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Total Beta: Free lunch for Private Equity

The impact of diversification on the valuation of private companies

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INTRODUCTION

The inspiration for this work arose from my own realization that the valuation practices adopted within the private equity industry often present several flaws and approximations.

My interest in valuation first manifested itself while attending a course on Financial Policies and Investment Strategies led by Professor G. M. Mantovani and Professor A. Moro. I then started expanding my knowledge on the valuation of companies and refining my valuation techniques. I was following a lecture by Professor A. Damodaran when I first became aware of firm-specific risk and the concept of Total Beta. I got suddenly intrigued by the intuition behind this concept and started wondering about its possible applications.

During a later experience working in a company that provides Risk and Valuation services to Private Equity funds, I realized that the firm-specific component of risk was completely disregarded, and that valuations were generally produced with an unacceptable degree of flaws and simplifications. Despite the purpose of these valuations being mostly related to upfront management fees calculation and monitoring, rather than ultimately assessing the funds performances, I could not wrap my head around why regulators, or the best practice guidelines, namely IPEV guidelines, would allow valuation processes that lack both statistical and economical foundations.

Thanks to this experience, I have managed to gain a broader understanding of the industry and the players involved. I have noticed that Private Equity funds are often purchasing stakes in private companies directly from the company's funders, in so-called Primary Buyouts. I suddenly realized that in such a situation there is a wide difference between the levels of diversification of sellers and buyers, and therefore that must translate in a different risk exposure between the two parties while holding the same assets.

By combining these concepts with the theory on Risk-Return models and asset pricing theory, foremost intrinsic valuation, it clearly appeared that different investors should expect different returns from the same assets, therefore discounting the same future cashflows by different rates, and obtaining different valuation of the same investment.

For such reason, there must be a gap between the intrinsic value of an asset when held by a completely undiversified investor, such as a funder of a company that has most of his wealth tied up into that asset, and the same asset when held by a Private Equity fund, where the investor is more, if not completely, diversified.

My claim is that, in this framework, Private Equity funds might be able to extract value for Limited Partners and fund managers when buying stakes of a private company directly from the undiversified funder of that company, and they can do so by paying a price that lies within these: the value perceived by an undiversified investor and the one perceived by a diversified one. In order to assess this misalignment in the perceived value, one can start from the estimation of the risk exposure of an undiversified investor when holding an asset and go on to compute the expected return that he would require, compared to a diversified investor.

Risk-Return models, such as the CAPM, are based on the assumption that the marginal investor is holding the "market portfolio" and therefore is completely diversified. On top of the risk of the market, other parameters, namely the Beta in the CAPM, are used to take into account the portion of risk that cannot be diversified, or the risk that the additional asset could add to the market portfolio. The Total Beta is a parameter that can be computed in order to bring in the specific risk of an investment, which is the one that can be diversified.

Once the different expected returns (one with Beta and one with Total Beta) are computed, they can be used to value firms obtaining different intrinsic values: a higher one for the diversified investor and a lower one for the undiversified. My claim is that Private Equity funds might be able to pay lower multiples when acquiring stakes in a private company directly from the undiversified funder of that company, due to

the lower value he should perceive in the asset.

The final chapter, that concludes the work, aims at assessing if this theoretical framework is reflected in the empirical observations, resulting in lower multiples for Primary Buyout deals (where the PE fund buys from individual entrepreneurs) compared to Secondary Buyouts (where the PE fund buys from institutional investors) when purchasing stakes in private companies.

ACKNOWLEDGEMENTS

I deem it mandatory to express my gratitude to my parents, Sandra and Mariano, who have always been supportive of the choices I have made for my life and career; nothing would have been possible without the serenity that they have always granted me.

I wish to thank Professor Nicola Chiaranda for his support and advice. His deep knowledge of the subject and intuition have been fundamental for shaping the work and delivering an output of which I feel proud.

A special thanks goes also to Professor Aswath Damodaran for providing world class contents on his YouTube channel, making his valuable knowledge and mentorship accessible to hungry finance students and professionals around the world, and profoundly contributing to my work and my understanding of valuation and finance.

Sincerely,
Alessandro Trevisin

Chapter 1
RISK AND RETURN

1.1 Mean-Variance model

"Diversification is the only free lunch" - Harry Markowitz.

In the early 1950s Harry Markowitz formulated the first axioms of Modern Portfolio Theory[1]. In his 1952 paper on Portfolio selection, he challenges the hypothesis that investors should only maximize discounted returns by introducing the idea that they should also care about volatility, and that therefore exists a diversified portfolio which is preferable to all non-diversified portfolios.

Following up on his previous work, in 1959 Markowitz published the book "Portfolio Selection: Efficient Diversification of Investment"[2] where he outlines his famous Mean-Variance model. Using statistical analysis to measure the risk and return of each asset in the portfolio, Markowitz is able to propose a mathematical framework that shows that the risk of an investment portfolio is not defined by the average riskiness of its underlying assets, but by the extent to which they move together, their correlations. The model proves that diversifying the portfolio across imperfectly correlated assets allows investors to increase their investment returns without increasing the investment's risks or decreasing the risk without decreasing the returns. Markowitz described diversification as the only free lunch in investing, because increasing expected returns typically require taking more risk. Under the assumptions of the model every investor is able to determine the optimal mix of assets and therefore construct an optimal investment portfolio that will maximize investment returns while minimizing risk. This portfolio is referred to as the "market portfolio" and represents a value-weighted portfolio of every risky asset

[1] H. Markowitz, "Portfolio Selection" The Journal of Finance 7, no. 1 (1952): 77-91.

available in the market. As a result, an investor should choose "mean variance-efficient" portfolios, that 1) minimize the variance of portfolio return, given expected return, and 2) maximize the portfolio expected return, given variance.

The risk-return relationship explained by Markowitz has served as a cornerstone for all the upcoming literature and has also influenced the development of financial instruments such as index funds, which are designed to track the performance of a particular market index, and exchange-traded funds, which are investment vehicles that track the performance of a particular asset or group of assets. Despite the intuition behind the model, it is important to recognize that Markowitz' model ignores many market imperfections by assuming that investors are risk averse and, care only about the mean and variance of their one-period investment return.

1.2 Capital Asset Pricing Model

In their 1964 paper William F. Sharpe and John Lintner bring together asset pricing theory and the model of portfolio selection developed by Markowitz (1959) by proposing the "Capital Asset Pricing Model"[3]. The CAPM turns the mean-variance algebraic statement into a testable prediction about the relation between risk and expected return for a given asset or portfolio.

Sharpe-Lintner's model adds two key assumptions to the Markowitz model. The first assumption is complete agreement: given market clearing asset prices at t-1, investors agree on the joint distribution of asset returns from t-1 to t. The second assumption is that all investors can borrow and lend at a risk-free rate, regardless of the amount. The intuition of the model is that, with complete agreement about distributions of returns, all investors see the same opportunity set, and the resulting efficient portfolios are combinations of the same risky portfolio (equal to the market portfolio) with risk-free asset (lending or borrowing).

[2] H. Markowitz, "Portfolio Selection: Efficient Diversification of Investment" Yale University Press (1959).

The result of the analysis proves that the following Minimum Variance Condition exists for M (the market portfolio):

$E(R_i) = E(R_{ZM}) + [E(R_M) - E(R_{ZM})] * B_{iM}$, $i = 1,\ldots,N$ *risky assets where:*

- $E(R_{ZM})$ → *expected return on assets that have market betas equal to zero*
- B_{iM} → *the market beta of asset i*
- $E(R_M)$ → *market expected return*

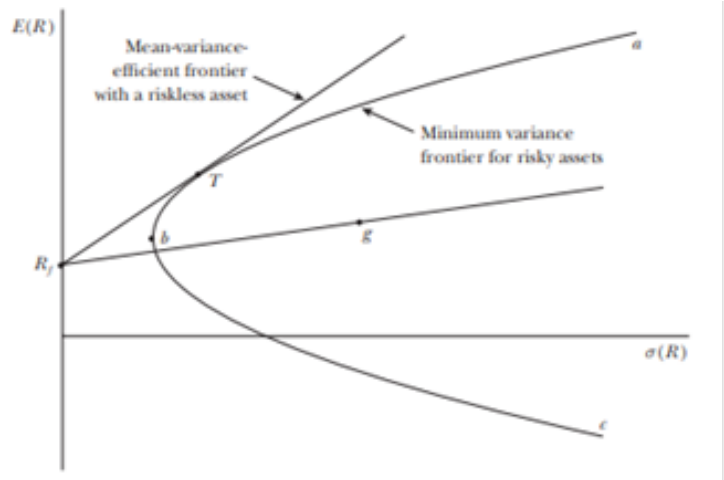
The Sharpe-Lintner model's final step involves determining the expected return on assets with Beta=0 ($E(R_{ZM})$) using the concept of risk-free borrowing and lending. An asset is considered to have Beta=0 and uncorrelated with the market return if its covariance with other assets' returns balances out its own return variance. Such an asset is riskless in the market portfolio as it does not increase the market return's variance. When risk-free borrowing and lending is possible, $E(R_{ZM})$ must equal the risk-free rate (R_f), leading to the well-known Sharpe-Lintner CAPM equation connecting expected return and beta.

$E(R_i) = R_f + [E(R_M) - R_f] * \beta i_M$, $i = 1,\ldots,N$ *risky assets*

The linear relationship shows that the expected return on an asset (i) is equal to the risk-free interest rate (R_f) plus a risk premium, calculated as the product of the asset's market beta (B_{iM}) and the additional return per unit of beta risk ($E(R_M) - R_f$).

[3] W. F. Sharpe, "Capital asset prices: a theory of market equilibrium under conditions of risk," The Journal of Finance 19, no. 3 (1964): 425-442.

Figure 1 *– Frontier of Investment opportunities*



Source: The Capital Asset Pricing Model: Theory and Evidence, Fama and French

In figure 1 the risk-return tradeoff is evident. The points from *a* to *b* on the *abc* curve represent the efficient combinations of risky assets. When the risk-free asset is introduced, the new efficient frontier becomes linear and the mean variance-efficient portfolios result from an allocation in the tangency portfolio T and either risk free lending or borrowing, depending on the risk appetite of the investor.

More than 50 years after its first formulation, the CAPM remains popular in various applications such as determining a firm's cost of capital and evaluating portfolio performance. This is due to its straightforward and appealing predictions about risk measurement and the relationship between expected return and risk. Sharpe and Markowitz were jointly awarded the Nobel Prize in Economical Sciences in 1990 for their "pioneering work in the theory of financial economics".

1.3 Beta

Beta is a measure of the volatility, or the portion of systematic risk[4] of a security or a portfolio in comparison to the overall market and can be calculated by taking the covariance of the security or portfolio's returns

[4] R. Genay , F. Seluk & B. Whitcher Systematic risk and timescales, Quantitative Finance, 3:2, (2003), 108-116

with the market's returns and dividing it by the standard deviation of the market's returns.

$\beta = \sigma_{s,m} / \sigma_{m,}$ *where:*

- $\sigma_{s,m}$ → *covariance of the security with the market*
- σ_m → *standard deviation of the market*

The same result can be obtained by dividing the standard deviation of the security by the standard deviation of the market and multiplying the correlation coefficient between the security and the market.

$\beta = (\sigma_s / \sigma_m)\, \rho_{sm}$, *where:*

- σ_s → *standard deviation of the security*
- σ_m → *standard deviation of the market*
- ρ_{sm} → *correlation coefficient of the security with the market*

Beta is useful for determining the required rate of return on an investment because it allows investors to adjust the expected return for the level of risk involved. For example, we have seen that if a security's returns are completely uncorrelated to the market, it will have a beta equal to 0. If, instead, is perfectly correlated with the market's returns, then its beta will be 1. If the security's returns are less volatile than the market's returns, then its beta will be less than 1. If the security's returns are more volatile than the market's returns, then its beta will be greater than 1.

Regression beta is a measure of the relationship between the returns of a security or portfolio and the returns of the overall market. It is calculated using a statistical technique called regression analysis, which involves fitting a line to a scatterplot of the data in order to identify the relationship between the two variables. In this regression, the independent variable is the returns of the overall market, and the dependent variable is the returns of the security or portfolio. The beta coefficient is the slope of the line of best fit, which represents the

relationship between the two variables. In other words, regression beta is calculated in the same way as traditional beta, by taking the covariance of the security or portfolio's returns with the market's returns and dividing it by the standard deviation of the market's returns.

The beta of a portfolio of assets is a measure of the portfolio's volatility, in comparison to the overall market. It is calculated by taking the weighted average of the beta of each individual asset in the portfolio, where the weight of each asset is equal to the proportion of the portfolio's total value that is invested in that asset.

The way beta is computed explains why the beta of an asset (or portfolio) is often associated with a measure of sensitivity of the assets (or portfolio) returns to variations in the returns of the market. In economic terms, therefore, Beta can be considered as proportional to the risk that each dollar invested in the asset contributes to the overall risk of the market portfolio. This interpretation of beta aligns with the principles of the CAPM, which seeks to provide a framework for understanding and pricing the risk of individual assets in the context of the overall market.

Though useful in assessing individual stocks behavior, often the outcome of linear regressions on single stocks returns against the market can have really small significance. This is because prices of stocks are influenced by a lot of noise and price movements which are not tightly related to actual business performance but rather on certain isolated events such as bad news in the press[5], which are not expected to repeat themselves in the future. Another limitation can arise when the stock object of the analysis represents a large part of the referenced index. In this case the standard error of the Beta would be lower, but the indicator would not be representative of the true risk in the security.

To avoid these issues, a possible solution is to compute a so-called Bottom-up Beta[6], which is a built-up indicator that is based on three fundamentals: the business or businesses in which the firm operates, the

[5] Brogaard, Jonathan, et al. "What moves stock prices? The roles of news, noise, and information." The Review of Financial Studies 35.9 (2022): 4341-4386.

[6] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 8

firm's operating leverage, and its financial leverage. This approach, rather than focusing on the returns of a single firm, takes into account the performances of other firms which have been operating in the same business and then builds up the specific characteristics of the firm. In the case of two firms operating the same business, all other things being equal, we expect the firm who has a higher financial leverage to be exposed to higher risk, thus having a higher Beta. In the same way, between two otherwise identical firms, the one who has a higher ratio of fixed costs over variable costs will have a higher volatility in returns, translating in higher risk and higher Bottom-up Beta.

The use of bottom-up betas represents a substantial improvement over regression betas in several key aspects. Firstly, it improves precision: regression betas are estimated with standard errors, but the average of multiple regression betas yields a significantly lower standard error. By averaging individual beta estimates, the resulting average beta becomes more precise than its constituent betas. This improvement is particularly evident when the estimation errors on individual firm betas are uncorrelated across firms. In such cases, the reduction in standard error can be quantified in relation to the average standard error of beta estimates and the number of firms in the sample. Bottom-up Beta also allows for flexibility in reflecting changes in the business mix. For instance, if a firm divests a significant portion of its operations or undergoes acquisitions, the weights assigned to different businesses can be adjusted accordingly in the beta calculation. Additionally, strategic plans for entering new businesses can be incorporated into future beta estimates. This adaptability ensures that the beta accurately captures the changing dynamics of a firm's operations.

Another advantage is the usage of a current Debt-to-Equity ratio compared to an historic one. Regression betas reflect in fact the average debt-to-equity ratio maintained by the firm during the regression period. In contrast, bottom-up betas utilize the current debt-to-equity ratio. This enables adjustments in the beta to account for planned changes in the firm's debt structure. By considering the firm's expected shifts in the debt-

to-equity ratio, the beta can provide a more accurate representation of the firm's risk profile. One last improvement is the reduction in dependence on historical stock prices. While historical prices are necessary to obtain betas for comparable firms, they are not a prerequisite for analyzing the firm of interest. Instead, all that is needed is a breakdown of the firm's business segments. Consequently, bottom-up betas can be estimated for various entities, including private firms, business divisions, and newly listed stocks in financial markets.

1.4 Critiques to the CAPM

Despite its contribution, the CAPM has been criticized on both theoretical and empirical grounds for a number of reasons and mostly related to the fact that it is based on unrealistic assumptions and to the failure in providing meaningful empirical results[7].

On the theoretical side one criticism is that it relies on the assumption of rational investor behavior, which may not always be accurate. In reality, investors may not always act in their own best interests or make decisions based on complete information. This can lead to deviations from the assumptions of the CAPM, such as the existence of bubbles or overreactions to news events. Other assumptions that raise concern are the complete agreement among investors, the availability of free and unrestricted risk-free borrowing and lending, and the absence of transactions costs. In truth the costs associated with borrowing or lending or with buying an additional security could eventually result in a disincentive towards building the market portfolio.

Another criticism of the CAPM is that it assumes that financial markets are efficient, meaning that prices reflect all available information and that it is not possible to consistently outperform the market. However, many investors and analysts believe that markets are not always efficient, and that it is possible to identify mispricings and opportunities for above-average returns.

[7] Eugene F. Fama and Kenneth R. French, "The Capital Asset Pricing Model: Theory and Evidence," Journal of Economic Perspectives 18, no. 3 (Summer 2004): 25-46.

A third criticism of the CAPM is that it does not take into account unsystematic risk, which is the risk that is specific to a particular investment or company and can be diversified away by holding a diverse portfolio. This means that the model may not accurately reflect the risk of individual investments or portfolios. This point is critical to this work, as it highlights the gap between the risk exposure of an undiversified investor to one that is completely diversified.

The empirical performance of the model has also been poor, raising doubts about its practical use. The reasons for these empirical difficulties may be due to the numerous simplifying assumptions inherent in the theoretical framework of the CAPM, or they may stem from difficulties in conducting valid tests of the model.

One issue in implementation is the definition of the "market portfolio" used to measure the risk of a stock. The CAPM states that the risk of a stock should be evaluated in relation to a comprehensive market portfolio that, in theory, can encompass not only traded financial assets, but also tangible assets such as real estate and human capital. However, when it comes to practical application, the definition of the market portfolio is often limited to U.S. common stocks, excluding other financial assets such as bonds and ignoring international markets.

1.5 Alternatives to the CAPM

In light of these criticisms, other approaches to the estimation of returns for risky assets have been developed by academics and practitioners.

Multi-factor models are considered to be an improvement over the CAPM as they provide a more comprehensive and accurate assessment of expected returns by considering multiple sources of systematic risk. Eugene Fama and Kenneth French developed a multi-factor model which includes additional factors other than the Beta in the CAPM model. They first published the three-factor model in their 1992 paper, "The Cross-Section of Expected Stock Returns". In this paper, they presented evidence that size and book-to-market equity were important in explaining the

cross-section of stock returns and proposed a new asset pricing model that included these two factors, in addition to the market factor. In 2015 they released their five-factor model[8] adding the profitability and investment factors to those in the three-factor model, where the profitability factor captures the excess returns of companies with high profitability over those with low profitability, while the investment factor captures the excess returns of companies with low levels of investment over those with high levels of investment.

The Barra Multi-Factor Model[9] (also known as the Barra risk model or the Barra optimizer) is a quantitative investment tool used by asset managers to analyze and manage portfolio risk developed in the 1970s by Barra Inc. (now a part of MSCI Inc.), a provider of investment decision support tools and services. It is based on the idea that there are several underlying factors that drive stock returns, such as macroeconomic variables, industry trends, company-specific characteristics, and market conditions. The Barra model uses a statistical approach to estimate the sensitivity of a portfolio's returns to each of these factors, and then quantifies the risk of the portfolio by measuring its exposure to these factors.

The Post-Modern Portfolio Theory[10] (PMPT) expands on the traditional framework of Modern Portfolio Theory by incorporating additional factors that affect investment decision-making such as goals, and risk tolerance of investors and acknowledging that investor decision-making may deviate from rationality assumptions and considers behavioral biases, such as loss aversion or herding behavior. PMPT also extends the traditional asset universe beyond stocks and bonds by including alternative investments and recognizes that financial returns often deviate from normal distributions and exhibit non-linear relationships. While PMPT offers promising advancements in portfolio

[8] K. R. French, "Presidential address: The five-factor asset pricing model," Journal of Finance 70, no. 2 (2015), 539-580.
[9] Barra, Inc., "The Barra Risk Model Handbook: Using the Barra System for Equity Risk Management and Portfolio Strategy", (1999).
[10] B. M. Rom and K. W. Ferguson. "Post-modern portfolio theory comes of age." Journal of investing 3.3 (1994): 11-17.

management, it is not without its criticisms and limitations related to requirements and estimation techniques, behavioral assumptions, computational complexity, and the lack of consensus and empirical evidence.

Behavioral Finance Models are another alternative which aim to incorporate in financial models' insights from psychology and economics to understand and explain why individuals make certain financial decisions and how their behavior impacts market prices and expected returns. Prospect Theory[11] and the Heuristics and Biases[12] approaches are among the most popular behavioral finance models and they try to understand and evaluate some common behavioral biases such as an excessive belief in one's ability to make accurate predictions or to control events (Overconfidence), the tendency to rely too heavily on initial information or impressions when making decisions (Anchoring), the tendency for individuals to follow the actions of others, rather than relying on their own independent judgment (Herding), letting the way information is presented influence decision making (Framing).

1.6 The CAPM argument

Corporate finance professor and valuation expert Aswath Damodaran[13], suggests that the Capital Asset Pricing Model (CAPM) is still to be considered the best risk-return model due to its simplicity, requiring a single firm-specific input, and the failure of other models in providing any substantial improvement in forecasting future performances. Damodaran also argues that the models' limitations do not necessarily make CAPM a bad model, but rather highlights the need for caution and careful interpretation of the results obtained from the model.

I personally agree with this view, for a number of reasons. Above all the CAPM succeeds in serving the main purpose of a risk-return model

[11] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk", Econometrica 47, no. 2 (1979), 263-291.
[12] A. Tversky and D. Kahneman, "Judgment under uncertainty: Heuristics and biases", Science 185 (1974), 1124-1131.
[13] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 4

which is to capture the systematic risk of an asset. At the same time, it is a simple and straightforward model, whereas other multi-factor models can lack this feature making their results difficult to understand and exploit. The CAPM is also a market-driven model, and therefore it is a reflection of the market's perception of the risk and return of an asset.

Furthermore, it is important to note that multi-factor models are also subject to various assumptions and limitations, such as the choice of factors, estimation error, and data availability. In addition, the factors can change through time, and the correlation between the variables can vanish, neglecting the effectiveness of the model.

For these reasons, like many academics and practitioners, I have decided to base my work on this model.

Summary 1

This first chapter. which opens the work, focuses on the development of asset pricing theory and constitutes the foundations for the theoretical derivation of a premium for non-diversification and for the literature on Total Beta. The Mean-Variance model by Harry Markowitz forms the basis of Modern Portfolio Theory by introducing the idea that investors should consider not just the expected return but also the volatility of their portfolio. He proposed a mathematical framework to determine the optimal mix of assets to construct a portfolio that maximizes expected returns while minimizing risk through diversification. Markowitz' work laid the foundation for Sharpe and Lintner's CAPM, which added the assumptions of complete agreement about the distribution of returns and the ability to borrow and lend at a risk-free rate. The CAPM equation states that expected return on an asset is equal to the risk-free interest rate plus a risk premium calculated as the product of the asset's market beta and the additional return per unit of beta risk. Although widely used, the CAPM has been criticized for its unrealistic assumptions and lack of empirical results. Some alternative models have been developed in the attempt to better explain differences in returns of risky assets, notably multi-Factor models such as the Three-Factor Model by Fama and French, and behavioral finance models. Nevertheless, the CAPM remains popular in applications such as determining a firm's cost of capital and evaluating portfolio performance.

Chapter 2
UNDIVERSIFIED INVESTORS

2.1 Portfolio diversification

In the previous chapter we have established the positive impact of diversification over an investment portfolio. In this framework, a rational investor should aim at diversifying as much as possible and then use borrowing and lending to balance between return and risk according to his risk aversion. The complete diversification expressed in Markowitz' equation, though, appears unachievable due to the amount of transaction costs that would incur, eventually reducing the appetite for further diversifying the portfolio[14]. The following question then comes naturally: if an investor cannot be completely diversified, how many assets should he include in his portfolio in order to resent the risk reduction of the market portfolio?

An empirical answer has been provided by Elton and Gruber in 1977 in their work published in The Journal of Business titled "Risk Reduction and Portfolio Size: An Analytical Solution". In the paper, the authors present a model for portfolio optimization that takes into account the trade-off between risk and return. They show that as a portfolio becomes larger, the risk associated with the portfolio decreases, but this advantage reduces while the number of assets grows. This result has important implications for portfolio construction and risk management, as it suggests that investors can significantly reduce the risk of their portfolios without holding the market portfolio. Their study is based on a sample of 3.290 securities and shows that, of the overall reduction in variance derived from diversifying from single asset portfolio to the market portfolio (which in this case is represented by the equally 3.290 securities weighted), 90% can be achieved by creating a random portfolio of 10 equally weighted securities while 95% can result from a random portfolio

of 50 securities.

Figure 2 *– Effect of Diversification*

Number of Securities	Expected Portfolio Variance	Variance in Variance	Total Risk
1	46.619	1,411.041	46.811
2	26.839	201.963	26.934
4	16.948	31.553	16.996
6	13.651	11.184	13.683
8	12.003	5.477	12.027
10	11.014	3.186	11.033
20	9.036	.623	9.045
50	7.849	.075	7.853
100	7.453	.013	7.455
200	7.255	.001	7.256
500	7.137	.000	7.137
1,000	7.097	.000	7.097
Minimum	7.070	.000	7.070

NOTE.—Parameters based on 3,290 securities values shown in table 5.

Source: Risk Reduction and Portfolio Size: An Analytical Solution, Elton and Gruber

Despite the evident advantage of diversification, several investors still choose to hold a smaller number of assets or to overweight certain assets within their investment portfolios. This is often the case for company's founders and employees who receive stock compensation. In some extreme cases, investors might have most of their wealth tied up to one investment and therefore give up completely on the advantages of diversification that we have observed both on a theoretical and empirical basis.

2.2 The undiversified investor

In the first chapter I have discussed how for the Mean Variance-model and the CAPM to stand, all the investors willing to maximize their utility, should decide to hold diversified portfolios of assets, similar to the market portfolio.

The diversified investor hypothesis seems to me the most frightening among the assumptions of the CAPM, as it can be extremely unrealistic and

[14] Goldsmith, D "Transactions Costs and the Theory of Portfolio Selection." The Journal of Finance 31, no. 4

misleading, in particular when dealing with private companies. In these instances, the outcomes of the models can considerably understate the effective risk exposure derived from these types of assets, as the model fails to consider the portion of risk that arises from the lack of diversification.

With the rise of alternative asset classes such as Private Equity, private capital transactions have grown significantly along with the opportunities to misuse the CAPM as a tool to estimate returns and estimate intrinsic value of private companies.

In 2020 the number of active enterprises within the EU economy was 26.3 million[15]. Among these, the large majority are Micro or Small firms, with Medium (above 50 employees) and Large (above 250 employees) accounting for less than 1% of the total number of businesses or about 250 thousand companies. In the US, a total of 6,1 million[16] firms were active in 2019, although the share of Medium and Large companies is considerably higher than the European one, making up 4,1% of the total or roughly 250 thousand companies. According to the World Bank, the listed companies in the EU domestic economy totaled 5.863 in 2018 while the date for the US was 4.266 in 2019. In both regions, therefore, the number of public companies amounts to roughly 2% of the total number of established Medium/Large companies. These figures highlight that, even when it comes to sized firms, equities are mostly held privately.

Most US listed equities belong to institutional investors, who owned 80% of all stocks in the S&P 500 as of 2019[17]. In Europe, the share of ownership attributable to institutional investor is still relevant but much smaller, representing the 26% of the major 100 non-financial companies listed in France, Germany, Italy, Spain, and the UK, in the period between 2010-2015[18]. It is important to mention that not all the Medium/Large

(1976): 1127–1139

[15] Eurostat

[16] US Census Bureau

[17] Harvard Business Review, How Big a Problem Is It That a Few Shareholders Own Stock in So Many Competing Companies? by Jacob Greenspon February 19, 2019, Updated February 22, 2019

[18] "Institutional Investors' Shareholdings in Large European Non-financial Listed Companies" F. Fancello and N. Linciano, 26 March 2018, Oxford Business Law Blog

companies would meet the requirements for listing. Public quotation is a complex and costly process, and thus, not appealing for many medium sized companies[19]. A company seeking to list must in fact meet all the financial requirements of the exchange it wishes to list on. These standards are, in most cases, related to number of shareholders, market capitalization, financial reporting, meeting certain corporate governance standards, having a certain number of shares outstanding, and maintaining a minimum share price, meeting certain trading volume thresholds, or having a minimum number of market makers. Companies seeking listing must furthermore comply with the regulation of the country of the aforementioned exchange. The relevant regulator for the US is the Security and Exchange Commission (SEC), while in Europe the main independent authority for the financial markets is the European Securities and Markets Authority (ESMA). Both regulators require a wide range of requirements for listing, including filing a registration statement, providing financial statements and other disclosures, as well as appointing an independent auditor[20].

Private companies, on the other hand, are owned in most cases by their founder, a small number of shareholders, the company's management and, in some cases, Private Equity Funds.

An article published by Forbes[21] shows estimates that almost 250 US private companies are able to generate a turnover above 2 billion USD. The largest US private company is Cargill Inc., a global firm operating in the food industry since 1865, employing more than 150 thousand people and, according to the same Forbes, realizing an annual revenue of 165 billion USD. Roughly 90 members of the Cargill-MacMillan family, who descended from the funder William Wallace Cargill, jointly own 88% of the company. This highlights how, even in large entities, private ownership is still a relevant feature.

[19] Ehrhardt, Olaf, and Eric Nowak. "The effect of IPOs on German family–owned firms: Governance changes, ownership structure, and performance." Journal of Small Business Management 41.2 (2003): 222-232.
[20] Geoffrey P. Miller, Regulation of Securities, Markets, and Transactions: A Guide to the New Environment," Wolters Kluwer, (2018).
[21] Murphy A., "America's Largest Private Companies," Forbes (December 2022)

The number of Private equity deals has increased dramatically in the past decades, in particular in the US where it reached 9.120 in 2021 according to PitchBook[22], while in Europe 7.076 deals took place the same year[23]. PE deals have therefore impacted about 3,6% of Medium/Large private companies in the US and roughly 2,8% of sized companies in Europe. A sample of 1.385 transactions concluded globally after 2010, shows that private investors were involved in less than 5% of the time (the full sample is presented in Chapter 6). Although the vast majority of private transactions are agreed between institutional investors, there are deals where either the seller or the buyer is private.

"Why does any of this matter?" a reader might ask. The interest, for the purpose of this work, lies in the identification and characterization of the marginal investor, who is defined as the investor who is most likely to trade the next share of the underlying asset. Arguably the marginal investor can be identified in one that holds the largest stake in an asset.

The market for privately traded stocks is dominated by the entrepreneurs. In the private market, therefore, entrepreneurs are often the marginal investors, and they set the prices for most private companies. This figure is different from that of the institutional investor, as the entrepreneur seeks and aims to have control of companies; moreover, more often than not the entrepreneur has limited resources such that the creation of a diversified portfolio is impossible.

A study by Barber and Odean (2000) found, that the mean household's portfolio contained only 4.3 stocks, while the median household holds as little as 2.6, which is far from the minimum number required for an effective diversification, as shown by Elton and Gruber, meaning that individual investors have been found to be consistently under diversified. Moskowitz and Vissing-Jorgensen in 2002 found that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth[24].

[22] PitchBook 2022 Annual US PE Breakdown
[23] Invest Europe, Private Equity Activity 2021
[24] Moskowitz, Tobias J., and Annette Vissing-Jørgensen. "The returns to entrepreneurial investment: A private equity premium puzzle?" American Economic Review 92.4 (2002): 745-778.

It is reasonable then to assume that when a private investor owns significant stakes in a private company, in the event of a transaction the marginal investors might not be completely diversified. For this reason, when estimating returns for a private company in the event of such a transaction, I deem it necessary to use an approach that considers the lower level of diversification that this type of investor might have.

2.3 Undiversified risk

While introducing Beta, I have already mentioned that this parameter reflects the portion of the systematic risk that an individual security might add to the market portfolio. This is the only risk a diversified investor should care about, because, thanks to the benefit of diversification, all the residual risk can be ignored. When it comes to an undiversified investor though, there is another component of risk that needs to be taken into account when assessing investment decisions within a portfolio. This additional layer of risk can be addressed as firm-specific or non-systematic risk and refers to the risk that could be diversified. Unlike systematic (or market) risk, which is related to a higher Beta, the non-systematic risk is not compensated by higher expected returns for the investment portfolio, and thus is undesirable for an investor to take on such risk.

Specific risk often is underestimated by investors who hold significant stakes within a single business that perceive their holding as safer just due to their proximity and familiarity. Huberman (2001) found evidence that investors are keener to invest in familiar stocks to an extent that makes them ignore the principles of portfolio diversification. Benartzi and Thaler (2001) found that employees tend to own a disproportionate amount of the employer's company stock in their pension plans.

This inefficient allocation results from a number of biases. Familiarity bias[25] refers to the tendency of individuals to favor what they are familiar with, rather than exploring other alternatives. This happens

[25] Guiso L., Sapienza P., and Zingales L., "The Familiarity Bias, Stock Market Participation, and Household Wealth," The Journal of Finance 63, no. 4 (2008), 2009-2050

because these investors often have a personal attachment to the business and may be resistant to change. Endowment bias[26] is the tendency of individuals to overvalue items they own or possess, often related to the fact that they may view their business as a part of their identity, making it difficult for them to objectively evaluate it. Both familiarity and endowment bias frequently affect family owners and employes who owns stocks in the company they work for and can have negative consequences on their financial decision such as understating the risk of the underlying investment or developing unrealistic expectations.

Despite the safety induced by these biases, the magnitude of the specific risk as a component of the total risk exposure on an asset can be significant and needs to be taken into account when dealing with a private company. The way specific risk is estimated will be discussed in the next chapter.

[26] Kahneman, D., Knetsch, J. L., & Thaler, R. H. "Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias," The Journal of Economic Perspectives, 5(1), (1991), 193-206.

Summary 2

This chapter constitutes a further building block towards the development of the work by laying down the advantages of portfolio diversification, introducing firm specific risk, and the undiversified investor.

Diversification is important for investment portfolios, but achieving complete diversification is challenging due to transaction costs. Elton and Gruber's study reveals that a random portfolio of 10 equally weighted securities can achieve 90% of the risk reduction of a market portfolio. Despite the benefits of diversification, some investors hold fewer assets or overweight certain ones, forfeiting their advantages.

The CAPM and Mean Variance-model assume diversified investors, but this assumption is unrealistic when dealing with private companies. CAPM fails to account for the risk arising from the lack of diversification. Private companies are predominantly owned by founders (or their families), management, and Private Equity Funds, while publicly listed equities are mostly owned by institutional investors. The complex process of public listing deters many medium-sized companies. Household portfolios often lack diversification, with individual investors consistently under-diversified.

Beta represents only the portion of systematic risk which a diversified investor should consider. Non-systematic risk (specific risk) can be diversified and is undesirable for investors but is unaccounted for by the CAPM framework. Investors often underestimate specific risk when holding significant stakes in a single business due to familiarity bias and endowment bias. Despite these biases, specific risk is significant and should be considered when dealing with private companies.

Chapter 3
TOTAL BETA

3.1 Firm-specific risk

While working at a fund service provider for Italian AIFMs, in particular Private Equity funds, I have obtained some practical experience in the endeavor of risk assessment. Complying with the duties of the risk management function for Private Equity funds in Italy, as laid down by the Directive 2011/61/EU of the European Parliament and of the Council of 8 June 2011 on Alternative Investment Fund Managers and enforced by the Banca d'Italia through the "Regolamento della gestione collettiva del risparmio", I was considering the specific risk as one of the 7 risk categories to estimate and disclose in the fund's mandatory reports. The risk categories include Market risk, Liquidity risk, Counterparty risk, Credit risk, Operational risk, ESG risk, and Specific risk. The definition of Specific risk is negative, meaning that the risk factors contributing to this risk where all those risks which cannot be traced back to any of the other risk categories. The assessment involved a qualitative judgment based on the number and magnitude of the specific risks identified by the risk manager.

The Beta in the CAPM measures the risk contribution of an investment to a diversified portfolio, and therefore is most suitable for firms with diversified investors[27]. However, in the past chapter, I have supported the argument that most private companies are held by individual investors and that this type of investor is generally far less diversified compared to institutional investors[28]. For this reason, in the case of private firms, betas may underestimate their risk exposure.

The risk of an investment, as already discussed, can be defined as the deviation standard of the return on said investment. The concept of diversification involves the placement of individual securities of risky

[27] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 24

[28] Chen, Hui, Jianjun Miao, and Neng Wang. "Entrepreneurial finance and non-diversifiable risk." The Review of Financial Studies 23.12 (2010): 4348-4388.

companies, which are not perfectly correlated[29], in a portfolio; if sufficient resources or securities, the firm-specific risk is diversified, i.e., the risk specific is removed from the volatility of the portfolio. Given the advantages and the possibility of diversification, for modern financial theory, investors only require to be compensated from the relative portion of systematic risk when holding an asset, which, as repeatedly stated, is captured by the Beta of the Capital Asset Pricing Model.

However, this assumption does not hold at all when the owner's entire wealth is invested in the private business, as it is often the case for entrepreneurs, resulting in the total exposure to firm-specific risk. In this case, entrepreneurs should be compensated for more than just systematic risk: in the private stock market, entrepreneurs need to seek compensation for total risk, both systematic and the specific one, as the allocation would otherwise be inefficient compared to the alternatives available in the market.

Specific or idiosyncratic risk is defined as the risk or volatility of an individual security which cannot be traced back to the two other main components of risk which are market and industry risk[30]. Empirical studies have highlighted the magnitude of this risk component as a determinant of overall volatility within publicly traded stocks[31], finding that it has become increasingly significant as more companies have been listed.

Nevertheless, estimating specific risk for a single firm is not an easy task, since it comprises a wide nexus of risk factors which may vary from firm to firm depending on the industry in which it operates. Unlike the market risk and the industry risk, the estimation of firm-specific risk is not based on an established framework, although being regarded as a fundamental feature by many regulators such as the European Parliament which make its assessment a mandatory requirement for funds investing

[29] Goetzmann, William N., and Alok Kumar. "Equity portfolio diversification." Review of Finance 12.3 (2008): 433-463.

[30] Gregory Brown, Nishad Kapadia, “Firm-specific risk and equity market development” Journal of Financial Economics, Volume 84, Issue 2, (2007) 358-388

[31] Campbell, John Y., et al. "Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk." The journal of finance 56.1 (2001): 1-43.

in private assets.[32]

3.2 Deriving Total Beta

Total beta represents an attempt to incorporate the firm-specific risk component, relevant for non-diversified investors, into the CAPM. This concept was first introduced by Stephen A. Ross, in the mid-1970s as a refinement of risk and return dynamics. In his influential paper titled "The Arbitrage Theory of Capital Asset Pricing," introduced the groundbreaking concept of total beta, recognizing that traditional beta failed to fully account for the risks faced by non-diversified investors who held significant positions in a limited number of stocks. By extending the beta framework to incorporate firm-specific risk, total beta offered a more holistic understanding of the risk profile of individual stocks for non-diversified investors.

In addition, other prominent academics and financial professionals have made significant contributions to the understanding and development of the concept of total beta. Most notably Professor Aswath Damodaran[33] has explored the concept of total beta extensively in his research and writings. His work has shed further light on the practical applications and implications of total beta in investment analysis, providing valuable insights into the calculation methodologies of total beta and its interpretation within the broader context of portfolio management. Butler and Pinkerton also support the Total Beta paradigm in the estimation of firm-specific risk[34] against the disputes and objection raised by other academicals, such as Kasper[35] and Helfenstein[36]

The argument for Total Beta starts from the acknowledgement that risk premium for public companies is usually greater than zero. Despite

[32] DIRECTIVE 2011/61/EU OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 8 June 2011 on Alternative Investment Fund Managers
[33] Damodaran, Aswath. "Estimating risk parameters." NYU (1999).
[34] Butler, Peter, and Keith Pinkerton. "Company-specific risk—A different paradigm: A new benchmark." Business Valuation Review 25.1 (2006): 22-28.
[35] Butler, Peter J., and Bob Dohmeyer. "The Total Beta Debate: A Real-World Analysis." Business Valuation Review 32.4 (2013): 227-230.
[36] Butler, Peter, Gary S. Schurman, and Andrew M. Malec. "Practical evidence and theoretical support for total beta." Biotechnology 11 (2011): 30-36.

this, experts begin their analysis from zero percent to determine a premium for the specific risk component when dealing with private companies[37]. In the Capital Asset Pricing Model, corporate valuation analysts compare and contrast their company with publicly traded benchmarks: this benchmark is used to select appropriate betas, industry premiums, equity risk premiums, and premiums for firm size; however, this comparison cannot be extended to determine the risk premium for private firms. Although modern theory states that a public company does not incorporate total risk, this does not preclude that it could have been used to evaluate private companies.

The Beta of the CAPM should, therefore, be improved or enhanced to be able to price the total risk. The original intent of the Beta is not to be intended to measure the absolute difference between the volatility of an individual investment and the market volatility, but it should express the degree of co-movement or change demonstrated by the expected returns of an individual investment in relation to the movement of the expected returns of the market.

The intuition behind the Total Beta is really simple. Let us consider the total risk in a security as the standard deviation of its returns. The Beta of the security (or portfolio) is the relative standard deviation of the security's returns against the market returns, multiplied by the correlation between the two variables, as per the formula below[38].

$\beta = (\sigma_s / \sigma_m)\, \rho_{sm}$, *where:*

- σ_s → *standard deviation of the security*
- σ_m → *standard deviation of the market*
- ρ_{sm} → *correlation coefficient of the security with the market [-1;+1]*

In the case of perfect correlation between the security (or portfolio) and the market, and therefore $\rho_{sm} = 1$, the Beta will only depend on the

[37] Butler, Peter, and Keith Pinkerton. "Company-specific risk—A different paradigm: A new benchmark." Business Valuation Review 25.1 (2006): 22-28.

relative standard deviation σ_s / σ_m. If the security (or portfolio) is more volatile than the market, Beta will be higher than 1, in case it is less volatile Beta will be lower than 1, and in case it is exactly as volatile Beta it will equal to 1.

Let us now start from this last scenario where the returns on a security (or portfolio) are just as volatile as the market. If the correlation coefficient is lower than 1, the Beta will also be lower than 1 and the lower the correlation the lower the Beta. In the case of a diversified investor, this lower Beta represents a lower risk contribution within the CAPM because he would only care about the portion of systematic risk which he is adding to its diversified portfolio and would not seek compensation for the portion of standard deviation in the returns of the security which are not correlated. An undiversified investor, in the same scenario, would need to consider the full standard deviation of the security relative to the market because the uncorrelated portion of standard deviation will not be diversified away[39]. In this way, for a security as volatile as the market, the resulting Beta will be $\sigma_s / \sigma_m = 1$, the same as if the security's returns were perfectly correlated to the market ones. The Beta so computed is defined as Total Beta and reflects the riskiness of an asset (or portfolio) relative to the market, without distinguishing between the portion of systematic and unsystematic risk in said asset.

Total beta can therefore be computed according to this formula.

$T\beta = \sigma_s / \sigma_m$, *where:*

- σ_s → *standard deviation of the security*
- σ_m → *standard deviation of the market*

Or also as starting from Beta follow.

$T\beta = \beta / \rho_{sm}$, *where:*

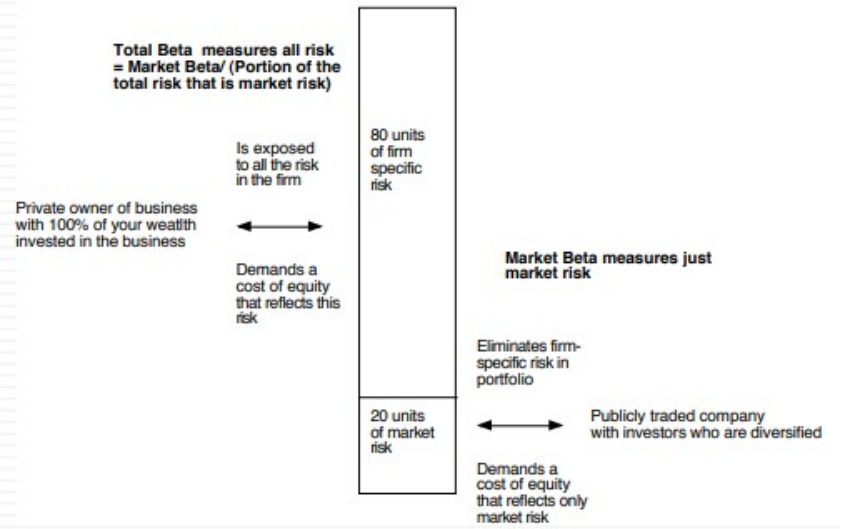
[38] Butler, Peter, and Keith Pinkerton. "Company-specific risk—A different paradigm: A new benchmark." Business Valuation Review 25.1 (2006): 22-28.

- ρ_{sm} → *correlation coefficient of the security with the market [-1;+1]*

The correlation coefficient, found in the formula, can be viewed as the percentage of total risk that is priced in the public market, consequently (1 - ρ_{sm}) is the percentage of total risk that is removed from diversification.

The figure below shows the relationship between Total Beta and Beta and the composition of total risk within a firm. In the hypothesis of correlation ρ_{sm} = 0,2, the market can explain 20 out of 100 units of risk, the remaining 80 units, therefore must derive from the firm itself.

Total Beta vs Market Beta



A. Damodaran "Valuing Private Companies"

Still considering the 0,2 correlation between the asset returns and the market, let us take the example of a relative volatility $\sigma_s/\sigma_m = 2$, where the returns of the securities are twice as volatile as the market's. In such scenario the Beta for a diversified investor would be 2*0,2 = 0,4, while the Total Beta for an undiversified investor would be 2, or five times as high as the respective Beta. Another way to put it is that Total Beta reflects the scaling of the firm's beta to account for all risks rather than just market

[39] Butler, Peter, and Keith Pinkerton. "Company-specific risk—A different paradigm: A new benchmark." Business Valuation Review 25.1 (2006): 22-28.

risk.

Although estimating a total beta for a private firm may appear challenging due to the absence of market prices, it can be achieved by utilizing publicly traded firms in the same sector to estimate the market beta and correlation coefficient[40]. These estimates serve as a basis for calculating the total beta for the private firm.

3.3 Using Total Beta

The decision of whether to make the total beta adjustment depends on the purpose of the estimation of expected returns. Estimated required rates of returns are a fundamental component in the valuation process for private firms. As discussed in the first paragraph, the CAPM is one of the most acknowledged approaches to estimate returns to equity holders. The CAPM, though, is designed to estimate the expected return for a diversified investor such as institutional investors which are in most cases the marginal investors in public equities. Applying the CAPM to estimate returns for an individual investor would therefore result in a misrepresentation, since it would understate the risk perceived and therefore reduce the required rate of return. To address this issue, the Total Beta can be used as a way to include the specific risk component in the CAPM, so that the model might provide a more accountable measure of expected return for an undiversified investor.

Even when dealing with private firms, the appropriate adjustment for non-diversification depends on the specific context and characteristics of the valuation. If the valuation is devoted to a sale, the adjustment of the market beta would depend on the levels of diversification of buyers and sellers. In the case of an initial public offering, no adjustment for non-diversification is necessary since potential buyers are mainly institutional investors, which are considered fully diversified. If the deal involves an individual or another private business, the extent of the adjustment would be influenced by his portfolio diversification. A more diversified buyer

[40] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 24

would have a higher correlation with the market and require a smaller total beta adjustment.

The Total Beta can be employed in the CAPM formula in the same way as the Beta in order to estimate expected returns for a completely undiversified investor.

$E(R_i) = R_f + [E(R_M) - R_f] * T\beta i_M$, $i = 1,\ldots,N$ *risky assets where:*

- R_f *is the risk-free rate*
- $E(R_M)$ *is the market's expected return*
- TB_{iM} *is the total beta of asset i*

According to Damodaran, the key point is to define the marginal investor, whose profile is dynamic over time, in the private company or project, and adjust the correlation with the market portfolio: for investors without any diversification, one can use the industry correlation coefficient, while for buyers and investors with a greater degree of diversification one can adjust the correlation. In this second case, essentially greater diversification results in greater correlation and consequently less change in Total Beta. Depending on the level of diversification, firm-specific risk will be assessed in differently: if the owner of a company has tied up in his company all his assets, the firm-specific risk will be priced entirely, whereas if one investor owns a small basket of uncorrelated stocks, he will be able to partially mitigate firm-specific risk, so the total risk will be very close to the systematic risk as measured by the CAPM, which, however, cannot fully represent all the risk present in the market private market.

To show how to compute in the estimation of expected return the degrees of diversification that range from completely diversified (where the investor owns the market portfolio), to completely undiversified (where the investors owns a single security or a small portfolio of perfectly correlated securities), one must therefore first define the

portfolio of an entrepreneur[41]. It can be assumed that it consists of two assets: the first asset is the market portfolio, which by definition does not contains company-specific risk, while the second is the stock of his private company; the percentage of equity allocated to the market portfolio is denoted by the symbol w, and accordingly (1 - w) indicates the percentage of the equity allocated to a private company.

The variance of the portfolio P is calculated according to the following formula[42].

$\sigma_P^2 = w_1^2 \sigma_1^2 + (1-w_1)^2 \sigma_2^2 + 2w_1 (1-w_1)^2 \rho_{1,2} \sigma_1 \sigma_2$, *where:*

- σ_1, *and* σ_2 *are the standard deviations of security 1 and security 2*
- w_1, *is the weight of security 1*
- $\rho_{1,2}$ *is the correlation coefficient of security 1 and 2*

Butler, Peter, et al.

The required rate of return considering the total risk in the portfolio P can be estimated as follows.

$E(R_P) = R_f + [E(R_M) - R_f] * (\sigma_P / \sigma_M)$ *where:*

- R_f *is the risk-free rate*
- $E(R_M)$ *is the market's expected return*
- σ_P, *is the standard deviation of the portfolio*
- σ_M *is the standard deviation of the market*

Butler, Peter, et al.

Since the Beta of the portfolio is the weighted average of the individual asset's Betas, the required rate of return can also be expressed in terms of average Beta.

[41] Butler, Peter, Gary S. Schurman, and Andrew M. Malec. "Practical evidence and theoretical support for total beta." Biotechnology 11 (2011): 30-36.
[42] Butler, Peter, Gary S. Schurman, and Andrew M. Malec. "Practical evidence and theoretical support for total beta." Biotechnology 11 (2011): 30-36.

$E(R_P) = R_f + [E(R_M) - R_f] * (w_1\beta_1 + (1-w_1)\beta_2)$ *where:*

- R_f *is the risk-free rate*
- $E(R_M)$ *is the market's expected return*
- w_1, *is the weight of security 1*
- β_1, *is the beta of security 1*
- β_2, *is the beta of security 2*

Butler, Peter, et al.

By comparing the two formulas, we can derive the following relationship, where the Total Beta of the portfolio is equal to the weighted average of the betas of the two assets composing the portfolio.

$$\sigma_p/ \sigma_M = w_1\beta_1 + (1-w_1)\beta_2$$

Given this relationship, let us now change the terms considering that security 1 is equal to the market portfolio M, while security 2 is the private firm S. Solving for Beta of the private firm.

$$\beta_S = (\sigma_p/ \sigma_M - w_M\beta_M)/(1-w_M)$$

Since the Beta of the market is equal to 1 by definition, we can further simplify the function.

$$\beta_S = (\sigma_p/ \sigma_M - w_M)/(1-w_M)$$

Let us now set the following parameters to see how the Beta of the private company behaves when changing the weights.

Standard deviation of the market returns (σ_M) = 0,2
Standard deviation of the private company returns (σ_S) = 0,4
Correlation between private company and market returns ($\rho_{S,M}$) = 0,5

Given these inputs, we can compute right away the original Beta and the Total Beta for the private company.

$\beta_S = (\sigma_S / \sigma_M) / \rho_{S,M} = (0,2/0,4) / 0,5 = 1$

$T\beta_S = (\sigma_S / \sigma_M) = (0,2/0,4) = 2$

The way to interpret this is that, when estimating returns in the private company S, a fully diversified investor will seek compensation for 1 unit of risk for every unit of market risk, while the investor who has all of his wealth tied up to the private company S will require twice as much compensation for risk, or 2 units for every unit of market risk.

An investor which is just partially diversified and owns a portfolio composed of the market and the private companies will only seek a compensation for risk that lies in between the two cases just highlighted. The Beta for such an investor, which I will refer to as Real Beta (Rβ) can therefore be computed according to the two formulas below.

$\sigma_p^2 = w_M^2 \sigma_M^2 + (1-w_M)2 \sigma_S^2 + 2w_M (1-w_M)\rho_{S,M} \sigma_M \sigma_S$

$R\beta_S = (\sigma_p / \sigma_M - w_M)/(1-w_M)$

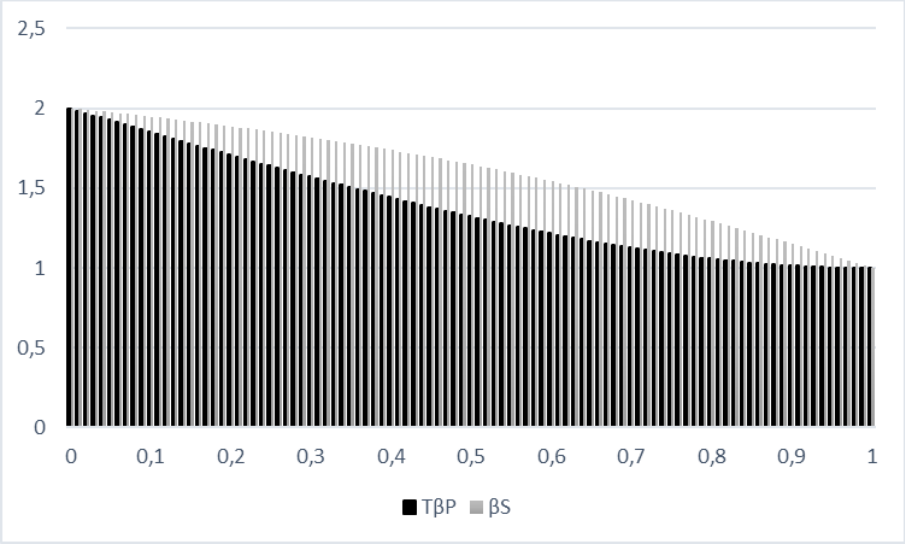
The results for some weightings in the portfolio P so computed are the following.

w_M	σ_P	$T\beta_P$	$R\beta_S$
0	0,40	2,00	2,00
0,1	0,37	1,85	1,95
0,2	0,34	1,71	1,89
0,3	0,31	1,57	1,82
0,4	0,29	1,44	1,74
0,5	0,26	1,32	1,65
0,6	0,24	1,22	1,54
0,7	0,23	1,13	1,42
0,8	0,21	1,06	1,29
0,9	0,20	1,01	1,15
1	0,20	1,00	1,00

As the weight of the market component in the portfolio P increases (which corresponds to a decrease of the weight of the private company S), we observe a reduction in the standard deviation of the portfolio, along with a reduction of the Total Beta of the portfolio and the private company's Real Beta.

In the graph below is possible to observe the relative behavior of Total Beta of the portfolio and Real Beta of the private company as the market's weight moves from 0 to 1 on the horizontal axis.

Total Beta and Beta estimates by market portfolio allocation



It is evident from the graph that, starting from a portfolio of a single private security where the Total Beta of the portfolio equals the Total Beta of the security, the reduction in the Beta of the portfolio is steeper as a little portion of market is introduced. The beta of the private company, meanwhile, starts declining faster when the portion of market within the portfolio outweighs the private company's share, until both betas converge to the market's Beta.

3.4 Total Beta estimates

We have seen how the estimation of Total Beta for a private company requires as an input the standard deviation of the returns of said company

and the correlation of those returns with the market returns. By their own nature, though, private companies are not traded on a listed exchange, and therefore do not have any historic prices, impairing the calculation of historic returns, which are necessary to compute their standard deviation and correlation with the market.

To cope with this issue, the concept of Bottom-up Beta which has been presented in the first chapter comes in handy. In the context of computing total betas, it is not only suggested, but even necessary to obtain a Beta which does not relate on the returns of a single asset, but rather takes into account the performances of other firms which have been operating in the same business[43].

In this way, by taking a sufficient sample of companies active in a certain business, a total beta can be computed for that business starting from the average Beta and the average correlation between the returns of the sampled firms and the market. Professor A. Damodaran publishes on a yearly basis the betas and total betas so computed. As of January 2023, the drafting of the dataset involves more than seven thousand publicly traded companies around the world, operating in 94 different businesses[44].

The results of the dataset show that the average levered beta (unlevered beta releveled by the average leverage) for the entire market, computed from returns of each of the stock within the sample against the S&P500, is equal to 1,16. It is important to notice that, since the betas are computed by using global stocks against a US index, the average levered beta is not equal to 1 as it should by definition be.

The average correlation between these stocks' returns and the index is 29%, meaning that, on average, the market movements explain less then a third of stock returns on. Such correlation corresponds to an average total unlevered beta of 3,99 for the market.

The same Professor Damodaran, on a semi-annual basis, provides estimates for a forward-looking measure of the Equity Risk Premium, which equals the expected return from the market minus the risk-free

[43] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 24

rate. At the start of 2023, the he has estimated a premium of 5,94%[45] for US equity market. Data collected from St. Luis Fred on U.S. Treasury Securities at 10-Year Constant Maturity show an average yield of 3,53% in January. Since the US is an AAA-rated country, the yield can be used as a proxy for risk free rate when estimating returns in dollar terms. With these inputs it is possible to use the CAPM to compute the expected returns for a diversified investor in the average global stock.

$$E(R_S) = R_f + [E(R_M) - R_f] * \beta_S$$
$$= 0{,}0353 + (0{,}0594) * 1{,}16$$
$$= 10{,}42\%$$

Computing the expected returns for the US market where Beta is equal to 1 by definition.

$$E(R_S) = R_f + [E(R_M) - R_f] * \beta_S$$
$$= 0{,}0353 + (0{,}0594) * 1$$
$$= 9{,}47\%$$

The expected returns for a non-diversified investor in a global stock would instead be calculated using the average Total Beta.

$$E(R_S) = R_f + [E(R_M) - R_f] * T\beta_S$$
$$= 0{,}0353 + (0{,}0594) * 3{,}99$$
$$= 27{,}23\%$$

Total expected returns for the US market can be computed from the Beta of the whole market divided by the average correlation.

[44] A. Damodaran, Total Betas by Sector, Stern NYU, Updated January 2023
[45] A. Damodaran, Country Default Spreads and Risk Premiums, Stern NYU, Updated January 2023

$E(R_S) = R_f + [E(R_M) - R_f] * T\beta_S$

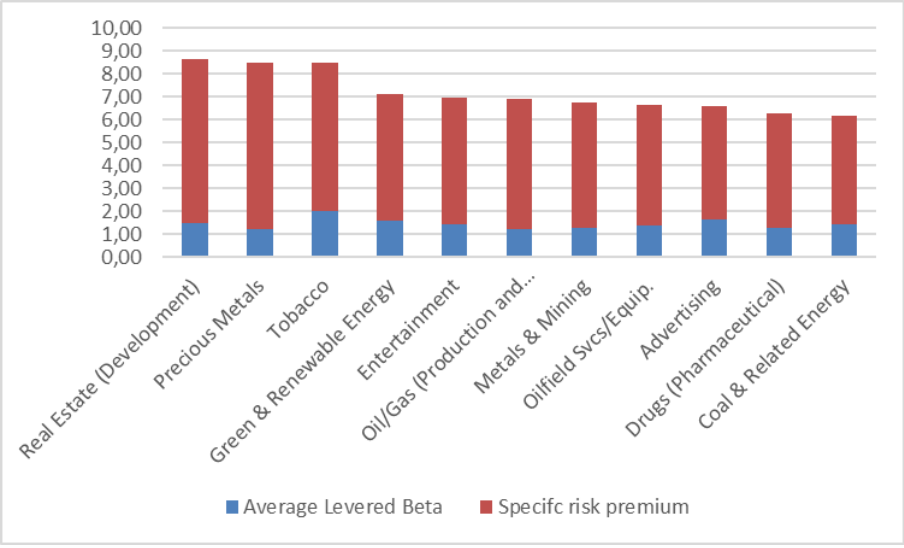
= 0,0353 + (0,0594) * 1/ 0,29

= 24,01%

Both globally and in the US, it is quite evident that the returns that a completely undiversified investor should expect are much higher than the ones required by the completely diversified investor. In particular, an investor who has all of his wealth tied up to a single global stock would expect 2,61 times more returns in that stock compared to a diversified investor, while in case of a US stock the required returns would be 2,54 times higher.

In the graph below are highlighted the businesses for which the highest total betas have been estimated. Real Estate Development ranks first with a Total Beta of 8,67, closely followed by Precious Metals which, despite the relatively low Beta of 1,23, is the least correlated business within the group, resulting in a Total Beta of 8,51. With a Total Beta of 8,47, Tobacco also occupies the podium of the highest total betas, but this is because, on top of the low correlation with the market, it has a Beta of 2, the highest across all the industries[46].

Top 10 Total Betas by Industry

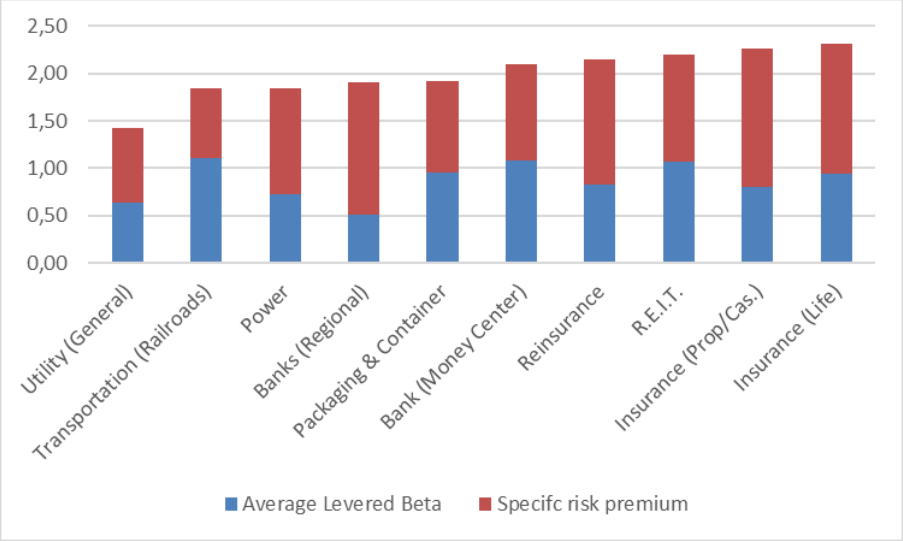


[46] A. Damodaran, Total Betas by Sector, Stern NYU, Updated January 2023

Source: elaboration of A. Damodaran dataset

On the low end of the spectrum, we observe instead that the Utility business has the lowest Total Beta, equal to 1,42, while the second lowest is Railroad Transportation which has a Total Beta of 1,84 thanks to the 60% correlation with the market, the highest across the all industries.

Bottom 10 Total Betas by Industry



Summary 3

Estimating specific risk is challenging and varies based on industry factors. Unlike market and industry risk, there is no established framework for estimating firm-specific risk, but regulators emphasize its importance for funds investing in private assets.

Total beta was introduced to incorporate asset pricing, the risk faced by investors with significant positions in a limited number of stocks. The argument for total beta stems from the need to price total risk, as traditional beta, while measuring the relative volatility of an investment compared to the market, excludes the non-correlated portion which is assumed to be diversified.

The Total Beta adjustment can be used for estimating expected returns in the valuation of private firms by including the specific-risk component into the CAPM, providing a more meaningful measure of expected return. The adjustment depends on the buyer's/seller's level of diversification and correlation with the market. Fully undiversified investors require a higher adjustment, while more diversified investors have a lower adjustment.

Estimating Total Beta for private companies is challenging due to the lack of historic prices. The concept of Bottom-up Beta suggests using the average Beta and correlation of sampled firms in the same business to compute a total beta. Professor Damodaran dataset reveals an average beta of 1.16 and an average correlation of 29%, which translates in an average total beta of 3.99.

Chapter 4

PRIVATE COMPANY VALUATION

4.1 Valuing a private company

Before addressing the topic of valuation, I deem it necessary to make a distinction between price and value. I think this dichotomy was best described by Warren Buffet who stated that "price is what you pay, value is what you get". This quote by the world's most successful value investor well describes his view on markets, which in his opinions are able to make mistakes in valuing companies, and therefore push the price of stocks away from where the value would be according to the fundamental resulting in an investment opportunity for him and other value investors around the world.

The International Private Equity and Venture Capital Valuation (IPEV) guidelines provide guidance on how to value private equity and venture capital investments. The guidelines are developed and maintained by the IPEV Guidelines Committee, which is made up of representatives from private equity and venture capital firms, accounting firms, and other industry experts. The IPEV guidelines provide guidance on how to implement valuation techniques as well as on how to account for certain factors that can impact the value of private equity and venture capital investments, such as the impact of dilution on value, the impact of options and warrants on value, and the treatment of goodwill and intangible assets.

The IPEV guidelines recommend the use of the DCF method as the primary method for valuing private equity and venture capital investments. In addition to the DCF method, the IPEV guidelines also recommend the use of comparable companies and transactions as a secondary method for valuing private equity and venture capital investments. This involves identifying comparable companies or transactions and using them as a basis for estimating the value of the investment.

While the DCF model can, indeed, be used to value private firms, the

process involves some additional challenges and adjustment compared to when applied to public firms. A first issue, intrinsic in the nature of private firms, is the absence of market values for debt and equity. This can impair the estimation of debt equity ratios needed to compute the WACC or to lever the Beta[47] and require making further assumptions. A second limitation is related to the absence of detailed financial information. In some instances, due to lower disclosure requirements, some private firms choose to produce less financial information, making it more difficult to produce cashflows estimates[48]. Furthermore, without a need to produce mandatory reporting private companies are encouraged to allocate fewer resources in areas such as finance and controlling. Expert financial operators can mitigate this deficiency by implementing extensive due diligence.

On top of these issues there are other risk components that need to be considered when dealing with a private firm. A primary source of risk arises from the illiquidity of the investment and is once again an intrinsic feature of private firms. Professor Damodaran defines illiquidity as the "cost of buyer's remorse" as the more illiquid the asset, the more expensive will be to revert the choice of the purchase[49]. In the context of private firms, illiquidity plays a fundamental role, since the market of potential buyers for private stocks is not as wide and established as it is in the public market, and each transaction involves a much larger magnitude of additional costs and time required by due diligences, negotiation, and deal structuring[50].

Another risk component is related to the size of the firm. While large firms are able to diversify their revenues across different markets, delivering a wide range of goods and services across different geographies, smaller firms often focus on a narrower segment, increasing the risk

[47] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 24

[48] Elfakhani, Said, and Tarek Zaher. "Differential information hypothesis, firm neglect and the small firm size effect." Journal of Financial and Strategic Decisions 11.2 (1998): 29-40.

[49] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 24

associated with the cashflows.

Within my experience working for an alternative fund's service provider, I have produced and reviewed several valuations. Concerning private equity funds, asset valuations can be performed for two purposes: new investments and periodic NAV calculation. Whenever fund managers consider the purchase of a private firm, they perform a valuation to get an arm's length measure of the price they could pay for the equity in the firm, and to estimate the returns that such asset could generate during the holding period. The other main reason for valuing private firms within a PE fund is the periodic monitoring of the value of the assets which is a mandatory requirement set by the European regulators through the AIFMD. From what I could observe during my experience, the primary method adopted to estimate value by a number of practitioners is relative valuation. This is performed mainly starting from panels of listed companies, and the multiple is estimated as a median or average of the multiple (usually EV/EBITDA). The obtained multiple can then be discounted due to liquidity or other factors. Occasionally the panel can be composted by recent similar transactions in private firms. Limited is instead the adoption of an intrinsic value approach, which usually takes its inputs from the forecasted business plan to compute the expected cashflows, while the discount rates are estimated with the CAPM. Even when the DCF method is exploited, the valuation officers compute in additional premiums to account for liquidity and size. These premiums can impact the cost of equity by a percentage equal or larger than 0.

4.2 Intrinsic Valuation: the DCF

Intrinsic valuation is based on the principle that the true value of an asset or investment can be estimated by analyzing its fundamental characteristics. The process of determining the intrinsic value of an asset requires an estimation of the future cash flows that the asset will generate, along with an appropriate discount rate that reflects the risk associated

[50] T.D. Nadauld, et al. "The liquidity cost of private equity investments: Evidence from secondary market transactions." Journal of Financial Economics 132.3 (2019): 158-181.

with those cash flows. Both are critical factors that can significantly impact the intrinsic value estimate and depend on a wide range of information and assumptions. To generate meaningful estimates, in fact, intrinsic valuation requires a thorough understanding of the asset's underlying fundamentals, such as the industry dynamics, competitive landscape, and growth prospects.

One of the most commonly used intrinsic valuation methods is the discounted cash flow (DCF) model, which was first introduced by J. B. Williams in 1938[51] and since then has been widely adopted and refined by academics and professionals. Before even starting the intrinsic valuation process, a decision must be made. The intrinsic equity value can be obtained by valuing the equity directly or by valuing the entire firm, intended as the value of operating assets, and then cleaning out net debt. In the first way the cashflows to be forecasted will be the FCFE (free cashflow to the equity) which has to be discounted by the Ke (cost of equity), meanwhile the second option will require to estimate the FCFF (free cashflow to the firm) which needs to be discounted in a way that also accounts for the debt holders. In both instances the result should be the same, but it is paramount to keep consistency throughout the process and avoid mixing and matching cashflows and discount rates to different stakeholders. The employment of the model can be broken down into three main steps: forecasting cash flows, estimating the appropriate discount rate, and calculating the present value of the cash flows.

The first step in the DCF model is to forecast the expected cash flows of the asset. For a company, this involves projecting the expected cash flows from its operations, such as revenues, expenses, and capital expenditures. The forecasting period typically ranges from 5 to 10 years, depending on the company's growth prospects and the industry dynamics. Once the forecasted cash flows are determined, they must be adjusted for factors such as inflation, growth rates, and one-time events that may affect the cash flows. Additionally, the cash flows must be normalized to reflect the company's operating performance, such as removing non-recurring

items or adjusting for changes in working capital.

The free cashflow (FCF) can be estimated directly to the holder (FCFE) or to the entire firm (FCFF). This second option is more straightforward as it reflects the after-tax cash flow available to all investors (both debt and equity holders) of a firm in a certain period, regardless of the capital structure[52].

The FCFF can be calculated from information in financial statements starting with NOPAT computed as follows[53].

*NOPAT = EBIT * (1 – t), where*

- t = effective tax rate

A. Damodaran

To move from NOPAT to FCFF the adjustment requires subtracting the changes in investment capital. To the NOPAT just computed depreciation and amortization (D&A) are added back, while changes in working capital (ΔWC), and capital expenditures (CAPEX) must be deducted.

FCFF = NOPAT - ΔReinvestment

= NOPAT + D&A - ΔWC – CAPEX

A. Damodaran

The second step in the DCF model is to estimate the appropriate discount rate that reflects the risk associated with the asset's cash flows. When estimating the discount rate in the process of valuing a firm, it is important to be consistent with the cashflows to be discounted in terms of monetary values, currency, and invested capital. The discount rate is typically composed of two components: the risk-free rate, represents the return an investor can earn from a risk-free investment, such as a U.S.

[51] W. J. Burr "The theory of investment value." No. HG4521 W48. (1938)

[52] Koller, Tim, Jack Murrin, and Tom Copeland. "Valuation: measuring and managing the value of companies." (2000)

Treasury bond, and the risk premium which reflects the additional return that an investor requires for taking on the risk associated with the asset. With regard to the risk of cashflows to equity holders, the risk premium can be estimated using a variety of approaches, such as the Capital Asset Pricing Model. When discounting cashflows to the firm, instead, both the required return by equity holders and the one form debt holders needs to be considered and this can be done by computing the WACC (weighted average cost of capital) as follows.

$WACC = (W_d * K_d) + (W_e * K_e)$, where
- W_d is the market weight of debt on total invested capital
- W_e is the market weight of equity on total invested capital
- K_d is the cost of debt
- K_e is the cost of equity

A. Damodaran

The final step in the DCF model is to calculate the present value of the asset's expected cash flows. This involves discounting each cash flow by the appropriate discount rate and summing the present values of all the cash flows. The present value of the future cash flows can be calculated using the DCF formula:

$DCF = CF_1 / (1 + r)^1 + CF_2 / (1 + r)^2 + ... + CF_n / (1 + r)^n$, where
- $CF_1, CF_2, ..., CF_n$ are the future cash flows
- r is the discount rate
- n is the number of periods till the last cashflow

A. Damodaran

When valuing a company as a "going concern" we expect it to be able to continue its operations and meet its financial obligations in the foreseeable future and therefore to continue generating cashflows. In other words, it assumes that the company will not go bankrupt or be

[53] Koller, Tim, Marc Goedhart, and David Wessels. "The right role for multiples in valuation." McKinsey on

forced to liquidate its assets in the near future. Since estimating cashflows forever would be impossible, assuming that the company reaches a steady state, it is possible to apply a "two stage" DCF approach and compute a terminal value for the company[54]. The terminal value, which is the present value of the cashflows after the forecasted cashflows, can be calculated using the Perpetual Growth Model, also known as the Gordon Growth Model, as follows.

*Terminal Value = (CF_n * (1 + g)) / (r - g), where:*

- *CF_n is the free cash flow of the last year of the forecast period*
- *g is the long-term growth rate of the business*
- *r is the discount rate.*

A. Damodaran

The resulting present value of the cashflows in the forecasted period plus the ones in the terminal period, represents the intrinsic value of the asset. If the intrinsic value is greater than the current market price, then the asset may be undervalued and may represent a good investment opportunity. On the other hand, if the intrinsic value is less than the current market price, then the asset may be overvalued and may represent a potential selling opportunity.

While intrinsic valuation and the DCF model can provide valuable insights into the true value of an asset, there are several challenges to consider. First, the accuracy of the intrinsic value estimate depends heavily on the quality of the cash flow projections and the choice of discount rate. If the projections are too optimistic or the discount rate is too low, then the intrinsic value estimate may be overstated, leading to potential investment mistakes. Second, intrinsic valuation requires a deep understanding of the underlying fundamentals of the asset, including the industry dynamics and competitive landscape. If the investor's analysis is incomplete or inaccurate, then the intrinsic value estimate may be

Finance 15 (2005): 7-11.

unreliable. Finally, intrinsic valuation is based on the assumption that market prices will eventually converge with intrinsic values over the long term. However, in the short term, market prices can be affected by a variety of factors, including market sentiment, news events, and behavioral biases, leading to potential market inefficiencies.

4.3 Relative Valuation: Multiples

Relative valuation is a fundamental aspect of financial analysis, providing insights into the fair value of a company's stock. Multiples are ratios that relate a company's market value or price to a specific financial metric, such as earnings, sales, or book value[55]. Multiples provide a straightforward and intuitive way to assess a company's value relative to a specific financial metric. By comparing a company's multiple to that of its peers or industry benchmarks, analysts can gain insights into its relative valuation and potential investment opportunities.

The use of multiples in equity valuation offers several advantages. Firstly, multiples are easy to calculate and interpret, making them accessible and understandable. Secondly, since they provide a relative measure, they enable analysts to compare the valuation of multiple companies within the same industry. This relative approach can provide a broader perspective on the company's value and potential performance. Lastly, multiples can capture various aspects of a company's financial health, such as profitability, growth potential, and asset quality, depending on the chosen multiple. While providing valuable insights, multiples also have limitations that need to be considered. Multiples are, in fact, highly dependent on the choice of financial metric used in their calculation. Different multiples emphasize different aspects of a company's financials, and choosing an inappropriate multiple can lead to misleading results. Additionally, multiples may be affected by market conditions, industry dynamics, and other factors that can distort their comparability.

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[54] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 15

[55] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 18,19

Furthermore, multiples do not capture qualitative factors, such as management quality, competitive advantages, or industry-specific risks, which are crucial for a comprehensive valuation analysis.

Multiples are widely used in practice for equity valuation across various industries. They serve as a benchmark for valuing initial public offerings (IPOs)[56] and M&A transactions, aiding in pricing negotiations and determining the fair value of a company. Moreover, multiples can assist investors in assessing the relative attractiveness of different investment opportunities and constructing diversified portfolios. Financial analysts often employ multiples to generate price targets for stocks, providing guidance to investors on whether a stock is overvalued or undervalued.

The most common multiples used in equity valuation include the price-to-earnings (P/E) ratio, the price/earnings-to-growth (PEG) ratio, the enterprise value-to-EBITDA (EV/EBITDA) ratio, the enterprise value-to-sales (EV/Sales) ratio, and the enterprise value-to-book value (EV/Book Value) ratio.

The price-to-earnings (P/E) ratio is the most widely recognized multiple[57]. It compares a company's market price per share to its earnings per share (EPS), providing an indication of how much investors are willing to pay for each dollar of earnings generated by the company. A higher P/E ratio suggests higher growth expectations or market optimism, while a lower P/E ratio may indicate undervaluation or market pessimism.

The price/earnings-to-growth (PEG) ratio builds upon the P/E ratio by incorporating the company's expected earnings growth rate. By dividing the P/E ratio by the earnings growth rate, the PEG ratio accounts for the company's growth prospects and helps assess whether the stock is overvalued or undervalued relative to its growth potential. A PEG ratio below 1 is often considered attractive, indicating an undervalued stock with growth potential.

The Enterprise Value-to-EBITDA (EV/EBITDA) ratio compares a

[56] Kim, Hugh H., and Jay R. Ritter. "Valuation of IPOs." In Handbook of Corporate Finance: Empirical Corporate Finance, edited by B. Espen Eckbo, 2 (2019): 37-77

[57] Liu, Yuting, and Qiaoqiao Wu. "Determinants and implications of P/E ratios in Chinese stock markets." Frontiers of Business Research in China 11 (1) (2017): 1-30.

company's enterprise value (market capitalization plus debt minus cash) to its earnings before interest, taxes, depreciation, and amortization (EBITDA). This multiple is commonly used in industries with significant capital investments or varying capital structures. The EV/EBITDA ratio provides a measure of the company's operating performance and is useful for comparing companies with different levels of leverage or capital intensity.

The enterprise value-to-sales (EV/Sales) ratio relates a company's enterprise value to its total revenue or sales. It indicates the market's valuation of each dollar of sales generated by the company. The EV/Sales ratio is particularly useful for evaluating companies with low or negative earnings, early-stage companies, or industries where profitability measures may be less relevant.

The enterprise value-to-book value (EV/Book Value) ratio compares a company's enterprise value to its book value (the net value of its assets minus liabilities as per accounting values). This multiple provides insights into how the market values the company's net assets. A ratio below 1 suggests the company is trading at a discount to its book value, which may indicate undervaluation or that the market sees a potential asset-related risks or a devaluation of the operating assets.

The choice of the multiple to apply depends on the availability of data and the specificity of the industry. In certain cases, different multiples can be computed in concert to provide additional insights on the firm in scope. Alternative measures can also be applied, like in the case of businesses with large platform benefits (i.e., social networks) where the number of users can be scaled to the enterprise value to compare similar businesses.

Other than performing general analysis, multiples can be enforced to determine a fair value, other than the current price, for the equity of publicly traded companies, or to estimate the value of private businesses. Multiple can be estimated on the basis of listed comparable firms or, as it often is the case with Private Equity, can result from other private transactions.

The process starts with the choice of the multiple to adopt and continues with the identification of a sample of peers. This step is particularly important because the sample has to be large enough to be representative, but also be composed of firms that are similar enough in terms of industry, business model, size and other characteristics. This leads to the two extremes where on the one side to have the larger sample you consider the whole market, while on the other side to have only firms that are identical to yours you only have one firm which is the one you are analyzing. The optimal number of peers within the group lies indeed in between these two extremes. While academics and practitioners have different opinions on this topic, it is common to use around 10 comparable firms. The most basic approach involves then computing either the mean or average of the peer's multiples and then multiplying the chosen multiple (e.g., the EV/EBITDA multiple), estimated from the comparable firms, by the corresponding value driver (e.g., EBITDA) of the firm being valued[58].

Despite its simplicity, this approach does not come without some strong assumptions. The selected value driver is the only parameter carrying information about the target company. When multiplying this driver for the average multiple one assumes that the firm in analysis, at its current state, deserves the same multiple as the typical or average firm within the group[59].

In case one is not comfortable with this assumption, a better approach is to deconstruct the multiples in their determinants and formulate a function that might explain differences across companies and computing a so-called synthetic multiple. This can be obtained by running a regression using as inputs the value fundamentals, and then multiplying them by the coefficients estimated in the regression to calculate a multiple for the target firm. Professor Damodaran suggests including 10 additional firms in the peer group for each regression input when estimating

[58] S. Bhojraj, and C. MC Lee. "Who is my peer? A valuation-based approach to the selection of comparable firms." Journal of accounting research 40.2 (2002): 407-439.
[59] S. Bhojraj, C. MC Lee, and D. T. Ng. "International valuation using smart multiples." Journal of Accounting Research 41 (2003): 745-774.

synthetic multiples.

As an example, I will decompose the EV/EBITDA multiple in the determinants of enterprise value to EBITDA. This process can be done by reverting back to the principle applied in the DCF model when computing the enterprise value from the estimated free cashflow to the firm and cost of capital[60].

Starting from the formula of Enterprise Value assuming the company is in steady state and projecting a single cashflow in perpetual growth.

$$EV = \frac{FCFF_1}{WACC - g}$$

A. Damodaran

The free cashflow can be reduced to its components, which include EBITDA, as follows.

$$\begin{aligned} FCFF &= EBIT(1-t) - (\text{Cap ex} - DA + \Delta \text{ Working capital}) \\ &= (EBITDA - DA)(1-t) - (\text{Cap ex} - DA + \Delta \text{ Working capital}) \\ &= EBITDA(1-t) - DA(1-t) - \text{Reinvestment} \end{aligned}$$

A. Damodaran

By rewriting the EV formula with the decomposed FCFF we obtain the following relationship.

$$EV = \frac{EBITDA_1(1-t) - DA_1(1-t) - \text{Reinvestment}_1}{WACC - g}$$

A. Damodaran

Dividing both sides by EBITDA we derive the formula to compute a bottom-up EV/EBITDA.

[60] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 18

$$\frac{EV}{EBITDA} = \frac{(1-t) - \frac{DA}{EBITDA}(1-t) - \frac{Reinvestment}{EBITDA}}{WACC - g}$$

A. Damodaran

By isolating the five determinants of EV/EBITDA from the equation, we derive the following relations:

- The tax rate is negatively related to the multiple, therefore we expected firms with higher tax rates to have lower EV/EBITDA multiples, other things being equal.
- Depreciation and amortization are negatively related to the multiple, therefore we expected firms with higher D&A to have lower EV/EBITDA multiples, other things being equal.
- Reinvestment requirements are negatively related to the multiple, therefore we expected firms with higher reinvestment needs to have lower EV/EBITDA multiples, other things being equal.
- Cost of capital are negatively related to the multiple, therefore we riskier expected firms to have lower EV/EBITDA multiples, other things being equal.
- Expected growth is positively related to the multiple, therefore we expected firms who are expected to grow more than their pears to have lower EV/EBITDA multiples, other things being equal.

An alternative way to break down an operating multiple in its components can be proposed by deconstructing reinvestment[61].

The return on invested capital (ROIC) at time t is the rate of return a firm earns on each dollar of capital invested in its core operations during the period t-1 to t.

$$ROIC_t = \frac{NOPAT_t}{invested\ capital_{t-1}}$$

A. Schreiner

The investment rate (IR) is the net relative amount of net operating profit after taxes (NOPAT) a firm invests back into its core operations each period t-1 to t.

$$IR_t = \frac{\Delta\ invested\ capital_t}{NOPAT_t}$$

A. Schreiner

Since we assume that firm has a stable return on capital and reinvestment needs, its expected growth in free cashflow is a product of the reinvestment rate, and the quality of these reinvestments, measured as the return on the capital invested[62].

$$g_t^{FCF} = ROIC_t \cdot IR_t$$

A. Schreiner

We can therefore substitute the above formulas to derive a new formula for free cashflow and EBITDA.

$$\begin{aligned} FCF_t &= NOPAT_t - \Delta\ invested\ capital_t \\ &= NOPAT_t - (NOPAT_t \cdot IR_t) \\ &= NOPAT_t \cdot (1 - IR_t) \\ &= EBIT_t \cdot (1 - tax\ rate) \cdot \left(1 - \frac{g_t^{FCF}}{ROIC_t}\right) \end{aligned}$$

A. Schreiner

Substituting the new free cashflow into the perpetual growth formula for the enterprise value and dividing both sides of the equation by EBIT we find the drivers of EV/EBITDA multiple.

[61] A. Schreiner "Equity Valuation Using Multiples: An Empirical Investigation" (2007)
[62] Damodaran, Aswath "Investment Valuation: Tools and Techniques for Determining the Value of Any Asset." 3rd ed. John Wiley & Sons (2012) Chapter 11

$$\frac{v_t^{entity}}{EBIT_t} = \frac{\left(1+g^{FCF}\right)\cdot\left(1-tax\ rate\right)\cdot\left(1-\frac{g^{FCF}}{ROIC_t}\right)}{r^{wacc}-g^{FCF}}$$

A. Schreiner

Based on this relationship we find that growth and profitability, which is hereby represented by ROIC, have a positive impact on the synthetic multiple, while tax rate and risk have instead a negative impact.

For the purpose of this work, the key takeaway of these multiple breakdowns is the effect of risk. A higher risk not only appears to be relevant in lowering firm value when performing intrinsic valuation, but also contributes to reducing the value estimates of relative valuation by lowering the synthetic multiples.

Summary 4

Valuation practices and regulation focus on two main methods: intrinsic valuation and relative valuation. Anyhow, this process involves challenges like absence of market values, limited financial information, and illiquidity risk. Furthermore, smaller firms focusing on a narrow segment face higher risk compared to larger diversified firms. When it comes to private firms' valuation, therefore, diligence and adjustments are necessary to account for these factors and generate accurate value estimates.

In this fourth chapter we have explained what the fundamentals of value are and broke them down into their drivers. I have identified that risk as a fundamental determinant of value, regardless of the way you estimate such value. All the discussion around Total Beta does therefore hold relevance in the valuation framework and should impact the estimated value of any private firm both when estimated through a DCF and when adopting comparable multiples.

Chapter 5
PRIVATE EQUITY

5.1 PE deal

Private Equity refers to a capital transaction through which specialized operators, the Private Equity funds, take over predominantly unlisted companies, acquiring majority or minority stakes, with the aim of contributing to their growth and monetizing upon divestment[63].

From a strictly terminological point of view, the concept of institutional investment in risk capital today takes on different connotations depending on whether one considers the practice most prevalent in the United States or Europe, that is, depending on whether one considers venture capital as distinct from private equity or as a subunit thereof. In the United States, private equity and venture capital are two autonomous asset class; the former includes all those operations that are aimed at companies in the early stages of life (early-stage financing) or at a later stage of development (expansion financing), while the latter focuses on interventions in mature companies (buy out)[64]. In addition to the different life cycle of the company being invested in, the clear separation between the two categories also concerns distinctions related to reputation, target selection process, value creation, and exit. This distinction is highlighted and evidenced in the US context by the presence of two different reference trade associations: the NVCA (National Venture Capital Association), which represents those operators who aim to invest in the transformation of new ideas into businesses, which could not be financed by the traditional banking channel and which require five to eight years to reach maturity[65], and the American Investment Council, which represents all those individuals who invest in mature companies with

[63] Wright Robbie, Mike, Ken. "Venture capital and private equity: A review and synthesis." Journal of Business Finance & Accounting 25.5-6 (1998): 521-570.
[64] Cumming, Douglas J., and Sofia A. Johan. Venture capital and private equity contracting: An international perspective. Academic Press, 2013.
[65] Maier II, John B., and David A. Walker. "The role of venture capital in financing small business." Journal of Business Venturing 2.3 (1987): 207-214.

development potential, with the goal of growing them by working alongside them. In contrast, in Europe, venture capital activity is an integral part of the private equity transaction category so much so that the European association called Invest Europe representing private equity, venture capital and infrastructure funds, formerly called the European Venture Capital Association (EVCA), defines private equity as "equity capital provided to enterprises not quoted on a stock market.", while venture capital is "a subset of private equity and refers to equity investments made to support the pre-launch, launch and early stage development phases of a business."[66] It can be inferred that private equity refers to all types of investment in the venture capital of unlisted companies concerning phases of the life cycle subsequent to the initial one (expansion, replacement, buy out, turnaround), while with the locution venture capital, it is usual to refer to a particular activity of private equity aimed exclusively at financing to support the birth and very early stages of development of companies (seed, start up, early stage financing).

Private equity and venture capital funds fall within the asset classes defined as alternatives, i.e., everything that is not identified with traditional asset classes such as cash, bonds, and equities. In particular, a more focused definition especially from the perspective of a domestic institutional investor, turns out to be the principle underlying the Directives regulating EU management companies[67], which, assuming the criterion of harmonization as the discriminating factor, consider "Alternative Investment Funds" all undertakings for collective investment of savings other than UCITS, i.e., undertakings for collective investment in transferable securities. In fact, by alternative investment funds (AIFs) the AIFMD Directive means collective investment undertakings, including sub-funds thereof, which raise capital from a plurality of investors for the purpose of investing it in accordance with a defined investment policy for the benefit of those investors and do not require authorization under

[66] OECD (2015), "International comparability of venture capital data", in Entrepreneurship at a Glance 2015, OECD Publishing, Paris.

[67] DIRECTIVE 2011/61/EU OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 8 June 2011 on Alternative Investment Fund Managers

Article 5 of Directive 2009/65/EC.

The introduction of a share of "alternatives" in the portfolio of investors, particularly institutional investors, is motivated by the primary need to optimize the risk-return combination. Empirical data shows that alternative asset classes are sufficiently decorrelated from traditional equity and bonds, to provide a significant reduction in the overall volatility of the portfolio. Private equity, as well as other alternative assets, allows for superior combinations of risk and return in the frontier of investors' financial portfolios.[68] There are some intrinsic aspects related to this asset class that contribute to the achievement of this performance objective. First, the investment is generally not liquid, and as a result investors obtain a premium for illiquidity that is discounted during the investment phase, i.e., at the time of the private equity fund's acquisition of the stake in the unlisted companies[69]; moreover, the investment is characterized by medium- to long-term time horizons that could lead to the enhancement of the investment to the extent that the stability of the funding sources made available to the investee companies contribute to the realization of medium- to long-term strategies capable of achieving better management results[70]. A second aspect concerns the ability of private equity fund managers to enhance the value of what they raise in terms of capital during the different stages that determine their management activities: if in the first phase the fund manager's ability to select the investments that will be most profitable is measured through the acquisition and processing of corporate information, usually not available to operators, during the investment enhancement phase become valuable his commitment to collaborate and contribute to the strategy and operational and financial management of the company[71]; finally, in the exit phase, a relevant aspect is the fund manager's ability to maximize cash flows in favor of investors,

[68] P. Jacques, and A. White. "The relative merits of alternative investments in passive portfolios." The Journal of Alternative Investments 10.4 (2008): 37-49.
[69] Lerner, Josh, and Antoinette Schoar. "The illiquidity puzzle: theory and evidence from private equity." Journal of Financial Economics 72.1 (2004): 3-40.
[70] Valkama, Petri, et al. "Drivers of holding period firm-level returns in private equity-backed buyouts." Journal of Banking & Finance 37.7 (2013): 2378-2391.

identifying the appropriate timing and the most appropriate disinvestment mode.

5.2 PE industry trends

While the birth of venture capital funds is difficult to pinpoint, most scholars agree that the American Research & Development Corporation, founded in Boston in 1946 by Harvard professors and local businessmen, was the first venture capital company. This company invested in high-tech post-war activities and was structured as a closed-end fund with a majority of individual investors.[72]

The first limited partnership with the same purposes as current funds was founded in 1958 by William Henry Draper, and this legal form is still one of the most commonly used in the United States for private equity and venture capital fundraising. In the same year, the US government passed the Small Business Investment Act, which created the legal framework for investment funds to buy shares in start-ups and led to the creation of Small Business Investment Companies (SBIC). Over the next 20 years, around $12 billion was invested in SBICs, creating over 100,000 new businesses.[73]

The growth of private equity and venture capital funds exploded in 1979 following a regulation from the US Department of Labor that amended the former Employment Retirement Income Security Act (ERISA)[74], thereon allowing pension funds to invest in private equity and venture capital. This practice was previously limited by the so-called "prudent man" rule, which stated that pension funds shall invest with the care of a prudent man, limiting the span of assets to allocate capital. This led to a massive influx of money into the fund industry, with numerous

[71] Leslie, Phillip, and Paul Oyer. Managerial incentives and value creation: Evidence from private equity. No. w14331. National Bureau of Economic Research, 2008.
[72] D. H. Hsu, and M. Kenney. "Organizing venture capital: the rise and demise of American Research & Development Corporation, 1946–1973." Industrial and Corporate Change 14.4 (2005): 579-616.
[73] B. Krumm "Understanding the new Tennessee small business investment company credit act: Stimulating economic growth at the intersection of free market capitalism and government intervention." Transactions: Tenn. J. Bus. L. 11 (2009): 93.
[74] Longstreth, Bevis. "Modern investment management and the prudent man rule." Oxford University Press (1986)

limited partnerships being formed.

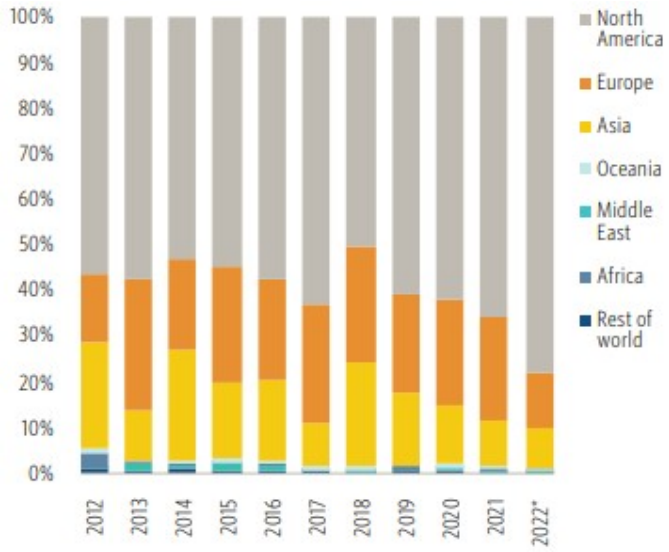
In the 1980s, the private equity industry developed significantly also in Europe where, in the same years, the European Venture Capital Association (renamed Invest Europe in 2015) was established with the aim to promote and encourage the private equity sector. The UK was the most active player in the industry among the European players[75], creating legislation to incentivize the sector and a parallel market for institutional funds, followed by France, Germany, and the Netherlands.

In the 1990s, the strong performance of global stock markets, particularly in the US, along with the rise of major technology companies such as Apple, Microsoft, and Cisco Systems, provided fertile ground for new initial public offerings (IPOs), which rapidly became a preferred exit route for Private Equity investments. However, the 2000s saw the collapse of the internet bubble, which resulted in significant losses for private equity and venture capital funds due to the devaluation of their portfolio companies. There was a recovery between 2004 and 2007, but the subprime mortgage crisis in 2007 affected institutional funds and the broader financial markets. Despite the challenges, the Private Equity industry has continued to grow significantly both in the number and in the size of funds raised, including the propagation of mega funds, funds that can raise above $5 billion in final closings according to PitchBook.

The most active region remains North America, where 55,9% of the global Private Equity funding raised since 2012 has converged, followed by Europe (22,9%) and Asia (18,8%).

[75] Tåg, Joacim. "The real effects of private equity buyouts." The Oxford handbook of private equity (2012): 271-299.

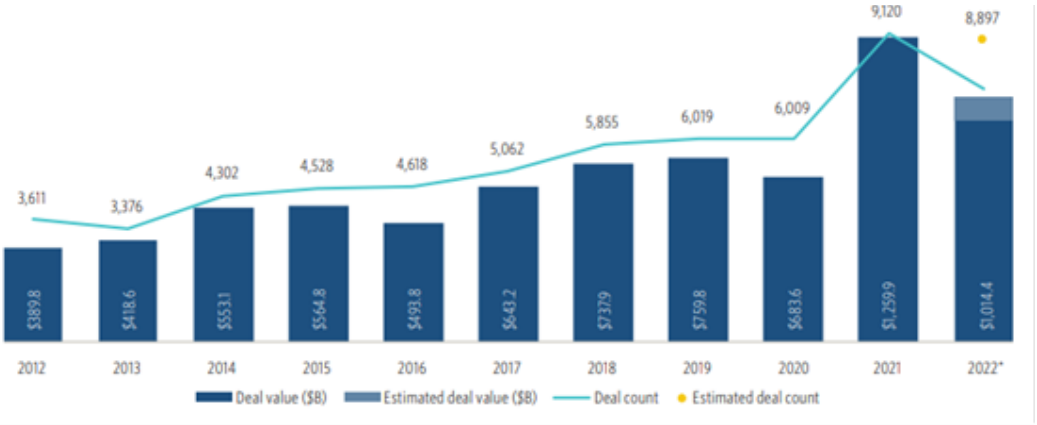
Figure 3.1 *– Share of PE capital raised by region*



Source: PitchBook, Global Private Market Fundraising 2022 Annual, 2023

Data collected by PitchBook shows a buoyant trend in the US where the deal activity has grown at a 10% CAGR in the last decade. The peak in dealmaking has been reached in 2021 with a total 1.259,9 billion dollars invested amid a low interest rate environment and optimism for a rapid recovery of the economy from the pandemic. In 2022 the increase of cost of borrowing along with the fear of recession have contributed to a slowdown of the activity, which still remain well above the figures registered pre-pandemic.

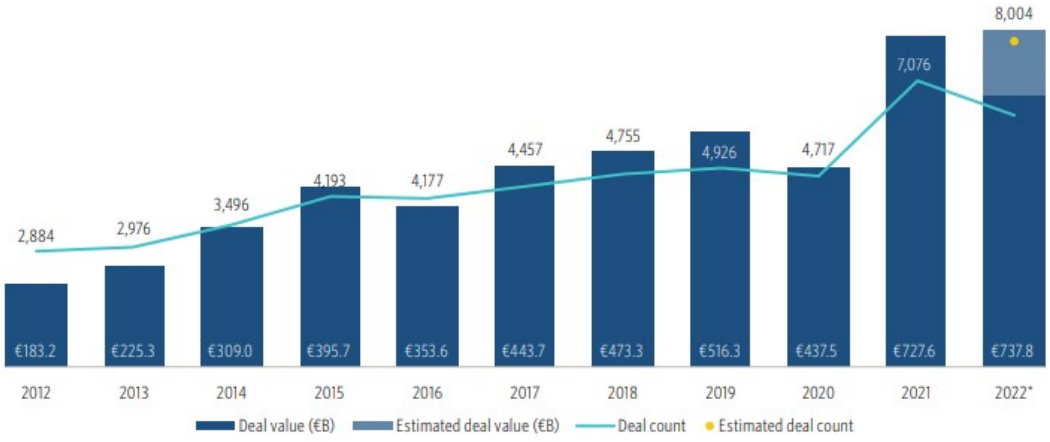
Figure 3.2 *- Private Equity deal activity, US*



Source: PitchBook, US PE Breakdown 2022 Annual, 2023

Even higher is the growth trend registered in Europe where the amount invested has grown by 15% annually in the past ten years to reach 737,8 billion euros in 2022 according to PitchBook.

Figure 3.3 *- Private Equity deal activity, Europe*



Source: PitchBook, Europe PE Breakdown 2022 Annual, 2023

Recent dealmaking activity has been particularly focused on two sectors. In 2022 Information Technology has attracted 28,1% of private capital in the US and 20% in Europe, while the Healthcare sector has received 12,1% and 9,1% of the investment flow respectively in the two regions[76].

5.3 PE structure and operations

A private equity fund can be identified as that vehicle that directs the funds collected from institutional investors toward investments in risk capital through the acquisition of shares in a private company, with the aim of divestment over a medium to long time horizon in order to realize a capital gain and remunerate the amount subscribed by investors.

At the international level, private equity and venture capital funds, are structured as limited partnership[77], which is characterized by the

[76] PitchBook, US PE Breakdown 2022 Annual, Europe PE Breakdown 2022 Annual, 2023

[77] P. Gompers, and J. Lerner. "The determinants of corporate venture capital success." Concentrated corporate ownership. University of Chicago Press (2000): 17-54.

involvement of three components: institutional investors such as pension funds insurance firms, and other investment funds, who provide the capital necessary for private equity funds to make investments (limited partners- LP), private equity fund managers, who invest clients' savings in order to achieve their financial objectives (general partners-GP), and the target companies, i.e., the companies that receives the investment capital. Thus, there is a clear distinction between those who invest in funds and those who manage the capital invested in those funds through investments in companies.

Following this structure, the main instrument by which private equity activity is conducted, is the closed-end securities investment fund[78], in which there is a fund where typically, in light of the high risk profile and reduced level of liquidity, capital is raised from institutional investors (pension funds and other investors of a pension nature, insurance companies, banks, funds of funds, sovereign wealth funds, etc.), and then investments are made in generally unlisted companies. The fund, being characterized by a closed-end structure, does not allow subscribers to redeem units at any time, but only at a predetermined maturity; at the same time, it does not allow the entry of new subscribers once the fundraising has been completed. The duration of closed-end funds must be consistent with the nature of the investments and, as a rule, ranges from 8 to 10 years[79]. Within this time period, a distinction can be made between the investment period and the time at which holdings are divested, or the disinvestment period, each of which has an average duration of 4 to 5 years. The presence of a formal timeline to complete the investment activity places an obligation for greater attention to the time planning of individual deals, including how to divest, thus leading to a greater level of accountability in management activity. Conversely, in the case of structured investment companies without a predefined term, i.e., so-called *evergreen operators*, there is greater flexibility in strategic choices due to

[78] S. Chertok and A. D. Braendel. "Closed-end private equity funds: a detailed overview of fund business terms, part i." The Journal of Private Equity (2010): 33-54.

[79] Valkama, Petri, et al. "Drivers of holding period firm-level returns in private equity-backed buyouts." Journal of Banking & Finance 37.7 (2013): 2378-2391.

less rigid temporal, as to being able, for example, to postpone the divestment process until the investee company achieves greater value.

Further classification generally used internationally is based on the distinction between captive operators and independent or autonomous operators[80]. The term captive generally refers to those entities that primarily use their own resources to make investments, thus without raising, or raising only a small amount of, capital in the market. Such operators arise as a direct emanation of other financial institutions, such as banks, industrial entities, or public entities, and have the parent company or other affiliated companies as their main source of supply, so called corporate venture capital. Independent operators, on the other hand, have completely opposite characteristics in that they offer their brokerage services to a plurality of investors, soliciting their subscription to shares of the investment company or closed-end fund they manage.

The lifecycle of a private equity fund consists of four phases[81]: fundraising, investing, managing the investments, and finally, divesting by selling the shares and returning capital to investors along with the realized gains and net of fees. Before fundraising, a planning phase is necessary to determine the strategy for attracting investors and investment goals. Fundraising involves soliciting investors to provide capital for investment and is a delicate and time-consuming process that typically takes around two years to compete. The fundraising process requires pre-marketing to potential investors, structuring the fund, preparing a placement memorandum, meeting, and agreeing terms with the investors, finalizing documentation, and closing fundraising once the target capital is reached. A good reputation is crucial to successfully concluding fundraising, and it is essential to first seek investors domestically before moving on to international ones. In the investment phase the PE Fund identifies potential investment opportunities and negotiates the deal. Factors influencing the process of generating deal flow include the operator's

[80] T. Tykvova "How do investment patterns of independent and captive private equity funds differ? Evidence from Germany." Financial Markets and Portfolio Management 20 (2006): 399-418.
[81] J. Gilligan & M. Wright “Private equity demystified: An explanatory guide.” Oxford University Press, USA (2020)

reputation, experience, and market knowledge, as well as the geographical location and investment goals[82]. To identify interesting investment proposals, private equity operators must regularly dedicate time and resources to research, selection, and targeting of potential companies. Ideal companies for private equity investment are those with dynamic growth prospects and explicit strategic goals[83]. The business plan, which outlines the company's goals, competitive strategies, and financial projections, serves as the primary tool for investor evaluation. The investor assesses the sustainability, reliability, and feasibility of the project, as well as the alignment of objectives between the investor and entrepreneur. Following the investment, private equity funds continually monitor their investment in a target company to enhance their participation value. The management monitoring and valorization of investments depends on the investor's attitude, type of intervention, and operational tools used. Investment types are categorized based on the company's life cycle, and the minority or majority ownership of the invested company determines the discriminant aspect. In the case of venture capital or growth capital with minority ownership, investors partner with the entrepreneur or shareholder to implement growth plans while monitoring management. In contrast, buyouts involve the operator's majority ownership and responsibility for monitoring the management team, while turnaround operations require dominant investor intervention. Geography and industry development are also critical factors influencing private equity operations, with Europe featuring more majority ownership investments compared to the US due to underdeveloped secondary markets and the significant role of banking institutions. Divestment is the final phase and undoubtably a crucial moment in the fund’s lifecycle since it represents the achievement of their institutional investor goal: monetizing the created value during the investment period. This phase is where theoretical value creation becomes

[82] Balboa, Marina, and José Martí. "An integrative approach to the determinants of private equity fundraising." Available at SSRN 493344 (2003).

[83] Capron, Laurence, and Jung-Chin Shen. "Acquisitions of private vs. public firms: Private information, target selection, and acquirer returns." Strategic management journal 28.9 (2007): 891-911.

tangible, transforming portfolio evaluation into price and, consequently, into market returns. Two critical factors in the exit phase are maximizing capital gain, which depends on the timing of disinvestment and the sale strategy chosen to valorize the exit asset. The timing of disinvestment depends primarily on the portfolio company's performance, although it can be influenced by fund term expiration or contractual exit clauses[84]. The optimal timing for disinvestment also depends on market conditions. The choice of exit strategy depends on various factors, including the company's size, sector, organizational characteristics, achieved or prospective results, and other equity holders' preferences. Investors can either be proactive, defining the exit strategy at the investment's outset, or passive, relying on dividends as the primary return. There are mainly five ways to disinvest[85]: Initial Public Offering (IPO), where a portion of the company's capital is offered on a public exchange forming the so-called "free float"; Trade Sale, which is the sale to an industrial group or company, usually referred to as the "strategic buyer" because has interest in integrating the newly acquired company in the ongoing business and often to generate synergies; Secondary Buy Out refers to a sale to another Private Equity fund or an equivalent financial institution; Buy Back refers to the re-purchase of the interests from the previous owner, i.e. the majority shareholder or the entrepreneur; finally a Write Off occurs when the value of the company has been permanently impaired.

Some work experience in the area of investment funds in Luxembourg have allowed me to identify what the key functions and players within the Alternative Investment Funds environment are in the EU jurisdiction. The key figure concentrating most responsibilities is indeed the Fund Manager. The Directive 2011/61/EU of the European Parliament and of the Council commonly known as the AIFMD, establishes a comprehensive regulatory framework governing Alternative Investment Fund Managers (AIFMs) operating within the European Union (EU) and

[84] Brown, Gregory, et al. "Can investors time their exposure to private equity?" Journal of Financial Economics 139.2 (2021): 561-577.
[85] D. Folus and E. Boutron. "Exit strategies in private equity." Private Equity: Opportunities and Risks 215 (2015).

outlines a set of responsibilities for AIFMs. The Directive aims to enhance investor protection, promote transparency, and ensure the sound management of alternative investment funds (AIFs). Firstly, AIFMs are mandated to obtain authorization from their home member state's competent authority before commencing their activities. Additionally, they must register with regulatory authorities in each member state where they operate or market their AIFs. To ensure the proper governance and operation of AIFs, AIFMs must also establish robust organizational structures, internal controls, and risk management systems. Furthermore, AIFMs are required to implement policies and procedures to address conflicts of interest that may arise in the course of their operations. A key responsibility of AIFMs is the management of the investment portfolios of AIFs. They are entrusted with acting in the best interests of the AIF and its investors, while diligently managing risks. Accordingly, AIFMs must implement adequate risk management systems and processes to monitor and mitigate risks effectively. The AIFMD imposes specific capital requirements on AIFMs to ensure their financial stability. AIFMs must possess an initial capital of at least €300,000, or an amount determined by the member state of establishment. Additionally, ongoing capital must be maintained in proportion to the assets under management (AUM) or the value of the AIFs they manage. Transparency and reporting are fundamental obligations for AIFMs. Regular reports must be provided to investors and regulatory authorities, encompassing various aspects of the AIFs, including investment strategy, financial results, risk profile, and material changes that may impact investors. AIFMs are also required to disclose information relating to leverage, liquidity management, and remuneration policies. The appointment of a depositary is a critical requirement for AIFMs. Each AIF must have a depositary overseeing its activities, including safekeeping of assets, monitoring cash flows, and verifying compliance with relevant regulations.

5.4 Types of PE transaction

Private Equity transactions can be classified in several ways although

a common and important discrimination can be drawn based on the parties involved in the transactions. A single firm can be the object of different types of transactions throughout its life, due to the different magnitude of its operations and the growing capital needs.

Following this logic of the firm lifecycle, a first transaction might involve the entrepreneur, who is seeking capital and expertise to expand its firm's operations, and a private equity fund, which is seeking to invest its capital and increase the value of its assets. This type of operation is referred to as "growth capital"[86]. Growth capital operations in private equity have gained significant prominence in recent years as a distinct investment strategy aimed at supporting the growth and expansion of established companies. While venture capital investments traditionally focus on early-stage start-ups, growth capital investments target mature companies that have achieved a certain level of success and are seeking capital to accelerate their growth trajectory. Growth capital operations serve as a catalyst for companies looking to expand their market presence, launch new products or services, enter new geographies, or invest in research and development initiatives. Private equity firms provide the necessary capital infusion, which can be utilized for various purposes such as funding sales and marketing efforts, upgrading infrastructure, strengthening operational capabilities, or acquiring complementary businesses. The motivations behind growth capital operations are rooted in the belief that these investments can unlock substantial value by accelerating growth, increasing market share, enhancing profitability, and improving the overall competitive position of the invested companies. By partnering with growth-stage companies, private equity firms bring more than just capital to the table. Their industry knowledge, operational expertise, and extensive networks enable them to actively support and guide the strategic decision-making of the invested companies. Private equity investors work closely with management teams to optimize operations, implement growth strategies, identify new market

[86] Demaria, Cyril. Introduction to private equity: venture, growth, lbo and turn-around capital. John Wiley & Sons, 2013.

opportunities, and drive efficiency improvements. Through this collaborative approach, growth capital operations in private equity offer the potential for significant value creation, which might generate substantial financial returns upon exit.

What if instead the same entrepreneur wanted to liquidate his stake in the private firm by selling controlling stakes to the PE Fund? This type of transaction is referred to as Primary Buyout and represents a cornerstone of the private equity landscape. Primary Buyout is the preferred route for entrepreneurs to dispose their stakes in private firms. A PWC survey covering 1.600 family-owned businesses estimated that around 35 percent of businesses globally consider ownership succession through a primary buyout[87]. The term Primary Buyout is often substituted by the simple "Buyout" or even "Leveraged Buyout" (or "LBO"). This is due to the fact that most often in this type of deals the private equity firms or a group of investors acquires the target company by using a significant amount of debt financing[88]. The acquired company's assets, cash flows, and sometimes even its future earnings are used as collateral for securing the debt. The objective of an LBO is to generate returns for the investors by improving the target company's financial performance, implementing operational efficiencies, and ultimately paying off the debt, generating additional returns on equity when selling the firm thanks to the lower cost of debt. Primary Buyouts provide private equity investors with the opportunity to obtain a controlling stake in a target company. This level of control allows them to actively participate in the strategic decision-making process and influence the direction of the business. By implementing operational improvements, strategic repositioning, and financial optimization, private equity investors seek to enhance the profitability and growth prospects of the acquired businesses. Private equity firms typically aim to align the interests of management and shareholders to drive long-term value creation, while also bringing industry-specific knowledge and operational expertise to the target companies. Through their extensive

[87] PWC. Kin in the Game: PWC Family Business Survey. (2011)

network of contacts and industry relationships, they provide access to resources, talent, and potential synergies that can help enhance the target company's competitive position and growth prospects.

Another connotation that one such Buyout can take is the Management Buyout (MBO). This refers to a transaction in which the existing management team of a company, often in partnership with external investors, acquires a controlling stake or the entirety of the company from its current owners. The management team, with their deep understanding of the business and industry, leverages their expertise to take ownership and assume managerial control of the company[89].

One important feature of most Private Equity funds is that they have limited vintage, meaning that their operations will eventually need to be concluded and their assets disposed to distribute the capital to the Limited Partners. The target firms though, operate as a going concern and, other than being liquidated or wound down, will be able to exist for longer than the fund. In such scenario, interests in a private firm which has already undergone a Buyout deal, can be sold to another Private Equity fund in a so-called Secondary Buyout deal. A secondary buyout (SBO) is in fact defined as a private equity transaction where a private equity firm sells stakes in a portfolio company to another private equity firm, and it usually involves the transfer of ownership and control of the portfolio company[90]. Secondary buyouts have emerged as a prevalent exit strategy in the private equity industry in recent years. This trend can be attributed to various factors, including an abundance of capital in the private equity industry, increased competition for attractive investment opportunities, and the desire for a faster exit process[91]. Secondary buyouts typically involve mature and established portfolio companies that have already

[88] Kaplan, S. N., & Stromberg, P. (2009). Leveraged buyouts and private equity. Journal of Economic Perspectives, 23(1): 121-146.
[89] Wright, M., Gilligan, J., & Amess, K. (2013). The role of management buyouts in entrepreneurial development: The case of management buyouts from divestitures. Journal of Business Venturing, 28(3): 466-484.
[90] Elitzur, R., Gavious, A., & Hauser, S. (2013). Secondary buyouts: The impact of repeat private equity investment. Journal of Corporate Finance, 21: 177-195.
[91] Wang, Yingdi. "Secondary buyouts: Why buy and at what price?" Journal of Corporate Finance 18.5 (2012): 1306-1325.

undergone a period of operational improvements and value creation under the previous private equity ownership.

Despite Private Equity operators being the designated buyers in most private equity deals, the appealing exit routes both for entrepreneurs and the same Private Equity funds are Trade Sales and IPOs.

Trade Sales, also known as strategic sales or corporate sales, are attractive exit routes for private equity investors due to various motivations. In the first place, the investor has the opportunity to realize substantial returns on investment by selling the portfolio company to a strategic buyer within the same or related industry, since such buyer would likely see value in synergies, such as operational efficiencies, access to new markets, or complementary product lines[92]. Trade sales also provide a faster and more certain exit compared to other options, such as IPOs, as the sale can be completed relatively quickly, and the investor can receive cash proceeds upfront.

Initial Public Offerings (IPOs) provide private equity investors with an opportunity to exit their investments by taking the portfolio company public. Going public allows investors to unlock the value of their investment by selling shares to public investors at a potentially higher valuation than in private markets. This is mainly the result of the increase in liquidity of the investment that comes with the public listing of company's shares. The IPO process involves various stages, including selecting underwriters, preparing the prospectus, conducting due diligence, pricing the offering, and allocating shares to investors. While being a rewarding and praised exit strategy, the requirements for listing can be really demanding and costly, and not all private companies are enough attractive to fully subscribe the capital offered[93].

[92] Fee, C. E., Hadlock, C. J., & Pierce, J. R. (2013). Managers with and without style: Evidence using trade sales. Journal of Corporate Finance, 19, 186-207.

[93] Ritter, J. R., & Welch, I. (2002). A review of IPO activity, pricing, and allocations. Journal of Finance, 57(4): 1795-1828.

Summary 5

Private Equity is the activity of dealmaking which has per object stakes in private firms. This type of deal is mostly executed by specialized financial operators which pool funds from institutional investors in an investment vehicle, the Private Equity Funds. For the purpose of this work, I have highlighted in this chapter the different types of transaction that might occur within the PE activity. A particular focus needs to go to Primary and Secondary buyouts. The two deals are identical in the rationale, both involve acquisition by a PE Fund of stakes in a private firm, usually granting control, The way they diverge is in the type of seller, In a primary buyout the PE fund is purchasing form the funder or entrepreneur itself, while in Secondary Buyouts both the buyer and the seller are PE operators.

During the work I have supported the argument that individual investors are on average far less diversified compared to the institutional ones. This difference in the level of diversification of the sellers might impact the valuation of private firms in Primary Buyouts compared to Secondary Buyouts.

Chapter 6
PRIMARY AND SECONDARY BUYOUTS

6.1 The rationale

Throughout this work I have remarked the importance of assessing the level of diversification of the investor when estimating the expected returns of on an asset, as well as the implications in estimating the asset's Intrinsic Value as perceived by the investor.

Most Private Equity transactions, including Secondary Buyouts, involve only institutional investors, which are assumed to be completely diversified. In this case both buyer and seller perceive the same intrinsic value in the private firm, and the negotiation leading to the deal price will depend on factors such as competition in the bidding, expertise of the buyer and time pressure on the seller[94].

Meanwhile, in the event of a Primary Buyout, as described in the previous chapter, the two parties involved in the transaction might have different levels of diversification. On the one hand the buyer is a Private Equity fund, whose Limited Partners are completely diversified; on the other hand, the seller is a private investor who is likely to be, to a certain extent, undiversified. In such a situation, the intrinsic values perceived by buyer and seller are not the same, suggesting that the resulting deal price should tilt towards a certain value that lies between the two intrinsic values.

By comparing the two types of transaction, one involving institutional seller and buyer and the other involving a private investor selling to an institutional investor, I expect that, all other conditions being the same, the deal price should be lower in the second type of transaction. This is because, in the negotiation process, is known to both parties that one, the private seller, perceives a lower intrinsic value in the asset compared to what two institutional investors might perceive and agree

[94] Ahlers, Oliver, et al. "Opening the black box: Power in buyout negotiations and the moderating role of private equity specialization." Journal of Small Business Management 54.4 (2016): 1171-1192.

upon, tilting the price downwards compared to an identical transaction on the same asset involving only institutional investors.

I make the argument that, by cleaning up for other factors that might affect the deal price, a positive relationship should emerge between the price and the level of diversification of the seller, arising from the lower risk and expected returns perceived on the traded asset. In this final chapter, I will therefore look for empirical evidence of this relationship by collecting data on the deal and performing some statistical analysis.

6.2 Data and methodology

To perform the analysis, I have collected data on capital transactions focusing on private companies through Bloomberg's terminal. The sample includes 2.491 transactions, successfully completed between the years 1992 and 2022. Among these transactions, I have selected only the deals valued above $100 million in Enterprise Value.

The targeted companies are located across the world in 81 countries, reaching every continent. Asia is the most represented region within the sample with 34% of targets established in the region, followed by North America (25%), Europe (22%), and the U.K. (10%). Remarkably Italy appears on top of the podium of European countries with 139 transactions or 6% of the sample.

The target companies composing the sample are also well distributed in terms of industry. The sectors displayed refer to the BICS (Bloomberg Index Methodology) and include Basic Materials, Communications, Consumer, Cyclical, Consumer, Non-cyclical, Diversified, Energy, Financial, Industrial, Technology, and Utilities. The classification used by the data provider for this dataset recalls the Global Industry Classification Standards (GICS) by MSCI, although it is not completely aligned.

For the purpose of the analysis, further discrimination is required to classify the type of deal into Primary Buyout, Secondary Buyout, and other transactions. To reach this objective I have first defined the type of seller within each deal, identifying as Individual investor the seller type of

transactions where, among the sellers, were included either the references "Private investor" or "Management group," as Unknown investor in the case of "Unknown seller" and empty fields, and finally Institutional investor in all other instances.

The next step in defining the type of deal requires the combination of two criteria: the type of seller and the acquirer's sector. In particular, I assume that acquirers whose sector is identified as "Financial" are to be considered equivalent to a diversified Private Equity operator. I therefore proceed to define as:

- Primary Buyout the deals involving an Individual investor as seller and a financial operator as buyer;
- Secondary Buyout the deals involving an Institutional investor as seller and a financial operator as buyer;
- Trade Sale/Unknown all other deals.

Finally, a relevant information about the deals which is available in the sample is the Enterprise Value of the target company agreed by the parties at the time of the transaction. It is important to consider the firm's size in the analysis due to the effect of size on the risk of the cashflows. As discussed in the fourth chapter, in fact, larger firms are expected to diversify their cashflows more than smaller firms, resulting in a lower risk and a higher valuation. Data on the size of the sampled firms are outlined in more detail in Annex 1.

The most observable and meaningful indicator that allows us to compare the pricing of these transactions is a relative measure such as the multiple EV/EBITDA (also called TV/EBITDA in the tables). In Annex 1 are highlighted the distribution of the multiple within the sample.

EV/EBITDA multiple has been decomposed in its fundamentals previously in this work. Using the same rationale, I isolated the relationship of the level of diversification of the seller with the "bottom-up multiple."

The determinants of EV/EBITDA have been identified in Tax Rate, Risk, Profitability, and Growth. The level diversification of the marginal investor, which affect the exposure to firm-specific risk, is captured by the

Cost of Capital, in the risk component to the equity holder, or rather the Cost of Equity (K_e in the following formula).

$$WACC = (W_d * K_d) + (W_e * K_e)$$

In the third chapter I have presented the Total-Beta method as a solution to incorporate firm-specific risk in the Cost of Equity estimation.

$$E(R_i) = R_f + [E(R_M) - R_f]\ T\beta i_M,\ i = 1,\ldots,N$$

$$T\beta = \beta / \rho_{sm}$$
$$= \sigma_s / \sigma_m$$

Since the correlation coefficient is by definition smaller than 1 (0,29 in on average according to Damodaran), other things being equal the expected returns shall be higher for a non-diversified investor. As a result, notwithstanding differences in the other value drivers, higher expected returns should lead to lower multiples in the event of a transaction involving a non-diversified investor, such as a Primary Buyout, compared to when both the seller and the buyer are institutional investor, such as a Secondary Buyout.

The business where the firm operates is a main driver of risk, as has many times discussed in this work. Whether it is captured by a Beta or by a Total Beta, the sector of a firm has a strong impact on the synthetic multiple, and therefore needs to be accounted for in my analysis. Different businesses also have different profitability and different growth rates, which are also important drivers of value, and can affect the EV/EBITDA multiple of a firm.

Since there is no numeric feature that might reflect all these features, I have used the sector of the target company as a proxy to account for the differences. I have therefore created a dummy variable for each business, which takes value of 1 if the firms operates in that business or 0 if it does not. After removing the sectors where no Primary Buyout have occurred

(in order to obtain cleaner results), the following 6 dummy variables are left: Basic Materials, Consumer Non-cyclical, Consumer Cyclical, Technology, Financial, and Industrial.

With the same rationale, to account for differences in tax rates, as well as better capturing growth rates, I have also considered the country of the target company in my analysis. By excluding the countries where no Primary Buyout have occurred the array of dummy variables for the target's country is composed by South Korea, U.K., Russia, U.S., India, Germany, and France.

Finally, the variable representing the type of transactions is also dummy. The variable is designed to assume value of 0 in case of Primary Buyout and 1 in case of Secondary Buyout.

The final sample comprises 251 observations, of which 240 recognized as Secondary Buyout and 10 as Primary Buyout. Each observation is characterized by 15 variables:

- Enterprise Value: real [from 100 million to 8.137 million]
- 6 Sector variables: dummy [1 or 0]
- 7 Country variables: dummy [1 or 0]
- Deal type: dummy [1 or 0]

With these inputs (x_1 to x_{15}), I have run a multiple linear regression to estimate the coefficients (k_0 to k_{15}) for the EV/EBITDA multiple (Y), across the each of the "i" observations.

$$Y_i = k_0 + k_1 * x_{i,1} + k_2 * x_{i,2} + + \ldots + k_{15} * x_{i,15}$$

6.3 Results

The outcome of the multiple linear regression analysis, conducted to examine the relationship between several independent variables and a dependent variable are the 16 coefficients highlighted in the table below. Among the independent variables, several exhibited significant associations with the dependent variable. Notably, the "Enterprise Value" variable showed a highly significant positive coefficient (Coefficient =

0.002, p < 0.001). This suggests that as the enterprise value increases, there is a corresponding increase in the dependent variable. Another significant variable was "India" (Coefficient = 15.470, p = 0.001), indicating that companies located in India have a strong positive impact on the dependent variable. On the other hand, variables such as Basic Materials, Consumer Non-cyclical, Consumer Cyclical, Technology, Financial, Industrial, Russia, Germany and France did not demonstrate significant associations with the dependent variable.

Regression coefficient outputs

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	2,355	6,222	0,378	0,705
Enterprise Value	0,002	0,001	4,001	0,000
Deal Type	1,202	4,208	0,286	0,775
Basic Materials	0,000	0,000	65535,000	NA
Consumer, Non-cyclical	0,171	4,569	0,037	NA
Consumer, Cyclical	0,279	4,422	0,063	0,950
Technology	-0,259	5,347	-0,048	0,961
Financial	-0,540	4,265	-0,127	0,899
Industrial	0,539	4,364	0,124	0,902
South Korea	5,465	4,181	1,307	0,192
U.K.	6,608	4,178	1,581	0,115
Russia	-0,512	9,718	-0,053	0,958
U.S.	3,764	3,892	0,967	0,334
India	15,470	4,486	3,448	0,001
Germany	0,000	0,000	65535,000	NA
France	5,584	4,361	1,281	NA

The main purpose of the analysis, though, concerns the relationship between the independent variable “Deal Type” and the dependent variable “EV/EBITDA.” Although the significance of the indicator being quite low (p=0,775), the resulting coefficient is equal to 1,202 which is higher than 0. This means that, within the sample, the independent variable which reflects whether the deal is a Primary Buyout, or a Secondary Buyout has a positive impact on the dependent variable of the multiple EV/EBITDA. Since the dummy variable Deal Type has been designed so that it takes a value of 0 when the deal is recognized as a Primary Buyout, and a value of 1 when the deal is recognized as a Secondary Buyout, a positive coefficient

for the variable is associated with a positive effect on the independent variable when the deal is a Secondary Buyout and no effect when it is a Primary Buyout. The multiple EV/EBITDA have been found therefore to be higher in case of Secondary Buyout compared to Primary Buyout, supporting the expectation that have been proven by the theoretical framework presented in this work.

The below indicators, though, suggest that the estimated regression function cannot be regarded as a solid predictor for the independent variable.

Regression Statistics	
Multiple R	0,341215109
R Square	0,116427751
Adjusted R Square	0,059522944
Standard Error	12,42538241
Observations	251

The results indicated a moderate positive correlation (Multiple R = 0.3412) between the predictors and the outcome. The regression model accounted for approximately 11.64% of the variance in the dependent variable (R Square = 0.1164). However, after adjusting for the number of predictors and the sample size, the adjusted R Square value (0.0595) suggested that some of the predictors may not be significantly contributing to the model.

Despite this, according to the analysis of variance "Anova" the regression is considered significant, as highlighted by the low significance coefficient (Significance F = 0.00296). This coefficient represents, in fact, the overall significance of the regression model and tests the null hypothesis that all of the regression coefficients are equal to zero. Since the value is lower than the conventional significance level of 0.05, it suggests that the regression model is statistically significant.

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	15	4821,501589	321,4334392	2,402256816	0,002956842
Residual	237	36590,46037	154,3901281		
Total	252	41411,96195			

In conclusion, while the overall regression model was statistically significant, caution should be exercised in interpreting the results due to the relatively low adjusted R Square value. The variables "Enterprise Value" and "India" emerged as the most influential factors in predicting the dependent variable

The lack of significancy in some of the variables, including all the sector-related variables, suggests that the data source was probably too narrow for the number of independent variables included in the analysis. This is indeed an intrinsic feature of a business such as the Private Equity where information about the deal are often kept undisclosed. In order to explore additional factors that may contribute to the model's predictive power and to validate these findings in different contexts, further data and research is required.

CONCLUSIONS

This work contributes to the literature on Total Beta in two ways. Firstly, it provides in-dept analysis of a practical application for the model by linking the underlying theory to the Private Equity practices. Secondly, through the empirical analysis, it provides some evidence of the existence of a premium for non-diversification.

The firm-specific component of risk is indeed influenced by a wide nexus of factors which make it extremely difficult to assess. Despite the lack of significance shown by empirical studies, the CAPM has persisted as the chosen tool for estimating returns by many practitioners, who have not found sufficient comfort in any other models. The concept of Total Beta springs up from the womb of an imperfect model and itself is not flawless. Its theoretical framework though is solid and intuitive, and the total expected returns estimated for private firms with the Total Beta seem by far more reasonable compared to those estimated by the common Beta.

Nevertheless, the framework itself has little to no use if not put in relation to transactions involving different levels of diversification, which is exactly the case for Primary Buyouts. The empirical results, though undermined by lack of available data, show some evidence that deals of this kind are closed at a lower multiple compared to Secondary Buyouts. These findings confirm that, in the context of Private Equity deals, operators consider the firm-specific risk that an undiversified investor, the seller in this instance, might bear when holding an asset, and therefore there is a risk premium that accounts for the lack of diversification of such investor, and it is larger than zero.

Appendix 1

- Deals by Target's Country

Target's Country	#Deals
U.S.	542
South Korea	257
U.K.	247
Japan	165
China	162
Italy	139
India	130
France	84
Canada	76
Germany	62
Australia	56
Sweden	37
Brazil	32
Hong Kong	32
Spain	28
Norway	27
Poland	24
South Africa	24
Vietnam	24
Netherlands	23
Singapore	17
Taiwan	17
Chile	16
Israel	16
Portugal	15
New Zealand	14
Denmark	13
Finland	13
Malaysia	13
Mexico	12
Switzerland	12
Thailand	11
Belgium	10
Indonesia	10
Russia	10
Czech Republic	9
Iceland	9
Ireland	8
Luxembourg	8
Peru	8
Hungary	6

Target's Country	#Deals
Argentina	5
Cambodia	4
Egypt	4
Greece	4
Philippines	4
Cyprus	3
Morocco	3
Slovakia	3
Ukraine	3
Croatia	2
Kazakhstan	2
Lithuania	2
Montenegro	2
Serbia	2
Tunisia	2
Turkey	2
U.A.E.	2
Zimbabwe	2
Austria	1
Bosnia And Herze	1
British Virgin Isla	1
Bulgaria	1
Cayman Islands	1
Colombia	1
Costa Rica	1
Estonia	1
Eswatini	1
Jersey	1
Kenya	1
Latvia	1
Macedonia	1
Mauritius	1
Oman	1
Pakistan	1
Panama	1
Paraguay	1
Puerto Rico	1
Slovenia	1
Venezuela	1
Zambia	1

- Deals by Target's Sector

Trarget's Sector	#Deals
Industrial	453
Consumer, Non-cyclical	448
Financial	427
Consumer, Cyclical	388
Communications	223
Utilities	159
Basic Materials	142
Technology	129
Energy	94
Diversified	28

- Deals by Seller Type

Seller Type	#Deals
Individual investor	42
Institutional investor	1.252
Unknown investor	1.197

- Deals by Type

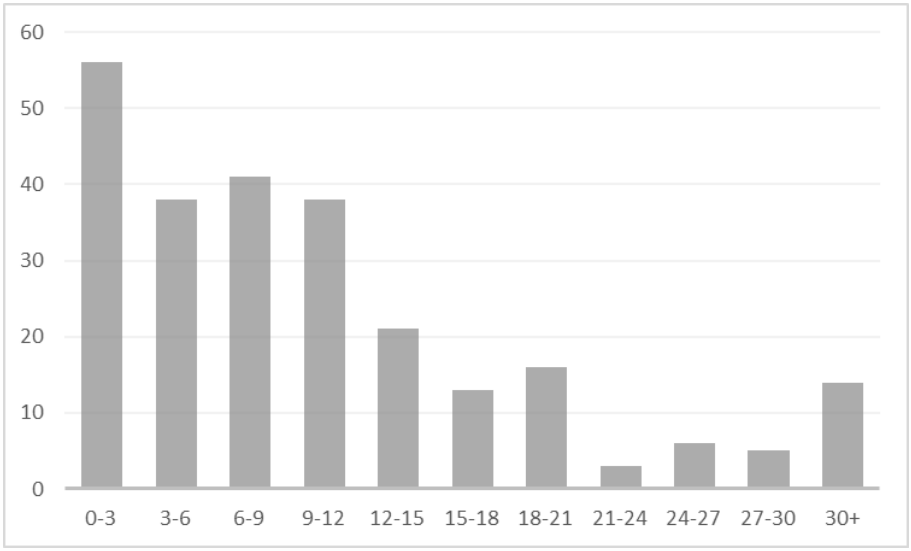
Deal Type	#Deals
Primary Buyout	14
Secondary Buyout	963
Trade Sale/Unknown	1.514

- Deals by Size (Enterprise Value)

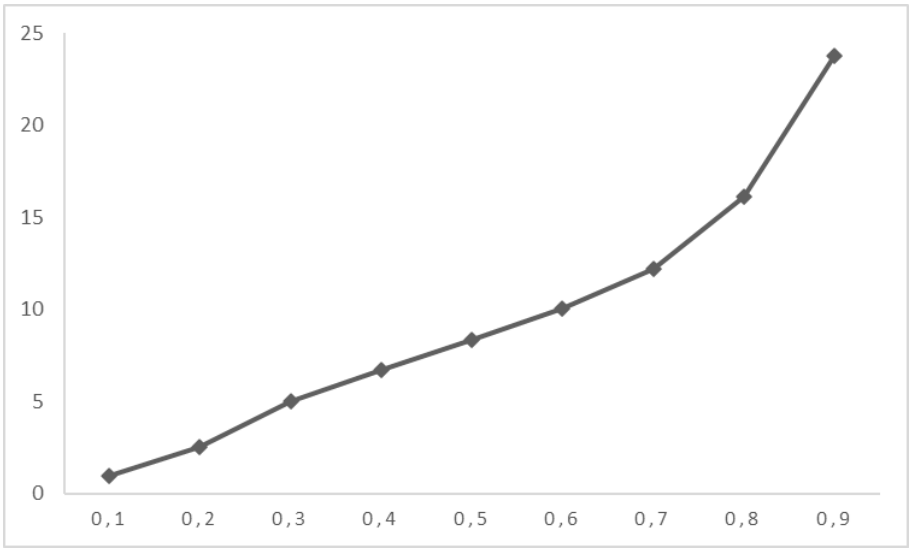
TV range (mln $)	#Deals
5 to 50	845
50 to 100	335
100 to 500	700
500 to 1000	256
Above 1000	355

Appendix 2

Distribution of EV/EBITDA (number of deals)



Percentiles of EV/EBITDA



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