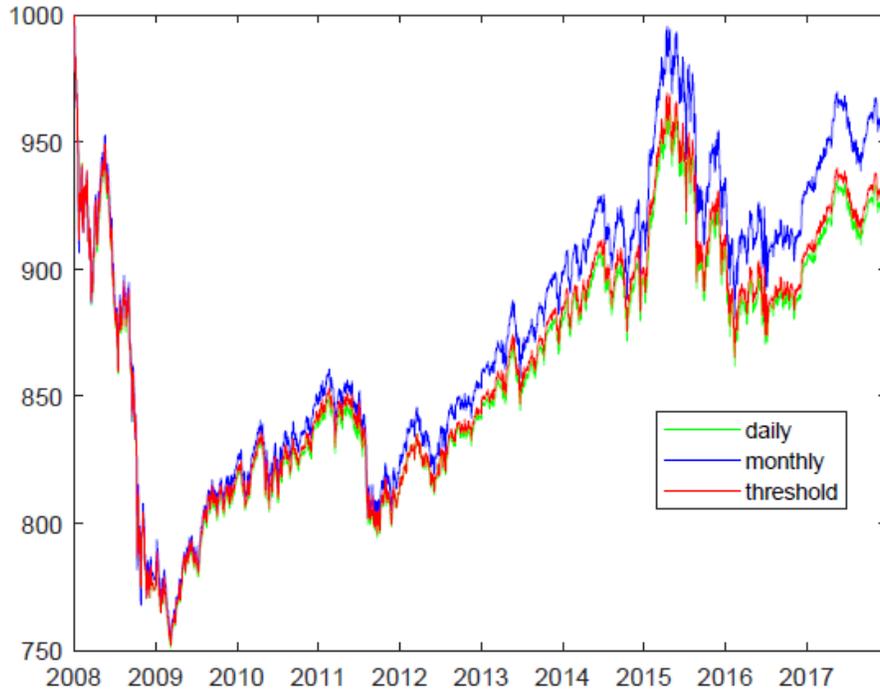
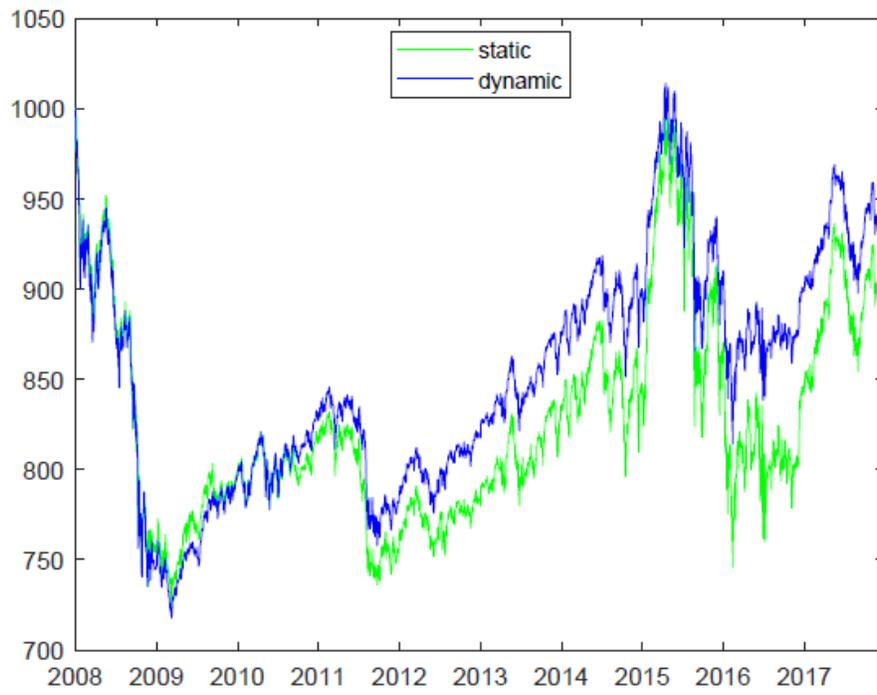


Figure C.15 OBPI with a protective put (static) and a synthetic put (dynamic) scheme.



Matlab Code

```
%% ETF PRICE STATISTICS
n = length(Dates);
years = 10;
%repeat for columns 1,2,3 representing the three funds
plot(Dates,ETFprice(:,2));
ETFret = price2ret(ETFprice(:,2)); %daily ETF log-returns
mret = mean(ETFret).*n/years; %annualized mean return
sd = std(ETFret).*sqrt(n/years); %annualized standard deviation
tret = log(ETFprice(n,2)./ETFprice(1,2)); %total return
[maxdd, maxddIndex] = maxdrawdown(ETFprice(:,2)); %maximum drawdown
minret = min(ETFret); %maximum daily drawdown

%% STARTING SCENARIO
Incapital = 1000;
re = ETFret;
rf = 0.01182*years/n; %daily risk-free return (12-months average Euribor)

%% BUY AND HOLD STRATEGY
wE = 0.3; %equity weight
wB = 0.7; %bond weight
E0 = Incapital*wE; %initial equity exposure
B0 = Incapital*wB; %initial bond exposure
E_bh = zeros(1,n);
B_bh = zeros(1,n);
E_bh(1) = E0;
B_bh(1) = B0;
for t = (2:n)
    E_bh(t) = E_bh(t-1).*exp(re(t-1)); %daily equity exposure
    B_bh(t) = B_bh(t-1).*exp(rf); %daily bond exposure
end
E_bh = E_bh';
B_bh = B_bh';
P_bh = E_bh+B_bh; %total portfolio daily value
rP_bh = price2ret(P_bh); %portfolio daily log-returns
%% STATISTICS
mret_bh = mean(rP_bh).*n/years; %annualized mean return
sd_bh = std(rP_bh).*sqrt(n/years); %annualized standard deviation
tret_bh = log(P_bh(n)/Incapital)/years; %annualized total return
[maxdd_bh, maxddIndex_bh] = maxdrawdown(P_bh); %maximum drawdown
Sharpe_bh = (mret_bh - 0.01182)./sd_bh; %Sharpe ratio
DR_bh = std(min(zeros(),rP_bh)).*sqrt(n/years); %downside risk
Sortino_bh = (mret_bh - 0.01182)./DR_bh; %Sortino ratio
Calmar_bh = (mret_bh - 0.01182)./maxdd_bh; %Calmar ratio
Information_bh = (mret_bh - mret)./(std(rP_bh-re).*sqrt(n/years));
%Information ratio with the sole ETF is the benchmark

%% CONSTANT MIX STRATEGY
transcost = 0.001; %10 bps
threshold = 0.08;
f = 22; %rebalance frequency in days
E_cm = zeros(1,n); %equity exposure after rebalancing
```

```

B_cm = zeros(1,n); %bond exposure after rebalancing
nrE_cm = zeros(1,n); %equity exposure before rebalancing
nrB_cm = zeros(1,n); %bond exposure before rebalancing
nrP_cm = zeros(1,n); %total portfolio before rebalancing
E_cm(1) = E0;
B_cm(1) = B0;
nrE_cm(1) = E0;
nrB_cm(1) = B0;
nrP_cm(1) = Incapital;
rebalance = 2; %1 = periodic, 2 = threshold
for t = (2:n)
    nrE_cm(t) = E_cm(t-1).*exp(re(t-1));
    nrB_cm(t) = B_cm(t-1).*exp(rf);
    nrP_cm(t) = nrE_cm(t) + nrB_cm(t);
    if rebalance == 1 %periodic
        if rem(t,f) == 0
            E_cm(t) = nrP_cm(t).*wE -abs(nrP_cm(t).*wE -
                nrE_cm(t)).*transcost;
            B_cm(t) = nrP_cm(t).*wB -abs(nrP_cm(t).*wB -
                nrB_cm(t)).*transcost;
        else
            E_cm(t) = nrE_cm(t);
            B_cm(t) = nrB_cm(t);
        end
    elseif rebalance == 2
        wEt = nrE_cm ./ nrP_cm;
        if abs(wEt(t)./wE-1) > threshold
            E_cm(t) = nrP_cm(t).*wE -abs(nrP_cm(t).*wE -
                nrE_cm(t)).*transcost;
            B_cm(t) = nrP_cm(t).*wB -abs(nrP_cm(t).*wB -
                nrB_cm(t)).*transcost;
        else
            E_cm(t) = nrE_cm(t);
            B_cm(t) = nrB_cm(t);
        end
    end
end
E_cm = E_cm';
B_cm = B_cm';
P_cm = E_cm+B_cm; %repeat for each rebalance frequency and assign to
rP_cm = price2ret(P_cm); %m = monthly, d = daily, t = threshold
%% STATISTICS
mret_cm = mean(rP_cm).*n/years; %annualized mean return
sd_cm = std(rP_cm).*sqrt(n/years); %annualized standard deviation
tret_cm = log(P_cm(n)/Incapital)/years; %annualized total return
[maxdd_cm, maxddIndex_cm] = maxdrawdown(P_cm); %maximum drawdown
Sharpe_cm = (mret_cm - 0.01182)./sd_cm; %Sharpe ratio
DR_cm = std(min(zeros(),rP_cm)).*sqrt(n/years); %downside risk
Sortino_cm = (mret_cm - 0.01182)./DR_cm; %Sortino ratio
Calmar_cm = (mret_cm - 0.01182)./maxdd_cm; %Calmar ratio
Information_cm = (mret_cm - mret)./(std(rP_cm-re).*sqrt(n/years));
%Information ratio where the sole ETF is the benchmark
%% reassign variables to m = monthly, d = daily, t = threshold
tP_cm= P_cm; %repeat with mP_cm, dP_cm, tP_cm
rtP_cm = rP_cm;
tmret_cm = mret_cm;
tsd_cm = sd_cm;

```

```

[tmaxdd_cm, tmaxIndex_cm] = maxdrawdown(P_cm);
tSharpe_cm = Sharpe_cm;
tSortino_cm = Sortino_cm;
tCalmar_cm = Calmar_cm;
tInformation_cm = Information_cm;

%% CONSTANT PROPORTION PORTFOLIO INSURANCE
m = 2; %multiplier
E_cp = zeros(1,n);
B_cp = zeros(1,n);
nrE_cp = zeros(1,n);
nrB_cp = zeros(1,n);
nrP_cp = zeros(1,n);
floor = zeros(1,n);
cushion = zeros(1,n);
nrP_cp(1) = Incapital;
floor(1) = Incapital*wB;
C0 = nrP_cp(1) - floor(1); %cushion at time zero
cushion(1) = C0;
nrE_cp(1) = C0*m;
nrB_cp(1) = nrP_cp(1)-nrE_cp(1);
E_cp(1) = nrE_cp(1);
B_cp(1) = nrB_cp(1);
rebalance = 2; %1 = periodic, 2 = threshold
threshold = 0.08; %threshold triggering the rebalance
f = 22; %rebalance frequency in days
for t = (2:n)
    nrE_cp(t) = E_cp(t-1).*exp(re(t-1));
    nrB_cp(t) = B_cp(t-1).*exp(rf);
    nrP_cp(t) = nrE_cp(t) + nrB_cp(t);
    floor(t) = floor(t-1).*exp(rf);
    cushion(t) = nrP_cp(t) - floor(t);
    if rebalance == 1
        if rem(t,f) == 0
            E_cp(t) = m.*cushion(t)-abs(m.*cushion(t)-
                nrE_cp(t)).*transcost;
            B_cp(t) = nrP_cp(t)-m.*cushion(t)-abs(nrP_cp(t)-
                m.*cushion(t)-nrB_cp(t)).*transcost;
        else
            E_cp(t) = nrE_cp(t);
            B_cp(t) = nrB_cp(t);
        end
    elseif rebalance == 2
        mt = nrE_cp ./ cushion(t);
        if abs(mt(t)./m-1) > threshold
            E_cp(t) = cushion(t).*m -abs(cushion(t).*m-
                nrE_cp(t)).*transcost;
            B_cp(t) = nrP_cp(t)-cushion(t).*m-abs(nrP_cp(t)-cushion(t).*m-
                nrB_cp(t)).*transcost;
        else
            E_cp(t) = nrE_cp(t);
            B_cp(t) = nrB_cp(t);
        end
    end
end
E_cp = E_cp';
B_cp = B_cp';

```

```

P_cp = E_cp+B_cp;
rP_cp = price2ret(P_cp);
%% STATISTICS
mret_cp = mean(rP_cp).*n/years; %annualized mean return
sd_cp = std(rP_cp).*sqrt(n/years); %annualized standard deviation
tret_cp = log(P_cp(n)/Incapital)/years; %annualized total return
[maxdd_cp, maxddIndex_cp] = maxdrawdown(P_cp); %maximum drawdown
Sharpe_cp = (mret_cp - 0.01182)./sd_cp; %Sharpe ratio
DR_cp = std(min(zeros(),rP_cp)).*sqrt(n/years); %downside risk
Sortino_cp = (mret_cp - 0.01182)./DR_cp; %Sortino ratio
Calmar_cp = (mret_cp - 0.01182)./maxdd_cp; %Calmar ratio
Information_cp = (mret_cp - mret)./(std(rP_cp-re).*sqrt(n/years));
%Information ratio with the sole ETF as the benchmark
%% reassign variables to m = monthly, d = daily, t = threshold
tP_cp = P_cp;
rtP_cp = rP_cp;
tmret_cp = mret_cp;
tsd_cp = sd_cp;
[tmaxdd_cp, tmaxIndex_cp] = maxdrawdown(P_cp);
tSharpe_cp = Sharpe_cp;
tSortino_cp = Sortino_cp;
tCalmar_cp = Calmar_cp;
tInformation_cp = Information_cp;

%% TIME INVARIANT PORTFOLIO PROTECTION
m = 2; %multiplier
E_ti = zeros(1,n);
B_ti = zeros(1,n);
nrE_ti = zeros(1,n);
nrB_ti = zeros(1,n);
nrP_ti = zeros(1,n);
floor = zeros(1,n);
cushion = zeros(1,n);
nrP_ti(1) = Incapital;
perfloor = 0.7; %fixed percentage floor
floor(1) = Incapital*perfloor;
C0 = nrP_ti(1) - floor(1); %cushion at time zero
cushion(1) = C0;
nrE_ti(1) = C0*m;
nrB_ti(1) = nrP_ti(1)-nrE_ti(1);
E_ti(1) = nrE_ti(1);
B_ti(1) = nrB_ti(1);
rebalance = 2; %1 = periodic, 2 = threshold
threshold = 0.08; %threshold triggering the rebalance
f = 22; %rebalance frequency in days
%periodic rebalance
for t = (2:n)
    nrE_ti(t) = E_ti(t-1).*exp(re(t-1));
    nrB_ti(t) = B_ti(t-1).*exp(rf);
    nrP_ti(t) = nrE_ti(t) + nrB_ti(t);
    if nrP_ti(t)*perfloor > floor(t-1)
        floor(t) = nrP_ti(t).*perfloor;
    else
        floor(t) = floor(t-1);
    end
    cushion(t) = nrP_ti(t) - floor(t);
    if rebalance == 1

```

```

    if rem(t,f) == 0
        E_ti(t) = m.*cushion(t)-abs(m.*cushion(t)-
            nrE_ti(t)).*transcost;
        B_ti(t) = nrP_ti(t)-m.*cushion(t)-abs(nrP_ti(t)-
            m.*cushion(t)-nrB_ti(t)).*transcost;
    else
        E_ti(t) = nrE_ti(t);
        B_ti(t) = nrB_ti(t);
    end
elseif rebalance == 2
    mt = nrE_ti ./ cushion(t);
    if abs(mt(t)./m-1) > threshold
        E_ti(t) = cushion(t).*m -abs(cushion(t).*m-
            nrE_ti(t)).*transcost;
        B_ti(t) = nrP_ti(t)-cushion(t).*m-abs(nrP_ti(t)-cushion(t).*m-
            nrB_ti(t)).*transcost;
    else
        E_ti(t) = nrE_ti(t);
        B_ti(t) = nrB_ti(t);
    end
end
end
E_ti = E_ti';
B_ti = B_ti';
P_ti = E_ti+B_ti;
rP_ti = price2ret(P_ti);
%% STATISTICS
mret_ti = mean(rP_ti).*n/years; %annualized mean return
sd_ti = std(rP_ti).*sqrt(n/years); %annualized standard deviation
tret_ti = log(P_ti(n)/Incapital)/years; %annualized total return
[maxdd_ti, maxddIndex_ti] = maxdrawdown(P_ti); %maximum drawdown
Sharpe_ti = (mret_ti - 0.01182)./sd_ti; %Sharpe ratio
DR_ti = std(min(zeros(),rP_ti)).*sqrt(n/years); %downside risk
Sortino_ti = (mret_ti - 0.01182)./DR_ti; %Sortino ratio
Calmar_ti = (mret_ti - 0.01182)./maxdd_ti; %Calmar ratio
Information_ti = (mret_ti - mret)./(std(rP_ti-re).*sqrt(n/years));
%Information ratio with the sole ETF as benchmark
%% reassign variables to m = monthly, d = daily, t = threshold
tP_ti = P_ti;
rtP_ti = rP_ti;
tmret_ti = mret_ti;
tsd_ti = sd_ti;
[tmaxdd_ti, tmaxIndex_ti] = maxdrawdown(P_ti);
tSharpe_ti = Sharpe_ti;
tSortino_ti = Sortino_ti;
tCalmar_ti = Calmar_ti;
tInformation_ti = Information_ti;

%% EXPONENTIAL PROPORTION PORTFOLIO INSURANCE
eta = 1.1; %constant > 1
m_ep = zeros(1,n); %variable multiplier
E_ep = zeros(1,n);
B_ep = zeros(1,n);
nrE_ep = zeros(1,n);
nrB_ep = zeros(1,n);
nrP_ep = zeros(1,n);
floor = zeros(1,n);

```

```

cushion = zeros(1,n);
m_ep(1) = eta + 1;
nrP_ep(1) = Incapital;
floor(1) = Incapital*wB;
C0 = nrP_ep(1) - floor(1); %cushion at time zero
cushion(1) = C0;
nrE_ep(1) = C0*m_ep(1);
nrB_ep(1) = nrP_ep(1)-nrE_ep(1);
E_ep(1) = nrE_ep(1);
B_ep(1) = nrB_ep(1);
rebalance = 2; %1 = periodic, 2 = threshold
threshold = 0.08; %threshold triggering the rebalance
f = 22; %rebalance frequency in days
for t = (2:n)
    m_ep(t) = eta + (1+re(t-1))^a;
    nrE_ep(t) = E_ep(t-1).*exp(re(t-1));
    nrB_ep(t) = B_ep(t-1).*exp(rf);
    nrP_ep(t) = nrE_ep(t) + nrB_ep(t);
    floor(t) = floor(t-1).*exp(rf);
    cushion(t) = nrP_ep(t) - floor(t);
    if rebalance == 1
        if rem(t,f) == 0
            E_ep(t) = m_ep(t).*cushion(t)-abs(m_ep(t).*cushion(t)-
            nrE_ep(t)).*transcost;
            B_ep(t) = nrP_ep(t)-m_ep(t).*cushion(t)-abs(nrP_ep(t)-
            m_ep(t).*cushion(t)-nrB_ep(t)).*transcost;
        else
            E_ep(t) = nrE_ep(t);
            B_ep(t) = nrB_ep(t);
        end
    elseif rebalance == 2
        mt = nrE_ep ./ cushion(t);
        if abs(mt(t)./m_ep(t)-1) > threshold
            E_ep(t) = cushion(t).*m_ep(t) -abs(cushion(t).*m_ep(t)-
            nrE_ep(t)).*transcost;
            B_ep(t) = nrP_ep(t)-m_ep(t).*cushion(t)-abs(nrP_ep(t)-
            m_ep(t).*cushion(t)-nrB_ep(t)).*transcost;
        else
            E_ep(t) = nrE_ep(t);
            B_ep(t) = nrB_ep(t);
        end
    end
end
end
E_ep = E_ep';
B_ep = B_ep';
P_ep = E_ep+B_ep;
rP_ep = price2ret(P_ep);
%% STATISTICS
mret_ep = mean(rP_ep).*n/years; %annualized mean return
sd_ep = std(rP_ep).*sqrt(n/years); %annualized standard deviation
tret_ep = log(P_ep(n)/Incapital)/years; %annualized total return
[maxdd_ep, maxddIndex_ep] = maxdrawdown(P_ep); %maximum drawdown
Sharpe_ep = (mret_ep - 0.01182)./sd_ep; %Sharpe ratio
DR_ep = std(min(zeros(),rP_ep)).*sqrt(n/years); %downside risk
Sortino_ep = (mret_ep - 0.01182)./DR_ep; %Sortino ratio
Calmar_ep = (mret_ep - 0.01182)./maxdd_ep; %Calmar ratio

```

```

Information_ep = (mret_ep - mret)./(std(rP_ep-re).*sqrt(n/years));
%Information ratio with the sole ETF is the benchmark
%% reassign variables to m = monthly, d = daily, t = threshold
tP_ep = P_ep;
rtP_ep = rP_ep;
tmret_ep = mret_ep;
tsd_ep = sd_ep;
[tmaxdd_ep, tmaxIndex_ep] = maxdrawdown(P_ep);
tSharpe_ep = Sharpe_ep;
tSortino_ep = Sortino_ep;
tCalmar_ep = Calmar_ep;
tInformation_ep = Information_ep;

%% OPTION BASED PORTFOLIO INSURANCE (STATIC)
%protective put+stock
%Black-Scholes put written on the index
in = length(iDates); %period from jan 2001 to dec 2017
in-n; %period form jan 2001 to dec 2007
hSt = Index(1:1826,2); %index value 2001-2007 (repeat for each column
vector)
St = Index(1827:in,2); %index value 2008-2017 (repeat for each column
vector)
ETF = ETFprice(:,2); %repeat for each column vector
hiret = price2ret(hSt);
hdvol = std(hiret); %historical daily volatility of returns on the index
%Black-Scholes price at time 1 of a put written on the index expiring in
10 years
iK = 0.9*St(1);
d1 = zeros(1,n);
d2 = zeros(1,n);
put = zeros(1,n);
d1(1) = (log(St(1)/iK) + (rf+0.5*hdvol^2)*(n-1))/(hdvol*sqrt(n-1));
d2(1) = d1(1) - hdvol*sqrt(n-1);
put(1) = iK*exp(-rf*(n-1))*normcdf(-d2(1)) - St(1)*normcdf(-d1(1));
%static portfolio construction
E_obs = zeros(1,n); %equity exposure in the static framework
x = St(1)/ETF(1); %ratio between the benchmark value and the ETF value
shares = Incapital/(put(1) + x*ETF(1)); %number of shares of the combined
portfolio (1 put + x ETF shares)
E_obs(1) = ETF(1)*x*shares; %initial exposure in the stock equals the
initial value of the ETF times x times shares
for t = (2:n)
    d1(t) = (log(St(t)./iK) + (rf+0.5*hdvol^2)*(n-t))/(hdvol*sqrt(n-t));
    d2(t) = d1(t) - hdvol.*sqrt(n-t);
    put(t) = iK.*exp(-rf.*(n-t)).*normcdf(-d2(t)) - St(t).*normcdf(-
d1(t));
    E_obs(t) = E_obs(t-1).*exp(re(t-1));
end
E_obs = E_obs';
put = put';
P_obs = E_obs + shares.*put;
rP_obs = price2ret(P_obs);
%% STATISTICS
mret_obs = mean(rP_obs).*n/years; %annualized mean return
sd_obs = std(rP_obs).*sqrt(n/years); %annualized standard deviation
tret_obs = log(P_obs(n)/Incapital)/years; %annualized total return
[maxdd_obs, maxddIndex_obs] = maxdrawdown(P_obs); %maximum drawdown

```

```

Sharpe_obs = (mret_obs - 0.01182)./sd_obs; %Sharpe ratio
DR_obs = std(min(zeros(),rP_obs)).*sqrt(n/years); %downside risk
Sortino_obs = (mret_obs - 0.01182)./DR_obs; %Sortino ratio
Calmar_obs = (mret_obs - 0.01182)./maxdd_obs; %Calmar ratio
Information_obs = (mret_obs - mret)./(std(rP_obs-re).*sqrt(n/years));
%Information ratio with the sole ETF as the benchmark

%% OPTION BASED PORTFOLIO INSURANCE (DYNAMIC)
%synthetic put replicated by taking a long position both in the ETF and
in bonds, no short selling allowed
I0 = 890; %available capital after a portion z of the initial capital is
liquidated to buy the insurance
    %also the capital the synthetic put is theoretically written on
    %I0 set to 890 through the target seek function
K = 0.9*I0; %the guaranteed amount at maturity is the 90% of the initial
value as in the static scheme
%I+sPut is the portfolio to replicate through a position in ETF and bonds
sd1 = zeros(1,n); %synthetic d1
I = zeros(1,n);
deltap = zeros(1,n); %delta synthetic put
B_obd = zeros(1,n); %bond exposure to replicate the I+sPut portfolio
E_obd = zeros(1,n); %equity exposure to replicate the I+sPut portfolio
sd1(1) = (log(I0/K) + (rf+0.5*hdvol^2)*(n-1))/(hdvol*sqrt(n-1));
sd2 = sd1(1) - hdvol*sqrt(n-1);
I(1) = I0;
P_obd(1) = K*exp(-rf*(n-1))*normcdf(-sd2) + I0*normcdf(sd1(1));
%replicating portfolio
deltap(1) = normcdf(sd1(1));
E_obd(1) = deltap(1)*I(1);
B_obd(1) = P_obd(1) - E_obd(1);
f = 65; %rebalance frequency in days = 3months in business days
for t = (2:n)
    I(t) = I(t-1).*exp(re(t-1));
    sd1(t) = (log(I(t)./K) + (rf+0.5*hdvol^2).*(n-t))./(hdvol.*sqrt(n-
t));
    P_obd(t) = B_obd(t-1).*exp(rf)+E_obd(t-1).*exp(re(t-1));
    if rem(t,f) == 0
        deltap(t) = normcdf(sd1(t));
    else
        deltap(t) = deltap(t-1);
    end
    E_obd(t) = deltap(t) .* I(t);
    B_obd(t) = max(P_obd(t) - E_obd(t), 0); %no short selling allowed
end
E_obd = E_obd';
B_obd = B_obd';
P_obd = P_obd';
rP_obd = price2ret(P_obd);
%% STATISTICS
mret_obd = mean(rP_obd).*n/years; %annualized mean return
sd_obd = std(rP_obd).*sqrt(n/years); %annualized standard deviation
tret_obd = log(P_obd(n)/Incapital)/years; %annualized total return
[maxdd_obd, maxddIndex_obd] = maxdrawdown(P_obd); %maximum drawdown
Sharpe_obd = (mret_obd - 0.01182)./sd_obd; %Sharpe ratio
DR_obd = std(min(zeros(),rP_obd)).*sqrt(n/years); %downside risk
Sortino_obd = (mret_obd - 0.01182)./DR_obd; %Sortino ratio
Calmar_obd = (mret_obs - 0.01182)./maxdd_obs; %Calmar ratio

```

```

Information_obd = (mret_obd - mret)./(std(rP_obd-re).*sqrt(n/years));
%Information ratio with the sole ETF as the benchmark

%% PLOTS
%% CONSTANT MIX
SMCXcm = plot(Dates,dP_cm, 'g'); hold on; plot(Dates,mP_cm, 'b'); hold
on; plot(Dates,tP_cm, 'r'); hold off
legend('daily', 'monthly', 'threshold', 'Location', 'Best');
saveas(SMCXcm, 'SMCXcm.pdf');
%% CONSTANT PROPORTION
SMCXcp = plot(Dates,dP_cp, 'g'); hold on; plot(Dates,mP_cp, 'b'); hold
on; plot(Dates,tP_cp, 'r'); hold off
legend('daily', 'monthly', 'threshold', 'Location', 'Best');
saveas(SMCXcp, 'SMCXcp.pdf');
%% TIME INVARIANT
SMCXti = plot(Dates,dP_ti, 'g'); hold on; plot(Dates,mP_ti, 'b'); hold
on; plot(Dates,tP_ti, 'r'); hold off
legend('daily', 'monthly', 'threshold', 'Location', 'Best');
saveas(SMCXti, 'SMCXti.pdf');
%% EXPONENTIAL PROPORTION
SMCXep = plot(Dates,dP_ep, 'g'); hold on; plot(Dates,mP_ep, 'b'); hold
on; plot(Dates,tP_ep, 'r'); hold off
legend('daily', 'monthly', 'threshold', 'Location', 'Best');
saveas(SMCXep, 'SMCXep.pdf');
%% OPTION BASED
SMCXob = plot(Dates,P_obs, 'g'); hold on; plot(Dates,P_obd, 'b'); hold
off
legend('static', 'dynamic', 'Location', 'Best');
saveas(SMCXob, 'SMCXob.pdf');
%% COMPARISON
plot(Dates,P_bh, 'black'); hold on
plot(Dates,mP_cm, 'b'); hold on;
plot(Dates,mP_cp, 'g'); hold on;
plot(Dates,mP_ti, 'y'); hold on;
plot(Dates,mP_ep, 'r'); hold on;
plot(Dates,P_obd, 'm'); hold off;
legend('BH', 'CM', 'CPPI', 'TIPP', 'EPPI', 'OBPI', 'Location', 'Best');

```