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YIELD SPREAD NETWORKS AS INDICATORS FOR SHORT-RUN RECESSIONS

A New Keynesian Approach

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Abstract

The term spreads for thirty-five OECD countries are employed in a Granger Causality network to infer the connections between them. For each country in the network structure, a static probit model is fitted to assess the probability of an economic recession in the short run using as covariates: the country's lagged spread, the lagged spread of countries that Granger Cause the country in question and the in-degree of Granger causalities. The model diagnosis reveal the network structure as a statistical significant covariate, yielding an improvement with respect to the benchmark models fitted by Estrella and Trubin (2006) and the multi-country approach by Bernard and Gerlach (1998). Moreover, an advancement in the selection of foreign spreads is suggested analyzing how smaller economies may possibly affect the term spreads of their larger counterparts, specifically in the European Union.

Keywords: Macrofinance, Granger causality, networks, yield rates, term structure.

JEL classification: E43, E44, E52, E37.

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1 Introduction

There has been an increased attention to early warning signals after the financial crisis of 2007–2008. An optimal monetary policy that enables Central Banks to properly respond changes in the economy should account for a collection of different key economic indicators. The term spread, defined as the difference between long term yield rates and short term yield rates of government securities, is one of such monetary gauges that enable policy makers to maneuver smoothly in the presence of rational expectations of investors.

Under a rational expectations scheme, whenever a Central Bank targets inflation in a 'more strict' sense, i.e. where the Taylor Rule is applied, a higher weight will be given to the term structure as a predictor of the policy rule making. Hence, the spread between yield rates can be combined with other leading economic indicators to derive a powerful predictor of future economical activity.

Estrella and Hardouvelis (1991) proposed a probit model with quarterly data frequency in order to forecast future economic activity as a function of the spread between long rate an short rate interest rates. Later, Estrella and Trubin (2006) adapted the model for monthly observations. Both articles found that there is a positive correlation between such spread and future economic output; in this sense a negative yield spread would help to forecast recessions. The underlying theoretical concepts conclude that whenever the yield curve flattens, or even inverts, the future interest rates are predicted to drop significantly: a drop in the interest rates is associated with a lower level of real GDP.

Jorion and Mishkin (1991) expanded the term-structure forecast model to four economies: US, Britain (UK), (West) Germany and Switzerland and concluded that the term structure had strong and statistically significant power to predict future inflation rates. Bernard and Gerlach (1998) extended the benchmark probit model to include the spreads of different countries as covariates. The term spreads for the United States and Germany were chosen as covariates for the probit model of the other countries given their relatively larger GDP's.

Billio, Getmansky, Lo and Pelizzon (2012) measured the degree of connectedness between the returns of financial institutions by pairwise Granger causality. The relationships inside the financial system were measured through Granger tests to detect both the degree and direction of connections over a moving window. The authors found that a higher degree of connectedness inside the financial system can be associated to an increase in the level of systemic risk in the market. An increasing connectivity inside the financial market, thus, can be an early warning signal of an imminent crisis stimulated by systemic risk.

There has been an undeniable escalation of interdependency between sovereign countries in the current environment of financial and economical globalization. The scope of this thesis is

to generalize the models employed by Estrella and Trubin (2006) and by Bernard and Gerlach (1998) by innovating a term spread network covariate constructed in a similar way to the work of Billio et al. (2012). The generalized regression equation per country in the newly proposed model employs as covariates the domestic term spread, methodologically selected foreign term spreads and the in–degree of the Granger causality network.

The present work initiates by describing in a summarized way the mechanisms of monetary policy in section 2, placing an emphasis in the essential components of macroeconomic models. Understanding the procedure to build yield curves is the principal objective of section 3. Moreover, this same passage outlines the financial economics theory and assumptions.

Following after the literature review, section 4 details the statistical methods that are occupied in the model derivation, calibration and diagnosis. Right after the numerical techniques have been discussed, the quantitative justification for considering the term structure as a predictor of future output is probed by the derivation of a theoretical model in section 5.

Section 6 comprises the specific literature review on recession forecasting through term structure models. Moreover, the generalized proposed model is designed and interpreted in a comprehensive approach. The summary of results, diagnosis reports and visual aid in the form of graphical networks, correlation plots and the evolution in the number of connections constitute the core of section 7.

The conclusion of the current work encompasses the general and secondary findings while interpreting the roles of term spreads and the network components in the endeavor of short–run recession forecasting under a New Keynesian approach.

2 Literature Review on Monetary Policy

2.1 Monetary Policy in the Long-Run

The relationship between money and inflation has a long tradition in economics, dating back to the times of David Hume and Thomas Tooke. In order to define this phenomenon, we understand the rate of inflation as the sustained percentage variation in the overall price level of goods and services. Moreover, the price of a good or service is the amount of money that is needed to be exchanged for it. The two former definitions allow us now to have a glimpse of the connection between money and inflation.

Extending the definition of money, Keynes (1930) categorized the functions of money as a store of value (imperfect due to inflationary issues), as a unit of account to quote prices and as a medium of exchange to buy goods and services.

Measuring the amount of money may lead to ambiguous and vague definitions of which assets are money and which ones are not. The types of financial assets that are most commonly classified as money include: currency, demand deposits, saving deposits, money market mutual funds, Eurodollars, short-term Treasury securities and bonds.

The amount of money available in an economy is called 'money supply'. An important distinction must be made between the types of money: fiat money and commodity money. Fiat money is issued by the Government does not have an inherent value on its own, while commodity money has intrinsic value by itself (e.g. gold).

From the point of view of the Government, fiat money has an inherent advantage in the sense that the Central Bank can control the money supply: this is the cornerstone of monetary policy.

Developing the idea of money supply, we can link the amount of the money with the nominal GDP by means of the 'quantity theory of money'. The mathematical formula that allows the previous point receives the name of the Cambrdige equation, or quantity equation, and was first introduced by Pigou (1917). Let us define M as the quantity of money available in an economy; V is the velocity of money, defined as the speed with which money changes hands (circulates) in an economy; P is the price of the asset to be transacted; and T is the number of transactions of money in exchange for goods or services within an annual time period. The quantity equation goes as:

$$MV = PT$$

PT is the the amount of money exchanged in a transaction times the number of transactions that took place during a year, therefore PT is also he total amount of money exchanged in such time period. In this way, PV can be replaced by the annual nominal GDP defined as PY, where Y is the real GDP and P is the price used as the GDP deflator. The quantity equation

is reformulated as:

$$MV = PY$$

It is important to note that V is no longer the velocity of money, but it is now the velocity of income. It can be understood as the speed rate in which a unit of currency enters an individual's income (the number of times a unit of currency enters the individual's income within a defined time period).

So far money has not been measured as the purchasing power, in other words, the amount of goods and services that an individual can actually acquire by spending a certain stock of monetary units. Diving the amount of money M by the price level P the real money balance is obtained, which gives the relation between the quantities of money and the quantities of goods and services. Following the introduction of this notion, it might be desirable to measure as well the amount of money that individuals want to hold in their hands: the money demand. In this sense, a typical money demand function can have the following formulation:

$$\left(\frac{M}{P}\right)^{d} = kY$$

where k is a constant value that designates the proportion of money that individuals want to hold with respect to their income. Whenever a certain condition is met –the money supply equals the demand for money– then $k = 1/V$.

Carrying on the quantity equation, if allowed to assume that the velocity is constant, the the quantity theory equation is derived as:

$$M\overline{V} = PY$$

The importance of this equation relies in the implicit relation that a variation in the money supply will further cause a proportional variation in the nominal GDP. For a given real output in an economy, if the amount of money is modified, then the nominal GDP (which equals PY) will be analogously modified, and, therefore, the price level will have to adjust in order to maintain the relationship between real and nominal GDP. Recalling that a change in the price level is defined as the inflation rate, then it is possible to conclude that variations in the money supply have a direct impact in the inflation rate. Empirically, Friedman and Schwartz (1963) provided strong evidence that the quantitative theory of money works reasonably well in the long run. Mankiw (2010) remarks from the quantity theory of money that it is the Central Bank who controls the rate of inflation given that this same institution is the one that regulates the money supply.

After the relationship between money supply and inflation has been thoroughly explained, another link between economic variables can be assessed: interest rates and inflation.

The nominal interest rate is the amount of money that a borrower repays (the original loan plus an additional amount) to the lender divided by the value of the loan borrowed, minus one. In one hand, the nominal interest rate is the increase in the nominal amount of money. On the other hand, the real interest rate is the increase in the real purchasing power, which is defined as the nominal interest adjusted by the inflation rate. The formula that links this three variables together is:

$$(1 + r) = \frac{1 + i}{1 + \pi}$$

where, for r, i, and π reasonably small, it can be simplified to $r \approx i - \pi$.

All this concepts can be combined in a simple formula that carries by name the Fisher equation:

$$i \approx r + \pi$$

where i is the nominal interest rate, r is the real interest rate and π is the inflation rate. This formula yields a one-to-one relation between the inflation rate and the nominal interest rate, which goes by the name of the Fisher effect.

Since it is impossible to foretell the true inflation rate that will prevail in the future, both lenders and borrowers can not know for sure which will be the real interest rate over a loan or contract to be made. In this way, a distinction must be made between the ex ante real interest rate, which is the expected real interest rate, and the ex post real interest rate, which is the real interest rate that actually took place. Following the previous notation, $i - \pi$ is the ex post real interest rate, since π denotes the true inflation rate. In the other hand, we shall define the ex ante real interest rate as $i - \pi^e$, where π^e is the expected inflation rate.

Now, to be perfectly clear, the nominal rates cannot be set using the real interest rate, since it is not known beforehand. Therefore, the most accurate version of the Fisher effect becomes:

$$i = r + \pi^e$$

This nominal interest rate can be seen as the cost of opportunity of holding money. By keeping the money in your hands instead of investing or saving, you do not earn the real interest rate r and, additionally, your money loses purchasing power at a rate of π^e, therefore the cost of holding money is exactly i.

Recalling the money demand function established previously, it can be expanded to include both the level of income and the interest rate as in Keynes (1936):

$$\left(\frac{M}{P}\right)^d = L(i, Y)$$

where $L(i, Y)$ is a function such that:

$$\frac{\partial L(i,Y)}{\partial i} < 0$$

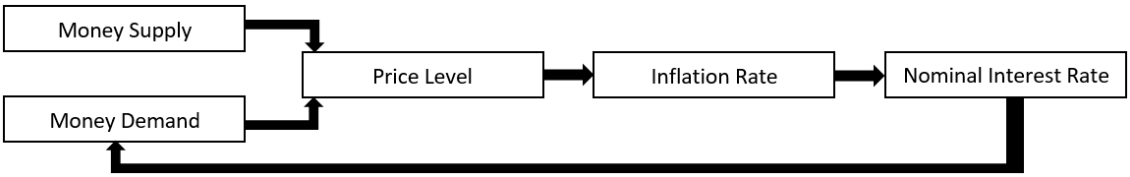
and

$$\frac{\partial L(i,Y)}{\partial Y} > 0$$

Substituting i for $r - \pi^e$ it is inferred that, in equilibrium, the expected level of inflation rate can affect the real money balances:

$$\left(\frac{M}{P}\right) = L(r + \pi^e, Y)$$

Mankiw (2010) provides a very simple, yet detailed, figure that enables us to understand in a simple manner how connections flow among money supply and demand, price levels, inflation rates and interest rates.



Given that the nominal interest rate depends on the expected inflation level, it cannot be assumed to be constant. Furthermore, the nominal interest rate is now another channel throughout which the money demand affects the price level. In this sense, the price level depends not only on today's money supply, but also on tomorrow's (future, to be more precise) expected money supply.

Exploiting the property that an expected growth in money supply is proportionally related to the price level, then the price level today is a weighted average of the current money supply and the expected money supplies in future periods.

Prior to continuing, it is important to distinct between real and nominal variables. Real variables are measured in physical units, such as the real GDP (defined as the quantity of goods and services produced by an economy in a year), the real interest rate or prices measured in current US Dollars. Nominal variables are expressed in terms of money; examples of nominal variables are inflation rates and price levels. This separation of variable types, or dichotomy, allows us economists to study the real variables by assuming them to behave irrelevantly from from money, in other words, in the classical theory, real variables are money neutral.

In the classical theory it is assumed that in the long run prices are fully flexible and that the classical money neutrality holds. Notwithstanding, in the short-run, this theory fails to

convince most economists. Therefore, a more detailed analysis of short-run monetary policy is in order.

2.2 Monetary Policy in the Short-Run

A key difference underlying the need to make a distinction between short and long term models is the behavior of prices. While in the long run prices may be assumed to be fully flexible, in the short run they have a 'sticky' nature and do not respond to fluctuations immediately. These short run fluctuations in the economic output are commonly labeled as the business cycle (other variables, such as unemployment, are also needed to assess the business cycle). The importance of this remark on sticky prices comes from the notion that if the prices do not respond immediately, then it corresponds to the output or the employment to adjust instead. In this sense, it might be the case that in the short run nominal variables can affect real variables and that money neutrality is no longer applicable.

Quoting the words of Mankiw (2010):

> *"Over long periods of time, prices are flexible, the aggregate supply curve is vertical, and changes in aggregate demand affect the price level but not the output. Over short periods of time, prices are sticky, the aggregate supply curve is flat, and changes in aggregate demand do affect the economy's output of goods and services."*

In order to derive this conclusion the aggregate demand function is readily occupied. The aggregate demand is a function that links the quantity of goods and services that the individuals are willing to buy at a certain price level. By changing the price level and solving each equation, the entire curve can be obtained.

Recalling the quantity equation $MV = PY$ and keeping both the money supply and velocity constant, then its straightforward to notice that the price level and the nominal output are inversely proportional between each other. Accordingly, the aggregate demand (AD) can be represented as a concave function with negative slope, where the x-axis indicates Y and the y-axis denotes P. Thus, holding velocity constant, an increase in the money supply would push the AD curve to the right, will a decrease in the money supply would push the curve to the left.

Conceive the Aggregate Supply (AS) curve as the relation between the amount of goods and services provided and the price level. In the long run, the amount of goods and services provided is held fixed and constant. Given that the stocks of capital K and labor L are held fixed, and the output is a function of K and L, then $Y = F(\overline{K}, \overline{L}) = \overline{Y}$. Thus the long run AS curve is in fact a straight vertical line, where the x-axis Y is indifferent to any change in the price level P. Whenever unemployment lies on its natural rate and the economy is said to be

fully-employed, then $\overline{Y}$ receives the name of natural output level.

However, thinking now about the short run AS curve, where prices are completely sticky, it yields an extreme scenario where the short run AS curve is in fact an horizontal straight line were the y-axis P remains constant for any value of Y. This implies that whenever an external shock generates a decrease in the aggregate demand (shifting the AD to the left) the equilibrium will lie on the same price level but in a lower output level. Therefore, a diminished aggregate demand, in the short run, would produce a shrinkage in the economic output.

Finally, the relationship between the aggregate demand and both short run and long run aggregate supply curves is used to assess Mankiw's statement about how changes in aggregate demand affect the economy's output in the short run.

Whenever an exogenous shock shifts either one of these curves the monetary policy becomes an important tool in the pursuance of stabilization mechanisms given that it has a direct repercussion in the aggregate demand. Shocks may be both negative and positive. In the example of oil production and availability, a negative shock could be a trade restriction that reduces the amount of available oil augments the price of this asset, while a positive shock would be the elimination of such trade barrier, reducing the oil price.

Suppose that certain phenomenon takes place and an economy receives a negative supply shock. Given that supply has become diminished, the price level rises and the horizontal short run aggregate supply curve rises. Now, the Central Bank faces a serious dilemma. If the aggregate demand curve is kept constant, the new equilibrium will be reached at a smaller value of Y, bringing a recession to the economy. In the other hand, in the Central Bank pushes for a shift in the aggregate demand curve to the right, by increasing the money supply for example, then the adverse supply shock is accommodated but the inflation rate is due to increase as a consequence of the money supply increment and a permanent raise of the price level.

In scenarios of this nature, the Central Bank must decide to accommodate the adverse supply shock by suffering an economic recession or by experiencing a sustained inflation rate. This idea will become a crucial part of the model to be derived in Section 3 of this thesis, where the Central Bank may be lured to relieve the inflation expectation by means of sacrificing future economic output though resourcing short term interest rates as a monetary policy tool.

2.2.1 IS Curve and LM Curve

The Investment and Savings Curve, referred to as IS curve from now on, is a relationship present in the goods and services market that links the interest rate with the level of income.

Keynes (1936) revolutionized the conception of the economy in his famous book *The General*

Theory and introduced the notion of planned expenditure as a different account from actual expenditure. The planned expenditure is the amount that households would like to spend, which is not necessarily the same as the actual amount spent.

Assuming a closed economy and allowing the actual expenditure E to be the economy's GDP, then $E = Y = C + I + G$, where C is the consumption function that depends on the net income after taxes $Y - T$, I is the investment function that depends on the interest rate r and G is the government expenditure. Whenever both of these expenditures equalize to each other, then the equation becomes the IS curve:

$$Y = C(Y - T) + I(r) + G$$

where the investment and consumption functions behave in a such a way that:

$$\frac{\partial I(r)}{\partial r} < 0$$

$$\frac{\partial C(Y - T)}{\partial Y} > 0$$

and given that we can rewrite the equation above as:

$$I(r) = Y - C(Y - T) - G$$

Then it can be implied that there is a negative relationship between the interest rate r and the level of economic output Y. Simply put, the IS curve is an upward concave function with negative slope that links the interest rate r in the y-axis with respect to the level of output Y in the x-axis.

In the same book, John Maynard Keynes (1936) developed the idea of the Liquidity and Money curve, which will be referred to as LM curve from now on. The key concept that lies behind this relationship between interest rate and economic output is knows as the Liquidity Preference Theory. In the LM curve for the money markets the real money balances are equalized to a function of interest rate and level of income, yielding an inverse relationship between r and Y.

Consider the quantity of money from both the demand and supply point of view. In the short run the supply of money can well be assumed to be fixed; also, in the short run the level of price may be assumed to be sticky and ergo fixed. Dividing the amount of money by the price level we derive the real money balance. Thus, the supply of real balances is fixed and denoted as:

$$\left(\frac{M}{P}\right)^s = \left(\frac{\overline{M}}{\overline{P}}\right)$$

Given that both the price level and the supply of money are exogenous variables, the supply of real money balances is a vertical straight line were the real money balance (x-axis) $\overline{M}/\overline{P}$ is held constant for any value of interest rate r (y-axis).

From the point of view of the demand side, the demand for real money balances have a negative relationship with respect to the real interest rate. Recalling the concept of an interest rate as a cost of opportunity to holding the money, the higher the real interest rate is then the less willing individuals will be to holding the money in their hands. Therefore, we can formulate this relationship as:

$$\left(\frac{M}{P}\right)^d = L(r)$$

where

$$\frac{\partial L(r)}{\partial r} < 0$$

Analogous to the investment function $I(\cdot)$, the function $L(\cdot)$ yields a negative relationship between the interest rate and the amount of money demanded.

In this way, the demand for real money balances function is an upward concave curve with negative slope, linking the real money balances $\overline{M}/\overline{P}$ in the x-axis with the interest rate r in the y-axis.

Whenever the supply of real money balances equals the demand of real money balances we say that the money market is in equilibrium. Consider that an economy is in fact in equilibrium, then if the Central Bank decides to increase the supply of money, the real money balances would increase and the interest rate would decrease accordingly, shifting the supply curve to right. If, in the other hand, the Central Bank decided to diminish the money supply, then the real money balances would decrease as well, producing a rise in the interest rate. The underlying mechanism behind this notion is that whenever the equilibrium is altered, individuals would modify their portfolio of investments until the new equilibrium is reached.

The second component of the LM curve is the level of income Y. Let us modify the demand function for real money balances to include this second variable, becoming now:

$$\left(\frac{M}{P}\right)^d = L(r, Y)$$

where

$$\frac{\partial L(r, Y)}{\partial r} < 0$$

and

$$\frac{\partial L(r, Y)}{\partial Y} > 0$$

Hereby the positive slope of the demand of real money balances with respect to the level of income, suggesting the result that the demanded amount of real money balances is proportional to the level of income of the individuals.

Recalling the condition portrayed previously in which the supply of real money balances is fixed, therefore it is held constant for any level of income and for any value of interest rate. In this manner, there must be a relationship between the real interest rate and the level of income in order to meet the condition of a fixed supply of real money balances.

In this sense, the LM curve establishes that whenever the level of income increases, the real money balance would increase, therefore, there must be a contrary effect to offset this change in real money balances. Such contrary effect can only be caused by a rise in the interest rate, yielding a decrease in the real money balances, offsetting the increase previously mentioned. Ergo, given that the partial derivatives $L'(r, Y)$ with respect to each variable have opposite sign, then there must be a proportional relationship between r and Y. For an increasing level of income, there must be a rise in the interest rate; analogous for a decrease in the level of income.

Assuming that taxes, government expenditure, price level and money supply are given exogenous values, whenever the IS curve, representing the goods market, and the LM curve, representing the money market, intersect, then such a point designates the equilibrium level of income and interest rate.

It is noteworthy to distinguish the effects of fiscal policy and monetary policy in the IS-LM equilibrium. Fiscal policy, controlled by the Government, has an impact in the IS curve. Monetary policy, however, controlled by the Central Bank, has an impact in the LM curve. The following lines focus on the fluctuations in the LM curve that can be generated throughout the usage of an efficient monetary policy. Defining the interest rate r and the level of income Y as endogenous variables and let taxes T, government expenditure G and the money supply M as exogenous variables. Given that we are exploring the economic fluctuations in the short run, this is in no way an irrational assumption.

Suppose that a Central Bank decides to follow an expansionary policy and increases the supply of money M, which in turn raises the real money balances M/P. Following the liquidity preference theory, this effect will repercuss by lowering the interest rate and shift the LM curve down, moving the new equilibrium to a higher level of income. A necessary assumption from this theory is that passive and active interest rates are equivalent. The intuition behind this is simple: given that there is more money in the economy, individuals save the excess in banks and bonds, pushing the interest rate down. Thereafter, given that the investment function is higher whenever the interest rate declines, the economy will experience a boom in investment. This higher amount of resources invested will be reflected as an increase in the level of expenditure,

income and production Y.

It is important to discuss, albeit briefly, the effects of fiscal policy and the reactions of the Central Bank that might need to take place. Suppose for now that the Government decides to cut taxes, shifting the IS curve down. The Central Bank has now a choice to make: to keep either the interest rate or the level of income constant. If the bankers decide to contract the money supply, then the LM curve would shift left and the interest rate would remain the same, however, the level of income Y would be reduces. If, on the other side, the bankers prefer to increase the money supply, then the LM curve would shift to the right, keeping the level of income constant, while letting the interest rate diminish at the same time.

A distinction in the assumptions for the short run and long run IS-LM models can be swiftly mentioned. In the short run, an assumption is made on the price level $\overline{P}$ being constant (called the Keynesian assumption). In the long run, the belief made is that the income level is fixed $\overline{Y}$ (called the classical assumption). In the first scenario both endogenous variables r and Y must adjust in order to achieve the equilibrium; while in the second case the endogenous variables to be adjusted are r and P.

As a final remark of the IS-LM model it is important to acknowledge the fact that an expected change in prices, that is, an inflation expectation may also influence the equilibrium. Recall the IS curve but replacing now the real interest rate r with the already known expression $i - \pi^e$. The IS curve becomes now $Y = C(Y - T) + I(i - \pi^e) + G$ and the inflation expectation becomes now a variable to consider in the IS-LM model.

2.2.2 Mundell-Fleming model

The Mundell-Fleming model was developed in the 1960's through the contributions of Robert Alexander Mundell (1963) and John Marcus Fleming (1962). This framework provides an extension of the IS-LM model to an open economy. Nevertheless there is an strong assumption made about this economy, being that it is small and open, with perfect capital mobility. In this sense, the interest rate r must match the 'global' interest rate $r*$, which is an exogenous variable.

Following this idea of an open economy, the IS curve will need to include a new parameter $NX(e)$ which establishes the net exports (exports minus imports) given a certain nominal exchange rate e. The IS curve becomes:

$$Y = C(Y - T) + I(r^*) + G + NX(e)$$

where

$$\frac{\partial NX(e)}{\partial e} < 0$$

and therefore

$$\frac{\partial Y}{\partial e} < 0$$

and the LM curve becomes:

$$\frac{M}{P} = L(r^*, Y)$$

Where e is the nominal exchange rate between currencies. The real exchange rate can be formulated as $\epsilon = \frac{eP^*}{P}$, where P is the price level in the domestic country and P^* in the foreign country. For the short run models the price levels P and P^* are assumed to be exogenous and fixed. Also, the money supply M can be assumed to be exogenous variable.

In this new IS and LM curves we have lost the endogenous variable r, which was replaced by the global interest rate r^*. There is, however, a new endogenous variable e. In this way, the two endogenous variables are now e and Y.

For the IS case, e affects both $NX(e)$ and Y; in the LM case, nonetheless, e does not affect Y. Holding the world interest rate at a constant level $r*$, then the new IS curve is a upward concave curve with negative slope, that links Y in the x-axis and e in the y-axis. The new LM curve, on the other hand, is a straight vertical line, where Y remains constant for any value of e.

The Mundell-Fleming model gives a summarizing view of the results and consequences of fiscal, monetary and trade policies applied to economics with either floating or fixed exchange rates.

In the floating exchange rate scheme, an expansionary fiscal policy would shift the IS curve to the right, but the LM curve would stay the same. Therefore, the exchange rate e would increase, but the income level Y would stay intact. An expansionary monetary policy, on the other way, would shift the LM curve to the right, lowering the exchange rate e and raising the income level Y. Finally, an protectionist or restrictive trade policy would increase NX, shifting the IS curve to the right, raising the exchange rate e but keeping the income level Y constant.

In the fixed exchange rate scheme, an expansionary fiscal policy would shift the IS curve to the right, but given that the exchange rate $\overline{e}$ cannot augment while in a fixed rate, then the LM curve must shift to the right to meet the equilibrium condition, raising in this way the income level Y. An expansionary monetary policy would shift the LM curve to the right, but given that the exchange rate $\overline{e}$ cannot decrease, the LM curve must return back to its place being ineffective whatsoever. Finally, a protectionist trade policy would analogously push both the IS and LM curves to the right, keeping the exchange rate fixed and increasing the income level Y.

The next table, taken from Mankiw (2010), summarizes in a concise manner the relationships between the three policies and the two exchange rates schemes available, where $\nearrow$ denotes an increase and $\searrow$ a decrease.

Table 1: Mundell-Fleming Floating Exchange Rate

Policy	Y	e	NX
Fiscal expansion	0	↗	↘
Monetary Expansion	↗	↘	↗
Import Restriction	0	↗	0

Table 2: Mundell-Fleming Fixed Exchange Rate

Policy	Y	e	NX
Fiscal expansion	↗	0	0
Monetary Expansion	0	0	0
Import Restriction	↗	0	↗

This is the proof to conclude that under a floating exchange rate scheme, only monetary policy can affect the income level Y. Under a fixed rate, on the contrary, the income level Y can only be affected through the fiscal policy. As a remark, a floating exchange rate gives room for action to policy makers in order to seek goals beyond exchange rate stability.

Taking further the notion of a global interest rate r^* there might be some inconvenience with this assumption, given that it is indeed hard to believe that all the countries may keep the same interest rate. This is the reason why most economies (if not all) need to add a country risk differential. This spread can be thought about in the same was as a risk premium ρ such that the country's interest rate is $r = r^* + \rho$. In this sense, the interest rate of a country has a positive correlation with the country's perceived risk by domestic and foreign investors. This same country risk has on its own a negative correlation with the exchange rate prevailing in the economy.

2.2.3 Phillips Curve

Another elemental piece to build a macroeconomic model is the Phillips curve (1958), which links the relationship between inflation and unemployment through the usage of the aggregate supply curve. The notion of the Phillips curve is vital for central bankers given that two fundamental objectives of most policy makers is to keep both inflation and unemployment as low as possible. However, this two objectives are the opposite sides of the same coin.

The aggregate supply curve can be formulated as:

$$Y = \overline{Y} + \alpha(P - P^e)$$

where $\overline{Y}$ is the natural rate of output, P^e denotes the expected value of the price level in the future and $\alpha > 0$ is the response ratio of the output with respect to a difference between the current price level and the expected price level.

In order to obtain the Phillips curve from the aggregate supply the price level, P, must be computed. From now on the values of P, P^*, Y, and $\overline{Y}$ will be understood as logarithms and ν will denote a supply shock.

$$P = P^e + \frac{1}{\alpha}(Y - \overline{Y}) + \nu$$

Subtracting the previous price level P_{-1} from both sides and using the properties of logarithms $\pi \approx P - P_{-1}$ and $\pi^e \approx P^e - P_{-1}$ we derive:

$$\pi = \pi^e + \frac{1}{\alpha}(Y - \overline{Y}) + \nu$$

Lastly, Okun's law (1963) states the inverse relationship between the deviation of output and unemployment with respect to their corresponding natural rates:

$$\frac{1}{\alpha}(Y - \overline{Y}) = -\beta(u - \overline{u})$$

where u and $\overline{u}$ are the actual and natural unemployment rates respectively.

By combining both the aggregate supply and the Okun's law equations the Phillips curve is derived as:

$$\pi = \pi^e - \beta(u - \overline{u}) + \nu$$

This last equation allows to interpret the adaptive expectations of inflation and how the inflation rate accounts for an inertia component. This intuition suggests that there is a chance that the reason why we have inflation is in part because we expect it to happen.

Therefore, in the short run the expectations over the future inflation rate affect the Phillips curve. Also, for the short run, it is safe to assume that there is a negative correlation between inflation and unemployment, therefore the relationship graphed in an xy-axis results in a downward sloping curve.

3 Financial Economics Theory

The main goal of the current section is the construction and interpretation of the yield curve. Notwithstanding, a comprehensive guideline is established to allow for the understanding of financial economics: financial securities, market characteristics and assumptions, and the theories behind the term structure composition.

In a rather simplistic definition, financial economics may be classified as the inter-temporal optimal allocation of resources in an uncertain environment. In order to achieve such objective, individuals and firms make use of the financial markets. The financial markets on its own can be subdivided in capital markets, which deal mainly stocks and bonds, the commodity markets, derivatives markets, money markets, futures markets and foreign exchange markets among others.

There are two key ideas behind financial economics and financial markets. The first one is the inter-temporal approach, which means that there are at least two time periods to account for in this economy. The second one is the notion of uncertainty about the future development of the asset prices, interest rates, yield rates and diverse financial and economic variables.

Most financial economist would agree with the statement that financial theory had its starting point with Bacherlier's *'Theory of Speculation'* (1900) applying stochastic processes in order to study the behavior of financial and economic variables. A collection of brilliant authors have carried on the development of modern financial theory, including for example, Markowitz, Sharpe, Miller, Black, Scholes, Arrow, Debreu, Modigliani, Merton and many others.

A simple, yet necessary, distinction must be made in order to be able to differentiate real investments from financial investments. A real investment represents the creation of a long-lasting physical asset, while a financial investment is a right over such real investment. A simple example would have the creation of a company as a real investment, while the emission of stocks over such company's equity would become a financial investment.

Marin and Rubio (2001) state that both theoretical and empirical studies have confirmed the ability of developed financial markets to help forecast future economical activity. The mechanism behind this suggests that in the short run the prices of Treasury Bonds can be used to deduce expected inflation rates. The term structure (the differential spread between long term and short term government bond yields) can be used as well to help plan an optimal monetary policy.

Financial institutions provide a possibility to individuals and firms to allocate in an efficient manner their resource in order to satisfy their intertemporal consumption functions. Another important usefulness of financial institutions is their inherent ability to provide information

regarding the financial markets. Fama's Efficient Market Hypothesis (1970) asseverates in fact that, in an efficient market, prices reflect all available information.

Throughout the extent of this work we will assume that the markets are frictionless. Two relevant characteristics of frictionless markets are: securities can be bought and sold on arbitrary quantities and the price rule of securities is linear, in the sense that unitary prices do not change as quantities traded vary. In other words, there are no transaction costs nor fees of any nature and no short–selling is allowed.

3.1 Financial Markets

3.1.1 No arbitrage assumption

The absence of arbitrage is, without a doubt, a crucial assumption of modern financial theory. Arbitrage can be defined as an investment strategy such that an investor can earn a positive amount of returns by investing zero. This idea is analogous to the economic concept of *no free lunch*.

A security, financial asset, or simply an asset can all be understood as a contract sold or bought at time $t = 0$ that promises to deliver a payoff in the case that an event is met. Such an event may be time to maturity as in bonds, or a contingent situation as insurance or call/put options.

The no arbitrage condition is actually not a very strong assumption. Suppose for now that there is an arbitrage opportunity somewhere inside a financial market, and an investor is able to buy an asset in market A at a price p^- and sell the same asset in market B at a price p^+, where $p^- < p^+$, earning a risk-free positive return. If one or more investors decide to repeat this strategy, the price of the asset in market A would rise and the price of the asset in market B would decrease obeying the laws of supply and demand until $p^- = p^+ = p$. In other words, even if an arbitrage opportunity may arise, it will disappear within a very short time span.

3.1.2 Arrow securities

Considering now an economy with two time periods and uncertainty, which are the basic characteristics of financial economics. Think for now that the present t_0 is known with certainty and the future t_1 is uncertain; in time t_1 there are a set of possible states, or events, that might take place. We call this set of events $\Omega = \{1, 2, ..., S\}$, where ω denotes the event that took place. In this framework, an Arrow security $\mathbb{1}_{[\omega]}$ would pay 1 unit if state ω prevails at time t_1. In this sense, an Arrow security may be seen as an insurance policy that pays a sum of 1 in case a certain state ω prevails.

Reformulating the concept of Arrow securities to include n time periods and modify the set of possible states to include the different time periods as $\Omega = \{1, 2, ..., T\}$. We will call $\mathbb{1}_{[\tau]}$ the Arrow security that pays 1 unit in period t_τ and zero in any other time. Analogously, μ_τ would be the discount factor that relates the Arrow security $\mathbb{1}_{[\tau]}$ with its net present value.

In this way, following mathematical statement is derived:

$$\mathbb{1}_{[\tau]}(t) = \begin{cases} 1 & \text{if} \quad \tau = t \\ 0 & \text{if} \quad \tau \neq t \end{cases}$$

3.1.3 Discount Factors

The net present value of an Arrow Security can be obtained by using the payoff and its corresponding discount factor. For simplicity its convenient to start with a zero-coupon bond that pays 1000 unit with maturity of one year. Then, the net present value, denoted as P_1 becomes:

$$P_1 = (1000)(\mu_1)$$

Noticing that the face value of the bond, 1000, can be nominally changed to become a basic bond paying 1 unit at maturity. This assumption is commonly known as perfect divisibility in financial markets. We shall rephrase this last equation by denoting p_1 the net present value of a basic bond paying one unit at maturity, one year into the future, as:

$$p_1 = \mu_1$$

In this last equation is rather obvious to notice that the net present value of a zero-coupon bond is nothing else than a discount factor. This relationship is exploited further on to construct the zero-coupon yield curves that are frequently used to price diverse financial assets.

From now on it will be understood that p_i is the net present value of an Arrow Security that pays one unit at time $t = i$ and zero in any other time $t \neq i$.

For the current usage of Arrow Securities assume that the unit would be payed with certainty, meaning that the discount rate is not stochastic. In this same framework we can introduce the notion of a risk-free interest rate r_i and its relationship with the net present value p_i:

$$p_1 = \frac{1}{1 + r_1} = \mu_1$$

$$p_2 = \frac{1}{(1 + r_1)(1 +_1 r_2)} = \mu_2$$

where r_1 is the risk-free interest rate to be payed for an investment with maturity one made today and where ${}_1r_2$ is the risk-free interest rate of an investment with maturity one made one

year from now. It is noteworthy to acknowledge that ${}_1r_2$ is not known at time $t = 0$.

Another relationship can be obtained by dividing the first equation over the second we derive:

$$\frac{\mu_1}{\mu_2} = 1 +_1 r_2$$

The sequential no arbitrage condition for risk-free assets states that $1 = \mu_0 > \mu_1 > \mu_2 > ... > \mu_T$, giving as a result that $1 = p_0 > p_1 > p_2 > ... > p_T$. If this condition would not hold, then an investor would be able to sell an asset at a larger price today and buy it a lower price tomorrow with certainty. Such a strategy would counterfeit the common logic that one unit of currency today is worth more than one unit of the same currency tomorrow.

The no arbitrage price of any bond can be computed as the present value of all the future payments discounted at their corresponding discount rate. Extending the notion of bonds to include coupon payments, we can price these assets as the sum of the net present value of the stream of future coupon payoffs. In this sense, assume we have the price of zero-coupon bonds $p_1, p_2, ..., p_T$ and we want to price a coupon that pays 1 unit at maturity and C units on annual coupons. The net present value B of the bond would become:

$$B = p_T + \sum_{t=1}^{T} p_t C_t$$

3.1.4 Interest Rates

Interest rates are an equivalent method of the discount rates. The key difference between both techniques lies in the application of the rates in order to bring future payments to net present value: interest rates divide the quantities (adding one unit to the rate) while discount rates multiply the payments.

In this framework, assume that r_1 is the interest rate payable for an investment made today for a zero-coupon bond with maturity of one year. Then we can write the relationship of the basic bond price and the one year interest rate as follows:

$$p_1 = \mu_1 = \frac{1}{1 + r_1}$$

Assume now that there is zero-coupon bond with maturity of two years, which pays a total interest rate of ρ_2; then r_2 would be the annualized compound interest rate such that, if held constant throughout the whole time to maturity, would match the price of the bond. The mathematical formulation is shown:

$$p_2 = \mu_2 = \frac{1}{1 + \rho_2} = \frac{1}{(1 + r_2)^2}$$

Analogous representations can lead to formulate a basic bond for any time to maturity with respect to the corresponding interest rate:

$$p_\tau = \mu_\tau = \frac{1}{1+\rho_\tau} = \frac{1}{(1+r_\tau)^\tau}$$

where r_τ is the geometric mean of the interest rates that is assumed will prevail in the time to maturity of the financial asset.

$$(1+r_\tau)^\tau = (1+r_1)(1+_1 r_2)...(1+_{\tau-1} r_\tau)$$

and

$$r_\tau = [(1+r_1)(1+_1 r_2)...(1+_{\tau-1} r_\tau)]^{1/\tau} - 1$$

Once that an investor knows a constant interest rate, the valuation of financial assets becomes a standard routine. Suppose an investor wants to price a security that pays yearly coupons of C and pays a face value FV at maturity T years from now. The mathematical derivation is for the security price P is:

$$P = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + ... + \frac{C}{(1+r_T)^T} + \frac{FV}{(1+r_T)^T}$$

If the interest rate is assumed to be held constant for the entire time to maturity at a rate r, the expression can be simplified algebraically as the pricing equation:

$$P = \frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{FV}{(1+r)^T}$$

3.1.5 Complete markets

An important term to define is replicable assets. An asset is said to be replicable whenever it can be exactly composed by the linear combination of other available assets in its same financial market. One simple example is to think of a bond that pays one unit at maturity (two years from now) and c units for concept of annual coupons. This same bond can be replicated by buying c one year bonds and $1+c$ two year bonds.

The application of the latter asset pricing formula has an inherent limitation: we assume that the market includes all the basic zero-coupon bonds $p_1, p_2, ..., p_T$.

A market is considered to be complete whenever there are at least the same number of linearly independent assets as there are possible states $\Omega = \{1, 2, ..., T\}$. A linearly independent asset is one that cannot be replicated. Therefore a complete market is such that contains as many non-replicable financial assets as future payment dates.

Assume that there are three securities traded in the market and our economy consists of the present plus three future periods. The first security is a zero-coupon bond that pays 1000 units and is traded for 985 units. The second security has a two year maturity and pays annual coupons of 50 units plus 1000 units at maturity; it is priced at 1080 units. Finally, the third security is a three year bond that pays annual coupons of 70 units plus 1000 units at maturity; the price of this security is 1175 units.

The procedure to find the price of the basic bonds p_1, p_2, p_3 is to replicate the securities in the following way:

$$1000p_1 = 985$$

$$50p_1 + (1000 + 50)p_2 = 1080$$

$$70p_1 + 70p_2 + (1000 + 70)p_3 = 1175$$

Applying basic linear algebra an investor can solve the system of three equations with three variables. A matrix representation can simplify the system as:

$$\begin{bmatrix} 0 & 0 & 1000 \\ 0 & 50 & 1050 \\ 70 & 70 & 1070 \end{bmatrix} = \begin{bmatrix} 985 \\ 1080 \\ 1175 \end{bmatrix}$$

where the columns of the matrix on the left side are p_3, p_2, p_1 respectively; the matrix in the right side contains the market prices of the securities.

As a result, $p_1 = 0.985$, $p_2 = 0.915$, and $p_3 = 0.814$, which is in line with the sequential no arbitrage rule $p_1 < p_2 < p_3$.

Notice that whenever we have the prices for all basic bonds $p_1, p_2, ..., p_T$ we can conclude that the market is complete. Nonetheless, there are generally no zero-coupon bonds for maturities larger than two years. The bonds that are actually available for longer maturities are coupon paying bonds. Nonetheless, we can use all the longer maturity coupon bonds as long as they are linearly independent among themselves.

The advantage of complete markets is their capacity of replicating all basic bonds $\{p_1, p_2, ..., p_T\}$; in turn, by linearly combining the complete set of basic bonds, an investor can replicate and price any financial asset traded in the market.

3.2 Yield rates and Term structure

There are two key aspects that must be in every investor's mind whenever judging the level of a yield rate: the intertemporal value of money and the risk-adjustment that must be adjudicated

to the interest rate.

Section 2.3 dealt with risk-free rates in an environment of certainty. Even when the notion of a completely risk-free asset is not universally accepted, we can work for now with this assumption whenever we are dealing with government securities. For example, in the US, Treasury Bills are understood as practically being risk-free assets; this is given by the idiosyncrasy that the United States is not going to default on its debt. Nonetheless we can explore ancient and recent events where economies were in the verge of defaulting their debt, such as the cases of Mexico in 1994 or Greece in 2010.

The importance of such explanation is a stepping stone for defining the term structure. It is possible to understand term structures as the evolution of yield rates as a function of their time to maturity. Interest rates respond both risk and time to maturity, therefore it is crucial that whenever constructing a term structure, all yield rates are equally risk-adjusted.

Consider a term structure derived from government bonds. These yields differ only in the time to maturity, while the risk is considered in this case to be null. In this manner, there is no credit risk spread between yields, making them perfectly comparable. Moreover, public debt inside a financial market is considered to be one of the most liquid and most highly traded securities; therefore the price of debt reflects all available information in the market. As a matter of fact, the yield rate on public debt is taken as a benchmark to assess the profitability of diverse financial investments in private and corporate debt. As a consequence of all the previous asseverations, the yield rate on government bonds in a fundamental tool in asset pricing.

It is important to acknowledge that the term structure can possess any shape and it may variate on a daily basis. Marin and Rubio (2001) consider the term structure to be one of the most appropriate variables in order to predict the real output on an economy. The shape of the yield curve may hide significant hints in the future development of the economic activity. For instance, empirical evidences from Estrella and Hardouvelis (1991) and subsequent authors suggest that a flattened or inverse yield curve may accurately predict recessions in the short run.

The most common scenario in interest rates and yield curves is that $\mu_1 > \mu_2 > ... > \mu_t$ and so $r_1 < r_2 < ... < r_\tau$, giving as a result that ${}_{\tau-1}r_\tau > 0$ for any $\tau \in \{1, 2, ..., t\}$. Nevertheless, it is not uncommon to see that the yield curves flattens or even invert ts shape, making the relation $\mu_1 < \mu_2 < ... < \mu_t$ invalid and not necessarily true (the foundations of this phenomenon are explained later on this work). Whenever a case like this prevails, the future interest rate ${}_{\tau-1}r_\tau$ is no longer assumed to be strictly positive, which can be used by financial economists as a warning signal that the future output of the economy will have a negative growth rate. It is also worthwhile to recognize the fact that short term interest rates have higher volatility than their long term counterparts. Such a condition is a result of the first type of rates being set by the Central Bank, while the former are smoothed by the market forces.

The notion behind the link between yield curves and future economic output comes from the conceptualization of central banks to increase the short run interest rates whenever attempting to follow a restrictive monetary policy in order to control future inflation. Following this example, suppose there are expectations of a future increase in the inflation rate. This means that the long end of the curve will rise as investors expect higher yields in order to be profitable for them to invest in the long run. Facing this situation, the Central Bank might be inclined to raise the short term interest rate by decreasing the money supply. In this case, the short term interest rates will move up, naturally; the long term interest rates, however, will decrease given that the expectation of inflation has been addressed, therefore the long end of the curve will go down. The result is a flattened, or inverse, curve that is linked to a restrictive economic policy and a short-run decline in the economy's output.

3.2.1 Dilemma: Zero-Coupon Curve or Yield to Maturity?

The Yield To Maturity (YTM), or Internal Rate of Return (IRR), is the interest rate such that, when applied to discount all future cash flows, matches the net present value of the future payments with the trade price of the financial investment. In this sense, assume we have an asset that is priced in the market at P and it presents yearly coupon payments of C for the next T years and a face value of FV at maturity. There are at least three different styles to represent this investment:

$$P = p_1 C + p_2 C + ... + p_T C + p_T FV$$

where p_i represents the basic bond p_1 that pays one unit whenever $t = i$.

$$P = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + ... + \frac{C}{(1+r_T)^T} + \frac{FV}{(1+r_T)^T}$$

where r_t is the compound interest rate for different times to maturity. It can be understood that r_t is the annualized interest rate earned on a zero-coupon bond traded today with a time to maturity of t years. And finally

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^T} + \frac{FV}{(1+i)^T}$$

where i is the yield to maturity or IRR of the security.

A distinction must be made between the geometric mean of interest rates r_t and the yield to maturity (IRR) rate i, which is in fact a weighted geometric mean; where the weights are the cash flows payed at each point in time.

A simple, yet clarifying, example can be made considering one market with two bonds. Bond A pays annual coupons of 40 units plus 1000 units at maturity and Bond B pays annual

coupons of 80 units plus 1000 units at maturity, both maturing three years from now. Bond A is priced today at B_A and Bond B is traded for B_B. Assume for now that the future interest rates are known and are such that $r_1 = 1\%$, ${}_1r_2 = 2\%$, and ${}_2r_3 = 3\%$.

The price of the basic bond p_3 is given by:

$$p_3 = \frac{1}{(1.01)(1.02)(1.03)} = 0.9424$$

and the interest rate geometric mean can be derived as:

$$p_3 = 0.9424 = \frac{1}{(1 + r_3)^3}$$

and so

$$r_3 = \left(\frac{1}{p_3}\right)^{1/3} - 1 = 1.9967\%$$

The no-arbitrage price of the bonds can be derived as follows:

$$B_A = \frac{40}{(1.01)} + \frac{40}{(1.01)(1.02)} + \frac{40}{(1.01)(1.02)(1.03)} + \frac{1000}{(1.01)(1.02)(1.03)} = 1058.5408$$

$$B_B = \frac{80}{(1.01)} + \frac{80}{(1.01)(1.02)} + \frac{80}{(1.01)(1.02)(1.03)} + \frac{1000}{(1.01)(1.02)(1.03)} = 1174.6687$$

Therefore, we shall now proceed to derive the Yield to Maturity rate i_A for Bond A and i_B for Bond B.

$$B_A = \frac{40}{(1 + i_A)} + \frac{40}{(1 + i_A)^2} + \frac{40}{(1 + i_A)^3} + \frac{1000}{(1 + i_A)^3} = 1058.5408$$

$$\Rightarrow i_A = 1.970945\%$$

$$B_B = \frac{80}{(1 + i_B)} + \frac{80}{(1 + i_B)^2} + \frac{80}{(1 + i_B)^3} + \frac{1000}{(1 + i_B)^3} = 1174.6687$$

$$\Rightarrow i_B = 1.949266\%$$

A very important relationship can be inferred from the previous results: there is a negative relationship between the Yield to Maturity of the bond and the Coupon Rate it bears. In this sense, the large the coupon the lower the YTM. This result is the fundamental argument of financial economists to construct the yield curve from basic bond prices, i.e. zero-coupon bonds.

$$\frac{\partial B}{\partial C} > 0$$

and

$$\frac{\partial i}{\partial B} < 0$$
$$\Rightarrow \frac{\partial i}{\partial C} < 0$$
$$\Rightarrow arg\max_{C \geq 0} i(C) = 0$$

The logical argumentation of why the relationship is negative can be addressed from the fact that a larger coupon rate would push the price of the bond up. A higher price for a same bond would then yield a lower Internal Rate of Return. Following this reasoning, it is logical to assume that zero-coupon bonds offer the highest YTM rate for a certain maturity.

3.3 Construction of the Yield Curve

Building the term structure through the yield curve is the core of the present financial economics literature review. The cornerstone that enables a yield curve to be built is the set of basic bond prices $\{p_1, p_2, ..., p_T\}$ and their corresponding zero-coupon yield to maturities $\{r_1, r_2, ..., r_T\}$.

3.3.1 Iterative procedure

Assume for now that a market has T securities that are traded for T different maturity dates. More specifically, there is an asset traded today that matures at the end of each year from $t \in \{1, 2, ..., T\}$. For simplicity, consider each of these securities as a par-value bond. A par value bond is a security which emission price is the same as its face value payable at maturity. A very useful condition of par-valued bonds is that the coupon rate r_c is the same as the yield to maturity i. In case that the bonds are not par-valued the YTM can be computed for every single security using a variety of methods (e.g. Newton-Raphson, Secant or Bisection).

Consider a set of par-valued bonds with face value of 1000 units and the following coupon rates or yield to maturities.

Time to Maturity	YTM
1	5%
2	5.5%
3	6%
4	6.5%
...	...
T	10%

Recall the no arbitrage net present value formula:

$$B = p_T + \sum_{t=1}^{T} p_t C_t$$

In order to compute the price of the first basic bond, p_1 we apply:

$$1000 = p_1 50 + p_1 1000$$
$$p_1 = \frac{1000}{1050} = 0.9524$$

its corresponding zero-coupon YTM is:

$$r_1 = \frac{1}{p_1} - 1 = 5\%$$

For the second basic bond, p_2:

$$1000 = p_1 55 + p_2 55 + p_2 1000$$
$$p_2 = \frac{1000 - (0.9524)(55)}{1055} = 0.8982$$

whose zero-coupon YTM is:

$$r_2 = \left(\frac{1}{p_2}\right)^{1/2} - 1 = 5.5138\%$$

The third basic bond price p_3 is computed analogously:

$$1000 = p_1 60 + p_2 60 + p_3 60 + p_3 1000$$
$$p_3 = \frac{1000 - (0.9524)(60) - (0.8982)(60)}{1060} = 0.8386$$

giving a zero-coupon YTM of:

$$r_3 = \left(\frac{1}{p_3}\right)^{1/3} - 1 = 6.0410\%$$

The process can be further iterated until every single basic bond price p_t with maturity $t \in \{1, 2, ..., T\}$ is computed from the securities available in the market. The price of the T-th basic bond p_T can be computed as:

$$p_T = \frac{FV - \sum_{t=1}^{T-1} p_i C_T}{FV + C_T}$$

where FV is the face par-value and C_T is the coupon rate payable annually for bond B_T and remains constant throughout the life of the security.

The T-th zero-coupon yield to maturity is swiftly obtained from p_T as:

$$r_T = \left(\frac{1}{p_T}\right)^{1/T} - 1$$

The final result of this iterative process is a zero-coupon yield curve composed by the nodes $r_1, r_2, ..., r_T$ which can be used to price any financial asset whose cash flows are considered to risk-free. In other sense, this zero-coupon yield curve can be understood as the risk-free rate curve of the economy we are studying.

3.3.2 Ordinary Least Squares (OLS) Method

A more elaborate and sophisticated procedure that was proposed by McCulloch (1971) to compute the zero-coupon yield curve consists in applying OLS to the sample of K bonds that are traded in the same market at certain point in time. Given that the procedure compute as many basic bond prices as future dates of coupon payment, it is required that $K \geq T$, where T is the largest time to maturity.

Following the notation of Marin and Rubio (2001) denote B_{kT_k} the price of the K-th coupon-paying bond with with coupon C_k and maturity date T_k. Let ε_k be a random white noise disturbance. For $k \in \{1, 2, ..., K\}$:

$$B_{kT_k} = p_1 C_k + p_2 C_k + ... + p_{T_k}(FV + C_k) + \varepsilon_k$$

In this model it is required to estimate T parameters. In order to reduce the number of coefficients to be estimated, the price of all generic basic bonds p_t will be replaced with a discount function that depends on time to maturity, i.e $p_t = p(t)$. Such function $p(t)$ is a linear combination of known functions $f_j(t)$, $j \in \{1, 2, ..., J\}$ which will be later specified, in the sense that any basic bond p_t can be represented as:

$$p_t = p(t) = 1 + \sum_{j=1}^{J} a_j f_j(t),$$

where a_j, $j \in \{1, 2, ..., J\}$ are the coefficients to estimate for the zero-coupon yield curve. The face value FV is known and C_k is constant along time. Substituting this new concepts into the no arbitrage net present value formula:

$$B_{kT_k} = FV p_{T_k} + \sum_{t=1}^{T} p_t C_k = C_k \sum_{t=1}^{T} \left(1 + \sum_{j=1}^{J} a_j f_j(t)\right) + FV \left(1 + \sum_{j=1}^{J} a_j f_j(T_k)\right)$$

then

$$B_{kT_k} - T_k C_k - FV = \sum_{j=1}^{J} a_j \left(C_k \sum_{t=1}^{T} f_j(t) + FV f_j(T_k)\right)$$

Denote the two following identities, where all components are either known values or known functions:

$$q_k \equiv B_{kT_k} - T_k C_k - FV$$

$$X_{kj} \equiv C_k \sum_{t=1}^{T} f_j(t) + FV f_j(T_k)$$

And so for $k \in \{1, 2, ..., K\}$

$$q_k = \sum_{j=1}^{J} a_j X_{kj} + \varepsilon_k$$

There is considerable statistical advantage with this representation whenever applying the OLS method, given that in this model we only need to estimate J coefficients instead of T as in the previous representation.

Extending the definition of the $f_j(t)$ functions, McCulloch (1971) proposed piecewise spline techniques as good candidates. Other options proposed as suitable candidates are polynomial functions of degree-j . An advantage of polynomial functions is that, given a sufficiently large degree, they can accurately approximate any function. In this sense, Marin and Rubio (2001) suggest:

$$f_j(t) = t^j; j = \{1, 2, ..., J\}$$

where empirically a value of $J = 3$ has been suggested as a sufficiently large degree to do suitable cubic spline approximations.

Summing up this OLS method, assume we choose a polynomial function of degree $J = 3$ as a candidate for $f_j(t)$. Then, the linear regression system whose parameters must be estimated for $k \in \{1, 2, ..., K\}$ becomes:

$$B_{kT_k} = a_0 \left(\sum_{t=1}^{T_k} C_{kt} \right) + a_1 \left(\sum_{t=1}^{T_k} tC_{kt} \right) + a_2 \left(\sum_{t=1}^{T_k} t^2 C_{kt} \right) + a_3 \left(\sum_{t=1}^{T_k} t^3 C_{kt} \right) + \varepsilon_k$$

3.3.3 Theories regarding the interest rate Term Structure

Prior to defining the three broader hypothesis that attempt to explain the constitution and behavior of the term structure, an important concept must be introduced: forward rates.

A forward is a financial derivative that can be defined as a contract made today to agree a transaction that will take place in the future; the underlying asset, quantity, price and date are all determined at time zero.

Suppose that such type of contract is agreed upon and the terms state that on time $t - 1$ a basic bond p_t that matures at time t must be delivered. What the receiver of the bond has secured is an interest rate to be earned from time $t-1$ to time t. This rate is called the forward rate f_t. The no arbitrage mathematical formulation for f_t can be expressed as:

$$(1 + r_t)^t = (1 + r_{t-1})^{t-1}(1 + f_t)$$

and can be iterated to become

$$(1 + r_t)^t = (1 + r_1)(1 + f_2)(1 + f_3)...(1 + f_t)$$

Using basic bonds, it can be defined as

$$(1 + f_t) = \frac{p_{t-1}}{p_t}$$

I. Pure Expectation Hypothesis

The first theory suggests that the implied forward rates f_t are actually the same as the expectation of the interest rates that will prevail in the future taken at time zero, i.e. ${}_{t-1}r_t$. So, for any t:

$$f_t = \mathbb{E}_{\{0\}}[{}_{t-1}r_t]$$

According to this theory, the reason why the yield curve has an increasing behavior with respect to time is given by the expectation of investors that future interest rates will be higher than current interest rates.

Generalizing the former expression, for any times s, t with $s < t$, we can express:

$${}_sf_t = \mathbb{E}_{\{0\}}[{}_{t-s}r_t]$$

where ${}_sf_t$ is the forward rate between dates s and t and ${}_{t-s}r_t$ is the interest rate that will prevail during this same period.

In the expectation hypothesis, long term rates are determined by current short rates and expected future short rates. It is assumed that various maturities are perfect substitutes between each other and money can be invested in a short term asset and further reinvested without risk; this becomes a natural drawback for this hypothesis.

II. Liquidity Preference Hypothesis

The easiest way to understand liquidity preference is to recognize that investors want to be compensated for investing in long term securities. Given that interest rate risk and price volatility increases with time, an extra yield is demanded by investors.

Departing from the expectation hypothesis, for any times s, t with $s < t$, we can express:

$${}_sf_t = \mathbb{E}_{\{0\}}[{}_{t-s}r_t] +_s L_t$$

where ${}_sf_t$ and ${}_{t-s}r_t$ are the same as previously defined and the new variable ${}_sL_t > 0$ is the liquidity premium (or risk premium) demanded by investors between times $t - s$ and t.

Under this framework, investors prefer short term assets and recursive investment techniques over long term securities hold until maturity. The underlying motive is the opportunity to dispose of certain amount of cash with certainty at the end of each period for the contingent case that they might need liquidity. To break the stigma of short term investments being preferred to their long term counterparts, a positive liquidity premium must be offered to investors in order for them to hold long term assets until maturity.

III. Segmented Markets Hypothesis

The third and last commonly supported hypothesis establishes a distinction between the market for short term securities separately from the market of long term securities. Under this scope there is no relationship between short and long term rates whatsoever.

3.3.4 Summarizing the Term Structure Theories

A different approach is to sum up the general theories in two broader categories: expectation and segmentation hypothesis. Moreover, it is evident that in practice expected interest rates are not the same as the actual interest rates that prevail in future periods. This result conveys to asseverate that the Pure Expectation Hypothesis is not satisfied in the general scenario. Nevertheless, it is a strong anchor for the Liquidity Hypothesis to use as a starting point.

There is, however, a particular case when the long term interest rates are equal or even lower that the short rate interest rates; such a phenomenon is a deviation from the otherwise acceptable Liquidity Preference. Estrella and Hardouvelis (1991) studied such deviations and found empirical evidence that strongly supports the connection between short and long term rates as indicators for future economic output in the short-run.

4 Statistical Methods

4.1 VAR Models

A Vector Autoregression is an econometric technique which represents a generalization of the Autoregressive 'AR' processes to more than one time series. VAR's are linear time series models designed to capture the joint dynamics of multiple time series. All endogenous variables are formulated by an equation including its own lagged values, the lagged values of all other endogenous variables and an error term. The VAR can be seen as a special case of Seemingly Unrelated Regression 'SUR' models where the covariates are the same for all endogenous variables. It is a crucial remark that all the time series inputed in the VAR model must have the same order of integration $I(d)$.

The popularity of VAR models in economics is greatly due to the work of Sims (1980) and the work of Granger (1969) who explored the concept of causality in bivariate systems. The usefulness of this method exploits the notion that barely any prior knowledge must be known regarding the time series; it suffices with having a reasonable hypothesis of intertemporal relation between the time series.

The specification of a (bivariate) VAR model can be expressed as equations.

$$y_{1,t} = \alpha_1 + \beta_{1,1} y_{1,t-1} + \beta_{1,2} y_{2,t-1} + \varepsilon_{1,t}$$

$$y_{2,t} = \alpha_2 + \beta_{2,1} y_{1,t-1} + + \beta_{2,2} y_{2,t-1} + \varepsilon_{2,t}$$

Denote $\boldsymbol{Y_t}$ a $(n, 1)$ vector of variables at time t, $\boldsymbol{B_t}$ a matrix of parameters, $\boldsymbol{X_t}$ the matrix of lagged values and $\boldsymbol{\varepsilon_t}$ matrix of errors, where $\boldsymbol{\varepsilon_t} \sim N(0, \Omega)$. It is straightforward to represent the former equations by the means of a VAR(1) representation.

$$\boldsymbol{y_t} = \begin{bmatrix} \alpha_1 & \beta_{1,1} & \beta_{1,2} \\ \alpha_2 & \beta_{2,1} & \beta_{2,2} \end{bmatrix} \begin{bmatrix} 1 \\ y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

Simplified to matrix notation as

$$\boldsymbol{Y_t} = \boldsymbol{B X_t} + \boldsymbol{\varepsilon_t}$$

The generalized form of a *p-th* order VAR(p) model can be expressed as follows.

$$\boldsymbol{y_t} = \boldsymbol{c} + \boldsymbol{A_1 y_{t-1}} + \boldsymbol{A_2 y_{t-2}} + ... + \boldsymbol{A_p y_{t-p}} + \boldsymbol{\varepsilon_t}$$

where $\boldsymbol{c}$ is a vector matrix of intercepts (constants), the matrix of error terms $\boldsymbol{\varepsilon_t}$ is such that $\boldsymbol{\varepsilon_t} \sim N(0, \Omega)$ and matrices $\boldsymbol{A_i}$ are time invariant $\forall i \in \{1, 2, ..., p\}$.

Moreover, the vector of error terms satisfies the three following conditions:

$$\mathbb{E}[\boldsymbol{\varepsilon_t}] = 0$$

$$\mathbb{E}[\boldsymbol{\varepsilon_t}\boldsymbol{\varepsilon_t}'] = 0$$
$$\mathbb{E}[\boldsymbol{\varepsilon_t}\boldsymbol{\varepsilon_{t-k}}'] = 0, \forall k \neq 0$$

Lastly, it may be useful to specify the property of this method to represent any VAR(p) model as a VAR(1) model.

$$\begin{bmatrix} \boldsymbol{y_t} \\ \boldsymbol{y_{t-1}} \\ \vdots \\ \boldsymbol{y_{t-p+1}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{c} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{A_1} & \boldsymbol{A_2} & \cdots & \boldsymbol{A_{p-1}} & \boldsymbol{A_p} \\ \boldsymbol{I_n} & \boldsymbol{0_n} & \cdots & \boldsymbol{0_n} & \boldsymbol{0_n} \\ \boldsymbol{0_n} & \boldsymbol{I_n} & \cdots & \boldsymbol{0_n} & \boldsymbol{0_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0_n} & \boldsymbol{0_n} & \cdots & \boldsymbol{I_n} & \boldsymbol{0_n} \end{bmatrix} \begin{bmatrix} \boldsymbol{y_{t-1}} \\ \boldsymbol{y_{t-2}} \\ \vdots \\ \boldsymbol{y_{t-p}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon_t} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix}$$

4.2 Granger Causality

Granger (1969) proposed a method to assess the ability of one time series to forecast another one. A time series y is said to Granger-cause a time series x if the lagged values of y provide a statistical significant basis to forecast the future values of x.

Following the notation by Hamilton (1994) and limiting our attention to linear time series, we say that a time series y fails to Granger cause a time series x if:

$$MSE[\hat{\mathbb{E}}(x_{t+1}|x_t, x_{t-1}, \dots)] = MSE[\hat{\mathbb{E}}(x_{t+1}|x_t, x_{t-1}, \dots, y_t, y_{t-1}, \dots)],$$

where MSE stands for the mean standard error. In this sense the information on the past values of y brought no gain to the expectation of future values of x, making y non-informative about x, or analogously, making x exogenous of y.

Hamilton (1994) denotes a slightly more mathematically demanding definition of a Granger causality test:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} \phi_{1,1}^{(1)} & 0 \\ \phi_{2,1}^{(1)} & \phi_{2,2}^{(1)} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_{1,1}^{(2)} & 0 \\ \phi_{2,1}^{(2)} & \phi_{2,2}^{(2)} \end{bmatrix} \begin{bmatrix} x_{t-2} \\ y_{t-2} \end{bmatrix} + \cdots + \begin{bmatrix} \phi_{1,1}^{(p)} & 0 \\ \phi_{2,1}^{(p)} & \phi_{2,2}^{(p)} \end{bmatrix} \begin{bmatrix} x_{t-p} \\ y_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix},$$

Employing this representation, y fails to Granger cause x whenever the coefficient matrices Φ_j are lower triangular, where

$$\Phi_j = \begin{bmatrix} \phi_{1,1}^{(j)} & \phi_{1,2}^{(j)} \\ \phi_{2,1}^{(j)} & \phi_{2,2}^{(j)} \end{bmatrix}, \forall j \in \{1, 2, ..., p\}.$$

The lower triangular coefficient matrix representation is very convenient to express the expected value of the time series x on period ahead as:

$$\hat{\mathbb{E}}(x_{t+1}|x_t, x_{t-1}, \dots, y_t, y_{t-1}, \dots) = c_1 + \phi_{1,1}^{(1)} x_t + \phi_{1,1}^{(2)} x_{t-1} + \cdots + \phi_{1,1}^{(p)} x_{t-p}.$$

The expected value of x_{t+1} does not include any current or lagged values of y. Therefore, if all coefficient matrices Φ_j are lower triangular $\forall j \in \{1, 2, ..., p\}$ then y fails to Granger cause x.

A particular benefit of the former representation is the swift application of an econometric test to formally assess whether y is statistically significant to Granger cause (or not) x. Proceed to express x_t as a bivariate autoregressive equation of lag p and afterwards to do the same with an univariate autoregressive equation.

$$x_t = c_1 + \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + ... + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \beta_2 y_{t-2} + ... + \beta_p y_{t-p} + u_t$$

$$x_t = c_0 + \gamma_1 x_{t-1} + \gamma_2 x_{t-2} + ... + \gamma_p x_{t-p} + e_t.$$

Define RSS_1 and RSS_0 as the sums of squared residuals of the bivariate and the univariate autoregressive equations of x_t respectively for T number of observations.

$$RRS_1 = \sum_{t=1}^{T} \hat{u}_t^2, \qquad RRS_0 = \sum_{t=1}^{T} \hat{e}_t^2$$

The null hypothesis imposed on the coefficients of the lagged values of y_t in the bivariate autoregression and conduct an Ordinary Least Squares F test:

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_p = 0,$$

where the test statistic S is

$$S = \frac{(T - 2p - 1)(RSS_0 - RSS_1)}{(p)(RSS_1)}.$$

The null hypothesis, y fails to Granger cause x is rejected whenever S is larger that the 5% critical value of the $F(p, T - 2p - 1)$ distribution.

Critique of Granger causality

Hamilton (1994) provides a clear example in which the Granger causality runs in the inverse relation as the true causation for a model of stock prices and dividends. Thus, there are cases when Granger causality cannot be used as a direct causation. A second point to consider is that the test assesses whether or not a time series helps to forecast another, not necessarily meaning that it directly causes the other. Finally, one should never forget the omitted variable bias (OVB) when addressing Granger causality tests and the optimal selection of lags.

4.3 Factor Analysis

Factor analysis (FA) is a statistical technique of data reduction that aims to explain the underlying correlation between observable variables in terms of a smaller number of non-observable

factors. The FA method searches for independent latent factors such that the observable variables can be explained as a linear combination of the factors plus an error term by analyzing the joint variations of variables as responses to the unobservable factors. The name given to the consequence that follows the orthogonality between factors is called rotation.

Factor Analysis may be either an exploratory or confirmatory method that 'clusters' variables into groups with similar characteristics. A loading λ_{ij} for variable x_i and factor FA_j can be understood as a standardized regression coefficient between each other. Given that we are dealing with a standardized coefficient, the value of the parameter denotes the number of standard deviations that the dependent variable will change as a response to an increase of one standard deviation of the independent variable. For a more detailed explanation of Factor Analysis see [Wooldridge].

In order to derive the mathematical model denote x_i as the observed variables for $i = \{1, 2, ..., N\}$, F_j as the latent factors for $j = \{1, 2, ..., J\}$, F as the set of factors $\{F_1, F_2, ..., F_J\}$, λ_{ij} as the loading for variable x_i and factor FA_j and ε_i as the error term associated to the equation of variable x_i.

The following assumptions on the error terms are made:

$$\mathbb{E}[\varepsilon_i] = 0, \forall i \in \{1, 2, ..., N\}$$

$$var[\varepsilon_i] = \sigma_i^2, \forall i \in \{1, 2, ..., N\}$$

$$cov(\varepsilon_i, \varepsilon_k) = 0, \forall i \neq k$$

$$cov(F_j, \varepsilon_i) = 0, \forall i \in \{1, 2, ..., N\}, \forall j \in \{1, 2, ..., J\}$$

Moreover, x_i and x_k are conditionally independent

$$cov(x_i, x_j|F) = 0, \forall i \neq k$$

The expected value of the error is zero, the error term has constant variance and there is no correlation between error terms. Moreover there is no correlation between error terms and factor variables. Finally, observable variables are conditionally independent given the set of factors F; this means that observable variables are only related through their common relationship with the set of factors.

The model written as a set of equations can be understood as

$$x_1 = \lambda_{1,1}F_1 + \lambda_{1,2}F_2 + \cdots + \lambda_{1,J}F_J + \varepsilon_1$$

$$x_2 = \lambda_{2,1}F_1 + \lambda_{2,2}F_2 + \cdots + \lambda_{2,J}F_J + \varepsilon_2$$

$$\vdots$$

$$x_N = \lambda_{N,1}F_1 + \lambda_{N,2}F_2 + \cdots + \lambda_{N,J}F_J + \varepsilon_N$$

Two conditions must hold in the multiple factor analysis model:

$$corr(x_i, F_j) = \lambda_{i,j} \Rightarrow \lambda_{i,j} \in [-1, 1], \forall i, j$$

$$corr(F_j, F_k) = 0, \forall j \neq k$$

Lastly, the matrix notation can express the system of equations as

$$\boldsymbol{X} = \boldsymbol{\Lambda F} + \boldsymbol{\varepsilon}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & \cdots & \lambda_{1,J} \\ \lambda_{2,1} & \lambda_{2,2} & \cdots & \lambda_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{N,1} & \lambda_{N,2} & \cdots & \lambda_{N,J} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_J \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{bmatrix}$$

Commonality

The commonality of a variable x_i denotes how informative this is: a higher commonality represents a larger informative degree. This measure is computed as the sum of the squared loadings that correspond to the variable x_i. Denote $C(x_i)$ as the commonality of x_i.

$$C(x_i) = \sum_{j=1}^{J} \lambda_{i,j}^2$$

Variables with high commonality can be viewed as covariates that 'share' things in common with other covariates. The lower the commonality of a variable, then the higher the 'uniqueness' of it.

Number of Factors

The Kaiser rule is a method to obtain the optimal number of factors. Denote λ_i as the *i-th* eigenvalue of the correlation matrix of the observed variables $corr(\boldsymbol{X})$ and let $\overline{\lambda} = 1$. Then the number J_{kaiser} of factors according to the Kaiser rule is:

$$J_{kaiser} = \sum_i (\lambda_i > \overline{\lambda})$$

4.4 Methods for Identifying the Business Cycle

The economic business cycle is the upward and downward movement of the real Gross Domestic Product. Peak is the name given to a local maximum (turning point) and trough is the name for a local minimum (turning point). The length of the business cycle can be defined from

either peak to peak or trough to trough; the length of a phase is measured from peak to trough or from trough to peak. Whenever a phase goes from trough to peak we say the economy is in expansion; the other possible scenario is going from peak to trough, in which case it is said the economy is in recession. As a thumb-rule, business cycle phases must last a minimum of 2 quarters.

4.4.1 Bry-Boschan

Bry and Boschan (1971) developed an algorithm to define peaks and troughs in economic time series with monthly frequency. The following conditions state the conditions of the Bry-Boschan algorithm.

- Data frequency: Monthly
- Minimum length of a business cycle: 15 months
- Minimum length of a phase: 5 months
- Peaks and troughs are required to alternate: if two peaks or troughs are consecutive, then the one with largest absolute magnitude is chosen
- Any turning point in either the first or the last six months is not taken into account

Ahking (2013) finds that the Bry-Boschan algorithm is a consistent and suitable option to replicate the National Bureau of Economic Research (NBER) business cycle dates.

4.4.2 Harding-Pagan

Harding and Pagan (2002) generalized the Bry-Boschan (BB) algorithm for quarterly data; this method is also known as Quarterly-BB or BBQ. This modification is particularly helpful whenever trying to find the turning points in a quarterly times series of real GDP.

For a given quarterly time series y_t (which is generally the logarithm of the real GDP) and a phase length $k > 0$:

- y_t is a peak $\Leftrightarrow max(y_{t-k}, y_{t-k+1}, \cdots, y_{t-1}, y_{t+1}, y_{t+2}, \ldots, y_{t+k}) < y_t$.
- y_t is a trough $\Leftrightarrow min(y_{t-k}, y_{t-k+1}, \cdots, y_{t-1}, y_{t+1}, y_{t+2}, \ldots, y_{t+k}) > y_t$.

where k is usually fixed as $k = 5$ for monthly data and $k = 2$ for quarterly data. These values of k go in line with the Bry-Boschan algorithm and also with the NBER thumb-rule stating that business cycle phases should last at least two quarters.

A convenient way to represent business cycles is to use a binary variable, where 1 indicates that the economy is in recession and 0 whenever the economy is experiencing an expansion.

One advantageous characterization of the time series would allow to measure the movements of the real GDP Y_t in percentage terms by applying the transformation $y_t = 100 \log(Y_t)$. This technique comes to be quite handy when the researcher needs to do manual overrides on the length of a phase whenever the absolute value of a quarterly observation trespasses a predefined threshold.

4.5 Probit Models

Probit models belong to the family of Generalized Linear Models (GLM), which are an extension of the Ordinary Least Square Models. There are three characteristics inherent of the GLM models.

- The linear predictor $\eta_i = x_i^T \beta$ depends on k covariates x_i and β parameters.
- The expected response $\mu_i = \mathbb{E}[y_i|x_i]$ is linked to the linear predictor η_i through monotonic transformation $g(\mu_i)$ such that $g(\mu_i) = \eta_i$
- The distribution of $y_i|x_i$ is linear exponential function and not necessarily $N(0, \sigma^2)$ as in the OLS case

The function $g(\mu_i)$ is called the link function and for the Bernoulli distribution (Binomial family) we can define the link function as $g(\mu) = log\left(\frac{\mu}{1-\mu}\right)$.

The distribution of $y_i|x_i$ for binary dependent variables can be defined as in Kleiber and Zeileis (2008):

$$f(y;p) = \left\{(y) log\left(\frac{p}{1-p}\right) + log(1-p)\right\}, y \in \{0,1\}$$

The probit model can be expressed by the following equation, where Φ is the standard normal cumulative distribution function.

$$\mathbb{E}[y_i|x_i] = p_i = \Phi(x_i^T \beta), i = \{1, 2, ..., n\}$$

The derivation of the probit model is rather straightforward. Denoting as an auxiliary variable $z_i = x_i^T \beta + \varepsilon_i$ and defining the variable y_i in terms of the auxiliary variable as

$$y_i = \begin{cases} 1 & \text{if} \quad z_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

or as in terms of the covariates

$$y_i = \begin{cases} 1 & \text{if} \quad x_i^T \beta > -\varepsilon_i \\ 0 & \text{otherwise} \end{cases}$$

Then the following expressions can be derived:

$$P(y_i = 1|x_i) = P(z_i > 0)$$
$$P(y_i = 1|x_i) = P(x_i^T\beta > -\varepsilon_i)$$
$$P(y_i = 1|x_i) = P(\varepsilon_i < x_i^T\beta)$$
$$P(y_i = 1|x_i) = \Phi(x_i^T\beta)$$

4.5.1 MLE and marginal effects

The maximum likelihood estimator for β can be obtained as

$$\text{Ln}\mathcal{L}(\beta) = \sum_{i=1}^{n} (y_i \text{Ln}[\Phi(x_i'\beta)] + (1 - y_i)\text{Ln}[1 - \Phi(x_i'\beta)])$$

Kleiber and Zeileis (2008) demonstrate a way to define the average marginal effects as a function of the regressors. The effects in a probit regression are:

$$\frac{\partial \mathbb{E}[y_i|x_i]}{\partial x_{ij}} = \frac{\partial \Phi(x_i^T\beta)}{\partial x_{ij}} = \phi(x_i^T\beta) \cdot \beta_j$$

Thus, the sample mean marginal effect is:

$$\frac{1}{n}\sum_{i=1}^{n} \phi(x_i^T\hat{\beta}) \cdot \hat{\beta}_j$$

4.5.2 Goodness of fit

Pseudo–R^2

As mentioned by Kleiber and Zeileis (?) there is no generally accepted definition of R^2 measure for the family of generalized linear models. It is possible, however, to build a pseudo-R^2 by specifying a model with covariates and a model without covariates.

The McFadden (1973) pseudo R^2 can be obtained as a function of the log-likelihood of the model with predictors $\ell(\hat{\beta})$ and the log-likelihood of the model without predictors, only the constant term of the model, $\ell(\hat{\alpha})$.

$$R^2_{MF} = 1 - \frac{\ell(\hat{\beta})}{\ell(\hat{\alpha})}$$

Whenever working with more than one predictor the R^2 values tends to be overestimated, therefore it may be useful to implement an adjusted measure of the McFadden R^2 to penalize a model with K covariates for having too many predictors.

$$R^2_{MF-adj} = 1 - \frac{\ell(\hat{\beta}) - K}{\ell(\hat{\alpha})}$$

Youden's J-statistic

A second index for assessing the goodness of fit is the Youden's J-statistic, which combines the sensitivity and the specificity of the fitted model: $J = (\text{sensitivity} + \text{specificity} - 1)$.

$$\text{sensitivity} = \frac{\text{true positives}}{\text{true positives} + \text{false negatives}}$$

$$\text{specificity} = \frac{\text{true negatives}}{\text{true negatives} + \text{false positives}}$$

The higher the value of the Youden index $J \in [-1, 1]$ then the better the model fits the reality. A value of $J = 1$ means that the model predicts to perfection and a value of $J = 0$ means that the model is as good as randomizing.

Receiver Operating Characteristic: ROC curve

Another way to assess both in–sample and out–of–sample goodness of fit is the ROC curve. This method is a graphical depiction of the goodness of fit over an xy-plane where the x-axis denotes the false positive rate (FPR) and the y-axis denotes the true positive rate (TPR). In this sense, the closer the curve can be to the upper left corner, where $FPR = 0$ and $TPR = 1$, the better the model fits the sample data. Usually the ROC curve is compared against a diagonal line that goes from coordinates (0,0) to (1,1). This diagonal functions is understood as a model that performs 'as good as randomizing'. Ergo, if the ROC curve of the fitted model revolves around, or relatively close, to this diagonal then one can conclude that the model is a poor fit.

The Area Under the Curve (AUC) is a common usage of the ROC curve that enables researchers to compare between two or more probit models. In this sense, the model with the largest AUC can be assumed to have a better performance over the alternative proposals.

Quasi-complete separation

Although this is not a goodness of fit test, it is an important remark to establish. There is a possibility that a maximum likelihood estimator does not exist, as pointed by Kleiber and Zeileis (2008). Instead, there is a vector of parameters β_0 such that the dependent variable y_i can be (quasi) perfectly separated as

$$y_i = \begin{cases} 1 & \text{if} \quad x_i^T \beta_0 \leq 0 \\ 0 & \text{if} \quad x_i^T \beta_0 \geq 0 \end{cases}$$

5 Term Structure as predictor of Output

5.1 Building the model

A real-life example taken from Mankiw (2010) explores the events around the decision of Paul Volcker and the monetary tightening that took place in the 1980's. Volcker announced that the Federal Reserve would follow a monetary policy aimed to reduce inflation. In this sense, in the short run, the real money balances decreased and the nominal interest rates experienced an increase. This phenomenon in the short run goes in line with the liquidity preference theory. In the long run, however, the Fisher effect prevailed: given a diminished inflation rate and a lower expected inflation rate, the future inflation actually decreased; thereafter the nominal interest rates in the long run also experienced a favorable decline.

One of the most recurred explanations about the reason why yield spreads have a predictive power over the future output, as explored in Section 3, is given by the notion that yield curves tend to 'flatten' whenever there is a monetary policy tightening, which is itself related to a slowdown in the future levels of economic activity and inflation.

There are three key components to the construction of the model. The first piece is the IS curve, which can be derived from the optimization of the Euler Equation from a consumption problem faced by households. The IS model was introduced in Section 2 as the downward slopping curve which considers the Gross Domestic Product as the dependent variable and the interest rate as an independent term. The second part is a Phillips curve relating the current inflation to the expectation of future inflation and the current output. The Phillips curve, introduces as well in Section 2, shows an inverse relation between inflation rate and unemployment. In the New Keynesian Phillips Curve the responsiveness parameter of inflation with respect to current output is an inversely proportionate function of nominal price rigidity as in Calvo (1983). Lastly, the model requires a rule, or reaction function, of the nominal interest rate with respect to deviations from target inflation and the current output gap as in Taylor (1993).

The following model, derived by Estrella (2005), is based on analytical rational expectations. Despite the fact that the model does not imply that there is a structural relationship between term structures and future economic output, the yield curve still holds predictive power under most circumstances.

5.1.1 Backward-looking IS and Phillips curves

Svensson (1997) explores the mechanisms of inflation targeting used by regulatory institutions and states that Central Banks have imperfect control over inflation and can only effectively affect future rates of inflation. Under this line of action, the actual maneuver employed by the Central Bank is inflation forecast targeting, and not addressing the inflation itself. As a consequence, the inflation forecast becomes now an intermediate point. Svensson, in this same

article, proposes the usage of a simple model which relates the inflation rate and the output gap in a Vector-Autoregressive framework:

$$\pi_{t+1} = \pi_t + \alpha_1 y_t + \alpha_2 x_2 + \varepsilon_{t+1}$$

$$y_{t+1} = \beta_1 y_t - \beta(i_t - \pi_t) + \beta_3 x_t + \eta_{t+1}$$

$$x_{t+1} = \gamma x_t + \theta_{t+1},$$

where variables are in yearly basis; π_t stands for the inflation rate, y_t is the endogenous output gap (actual minus potential output), x_t is an exogenous variable, i_t is the interest rate used as monetary instrument and $\varepsilon_t, \eta_t, \theta_t$ are all independent and identically distributed (i.i.d.) shocks.

An important remark from this model is that the monetary instrument i_t affects the output y_{t+1}, which implies a one year lag between i and y, which in turn, has a one year lag between y and π. A second remark, suggested by Svensson, is the notion that today's inflation forecast two years into the future $\pi_{t+2|t}$ is more 'controllable' than the inflation two years ahead π_{t+2} itself. Given that the inflation forecast is highly correlated to the actual future inflation rate, it is an ideal intermediate measure or 'implicit instrument' to be used.

Rudebusch and Svensson (1998) extended the study of optimal policy rules to target inflation. They engage into a model consisting of two equations that relate the inflation and output of the United States in a reduced-VAR form:

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \alpha_4 \pi_{t-3} + \alpha_y y_t + \varepsilon_{t+1}$$

$$y_{t+1} = \beta_1 y_t + \beta_2 y_{t-1} - \beta_r(\overline{\iota_t} - \overline{\pi_t}) + \eta_{t+1},$$

where π_t is the annualized quarterly inflation of the GDP (chain-weighted) $\overline{\pi_t}$ is the average quarterly inflation rate of the last year (4 quarters), y_t is the output gap, i_t is the annualized quarterly federal fund interest rate and $\overline{\iota_t}$ is the average quarterly federal fund rate of the last year.

Estrella (2005) derives the *backward looking* IS and Phillips curves with two equations consistent with the VAR framework. As in Svensson (1997) removing the exogenous variable x_t and denoting $\rho_{t-1} = (i_t - \pi_t)$ or as in Rudebusch and Svensson (1998) limiting to a one period lag and replacing $\rho_{t-1} = (\overline{\iota_t} - \overline{\pi_t})$.

IS curve [Backward-looking]:

$$y_t = b_1 y_{t-1} - b_2 \rho_{t-1} + \eta_t$$

Phillips curve [Backward-looking]:

$$\pi_t = \pi_{t-1} + a y_{t-1} + \varepsilon_t$$

where ρ_{t-1} is now the long-term (two-period) real interest rate, y_t is the log-difference output gap, π_t is the one-period inflation rate and both ε_t and η_t are i.i.d. shocks. Considering as well the constraints on coefficients $0 < b_1 \leq 1$, $b_2 \geq 0$ and $a > 0$.

5.1.2 Forward-looking IS and Phillips curves

The formulation of the *forward-looking* IS and Phillips curves, as shown in Estrella (2005) can be derived from the work of Woodford (1996) and Roberts (1995).

Woodford (1996) expands the staggered prices model by Calvo (1983) where each period a percent α of producers are stuck with their prices and the remaining $1 - \alpha$ fraction set a new price. In this economy the probability of being chosen to change prices is independent and randomly computed. Each household seeks to maximize

$$\mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t [u(C_t^j + G_t) + \upsilon(M_t^j / P_t) - \omega(y_t(j))] \right]$$

where u and υ are increasing concave functions, ω is increasing convex, β is a discount factor between zero and 1, C_t^j is the consumption of household j at time t, M_t^j is the households' money balance, P_t is the price index of goods at time t, G_t is the index of public goods and $y_t(j)$ the household's supply of its product.

Woodford solves the model optimizing from the point of view of the households and derives the following four equations for the aggregate demand:

The intertemporal budget constraint:

$$\frac{\upsilon'(M_t/P_t)}{u'(Y_t)} = \frac{i_t}{1 + i_t}$$

The risk-free short term nominal interest rate must condition:

$$\beta \mathbb{E}_t \left[\frac{u'(Y_t + 1)}{u'(Y_t)} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1 + i_t}$$

A monetary policy regime that postulates a feedback rule to guide short-term (nominal) interest rates using as covariates the variations in inflation and output.

$$i_t = \Gamma(\pi_t, y_t)$$

And finally the law of motion for B_t

$$B_{t+1} = (1 + i_t)[B_t + P_t \Delta_t - (M_t - M_{t-1})]$$

where B_t is the nominal value of a portfolio composed by bonds owned by the household and Δ_t is the real primary deficit. $\Delta_t \equiv G_t - (T_t/P_t)$, where T_t is the net nominal tax obligations.

The linearized aggregate demand block can be derived solving the last four equations for $\{Y_t, i_t, M_t, B_t\}$ as deviations from their stationary values, yielding:

$$\hat{m_t} = \chi \left[\sigma^{-1} \hat{Y_t} - \frac{\beta}{1-\beta} \hat{i_t} \right], \tag{1}$$

$$\hat{Y_t} = \mathbb{E}_t[\hat{Y}_{t+1}] - \sigma(\hat{i_t} - \mathbb{E}_t[\hat{\pi}_{t+1}]), \tag{2}$$

$$\hat{i_t} = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y_t}, \tag{3}$$

$$\hat{b}_{t+1} = \hat{i}_t + \beta^{-1}(\hat{b}_t - \hat{\pi}_t) + (\beta^{-1} - 1)\hat{\Delta}_t + \gamma(\hat{m}_{t-1} - \hat{m}_t - \hat{\pi}_t), \tag{4}$$

where a circumflex over a variable denotes percent deviation of the variable with respect to its stationary value; σ is the elasticity of substitutions of intertemporal consumption, χ is the elasticity of demand for money with respect to the cost of holding money, ϕ_π and ϕ_y are the elasticities of the $\Gamma(\pi_t, Y_t)$ function (defined previously) with respect to both arguments and γ is the relative importance of money and bonds with respect to financial wealth.

Further more, we can defined the elasticities σ, χ and γ:

$$\sigma \equiv -\frac{u'(Y^*)}{u''(Y^*)Y^*}, \quad \chi \equiv -\frac{v'(m^*)}{v''(m^*)m^*}, \quad \gamma \equiv \frac{m^*}{\beta b^*}$$

where Y^*, m^* and b^* are the stationary values of each respective variable.

Roberts (1995) published a New Keynesian Phillips Curve obtained from a model with sticky prices and staggered contracts. In this framework, where prices are not fully-flexible, it is crucial to deal with the expectation of future prices.

Following the quadratic price adjustment model employed by Rotemberg (1982), where Ω is the adjustment cost, p_t is the log-price at time t and p_t^* is the log-price if adjustment prices were zero (ideal price set by the firm in a fully-flexible scenario), θ is a discount factor and c is a parameter that measures the ratio between the cost of adjusting the price versus the price of staying deviated from the optimum; the firms will attempt to minimize Ω_t:

$$\min_{\{p_t\}} \Omega = \mathbb{E}_t \sum_{\tau=t}^{\infty} \theta^{\tau-t}[(p_\tau - p_\tau^*)^2 + c(p_\tau - p_{\tau-1})^2]$$

where the FOC$[p_t]$ equalized to zero is

$$\mathbb{E}_t\left[(p_t - p_t^*) + c[(p_t - p_{t-1}) - \theta(p_{t+1} - p_t)]\right] = 0$$

Roberts (1995) assumes that $\theta = 1$ (which is approximately true for high frequency data) and the first order condition simplifies to

$$\Delta p_t = \mathbb{E}_t[\Delta p_{t+1}] - \frac{p_t - p_t^*}{c}$$

First, redefine $\pi_t = \Delta p_t$ as the inflation rate at time t. Then, define $p_t^* = \hat{p}_t + \beta y_t + \varepsilon_t$, where y_t is the percent output gap of the economy; $\hat{p}_t$ is the priced charged by other firms, which will be assumed to be $p_t = \hat{p}_t$, $\beta > 0$ and ε_t is an i.i.d. shock.

As a result:

$$\pi_t = \mathbb{E}_t[\pi_{t+1}] + (\beta/c)y_t - \varepsilon/c$$

Gali and Gertler (1999) studied the dynamics of short-run inflation and concluded that a New Keynesian Phillips Curve had the ability to accurately model the behavior of inflation. Clarida, Gali and Gertler (1999) explore the role of nominal price rigidities in monetary policy and Clarida, Gali and Gertler (1998) propose a sticky price baseline model for a forward-looking monetary policy. The equilibrium conditions of such model, after log-linearizing around the steady state as in Woodford (1996) are:

$$\pi_t = \delta \mathbb{E}_t[\pi_{t+1}] + \lambda(y_t - z_t), \tag{5}$$

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\sigma}(r_t - \mathbb{E}_t[\pi_{t+1}]) + g_t, \tag{6}$$

$$r_t^* = \beta \mathbb{E}_t[\pi_{t+1}] + \gamma x_t, \tag{7}$$

$$r_t = \rho r_{t-1} + (1 - \rho) r_t^*, \tag{8}$$

where π_t is the inflation rate at time t, y_t is the log output and z_t is the natural rate of output at time t, x_t is the average output gap, r_t^* is the target nominal rate when both inflation and output are at their corresponding target levels, r_t is the actual Fed Fund interest rate, g_t is an exogenous demand factor, δ and β are coefficients associated with the expected inflation and γ is the coefficient that measures the cyclical variable sensibility.

The latter block of equations is easy to analyze. Equation (5) defines the inflation rate as a function of expected future inflation and the deviance between the actual output y_t and the fully-flexible prices level of output z_t (natural rate). Equation (6) is the IS curve that links the output gap with the expected future output and the ex-ante real rate plus an exogenous demand. Equations (7) and (8) define the monetary policy rule of reaction.

Estrella (2005) takes the forward-looking IS curve as in Woodford (1996) defining $\rho_t = (i_t - \pi_t)$. The New Keynesian Phillips Curve is taken as in Roberts (1995) and renaming parameters the output gap as y_t:

IS curve [Forward-looking]:

$$y_t = \mathbb{E}_t[y_{t+1}] - \beta\rho_t$$

Phillips curve [Forward-looking]:

$$\pi_t = \mathbb{E}_t[\pi_{t+1}] + \alpha y_t$$

where ρ_t is the long-term (two-period) real interest rate, y_t is the log-difference output gap, π_t is the one-period inflation rate, $\beta > 0$ and $\alpha > 0$.

5.1.3 Reaction Function

The trade-off between Forward-Looking and Backward-Looking, as pointed by Estrella (2005) and Gali and Gertler (1999) is that the first ones have stronger theoretical argumentation, while the second approach fits better the time series.

Estrella (2005) generalizes the popular Taylor Rule (1993) by employing the following reaction function:

$$r_t = g_r r_{t-1} + g_\pi \pi_t + g_y y_t + (1 - g_r - g_\pi)\pi^*$$

The Taylor rule can be considered as a specific case of this reaction rule whenever $g_r = 0$, $g_y = 0.5$ and $g_\pi = 1.5$ and substituting into the function

$$r_t = \pi_t + \frac{y_t}{2} + \frac{\pi_t - \pi^*}{2},$$

where y_t is the log output gap, $\pi_t - \pi^*$ is the deviation of inflation with respect to its target value and r_t is the nominal interest rate i_t minus the target interest rate r_t^*. However, the target rate of inflation π^* is chosen in such way that the implied equilibrium real interest rate is zero; $r_t = (i_t - r_t^*) = i_t$.

It is clear to see that the chosen reaction function of nominal interest rate reacts to current gaps in output and also to deviations from the target inflation rate. The coefficients g_r, g_y and g_π measure the responsiveness of the rule with respect to each variable r_t, y_t and π_t. The expected values of these parameters are roughly defined as $0 \leq g_r \leq 1$, $g_y > 0$ and $g_\pi > 0$.

The Fisher equation relates the real interest rate and the nominal interest rate under an environment of inflation. Denote R_t as the two-period nominal rate. The Fisher equation links

the two-period nominal rate R_t as the sum of the real interest rate ρ_t plus an average of the of the expected inflation over the next to periods:

$$R_t = \rho_t + \frac{1}{2}\left(\mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[\pi_{t+2}]\right)$$

The expectations hypothesis can express the two-period nominal rate R_t as an average of the current nominal rate r_t and the future expected nominal rate $\mathbb{E}_t[r_{t+1}]$

$$R_t = \frac{1}{2}\left(r_t + \mathbb{E}_t[r_{t+1}]\right)$$

5.2 Solving the model

5.2.1 Backward-looking model

Employing the Phillips Curve as in Svensson (1997) the inflation expectation for the two following periods is derived, where by construction $a > 0$ and $b_2 > 0$.

$$\mathbb{E}_t[\pi_{t+1}] = \pi_t + ay_t$$

$$\mathbb{E}_t[\pi_{t+2}] = \mathbb{E}_t[\pi_{t+1}] + a\mathbb{E}_t[y_{t+1}]$$

Extending the IS curve from Rudebusch and Svensson (1998) one-step ahead we obtain

$$\mathbb{E}_t[y_{t+1}] = b_1 y_t - b_2 \rho_t,$$

taking the expected reaction function by Estrella (2005) one-step into the future as well

$$\mathbb{E}_t[r_{t+1}] = g_r r_t + g_\pi \mathbb{E}_t[\pi_{t+1}] + g_y \mathbb{E}_t[y_{t+1}] + (1 - g_r - g_\pi)\pi^*,$$

lastly, combining the Fisher Effect equation and the expectations hypothesis equation and solving for the two-period real interest rate ρ_t we can compute

$$\rho_t = \frac{1}{2}\left(r_t + \mathbb{E}_t[r_{t+1}] - \mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_t[\pi_{t+2}]\right).$$

The model must be solved for the five terms $\mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[\pi_{t+2}], \mathbb{E}_t[y_{t+1}], \mathbb{E}_t[r_{t+1}]$ and ρ_t in terms of the contemporaneous variables π_t, y_t, r_t and the inflation target π^*. The model is in equilibrium when the following conditions meet: $y_t = 0$, $\rho_t = 0$, $R_t = r_t$, $r_t = \pi_t$, and $\pi_t = \pi^*$; therefore, if all conditions are met, also $r_t = \pi^*$ must hold in equilibrium.

A vector of the deviations from the variables with respect to their corresponding equilibrium values, previously defined, can be constructed as $\boldsymbol{s_t} = (\pi_t - \pi^*, y_t, r_t - \pi^*)'$ and the vector of shocks is defined as $\boldsymbol{\xi_t} = (\varepsilon_t, \eta_t, g_\pi \varepsilon_t + g_y \eta_t)'$.

The system can be represented as a Vector Autoregression by the means of a transition matrix $\boldsymbol{T}$ derived from the expectations in terms of $\boldsymbol{s_t}$. Therefore, the VAR representation is

$$\boldsymbol{s_t} = \boldsymbol{T}\boldsymbol{s_{t-1}} + \boldsymbol{\xi_t}$$

which can be expanded into the exact expression

$$\begin{bmatrix} \pi_t - \pi^* \\ y_t - 0 \\ r_t - \pi^* \end{bmatrix} = \begin{bmatrix} 1 & a & 0 \\ \dfrac{(2-g_\pi)b_2}{2+(g_y-a)b_2} & \dfrac{(2-g_\pi)b_2 a + 2b_1}{2+(g_y-a)b_2} & \dfrac{-(1+g_r)b_2}{2+(g_y-a)b_2} \\ \dfrac{\lambda + 2g_y b_2}{2+(g_y-a)b_2} & \dfrac{a\lambda + 2g_y(ab_2+b_1)}{2+(g_y-a)b_2} & \dfrac{g_r(2-ab_2)-g_y b_2}{2+(g_y-a)b_2} \end{bmatrix} \begin{bmatrix} \pi_{t-1} - \pi^* \\ y_{t-1} - 0 \\ r_{t-1} - \pi^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \eta_t \\ g_\pi \varepsilon_t + g_y \eta_t \end{bmatrix}$$

where $\lambda = g_\pi(2 - ab_2)$ for display reasons.

Following the Schur-Cohn stability test conditions to assess whether all roots of the characteristic polynomial from $\boldsymbol{T}$ are inside the unitary circle, Estrella (2005) finds the necessary conditions for the system to be stationary.

Therefore, whenever $a > 0$ and $b_2 > 0$, the process $\boldsymbol{s_t} = \boldsymbol{T}\boldsymbol{s_{t-1}} + \boldsymbol{\xi_t}$ is stationary if and only if:

$$g_r > -1,$$
$$1 - g_r < g_\pi < B_\pi(g_r, g_y),$$
$$g_y > B_y(g_r)$$

where B_π and B_y are the bounds of the parameters for inflation and ouput, respectively, in order for the system to be stationary. The bounds can be mathematically expressed as

$$B_\pi = 2b_1(2-2b_1-ab_2)g_r^2 + (2+2b_1+ab_2)(2b_1+ab_2-2-b_2g_y)g_r + \frac{(2-ab_2+b_2g_y)(2-2b_1-ab_2+2b_2g_y)}{ab_2(2-ab_2+2b_1g_2+b_2g_y)}$$

and

$$B_y = a - \frac{(1-g_r)(1-b_1)}{b_2}$$

5.2.2 Forward-looking model

The solution of the forward-looking model is analogous to the prior result once we substitute for the IS curve as in Woodford (1996) and the Phillips Curve for the next two periods as in Roberts (1995):

$$\mathbb{E}_t[y_{t+1}] = y_t - \beta\rho_t$$

$$\mathbb{E}_t[\pi_{t+1}] = \pi_t - \alpha y_t$$
$$\mathbb{E}_t[\pi_{t+2}] = \mathbb{E}_t[\pi_{t+1}] - \alpha \mathbb{E}_t[y_t]$$

In this system, as well as the other, both $a > 0$ and $b_2 > 0$ by construction. Moreover, the coefficients in both approaches can be linked by the definitions: $a = -\alpha$, $b_1 = 1$ and $b_2 = \beta$.

Analogous to the backward-looking approach the same expected reaction rule and the same two-period real interest rate function are employed.

$$\mathbb{E}_t[r_{t+1}] = g_r r_t + g_\pi \mathbb{E}_t[\pi_{t+1}] + g_y \mathbb{E}_t[y_{t+1}] + (1 - g_r - g_\pi)\pi^*$$
$$\rho_t = \frac{1}{2}\left(r_t + \mathbb{E}_t[r_{t+1}] - \mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_t[\pi_{t+2}]\right)$$

The new model involves the same five terms $\mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[\pi_{t+2}], \mathbb{E}_t[y_{t+1}], \mathbb{E}_t[r_{t+1}]$ and ρ_t in terms of π_t, y_t, r_t and π^*. The equilibrium conditions are identical and are summarized in $y_t = 0$, $\pi_t = \pi^*$ and $r_t = \pi^*$. Therefore we can reuse the VAR representation previously defined as the deviations from equilibrium $\boldsymbol{s_t} = (\pi_t - \pi^*, y_t, r_t - \pi^*)'$.

The forward-looking system can be represented as a Vector Autoregression by the means of a transition matrix $\hat{\boldsymbol{T}}$. Therefore, the VAR representation for the expectation of $\boldsymbol{s_{t+1}}$ taken at time t is

$$\mathbb{E}_t \boldsymbol{s_{t+1}} = \hat{\boldsymbol{T}} \boldsymbol{s_t}$$

which can be expanded into the exact expression

$$\begin{bmatrix} \mathbb{E}_t[\pi_{t+1} - \pi^*] \\ \mathbb{E}_t[y_{t+1} - 0] \\ \mathbb{E}_t[r_{t+1} - \pi^*] \end{bmatrix} = \begin{bmatrix} 1 & -\alpha & 0 \\ \dfrac{(g_\pi - 2)\beta}{2 - (g_y + \alpha)\beta} & \dfrac{(2 - g_\pi)\beta\alpha + 2}{2 - (g_y + \alpha)\beta} & \dfrac{(1 + g_r)\beta}{2 - (g_y + \alpha)\beta} \\ \dfrac{g_\pi(2 - \alpha\beta) - 2g_y\beta}{2 - (g_y + \alpha)\beta} & \dfrac{-\alpha g_\pi(2 - \alpha\beta) + 2g_y(\alpha\beta_2 + 1)}{2 - (g_y + \alpha)\beta} & \dfrac{g_r(2 - \alpha\beta) + g_y\beta}{2 - (g_y + \alpha)\beta} \end{bmatrix} \begin{bmatrix} \pi_t - \pi^* \\ y_t - 0 \\ r_t - \pi^* \end{bmatrix}$$

where the transition matrices $\boldsymbol{T}$ and $\hat{\boldsymbol{T}}$ are formally identical defining the parameters as $a = -\alpha$, $b_1 = 1$ and $b_2 = \beta$.

Pursuing an homological representation, the VAR system for the forward-looking model can be expressed as

$$\mathbb{E}_t \boldsymbol{s_{t+1}} = \boldsymbol{T} \boldsymbol{s_t}$$

There is a drawback in this representation, given that for a stable solution to exist the transition matrix $\boldsymbol{T}$ is required to have two large roots. Estrella (2005) adds two necessary conditions for the system to be stationary.

$$1 - g_r < g_\pi,$$

$$g_\pi > B_\pi(g_r, g_y)$$

where $g_\pi > B_\pi(g_r, g_y)$ is a violation of the stationary conditions for the backward-looking case. An important remark by Estrella (2005) is that for both approaches, the Taylor rule ($g_r = 0, g_\pi = 1.5, g_y = 0.5$) is consistent with the stationarity conditions.

5.3 Conclusion on the macrofinancial model

The model solved in the previous subsection does not involve the term structure directly. The next step is to introduce the yield curve spread, defined as the difference between the two-period interest rate minus the one-period interest rate and relate this new variable to the expectations of future inflation and output. In this sense the financial theory argumentation on the ability of term spreads as output predictors, developed in Section 3, can be mathematically supported by an analytical model.

The term real interest rate ρ_t is replaced by the two-period nominal rate R_t. In order to do so, R_t is computed from the Fisher Effect equation in the form of:

$$\rho_t = -R_t + \frac{1}{2}\left(\mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[\pi_{t+2}]\right)$$

Recalling the five equations employed to solve the backward-looking model and substitute ρ_t with R_t. In this manner the system will be solved in terms of R_t and r_t.

$$\mathbb{E}_t[\pi_{t+1}] = \pi_t + ay_t$$

$$\mathbb{E}_t[\pi_{t+2}] = \mathbb{E}_t[\pi_{t+1}] + a\mathbb{E}_t[y_{t+1}]$$

$$\mathbb{E}_t[r_{t+1}] = g_r r_t + g_\pi \mathbb{E}_t[\pi_{t+1}] + g_y \mathbb{E}_t[y_{t+1}] + (1 - g_r - g_\pi)\pi^*$$

$$R_t = \frac{r_t + \mathbb{E}_t[r_{t+1}]}{2} + \mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[\pi_{t+2}]$$

$$\mathbb{E}_t[y_{t+1}] = b_1 y_t + b_2 \left(R_t - \frac{\mathbb{E}_t[\pi_{t+1}] + \mathbb{E}_t[\pi_{t+2}]}{2}\right)$$

Defining a new vector of deviations from equilibrium $\boldsymbol{x_t}$

$$\boldsymbol{x_t} = (\mathbb{E}_t[r_{t+1}] - \pi^*, \mathbb{E}_t[\pi_{t+1}] - \pi^*, \mathbb{E}_t[\pi_{t+2}] - \pi^*, \mathbb{E}_t[y_{t+1}], R_t - \pi^*)'$$

The system can be rewritten in matrix form and equalized to the former vector of deviations $\boldsymbol{s_t}$ through two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ as

$$\boldsymbol{A x_t} = \boldsymbol{B s_t}$$

Matrix $\boldsymbol{A}$ is full-rank and it is constructed in such a way that there exists a permutation matrix $\boldsymbol{P}$, a lower triangular matrix $\boldsymbol{L}$ and an upper triangular matrix $\boldsymbol{U}$ such that the LU decomposition gives the following results

$$\boldsymbol{LU} = \boldsymbol{PA}$$

$$\Rightarrow \boldsymbol{PAx_t} = \boldsymbol{PBs_t}$$

$$\Rightarrow \boldsymbol{LUx_t} = \boldsymbol{PBs_t}$$

$$\Rightarrow \boldsymbol{Ux_t} = \boldsymbol{L}^{-1}\boldsymbol{PBs_t}$$

where $\boldsymbol{U}$ is the unique upper triangular matrix with ones in the diagonal that generates a recursive solution for the system.

The result is R_t in terms of $\boldsymbol{s_t}$ only; $\mathbb{E}_t[y_{t+1}]$ in terms of R_t and $\boldsymbol{s_t}$; $\mathbb{E}_t[\pi_{t+2}]$ in terms of $\mathbb{E}_t[y_{t+1}]$, R_t and $\boldsymbol{s_t}$; $\mathbb{E}_t[\pi_{t+1}]$ in terms of $\mathbb{E}_t[\pi_{t+2}]$, $\mathbb{E}_t[y_{t+1}]$, R_t and $\boldsymbol{s_t}$; $\mathbb{E}_t[r_{t+1}]$ in terms of $\mathbb{E}_t[\pi_{t+1}]$, $\mathbb{E}_t[\pi_{t+2}]$, $\mathbb{E}_t[y_{t+1}]$, R_t and $\boldsymbol{s_t}$.

The unique form of equations that relate $\{\mathbb{E}_t[y_{t+1}] - 0\}$ and $\{\mathbb{E}_t[\pi_{t+2}] - \pi^*\}$ in terms of $\{R_t - r_t\}$ and $\boldsymbol{s_t}$ as derived by Estrella (2005) are presented here

$$\mathbb{E}_t[y_{t+1}] = \frac{2}{g_y}\{R_t - r_t\} + \frac{1-g_r}{g_y}(r_t - \pi^*) + \frac{g_\pi}{g_y}(\pi^* - \pi_t - ay_t)$$

and

$$\frac{1}{2}\mathbb{E}_t[\Delta\pi_{t+2}] = \frac{a}{g_y}\{R_t - r_t\} + \frac{a(1-g_r)}{2g_y}(r_t - \pi^*) + \frac{ag_\pi}{2g_y}(\pi^* - \pi_t - ay_t)$$

The former two equations relate the expectation of future output and the expectation of future inflation in terms of the yield spread $\{R_t - r_t\}$.

Where $\mathbb{E}_t[\Delta\pi_{t+2}] = \mathbb{E}_t[\pi_{t+2}] - \mathbb{E}_t[\pi_{t+1}]$ as proposed by Mishkin (1988).

It is now possible to discuss the implications of the Term Structure Model. The two final equations denote that, in general, there is a relation between the yield rate spread and the expected future output and inflation rates. The weight of this relationship is governed by the parameter g_y. It is also worthwhile to notice that the effect of the yield spread on the expectation of future inflation is affected by the coefficient a, which measures the response to the output gap in the Phillips curve. Lastly, given the presence of g_y, g_π and g_r in the equations, it is understood that the predictive power of the yield spread is affected by all three monetary policy parameters.

In order to extend the familiarity with the resulting model two extreme case scenarios are explored. In the first scenario assume that $g_r = g_\pi = 0$ and $g_y > 0$. Following these assumptions all terms disappear with the exemption of $\{R_t - r_t\}$. In this case the yield spread is the sole and only predictor of expected future output and inflation. It is, however, implausible to believe that a Central Bank would have a zero response to inflation and the assumption $g_\pi = 0$ is not viable. The second extreme scenario is whenever $g_y \to \infty$, $g_\pi \to \infty$ and $\frac{g_\pi}{g_y} \to k$, where k is a constant. In this case the term $\{R_t - r_t\}$ is diluted. Notice that the third condition is required in order to have at least one term when the equation is valued in the limit.

Given that the yield spread affects both the expectations of future output and inflation, let us assume that the Central Bank of an economy has a monetary policy such that their objective function to minimize is a weighted average of the deviations from target inflation $(\pi_{t+1} - \pi_t)$ and target output $(y_{t+1} - 0)$. In this way, assume an objective function of the form

$$\min_{\{g_y, g_\pi, g_r\}} \mathbb{E}_t \sum_{t=1}^{\infty} \theta^i \frac{1}{2} \left[(1-\omega)(\pi_{t+1} - \pi^*)^2 + (\omega y_{t+1})^2 \right]$$

where $\theta > 0$ is a discount factor and $\omega \in [0, 1]$. Estrella (2005) derives in the Appendix of his paper the solution to this objective function by solving the associated Bellman equation.

Consider first the case when $\omega = 0$, then the minimization problem focuses only on the deviation from target inflation. Whenever $\omega = 1$ the only factor to be concerned about becomes the output gap. Nevertheless, the majority of scenarios should take place for a value of $\omega \in (1, 0)$, yielding a function expressed as a weighted average of the deviations.

Estrella (2005) concludes that as the inflation target becomes 'more strict', a higher weight will be given to the spread as a predictor of the policy rule making. In this sense, in an economy with a Central Bank that has a strict inflation target, or where the Taylor Rule is applied, the term structure of the yield rates would have a higher importance in the prediction of future economic activity in the short run. Notwithstanding, the predictive power of the term structure depends on the monetary parameters g_y, g_r and g_π, therefore we cannot claim that the effect of the yield curve on the policy rule is structural even when it is robust. Nevertheless, the latter does not imply that the term spread indicator should be overlooked whenever the Central Bank is pursuing a regime different from inflation targeting.

Finally, whenever the Central Banks' policy rule reacts only to output deviations, or when it responds to both output and inflation deviations (as long as g_y and g_π do not tend to infinity) then it is safe to conclude under a rational expectations scheme that the term structure of the yield rates can be combined with other leading economic indicators to derive a powerful predictor of future economical activity.

6 Yield Spread Network Model

Fama and Bliss (1987) found that forward rates have predictive power from two to four years ahead given to the presence of mean-reversion inherent to interest rates. Fama (1990) later found empirical evidence that gave strong support to acknowledge a relationship between current variations of the yield rate spread with future changes in both inflation and interest rates. The paper concluded that a higher spread is associated with an increase of future inflation and a decrease of future interest rates.

Estrella and Hardouvelis (1991) proposed a probit model with quarterly data frequency in order to forecast future economic activity as a function of the spread between long rate an short rate interest rates. The paper found that there is a positive correlation between such spread and future economic output; in this sense a negative yield spread would help to forecast recessions. The underlying theoretical concepts arrive to the conclusion that whenever the yield curve flattens, or even inverts, the future interest rates are predicted to drop significantly: a drop in the interest rates is associated with a lower level of real GDP.

The regression equation employed by Estrella and Hardouvelis is of the form

$$Y_{t,t+k} = \alpha + \beta SPREAD_t + \sum_{i=1}^{N} \delta_i X_{it} + \varepsilon_t,$$

The spread term can be defined as:

$$SPREAD_t = R_t - r_t,$$

where R_t is the long–run (two period) interest rate and r_t is the short–run (one period) interest rate defined in Section 5. The dependent variable is the annualized cumulative change in the real GNP level, k defines the forecasting horizon and the spread at time t is defined as the difference between the long term and the short term nominal interest rates. The proposed covariates X_{it} for this model were the real federal funds rate, annualized growth in the list of index of leading indicators, the lagged inflation π_{t-k} and a term with lagged changes in the log GDP (log y_t - log y_{t-k}).

6.1 Base Model: One predictor

The base model to measure the probability of the recession is of the form

$$P[B_t = 1 | SPREAD_{t-4}] = \Phi(\alpha + \beta SPREAD_{t-4})$$

where $B_t = 1$ whenever the economy is experiencing a recession in quarter t and $B_t = 0$ if it is an expansion.

The log-likelihood function can be expressed as

$$\log \mathcal{L} = \sum_{B_t=1} \log[\Phi(\alpha + \beta SPREAD_{t-4})] + \sum_{B_t=0} \log[\Phi(1 - \alpha - \beta SPREAD_{t-4})]$$

Estrella and Hardouvelis (1991) found that the coefficient $\beta < 0$ with statistical significance of 5%, which goes according to the theory. A negative value for β yields the understanding that whenever the spread increases, the probability of a recession diminishes. Moreover, if the spread inverts its usual shape, then the double negative signs would make the probability of a recession to increase whenever the yield rate inverts. In addition to this, the yield spread outperformed (in-sample and out-of sample) the survey forecasts. Moreover, the slope predicted consumption, consumer durables and investment; these three altogether compose the private sector index of the GNP.

The rooted reasoning to explain this finding comes from how monetary policy affects the yield curve. Whenever a monetary contraction takes place, the nominal short term interest rate increases. In the case of price rigidities, this effect will also affect the real short term interest rate, taking down the investments made into the economy and therefore diminishing the short-term economic output. Thus, the slope of the yield curve is negatively correlated with future economic activity.

Jorion and Mishkin (1991) expanded the term-structure forecast model to four economies: US, Britain (UK), (West) Germany and Switzerland and concluded that the term structure had strong and statistically significant power to predict future inflation rates. Their research concluded that for all four economies the term structure slope is a powerful predictor of future inflation pressures, particularly in the long run. Nevertheless, they warn that whenever monetary policy changes may arise the predictive power of the term structure may be affected.

6.2 Financial variables as predictors of output

Estrella and Mishkin (1996) investigated the out-of-sample performances of different financial variables as possible predictors of economic output, specifically for the case of the United States. The financial time series tested as predictors can be classified into five aggregated categories:

- Interest Rates and spreads:
  - 10-yr minus 3-mo Treasury spread
  - Comm. paper minus 6-mo spread
  - 10-year T bond
  - 3-month T-bill
- Stock prices
  - Dow Jones Industrial average
  - NYSE composite
  - S&P 500
- Monetary Aggregates
  - Monetary base
  - M1

M2

M3

Monetary base deflated by CPI

M1 deflated by CPI

M2 deflated by CPI

M3 deflated by CPI

- Individual Macroeconomic Indicators

  Growth in real GDP; prev. quarter

  Consumer Price Index

  Purchasing managers' survey

  Vendor performance

  Contracts and orders (equipment)

  Housing permits

  Consumer expectations

  Trade-weighted dollar

  Change in manufacturers' orders

- Indexes of Leading Indicators

  Commerce Department lead Index

  Stock-Watson (1989) leading index

  Stock-Watson (1993) leading index

For more than one covariate, the base probit model becomes generalized to the form

$$P[B_t = 1|x_t] = \Phi(\beta' x_t)$$

where$B_t = 1$ whenever the economy is experiencing a recession in quarter t and $B_t = 0$ if it is an expansion. The log-likelihood function can be expressed as

$$\log \mathcal{L} = \sum_{X_t=1} \log[\Phi(\beta' x_t)] + \sum_{X_t=0} \log[1 - \Phi(\beta' x_t)]$$

The scope of the analysis was the out-of-sample performance with horizons from 1 to 8 quarters. A probit model was fitted individually for each time series. Moreover, a joint probit model including the 10-year minus 3-month Treasury spread and each covariate (one at a time) was also fitted. The conclusion of the article was that, beyond one quarter ahead, the yield curve is not only the leading indicator but also performs better (out-of-sample) individually than whenever associated with other financial and macroeconomic variables. The complete set of out-of-sample pseudo R^2's and t-statistics to assess the statistical significance of the covariates can be seen in the Appendix of Estrella and Mishkin (1996).

Adrian and Estrella (2009) found that eleven of the past fourteen monetary tightening cycles were followed by a rise in unemployment since 1955. A monetary tightening in this sense is defined as a peak in the Federal Funds Rates (FFR). Such peaks take place whenever at least one of the following two conditions is met: a) the FFR is higher than its previous 12 and following 9 monthly values plus it is at least 50 basis points above the beginning of this time interval, b) the FFR is higher than its previous 6 and following 6 monthly values plus it is at least 150 basis points above the beginning of this time interval.

6.3 Benchmark model

Despite being the base model developed by Estrella and Hardouvelis (1991), the benchmark model will be the monthly frequency adaptation of the model by Estrella and Trubin (2006). In fact, in this same article the idea of implementing the yield curve as a real-time forecasting tool is contemplated. Financial markets today allow us to observe the long term and short term yield rates instantaneously at any given point in time. Before the last seven recessions in the United States, the yield spread between long and short term interest rates has been negative, i.e., the slope of the yield curve inverted previous to economic recessions.

The intuition behind the model lies in the idea that a tightening of the monetary policy, by increasing the short-term yield rates, intended to relieve pressures of future inflation, would have a collateral effect of slowing down the economy. This maneuver would be captured by a flattening phenomenon in the yield rates.

The benchmark model attempts to link the steepness of the yield curve to the probability of a recession one year in advance, which in monthly observations becomes twelve periods ahead. Recessions are considered from peak to trough, counting the trough but not including the peak. The spread is composed by the difference between the 10 year Treasury bonds minus the 3 month Treasury bills. Treasury securities are optimal components of the spread for two reasons: the availability of information of their past and current values, and the notion that such assets do not have an inherent credit risk premium that needs to be controlled for.

In this way, the independent variable s_t is the monthly average of the spread between the 10 year Treasury bond and the 3 month Treasury Bill, while the dependent variable B_t is the NBER definition of presence or absence of recession at time t.

$$P[B_t = 1|s_{t-12}] = \Phi(\alpha + \beta s_{t-12})$$

Estrella and Trubin (2006) find that whenever the yield curve inversion persisted for a longer amount of periods the reliability of the signal was consistently better. Also, all NBER recessions since 1968 have been anticipated by at least 3 periods with negative spread s_t prior to the crisis itself.

Monetary policy affects the short term of the curve, while investor expectations affect the long side of the curve. Expectations over longer durations are related to future interest rates and future credit demands. Whenever there is a monetary tightening, investors may feel that there should be a lower inflation expectation and a possible future easing in monetary policy. Thus, the right side of the curve would be pressured down while long term interest rates diminish. Estrella and Trubin suggest that there is a possibility that the inversion of the yield curve does not depend on the long-term interest rates, but that its main driver is the monetary policy.

Summary of the benchmark model: 1) recessions are defined by the NBER; from peak (not included) to trough (included); 2) monthly frequency; 3) interest rates are taken from Treasury securities; 3) short term rate is set for 3 months; 4) long term rate is set for 10 years.

6.4 Previous model proposals

6.4.1 Business cycle and autocorrelated errors

Chauvet and Potter (2005) state two important remarks about the usage of the yield curve as a predictor of future economic activity. The first thing to bear in mind is the fact that the predictive power of the yield curve is not necessarily stable along time. The second point made is that monetary shocks, oil supply shocks and other unexpected phenomenons that might impact the monetary policy reaction function need to be accounted for.

The authors assess the condition of binary (probit) models to be more successful than linear regression models and other type of continuous response models when it comes the stability of predictive power along time. Moreover, Chauvet and Potter find that binary models estimated through Gibbs sampling tend to be structurally unstable.

Chavet and Potter propose four extensions of the basic model equation. Hereby are presented the four linear equations that propose a new fit to model the economic activity Y_t as a function of the yield spread s_t and two innovation terms. The first newly added term is a volatility innovation term $\sigma(t)$ that may change along the different business cycles. The second original term is an autoregression coefficient θ. Clearly, by including or omitting these two terms we can get four different combinations. Therefore, there is a time invariant model, a business cycle model, a time invariant model with autocorrelated errors and a business cycle model with autocorrelated errors.

The general expression of the four models can be formulated as

$$Y_t = \alpha + \beta_1 s_{t-k} + \theta Y_{t-1} + \sigma(t)\varepsilon_t$$

where

$$|\theta| < 1 \quad \text{and} \quad \sigma_n = \sigma(t) \text{ if } t_{n-1} < t \leq t_n, n = 1, 2, ..., N$$

Regrettably, even when the inventive models had a moderate forecasting ability, the authors accepted that the more sophisticated models emitted less strong and more imprecise signals than the benchmark model.

6.4.2 Pure expectations hypothesis and term premium

Estrella and Wu (2008) proposed a model that decomposed the yield of the Treasury bills in two components: the 'pure hypothesis expectations' yield and the term premium representing

the risk-aversion of the investors. Denote the yield given by zero-coupon bonds as y_t^L and y_L^S for long-term and short-term rates at time t, e_t^L and e_t^S the expectation hypothesis terms and p_t^L and p_t^S the risk-premium component.

$$y_t^L = e_t^L + p_t^L \quad \text{and} \quad y_t^S = e_t^S + p_t^S$$

Define the term spread s_t as

$$s_t = [e_t^L - e_t^S] + [p_t^L - p_t^S] \equiv e_t + p_t$$

Using monthly data and a forecasting horizon of one year (twelve months) Estrella and Wu propose three models. The first model uses only the pure expectation hypothesis term as predictor

$$B_{t+12} = \Phi(\alpha_1 + \beta_1 e_t)$$

The second proposal employs the risk-aversion term as the sole covariate

$$B_{t+12} = \Phi(\alpha_2 + \beta_2 p_t)$$

Finally, both the pure expectation hypothesis and the risk-aversion terms are inputed as independent variables

$$B_{t+12} = \Phi(\alpha_3 + \beta_3 e_t + \beta_4 p_t)$$

Unfortunately, the authors concluded that the method of separating yield rates into components showed little improvement in the task of forecasting recessions.

Wright (2006) states that term premiums do exist and are time-varying, proposing that a term premium proxy should be accounted for in the model as well as the Federal Funds Rate. The model proposed in his article is

$$B_{t+h} = \Phi(\alpha + \beta_1 SPREAD_t + \beta_2 FF_t + \beta_3 RFF_t)$$

where h is the forecast horizon, FF_t is the Federal Funds rate and RRF_t is a proxy for the term premium at time t.

The two main findings by Wright are that the coefficient on the spread is statistically significant and that the probit model that controls for the Federal Funds rates and includes a proxy for the term premium has a better in-sample and out-of-sample performance than the benchmark model.

The reason given to explain this favorable circumstance for the proposed model is that the probit model with only the spread as covariate tends to give consistently higher recession probabilities than the new version. The underlying motive is that there are cases when the spread diminishes not because of an imminent recession but as a consequence of low term

premiums. Lower term premiums have been displaying in markets all around the globe as a response to an increased demand for long term assets, for example pension plans seeking financial securities with longer durations. Given a higher demand for such financial products, their prices naturally increase and their yield rates automatically diminish, pushing the spreads down and giving the yield curve a flattened appearance.

6.4.3 Multi-country models

Bernard and Gerlach (1998) empirically confirmed the predictive power of the yield curve for eight different countries: Belgium, Canada, France, Germany, Japan, Netherlands, UK and USA. This result, which comes before the monetary union experienced by a collection of European nations, confirms the ability of the term structure to forecast future recessions. The countries that experienced the best fits for their probit model were Germany, USA and Canada; Japan accounted for the lowest predictive ability.

Moreover, the authors extended the benchmark probit model to include the spreads of different countries as covariates. Specifically, the yield spreads of the United States and Germany were employed as covariates for the probit model of each nation. The model was compared with recession models using the Economic Leading Indicators as predictors, finding that yield spreads perform better in terms of forecasting; the leading indicators were useful only for immediate predictions and not for the short and mid time frames. A critique to Bernard and Gerlach inferred from the present work is that both countries were chosen as the universal independent variables given their relatively larger GDP's. Notwithstanding, Section 7 will explore the situations where small countries can significantly affect the term spreads of larger economies, specifically in the European Union.

The multi-country model employed the spreads of the USA and Germany as predictors for recessions in other countries from one to eight quarters ahead. The study found that the spread of the USA is statistically significant as a predictor for recessions in the UK for all horizons; the USA spread was also significant for larger time horizons in the Netherlads, Germany and Canada. Nevertheless, the US spread had no effect whatsoever in predicting the presence of recessions in the French economy. The spread of Germany, on the other hand, proved to be statistically significant for short and mid-term horizons for the USA, Japan and Canada, while being irrelevant for the British probit model.

The work by Bernard and Gerlach is an important anchor for the yield spread network model as it provided statistically significant evidence that the term structure of some countries can effectively by used as covariates to forecast recessions in other countries as well as on its own economy.

6.4.4 Dynamic probit models

Kauppi and Saikkonen (2008) and Nyberg (2010) proposed dynamic probit models by adding a lagged value of the dependent variable. Thus the regression equation would acquire the following structure

$$Y_t = \alpha + x'_{t-k}\beta + \delta GDP_{t-1} + \varepsilon_t$$

where the equation takes into account the lagged value of the dependent variable, i.e. the real GDP. The vector of covariates x_t may include the spread of more than one country analogously to Bernard and Gerlach (1998).

The generalized form of the dynamic probit model, however, includes an autocorrelated value for Y_t, yielding a dynamic and autoregressive model.

$$Y_t = \alpha + x'_{t-k}\beta + \theta Y_{t-1} + \delta GDP_{t-1} + \varepsilon_t$$

where $|\theta| < 1$ and the model can extend to more than one country's spread as predictive variables.

Nyberg (2010) finds that once the model was applied to the economies of the USA and Germany, the dynamic probit model outperformed in-sample and out-of-sample the static probit model. Moreover, both the spread of the USA was statistically significant as covariate for the predictive model Germany and vice versa.

6.5 New model proposal: Yield Spread Networks

The proposed model is a generalization of the probit models proposed by Estrella and Hardouvelis (1991), Estrella and Trubin (2006) and Bernard and Gerlach (1998). Taking as a starting point the general regression equation

$$Y_{t,t+k} = \alpha + \beta SPREAD_t + \sum_{i=1}^{N} \delta_i X_{it} + \varepsilon_t$$

and adding as covariates the spreads of countries that are interconnected with the dependent variable and also a network covariate.

Billio, Getmansky, Lo and Pelizzon (2012) measured the degree of connectedness between the returns of financial institutions by two methods: Principal Components Analysis and pairwise Granger causality. The scope of the network consisted in testing the increasing number of connections between financial stakeholders as a measure of systemic risk. The relationships inside the financial system were measured through Granger tests to discover both the degree and direction of connections over a moving window.

Billio et al. found that a higher degree of connectedness inside the financial system can be associated to an increase in the level of non–diversifiable (systemic) risk in the market.

Moreover, the empirical analysis observed the highest connectivity among financial institutions during the Financial Crisis of 2007-2008. An increasing connectivity inside the financial market, thus, can be an early warning signal of an imminent crisis stimulated by systemic risk.

6.5.1 Generalized form of the model

The new regression equation sets the forecasting horizon for k months and denotes the spread for country i at time t as $s_{i,t}$. The set of covariates X_{ij} is replaced with the spreads of all other countries inside the network that fulfill two conditions: 1) the spread of country j Granger causes the spread of country i; 2) the spreads of both countries i and j are explained by the same factor F in the Factor Analysis procedure. Lastly, the covariate $W_{i,t}$ denotes the in-degree of country i at time t in the Granger-causality network.

$$Y_t = \alpha + \beta_i s_{i,t-k} + \sum_{j=1}^{N} \beta_j s_{j,t-k} + \sum_{j=1}^{N} \delta_{ij} s_{i,t-12} s_{j,t-k} + \gamma_i W_{i,t-1} + \varepsilon_t$$

The spreads for each country are computed as the difference between the 10 year (long term) government bonds minus the 3 month (short term) interest rate. The frequency of observations is monthly and the source of information is the OECD Statistics Department. The Granger-causality network is practiced on the term structure controlled for shocks in oil and electricity prices of each country.

The probit model is of the form

$$P[B_t = 1|x_t] = \Phi(\beta' x_t)$$

and the log-likelihood function can be expressed as

$$\log \mathcal{L} = \sum_{X_t=1} \log[\Phi(\beta' x_t)] + \sum_{X_t=0} \log[1 - \Phi(\beta' x_t)]$$

where $B_t = 1$ whenever the economy is experiencing a recession in month t and $B_t = 0$ if it is an expansion.

Four different versions of the proposed model were tested. The first version is the standard method of Estrella and Hardouvelis (1991) and updated by Estrella and Trubin (2005), which is denoted as the Benchmark model.

$$Y_t = \alpha + \beta_i s_{i,t-12} + \varepsilon_t$$

Notice that in this version of the model, $\{\beta_j = 0, \forall j \neq i\}$, $\{\delta_j = 0, \forall j\}$, and $\gamma_i = 0$.

The second version is the generalization of the method employed by Bernard and Gerlach (1998) conditioned on the newly proposed technique for choosing the countries j to be included as covariates

$$Y_t = \alpha + \beta_i s_{i,t-k} + \sum_{j=1}^{N} \beta_j s_{j,t-k} + \sum_{j=1}^{N} \delta_{ij} s_{i,t-k} s_{j,t-k} + \varepsilon_t$$

Consider as well that the model by Bernard and Gerlach can be seen as a specific case of the generalized model where $\{\delta_j = 0, \forall j\}$ and $\gamma_i = 0$.

A third proposal is contemplated by adding an innovation term $W_{i,t}$ to the benchmark model. To the best of our knowledge, it is the first time the in-degree parameter of a Granger-causality network is employed as a covariate for a term structure probit model.

$$Y_t = \alpha + \beta_i s_{i,t-k} + \sum_{j=1}^{N} \beta_j s_{j,t-k} + \sum_{j=1}^{N} \delta_{ij} s_{i,t-k} s_{j,t-k} + \gamma_i W_{i,t-1} + \varepsilon_t$$

This is also a specific case of the generalized version of the model where $\{\beta_j = 0, \forall j \neq i\}$ and $\{\delta_j = 0, \forall j\}$.

The generalized version of the model is presented as the main proposal for the scope of this work. The regression equation is expressed as

$$Y_t = \alpha + \beta_i s_{i,t-k} + \sum_{j=1}^{N} \beta_j s_{j,t-k} + \sum_{j=1}^{N} \delta_{ij} s_{i,t-k} s_{j,t-k} + \gamma_i W_{i,t-1} + \varepsilon_t$$

6.5.2 Methodological specifications

The optimal selection of the forecasting horizon is of utmost importance. A second consideration is the selection of representative interest rate periods to construct the yield structure, particularly the optimal spread to be used as the main predictor.

Fabio Moneta (2003) solved this dilemma empirically by performing 80 different models combining the time horizons $k \in \{1, 2, ..., 8\}$ lags ahead with ten different yield spreads s_t, where $s \in \{(1Y - 3M), (2Y - 3M), (5Y - 3M), (10Y - 3M), (2Y - 1Y), (5Y - 1Y), (10Y - 1Y), (5Y - 2Y), (10Y - 2Y), (10Y - 5Y)\}$. The concluding remarks state that the optimal forecasting horizon is $k = 4$ quarters ahead and that the best performing yield spread is composed by the difference of the $10Y$ minus the $3M$ government securities.

Chauvet and Potter (2005) claim that parameter instability is not a relevant issue when it comes to monthly data given that the sample of observations is relatively large and it can be estimated by applying the standard statistical methodologies.

Following the findings of Moneta (2003) and the notion of a monthly frequency model by Chauvet and Potter (2005), the general equation can be specified for $k = 12$ months as

$$Y_t = \alpha + \beta_i s_{i,t-12} + \sum_{j=1}^{N} \beta_j s_{j,t-12} + \sum_{j=1}^{N} \delta_{ij} s_{i,t-12} s_{j,t-12} + \gamma_i W_{i,t-1} + \varepsilon_t$$

Interest rates can be obtained without trouble for a daily basis, therefore the average monthly value will be employed as the observation for each period. There is, however, no monthly binary variable that identifies whether the economy is experiencing either a recession or an expansion. Bernard and Gerlach (1998) point out that in order for multi-country comparisons to be allowed, the same rule to dictate the presence and absence of recessions must be applied to all economies equally. In the most common versions of the probit model the official NBER business cycle dates for the US had been employed, nevertheless it is now implausible to extend this NBER methodology for countries other than the USA. Following this line of logic, the business cycle assessment can computed according to the Harding Pagan (2002) procedure practiced on the real Gross Domestic Product constructed from data provided by the OECD Statistics Department. The Harding Pagan method is also know as the quarterly Bry-Boschan (1971) and can be employed as a proxy for monthly binary observations. A simplified, yet holistic, specification of the Harding Pagan method is found in Section 4.

A common unit of measurement must be applied to all the models being compared in order to have a legitimate identification of their performance. The determination coefficient R^2 provides a measure of the in-sample goodness of fit, while the *p-value* measures the statistical significance of the out-of-sample ability to predict future movements of the dependent variable.

Estrella and Hardouvelis (1991), Jorion and Mishkin (1991), Bernard and Gallach (1998), Moneta (2003), Estrella and Trubin (2005) and other authors have assessed the goodness of fit of their forecast models by employing a generalized pseudo–R^2 of the form

$$pseudo\text{–}R^2 = 1 - \left(\frac{\ell(\hat{\beta})}{\ell(\hat{\alpha})} \right)^{\frac{-2}{n}\ell(\hat{\alpha})}$$

Nevertheless, whenever more than one predictor is employed the R^2 values tends to be overestimated. Therefore, it may beneficial to implement an adjusted measure that penalizes models with a large number of K–covariates for having too many predictors. In this sense, the proposed adjusted generalized pseudo–R^2 measure can be expressed as

$$pseudo\text{–}R^2_{adj} = 1 - \left(\frac{\ell(\hat{\beta}) - K}{\ell(\hat{\alpha})} \right)^{\frac{-2}{n}\ell(\hat{\alpha})}$$

6.5.3 Validation of the Data

In order to allow for data to be homogenized across countries, a common source of information was occupied for all the long term and short term interest rates. The 10-year government bond zero-coupon yield rate was obtained as the 'Long-term interest rates' from the OECD. The 3-month government security zero-coupon yield was retrieved as the 'Short-term interest rates' from the same source, i.e. the OECD.

The Consumer Price Index for electricity and energy for the different nations that were analyzed in the network was, as the previous datasets, obtained from the OECD. The observed variable taken into account was the 'End-use energy prices in US Dollars', given that this measure has been adequately standardized to a common denominator. Lastly, the oil prices where retrieved from the OECD, whose original source of information is the International Energy Agency (IEA). Supply shocks for the oil and energy markets where isolated in order to avoid the stability of the theoretical model. Applying such procedure, the analytical model may be combined with the empirical results.

Hamilton (1994) warns about the uses and misuses of Granger causality. In order to avoid mistakes, the yield spreads used for the network are orthogonal to the oil and electricity shocks given that they are statistically significant for predicting real GNP. Moreover, systems with three variables are diagnosed in order to assess the role of omitted information and the possibility of an omitted variable bias (OVB). There is literature supporting the practice of performing orthogonalization to the time series prior to be used inside Granger causality networks. Billio et al. (2012), for example, also performed their network analysis in the returns of the institutions regressed on the returns of the S&P500 Index in order to separate any possible contagion factors.

6.5.4 Defining the Granger-causality network

Denote a network $\mathbb{G}$ composed by n countries for which there is a complete set of observations between times $t-h$ and t. The time series to be studied are the differenced term structures of the yield rates of government securities for country i orthogonal to the log-prices of oil and electricity of same country i denoted as $\hat{s}_i$ for $i \in \{1, 2, ..., n\}$.

Notice that whenever using the Granger-causality methodology, one must ensure that all the time series are stationary. Given that neither one from the term spread, oil log-price nor electricity log-price are stationary, first differences were applied to the time series. Slightly abusing from mathematical notation, but keeping in mind that this is not a panel data model, the actual proxy of the spread for country i used as input for the network can be computed from the regression equation:

$$\Delta(s_{it}) = \hat{\alpha}_0 + \hat{\alpha}_1 \Delta(oil_{it}) + \hat{\alpha}_2 \Delta(elec_{it}) + \hat{s}_{it}$$

where the Frisch-Waugh-Lovell theorem states that by properties of OLS, the residuals $\hat{s}_i$ contain all the information of the spread that is not explained by the covariates oil_i and $elec_i$ plus the advantage of being stationary [see Wooldridge].

Multi-country approach

Denote $\mathbb{G}$ as a (n, n) matrix.

$$\mathbb{G} = \begin{bmatrix} 1 & \mathrm{g}_{1,2} & \cdots & \mathrm{g}_{1,n} \\ \mathrm{g}_{2,1} & 1 & \cdots & \mathrm{g}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{g}_{n,1} & \mathrm{g}_{n,2} & \cdots & 1 \end{bmatrix}$$

where $\forall k \neq \{i, j\}$ and $i \neq j$

$$\mathrm{g}_{i,j} = \begin{cases} 1 & \text{if} \quad \hat{s}_i \xrightarrow{GC} \hat{s}_j | \hat{s}_k, \forall k \\ 0 & \text{otherwise,} \end{cases}$$

and by definition of the term structure probit model $\mathrm{g}_{i,i} \equiv 1, \forall i \in \{1, 2, ..., n\}$.

The Granger-causality network $\mathbb{G}$ is based in a tri-variate VAR(1) process. Therefore, $\hat{s}_i$ Granger-causes $\hat{s}_j$ if there exists no $k \neq \{i, j\}$ such that $\hat{s}_i$ fails to Granger-cause $\hat{s}_j$ given $\hat{s}_k$. The previous procedure is helpful for getting rid of all the spurious causalities that may arise due to the Omitted Variable Bias. In this way, the countries that are possible candidates for covariates were not in violation of the OVB.

Lastly, all spreads $\hat{s}_j$ for which $\mathrm{g}_{i,j} = 1$ are evaluated by a Factor Analysis procedure to assess whether or not countries i and j are explained by the same Factor F. If and only if both conditions are met, then the spread s_j of country j can be used as a covariate for the dependent variable $[Y_i]$ recessions on country i.

Number of connections

Denote $\mathbb{H}$ as a (n, n) matrix.

$$\mathbb{H} = \begin{bmatrix} 1 & \mathbb{h}_{1,2} & \cdots & \mathbb{h}_{1,n} \\ \mathbb{h}_{2,1} & 1 & \cdots & \mathbb{h}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{h}_{n,1} & \mathbb{h}_{n,2} & \cdots & 1 \end{bmatrix}$$

where $\forall i \neq j$

$$\mathbb{h}_{i,j} = \begin{cases} 1 & \text{if} \quad \hat{s}_i \xrightarrow{GC} \hat{s}_j \text{ , with Robust errors} \\ 0 & \text{otherwise,} \end{cases}$$

and by definition of the term structure probit model $\mathbb{h}_{i,i} \equiv 1, \forall i \in \{1, 2, ..., n\}$.

The covariate $W_{j,t}$ is measured as the in-degree of country j at time t in the network and it is computed as the sum of the j–th column of matrix $\mathbb{H}$ with the exemption of $\mathbb{h}_{j,j}$ (which is one by definition).

$$W_j = \sum_{j \neq i} \mathbb{h}_{i,j}$$

The length of the moving window to assess the number of connections at each time t is based on a 7 year framework (84 months). This number was computed as the mean duration of a complete business cycle for all countries sampled according to the Harding Pagan algorithm.

Note: There is a noteworthy difference between the way in which the networks $\mathbb{G}$ and $\mathbb{H}$ are computed. Network $\mathbb{G}$ is based in a tri-variate approach seeking to wipe out all fraudulent causalities. On the other hand, the actual nodes in network $\mathbb{H}$ are not particularly relevant, as the variable to be measured is the total number of connections.

The Granger causality network applying White standard errors allows us to narrow down the number of causalities to include only those that have a robust statistical probability of actually being true. In this sense, some of the biases in the number of causalities can be controlled for.

The Newey-West (1986) covariance matrix was implemented on the causality tests as well, nevertheless, problems were experienced whenever time series had shorter lengths than average. Countries with a relatively small amount of information (less than half the number of observations from the most complete time series) and that were, *a priori*, considered to be not very 'well connected' accounted for a large amount of connections.

The models for time series were ran using both White and Newey-West covariance matrices, finding the same statistical significance level for the in–degree network covariate in the generalized form model (which includes foreign spreads) for 21 out of 28 cases (the level of statistical significance decreased in 5 cases and increased in 2 cases). Nevertheless, there were differences between using White and Newey-West covariance matrices whenever fitting a probit model without using foreign spreads.

However, in order to homologize tests and pursue comparability across models [despite sacrificing a possible critique in statistical efficiency], the results presented in Section 7 are those computed from networks using White (robust) to test for Granger causalities.

7 Results

The universe of countries studied is composed by 35 nations. Sorting their code label by alphabetical order the collection of countries is:

- AUS (Australia)
- AUT (Austria)
- BEL (Belgium)
- CAN (Canada)
- CHE (Switzerland)
- CHL (Chile)
- COL (Colombia)
- CZE (Czech Republic)
- DEU (Germany)
- DNK (Denmark)
- ESP (Spain)
- FIN (Finland)
- FRA (France)
- GBR (United Kingdom)
- GRC (Greece)
- HUN (Hungary)
- IRL (Ireland)
- ISL (Iceland)
- ISR (Israel)
- ITA (Italy)
- JPN (Japan)
- KOR (Korea)
- LTU (Lithuania)
- LUX (Luxembourg)
- LVA (Latvia)
- MEX (Mexico)
- NLD (Netherlands)
- NOR (Norway)
- NZL (New Zealand)
- POL (Poland)
- PRT (Portugal)
- SVK (Slovakia)
- SVN (Slovenia)
- SWE (Sweden) and
- USA (United States)

Nevertheless, seven countries had to be taken out from the network model, leaving 28 time series for the Granger causality network. Japan and Korea were taken out given that they had not experienced a single recession during the time comprised by their interest rates time series, at least according to the Harding-Pagan method. Colombia, Chile, Slovenia and Slovakia were removed upon the lack of a sufficiently long time series to be analyzed in the network procedure. Lastly, Poland was excluded from the model given that the Granger causality network found it to be the only country that had no connection degree with any other economy.

Three problems arose in the time series. First, given the impossibility to find the specific spot price for each country with a single data source, the time series employed were the International Energy Agency's monthly average oil price per economic region: Asia/Oceania, Europe, North America and Worldwide. The second consideration is with respect to the end-use electricity prices for Australia. Given the lack of success to obtain a complete time series without interruptions, the OECD average electricity price was adjusted as the variable to be used to regress the term spread. Finally, the yield spread for Mexico had to be interpolated by means of spline approximation given missing data in the long term interest rates.

Table 4 shows the summary of results for the proposed model. From now on, for simplicity, we may employ the code labels found in Table 3 and not the official name to refer the each country.

The first column depicts the commencement of the time series for the country, represented in the second column. The third column shows the coefficient value and significance of the parameter β_i in the benchmark model. The fourth column analogously represents the coefficient value and significance for the network covariate W_i measuring the in-degree of country i. The next column symbolizes the countries which spreads have an effect on the recession probability of country i with a 5% level of significance. A subindex denotes the channel of effect: 1 = direct covariate effect, 2 = effect through interactive covariate, 3 = both direct and interactive covariates have an effect. An empty cell means that no country met the Granger causality criteria and the N/D code means that at least one country met the Granger causality criteria, however its effect was not statistically significant. Lastly, the Diagnostics 1 and 2 measure the differences in the adjusted pseudo–R^2 values between the Benchmark model and the Network and General models respectively.

It is very helpful for the analysis to always keep in mind the commencement year of the time series for each country. Bernard and Gerlach (1998) advocate that international differences may arise whenever one or more country have measurement errors in their reported data. There is a possibility for the presence of noise in the time series to dampen the actual relations between spreads and generates spurious linkages. An environment where such mistakes may take place is whenever a financial market faces deregulation, where the interest rates may not precisely reflect the expectations of financial investors about the future economic activity.

Given the comments by Bernard and Gerlach above, whenever a model was fitted for country i, only countries j with an equal or larger number of observations were eligible to be covariates and to be considered for the in-degree of the Granger causality network. There is a simple logic behind this condition. The number of observations is a clear measurement of how long have the interest rates been known for each country's market. This duration, or number of observations, is a proxy variable to measure the 'completeness' or 'maturity' of a market. Following the notion of 'maturity', we will say that young markets can be affected by mature markets, but not the other way around. Thereafter, there is a bias as a consequence of the delayed entry times for different countries. Future scrutiny could be in order to better assess the level of market development per country (e.g. the domestic stock exchange).

The first finding, which is actually quite straightforward, is the value and the statistical significance of the term spread to predict a recession in the economy. The a priori expected value of the term spread coefficient should be negative; the idea behind the model suggests that a flattened or inverted yield curve would give a negative value of β_i and augment the foretold probability of a future recession.

The term spread for 23 out of 28 countries empirically proved to be a statistically significant parameter: 18 with the expected negative sign and 5 with an unexpected positive sign.

The following 18 countries, out of 28 possible cases, have a β_i coefficient with the expected negative sign and a significance level of 1%: AUS, CAN, CHE, DEU, FRA, GBR, USA, NZL, NOR, AUT, FIN, SWE, ISL, ITA, ISR, LTU, LVA and MEX. While BEL had a low significance level of 10%.

Five countries: ESP, IRL, PRT, GRC, HUN, attained a significance level of 5% but the coefficient value had a positive sign. This finding is somewhat counterintuitive. Nevertheless, given that the countries are mostly part of the so called GIPSI group of nations (with Hungary replacing Italy), it might be the case that this collection of countries were subject to diverse factors whose dynamics are not captured by the term spread. (NOTE: the GIPSI group consists of GRE, IRL, PRT, ESP and ITA).

Only BEL, DNK, NLD, CZE, LUX differed from the general finding. BEL, DNK, NLD and LUX have a common characteristic inside the world market, as the three of them are quasi-perfect examples of small open economies. Such conditions may influence the monetary policy of the nations and therefore dilute the significance of the term spread. CZE, on the other hand had a rather short and volatile time series, which may have accounted for a non-statistically significant parameter.

The network parameter W_i had a favorable outcome in the statistical sense. From the 28 countries, 16 included the network parameter as a statistically significant covariate at a 5% level. The value of the coefficient was larger than zero for 15 countries: AUS, BEL, CAN, DEU, ESP, FRA, NZL, IRL, NOR, AUT, NLD, ISL, GRC, HUN and MEX; and negative for one: ITA.

Empirically, a possible conjecture about the effect of the in-degree network parameter could be that the more connected a country is, the more prone or vulnerable it is to suffer a recession in the short run if. If the parameter has a coefficient larger than zero, then whenever the number of in-degree connections rises, the probability of a future downturn in the output will also increase (given that the probability function is the cumulative standard Gaussian function).

Another innovative and interesting analysis can be made from the fifth column. It is perhaps to no one's surprise to see CHE as a covariate for DEU, AUT, FIN, NDL or LUX, given the strong and independent economy. FRA has an effect in BEL, while BEL pressures FRA, DEU and CAN. USA affects CAN while CAN alters both USA and DEU. While DEU influences FRA and a network starts to form. Lastly, GBR impacts both BEL and FIN, FIN affects SWE, and NDL has an effect AUT.

All the previous causalities seem to have a rather obvious interpretation, and perhaps little explanation is needed. However, it is fairly interesting to see how IRL has a statistically significant effect on NOR, DNK, ITA and PRT, while NOR influences IRL back. Without jumping into conclusions and picturing that IRL has a strong financial effect in continental Europe, it may be worthwhile to take a look at the Maastricht Treaty written by the European Union.

The Maastricht criteria includes the 'Long-term interest rate criterion', which establishes a clear objective for all EU nations to converge into a same long-term interest rate. Each two years the best performing nations are set as reference for the other countries to follow. The criteria employed can be revised in the European Union convergence reports (1998), (2000), (2002), (2004), (2006), (2008), (2010), (2012), (2014), (2016). Meanwhile, a brief summary is presented here depicting the two or three countries that were established as references for each two-year period since 1998 to 2018. (MAL = Malta and BUL = Bulgaria)

Table 3: Long-term interest rate reference countries

1998	AUT	FRA	IRL
2000	AUT	SWE	FRA
2002	FIN	DNK	SWE
2004	SWE	FIN	POL
2006	MAL	NDL	DNK
2008	BEL	PRT	
2010	SWE	SVN	
2012	SWE	LVA	IRL
2014	LVA	IRL	PRT
2016	SVN	ESP	BUL

Table 3 shows the countries that were chosen as reference for the long–term interest rate criterion to be applied to all the Maastricht Treaty signees for a period of two years.

Seven 'excluded' nations attained not a single possible covariate: CHE, NZL, ISL, ISR, CZE, LTU and MEX. These countries can be considered to be somewhat isolated, either for a good or a bad reason, and therefore not significantly affected by foreign term spreads.

Finally, Diagnostic 1 and Diagnostic 2 reveal the improvement achieved by the newly proposed models with respect to the benchmark. While the generalized pseudo–R^2 is the common practice in the literature, an adjustment was made to auto-penalize the general network model for adding more covariates. The distribution of Diagnostics 1 show that 15 out of 28 fitted models improved, with a mean improvement of 0.05 and a standard deviation of 0.11. The behavior of Diagnostics 2 is considerably more notorious: 25 out of 28 models improved its fitting, with a mean improvement of 0.15 and a standard deviation of 0.15.

7.1 Summary of General Results

Table 4: Summary of Results

Year	Country	Benchmark β_i	Network W_i	General $\beta_{j\neq i}$	Diagnostic 1	Diagnostic 2
1980	AUS	-0.184^{***}	0.734^{**}	N/D	0.01	0.02
1980	BEL	-0.069^{*}	0.239^{***}	$FRA_{1,2}$, ESP_2, GBR_2	0.02	0.18
1980	CAN	-0.501^{***}	0.463^{**}	BEL_2, USA_2	-0.00	0.03
1980	CHE	-0.402^{***}	0.026		-0.05	-0.05
1980	DEU	-0.249^{***}	0.231^{**}	$BEL_{1,2}$, $CHE_{1,2}$, CAN_2	-0.03	0.06
1980	ESP	0.114^{***}	0.285^{***}	$USA_{1,2}$	0.01	0.15
1980	FRA	-0.304^{***}	0.161^{**}	BEL_1, DEU_1	0.00	0.36
1980	GBR	-0.256^{***}	-0.078	N/D	0.16	0.15
1980	USA	-0.359^{***}	-6.441	CAN_1	0.24	0.24
1980	NZL	-0.309^{***}	0.695^{**}		0.01	0.01
1985	IRL	0.203^{***}	0.512^{**}	NOR_2	0.03	0.10
1985	NOR	-0.253^{***}	0.617^{***}	$IRL_{1,2}$	0.01	0.07
1990	AUT	-0.266^{***}	0.244^{***}	$NDL_{1,2}$, CHE_2	0.06	0.39
1990	DNK	-0.061	-0.043	IRL_1	0.00	0.18
1990	FIN	-0.596^{***}	0.028	$CHE_{1,2}$, $GBR_{1,2}$	-0.15	0.19
1990	NLD	-0.010	0.225^{**}	$CHE_{1,2}$	0.01	0.02
1990	SWE	-0.370^{***}	0.120	FIN_1	-0.06	0.10
1995	ISL	-0.236^{***}	0.173^{**}		0.07	0.07
1995	ITA	-0.143^{***}	-0.241^{***}	$IRL_{1,2}$	0.01	0.12
1995	PRT	0.063^{**}	-0.063	$IRL_{1,2}$	-0.02	0.64
1999	GRC	0.100^{***}	1.890^{***}	N/D	0.28	0.28
1999	ISR	-0.089^{***}	-4.700		0.04	-0.01
2002	CZE	0.183	0.013		0.00	0.00
2002	HUN	0.109^{**}	0.289^{***}	N/D	0.17	0.18
2002	LTU	-1.745^{***}	-0.051		0.09	0.09
2002	LUX	0.117	0.045^{*}	$CHE_{1,2}$	0.00	0.04
2002	LVA	-0.986^{***}	0.028		0.30	0.30
2002	MEX	-0.964^{***}	0.331^{**}		0.24	0.24
Note:						*p<0.1; **p<0.05; ***p<0.01

Diagnostic 1: Measures the difference between the generalized adjusted pseudo–R^2 for the network model minus the corresponding value of the benchmark model.

Diagnostic 2: Analogous to Diagnostic 1, but measuring the difference between the General model and the Benchmark Model

Four different versions of the general model were tested on each country:

- **Model B:** Benchmark model, analogous to Estrella and Trubin (2006), including only the spread of country i: s_i
- **Model M:** Multi-country model, analogous to Bernard and Gerlach (1998), adding the spreads of countries $j \neq i$ if and only if they met the Granger causality and Factor Analysis criteria: s_i and s_j
- **Model N:** Network model which employs the spread of country i plus the number of in-degree connections of the same country: s_i, W_i.
- **Model MN:** Generalized form of the model including the multi-country criteria and the network covariate as well: s_i, s_j, W_i

Tables 5 through 10 include basic indicators that are of considerable aid whenever assessing the goodness of fit for more than one model.

In Table 5. Generalized adjusted pseudo–R^2, in 21 out of 28 cases the either the general or the network model were the best fit. It is important to realize that in more than one case the general model is exactly the same as the network model; this happens whenever no covariate met the Granger causality and Factor Analysis conditions. Finally, only in 1 out of 28 cases was the benchmark model the best fit: the case for CHE. We can modestly conclude that the in–sample performance increased with the implementation of the new model.

Table 6. enlists the Youden's J–statistics, where a value of zero represents a model that is as good as randomly guessing. For 17 out of 28 cases either models N or M–N achieved the highest J–statistic. Nevertheless, in two cases the benchmark outperformed the new proposals: CHE and DEU. Lastly, in four countries there was no single model that achieve a value different from zero: AUS, NLD, CZE, LUX. Perhaps the Youden's J–statistic is not the best goodness of fit test.

Continuing to Table 7. Residual Deviance and Table 8. Pearson Residuals we can observe very affirmative results that favor the general model M–N. For 28 out of 28 models the residual deviance was minimized when applying the multi-country network model. The Pearson residuals are almost as supporting, where the general model outperformed (or tied) all models with exception of four countries were the network model posed as the best fit.

Table 9. AUC shows the in–sample test of the Area Under the Curve for the Receiver Operating Characteristic, where a value of 0.5 means the model is as good as randomization. For 18 out of 28 the general model outperformed the benchmark and multi-country models. However, in three cases the M model (without network) outperformed the M–N model (with

network) and in three more cases the outcome was a tie.

The in–sample performance overall by the generalized form of the multi-country network models seems striking, with its lowest value being 0.75 for CHE and a mean value of 0.85 versus a mean value of 0.71 for the benchmark model. However, the Lucas Critique (1976) dictates that the historical predictive power does not guarantee that the spread will continue to be a powerful predictor of future economic activity. The given argument suggests that once the policy makers are aware of the existence of an underlying relation, their behavior is subject to be accordingly modified.

In an attempt to give a response to the Lucas Critique, an out–of–sample AUC test was performed by forecasting the last 84 observations. Recall from previous section that according to the Harding Pagan algorithm, the mean duration of a business cycle among all sampled countries was of approximately seven years.

Table 10. shows the out–of–sample AUC of ROC and the results are not as straightforward to interpret as the in–sample outcomes. In 16 out of 28 cases the M–N model outperformed the competition. However, in nine cases the multi-country model, which can be roughly described as the Bernard and Gerlach (1998) approach, outperformed the generalized model that includes the in–degree of the network as a covariate. Even more, the benchmark model appeared to be the best performer for the case of CZE and was tied for the cases of LTU and LVA.

A conservative conclusion regarding the out–of–sample predictions is that time series with few available observations are not well-suited candidates for the application of the network covariates. CZE, LTU, LUX and LVA presented the largest problems, and are themselves four of the shortest time series available. Thus, in the forecasting sense, it is advisable to use the network and the generalized approaches for countries that have perhaps at least two or three complete business cycles observed.

The last table to be presented is Table 11. Factor Analysis (FA) for data contained from 2002 to 2017. The FA procedure clustered the term spreads of different countries into 5 factors or groups with a strong accuracy. Factor 1 is composed by the non-GIPSI European nations: AUT, BEL, CHE, DEU, DNK, FIN, FRA, GBR, LUX, NLD, NOR and SWE. Factor 2 comprises the developed non-European economies: AUS, CAN, ISR, KOR, MEX, NZL and USA. Factor 3 includes exactly the GIPSI nations: GRC, IRL, PRT, ESP and ITA. Factor 4 accounts for the 'periphery' European countries: CZE, HUN and ISL. Finally, Factor 5 incorporates the Baltic region: LVA and LTU.

As a final remark on the Factor Analysis, the countries with a commonality degree superior to an arbitrary high cut-off of 0.90 are: AUT, BEL, DEU, FIN, FRA and NLD; while CAN, GBR and USA are above the 0.80 line.

7.2 Goodness of Fit

Table 5: Generalized adjusted pseudo–R^2

Country	B	M	N	M-N
AUS	0.04	**0.12**	0.05	0.05
BEL	0.00	0.16	0.02	**0.18**
CAN	0.16	**0.22**	0.16	0.19
CHE	**0.19**	0.18	0.13	0.13
DEU	0.06	0.07	0.03	**0.12**
ESP	0.02	0.05	0.03	**0.17**
FRA	0.05	0.26	0.05	**0.41**
GBR	0.07	0.18	**0.23**	0.22
NZL	0.14	0.14	**0.15**	**0.15**
USA	0.10	0.15	**0.34**	**0.34**
IRL	0.10	0.15	0.13	**0.20**
NOR	0.04	0.08	0.06	**0.11**
AUT	0.03	0.33	0.09	**0.42**
DNK	0.00	0.14	0.00	**0.17**
FIN	0.26	0.44	0.10	**0.45**
NLD	0.00	**0.03**	0.01	0.01
SWE	0.12	**0.40**	0.06	0.21
ISL	0.16	**0.23**	0.22	0.22
ITA	0.02	0.12	0.03	**0.14**
PRT	0.01	0.46	0.00	**0.65**
GRC	0.19	0.14	**0.48**	0.47
ISR	0.04	**0.07**	**0.07**	0.03
CZE	0.00	0.00	0.00	0.00
HUN	0.01	0.09	0.18	**0.19**
LTU	0.59	0.60	**0.68**	**0.68**
LUX	0.00	0.00	0.00	**0.04**
LVA	0.54	0.57	**0.84**	**0.84**
MEX	0.15	0.17	**0.39**	**0.39**

Table 6: Youden's J–statistic

Country	**B**	**M**	**N**	**M-N**
AUS	0.00	0.00	0.00	0.00
BEL	0.00	0.20	0.00	**0.21**
CAN	0.21	**0.50**	0.40	0.36
CHE	**0.35**	0.32	0.22	0.22
DEU	**0.08**	0.05	0.00	0.06
ESP	0.00	0.00	0.00	**0.02**
FRA	0.00	0.47	0.00	**0.65**
GBR	0.00	0.26	**0.35**	0.32
NZL	0.13	0.13	**0.23**	**0.23**
USA	0.02	0.05	**0.77**	0.60
IRL	0.19	0.28	0.27	**0.30**
NOR	0.00	**0.11**	0.00	0.06
AUT	0.00	0.44	0.05	**0.67**
DNK	0.00	0.11	0.00	**0.20**
FIN	0.39	0.36	0.17	**0.58**
NLD	0.00	0.00	0.00	0.00
SWE	0.08	**0.56**	0.00	0.39
ISL	0.40	0.48	**0.49**	**0.49**
ITA	0.01	0.15	0.00	**0.20**
PRT	0.00	0.57	0.00	**0.76**
GRC	0.33	0.43	0.49	**0.58**
ISR	0.00	**0.30**	0.23	0.00
CZE	**0.00**	**0.00**	**0.00**	**0.00**
HUN	0.00	0.25	0.26	**0.39**
LTU	0.93	**0.99**	0.98	0.98
LUX	0.00	0.00	0.00	0.00
LVA	0.63	0.63	**0.90**	**0.90**
MEX	0.00	0.00	**0.57**	**0.57**

Table 7: Residual Deviance

Country	**B**	**M**	**N**	**M-N**
AUS	291	149	42	**36**
BEL	470	222	271	**202**
CAN	261	187	135	**118**
CHE	349	270	**232**	**232**
DEU	510	294	279	**236**
ESP	370	352	329	**275**
FRA	254	125	199	**81**
GBR	286	143	128	**127**
NZL	272	272	**266**	**266**
USA	316	151	71	**67**
IRL	314	268	244	**219**
NOR	290	270	222	**198**
AUT	223	124	167	**82**
DNK	289	227	230	**180**
FIN	294	201	218	**122**
NLD	247	221	222	**217**
SWE	269	162	170	**121**
ISL	214	169	**155**	**155**
ITA	357	266	248	**219**
PRT	266	139	230	**86**
GRC	262	247	108	**104**
ISR	130	216	217	**43**
CZE	142	137	**116**	**116**
HUN	176	145	120	**114**
LTU	28	28	**24**	**24**
LUX	169	169	111	**102**
LVA	81	76	**25**	**25**
MEX	62	58	**15**	**15**

Table 8: Pearson Residuals

Country	**B**	**M**	**N**	**M-N**
AUS	506	244	101	**65**
BEL	683	304	355	**269**
CAN	803	362	272	**267**
CHE	639	524	**394**	**394**
DEU	682	401	**349**	358
ESP	439	857	364	**314**
FRA	507	162	350	**95**
GBR	593	230	187	**182**
NZL	504	504	**499**	**499**
USA	441	227	142	**120**
IRL	408	350	337	**272**
NOR	366	337	**265**	266
AUT	303	173	250	**102**
DNK	347	277	**269**	290
FIN	314	176	236	**130**
NLD	360	296	287	**282**
SWE	505	235	247	**201**
ISL	304	335	**225**	**225**
ITA	302	240	214	**206**
PRT	269	219	196	**106**
GRC	216	201	**98**	**98**
ISR	198	219	215	**90**
CZE	191	169	**115**	**115**
HUN	196	176	**119**	125
LTU	257	244	**105**	**105**
LUX	202	202	121	**115**
LVA	138	117	**38**	**38**
MEX	66	59	**17**	**17**

Table 9: In–sample AUC of ROC

Country	**B**	**M**	**N**	**M-N**
AUS	0.72	0.89	0.93	**0.96**
BEL	0.57	**0.87**	0.64	0.85
CAN	0.83	0.87	0.86	**0.88**
CHE	0.76	**0.77**	0.75	0.75
DEU	0.66	0.77	0.69	**0.82**
ESP	0.58	0.75	0.65	**0.81**
FRA	0.77	0.95	0.73	**0.98**
GBR	0.73	0.90	**0.91**	**0.91**
NZL	0.77	0.77	**0.78**	**0.78**
USA	0.85	0.91	0.97	**0.98**
IRL	0.62	0.72	0.68	**0.80**
NOR	0.71	0.70	0.74	**0.80**
AUT	0.67	0.94	0.76	**0.96**
DNK	0.64	0.79	0.62	**0.83**
FIN	0.80	0.90	0.72	**0.94**
NLD	0.49	**0.64**	0.61	**0.64**
SWE	0.81	**0.89**	0.73	0.88
ISL	0.69	**0.76**	**0.76**	**0.76**
ITA	0.62	0.74	0.59	**0.77**
PRT	0.57	0.90	0.54	**0.96**
GRC	0.73	0.72	0.89	**0.91**
ISR	0.78	0.59	0.59	**0.83**
CZE	0.58	0.56	**0.60**	**0.60**
HUN	0.58	0.69	**0.82**	0.81
LTU	**0.99**	**0.99**	**0.99**	**0.99**
LUX	0.63	0.63	0.67	**0.75**
LVA	0.94	0.94	**0.99**	**0.99**
MEX	0.90	0.91	**0.98**	**0.98**

Table 10: Out–of–sample AUC of ROC

Country	**B**	**M**	**N**	**M-N**
AUS	0.72	0.89	0.93	**0.96**
BEL	0.57	**0.67**	0.64	0.65
CAN	0.83	**0.87**	0.85	0.86
CHE	0.76	**0.77**	0.75	0.75
DEU	0.66	0.75	0.69	**0.81**
ESP	0.42	0.43	0.45	**0.66**
FRA	0.77	0.95	0.73	**0.96**
GBR	0.73	0.90	**0.91**	**0.91**
NZL	0.77	0.77	**0.78**	**0.78**
USA	0.85	0.91	0.97	**0.98**
IRL	0.38	0.67	0.74	**0.79**
NOR	0.71	0.70	0.73	**0.77**
AUT	0.67	0.78	0.65	**0.79**
DNK	0.64	0.79	0.62	**0.83**
FIN	0.80	**0.87**	0.72	0.86
NLD	0.49	0.56	**0.57**	0.55
SWE	0.81	**0.83**	0.72	0.74
ISL	0.69	**0.76**	0.73	0.73
ITA	0.62	**0.75**	0.61	0.66
PRT	0.43	0.84	0.54	**0.95**
GRC	0.27	0.34	0.66	**0.81**
ISR	0.78	0.46	0.46	**0.83**
CZE	**0.58**	0.44	0.41	0.41
HUN	0.58	0.60	0.47	**0.80**
LTU	**0.99**	**0.99**	**0.99**	**0.99**
LUX	0.37	**0.63**	0.57	0.58
LVA	**0.94**	**0.94**	0.50	0.50
MEX	0.90	0.91	**0.98**	**0.98**

7.3 Factor Analysis

Table 11: Factor Analysis Loadings: 2002 to 2017

Country	Factor1	Factor2	Factor3	Factor4	Factor5	Commonality
AUS	0.45	**0.74**		0.10	-0.13	0.77
AUT	**0.71**	0.35	0.50	0.11	0.21	**0.94**
BEL	**0.62**	0.32	0.60	0.19	0.22	**0.93**
CAN	0.39	**0.79**	0.11		0.13	**0.80**
CHE	**0.45**	0.37	0.18	-0.18	-0.12	0.41
CZE	0.43	0.20	0.33	**0.55**	0.12	0.65
DEU	**0.86**	0.40	0.17		0.24	**0.98**
DNK	**0.78**	0.21	0.11	0.27	0.16	0.76
ESP	0.43	0.18	**0.69**	0.17	0.15	0.74
FIN	**0.80**	0.39	0.29	0.10	0.24	**0.95**
FRA	**0.72**	0.36	0.50	0.11	0.21	**0.96**
GBR	**0.60**	0.58	0.18	-0.14	0.26	**0.82**
GRC			**0.23**			0.05
HUN				**0.65**		0.43
IRL	0.27	0.23	**0.44**			0.32
ISL			-0.14	**0.15**	-0.52	0.31
ISR	0.12	**0.42**		0.17		0.22
ITA	0.36	0.17	**0.76**	0.15		0.76
KOR	0.28	**0.41**	0.16	0.34	0.34	0.51
LTU	0.25	0.12	0.13	0.19	**0.44**	0.33
LUX	**0.56**	0.29	0.22		0.28	0.53
LVA				0.12	**0.50**	0.26
MEX	0.14	**0.43**		0.11	-0.12	0.23
NLD	**0.84**	0.37	0.28		0.21	**0.95**
NOR	**0.72**	0.37	0.12			0.67
NZL	0.45	**0.67**	0.16			0.68
PRT	0.18	0.18	**0.40**	0.16		0.25
SWE	**0.74**	0.36		0.25	-0.16	0.76
USA	0.34	**0.76**	0.17	-0.14	0.27	**0.81**

7.4 Granger Causality Networks, Connections and Correlations

In section 7.4.1 we have four figures that represent the Granger causality network in different time periods. All four networks are composed as the tri-variate VAR(1) matrix $\mathbb{G}$ defined in section 6.

Figure 1 shows the ten countries with largest time series: AUS, BEL, CAN, CHE, DEU, ESP, FRA, GBR, NZL and USA from 1980 to 1999. Figure 2 is the same network but from years 2002 to 2017. Figure 2 is a sub-network of Figure 4.

It is straightforward to notice how the network became more linked in the last 15 years; Figure 2 includes notoriously more vectors than Figure 1. Figure 3 shows the network for the complete time series 1980 to 2017.

Lastly, Figure 4 conceals a several interesting comments. The first one thing that is undeniable is the large amount of connections inside the network. The second one is finding POL completed excluded from the network, which was previously mentioned that this same condition discarded POL from the scope of this study.

The third and most interesting result is the fact that SWE, DNK, IRL and CHE are in the center of the network (with LUX, GBR and BEL to a minor extent). Recall the Maastricht long-term interest rate criterion; these were the same countries that were used as reference in most occasions. It is not implausible, thus, that as a consequence of being the referred nations, they became the center of the network themselves.

Proceeding to section 7.4.2, the evolution in the number of connections is viewed in two different ways. Figure 5 and Figure 6 represent the number of connections at each time t, over moving windows of 84 observations and keeping the number of countries fixed. This enables the graph to be comparable along time and see how the number of connections increased in the early 2000's and then expanded steeply in the 2007-2008 scene and kept high values until 2014.

The number of connections can be humbly associated with the presence of economic recessions. There is, however, a pronounced decrease in the observations for 2015. A very plausible reason may be that the quantitative easing implemented by the European Central Bank took effect quickly and swiftly, diluting the connections between term spreads.

Figure 7 represents the number of connections in a scope of increasing availability of countries. In order for the time series to be easily interpreted, the number of connections is divided by $(N_t)(N_t - 1)$, where N_t is the number of countries at time t. The red line denotes the mean value of the time series. Since the start of 2007 to the end 2014 the percentage of connections with respect to the possibilities has been above average. Even after a considerable decline on

2015, the percentage is still above the average line.

The last network plot is Figure 8, which shows the evolution of the number of connections for an increasing scope of nations. This graph cannot be precisely analyzed without an ad-hoc transformation, such as Figure 7. Nonetheless, it still grasps the appalling notion of an overwhelming amount of interconnectedness between foreign term spreads.

The correlation between term spreads is scrutinized in section 7.4.3. with four plots. Figure 9 shows the correlation of ten fixed countries from 1980 to 1999, while Figure 10 shows how the term spread correlation for those same countries behaved from 2002 to 2017.

Despite Figure 11 showing that over the complete horizon of observations 1980 to 2017 the correlation between term spreads is not staggering, Figure 10 portrays a very different story about the evolution of interconnectedness in the last 15 years.

Last but not least, the complete correlation plot for all available countries from 2002 to 2017 shows that there is a very high concentration of positive correlations inside the European Union. FRA, AUT, BEL, NLD, DEU and FIN have the highest correlation coefficients; which is not surprising at all, giving that they belong to a same monetary union and follow several convergence criteria. Figure 12 helps asseverate the fact that concurrence in the European Union is starting to become a visible reality.

Model Fitting and Output (see Annex)

The complete list of regression model specifications for all 28 countries included in the network is provided in the Annex of this work. Moreover, the Benchmark and the Network Multi-country fitted probabilities were portrayed with respect to the presence (1) and absence (0) or recessions in the economy according to the Harding Pagan method; the extensive collection of time series is provided for all available countries in the Annex.

Note: There is a trade-off in the advantage of employing White standard errors instead of Newey-West standard errors to test Granger causality inside the networks. The following changes in statistical significance of the in–degree network variable change in the following sense: CHE becomes significant at a 5% level and ISR at a 10% level. NZL goes from 1% to 5% significance level. In–degrees for IRL and HUN are no longer statistically significant.

7.4.1 Granger Causality Networks

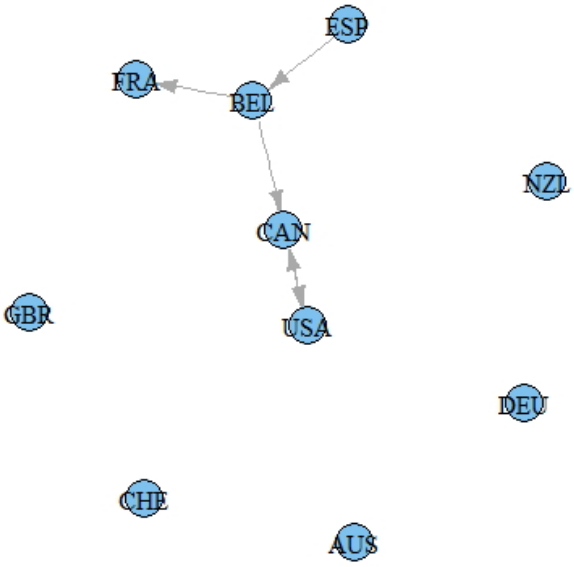


Figure 1: Granger causality network 1980-1999: 10 countries

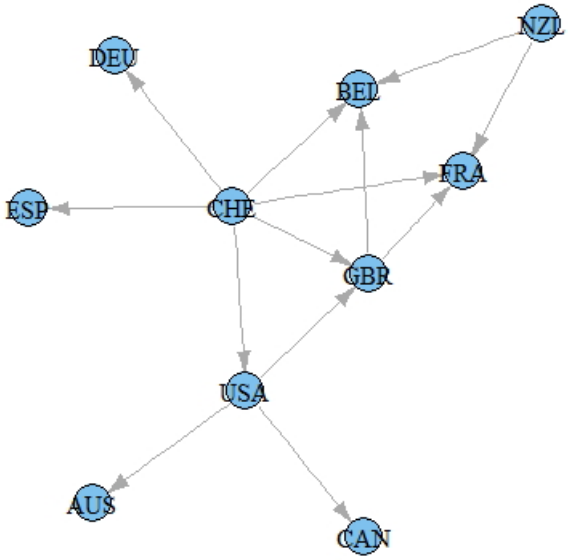


Figure 2: Granger causality network 2002-2017: 10 countries

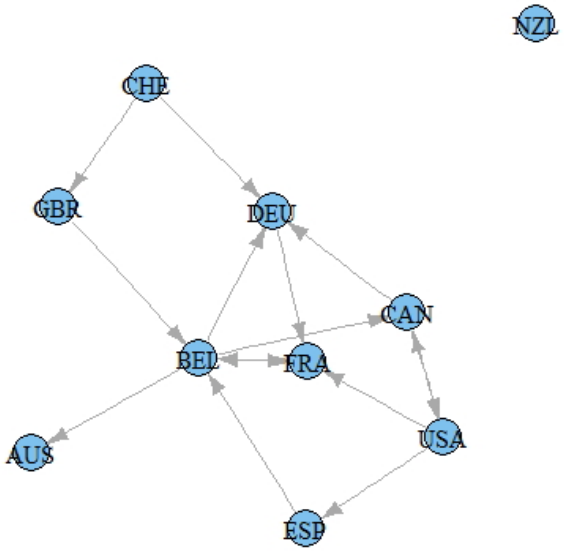


Figure 3: Granger causality network 1980-2017: 10 countries

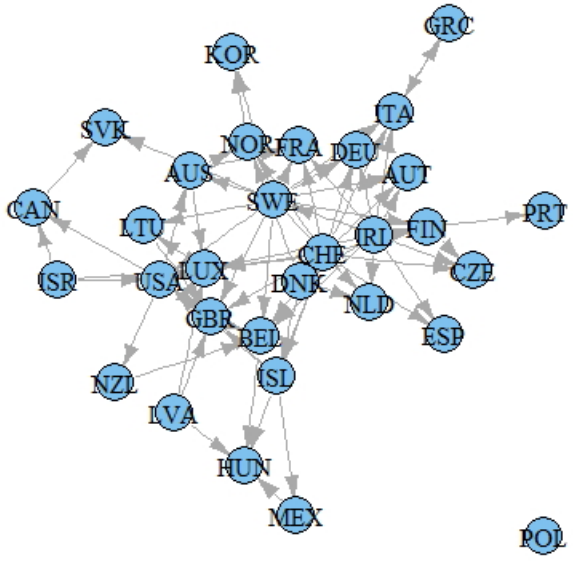


Figure 4: Granger causality network 2002-2017: All available countries

7.4.2 Evolution of the Number of Connections

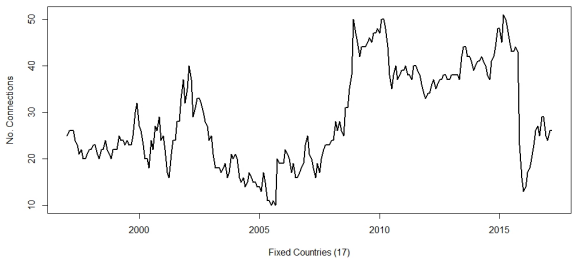


Figure 5: Number of Connections
Since 1990: Fixed countries (17)

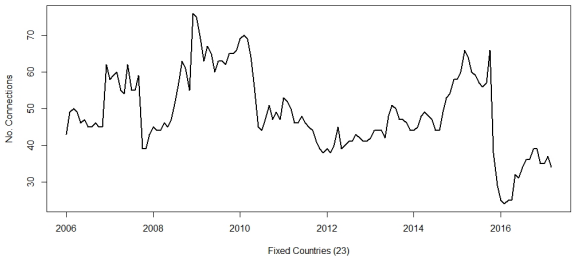


Figure 6: Number of Connections
Since 1999 Fixed countries (23)

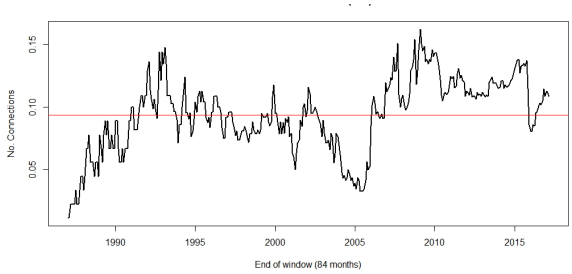


Figure 7: Connections over possibilities
Since 1980: All available countries (35)

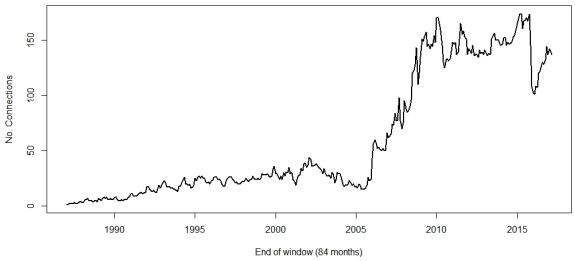


Figure 8: Total Number of Connections
Since 1980: All available countries (35)

7.4.3 Evolution of Term Spread Correlation

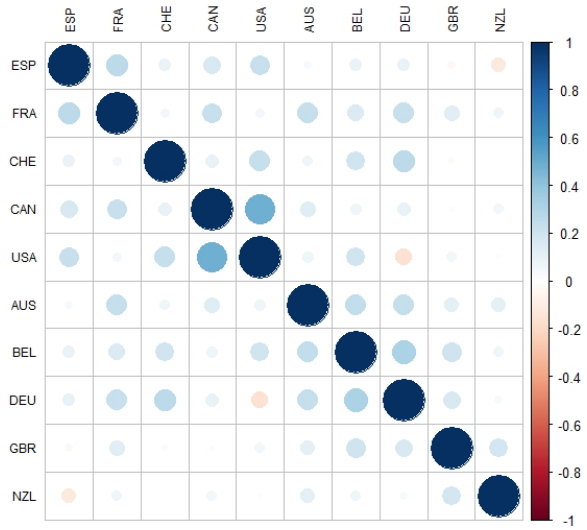


Figure 9: Correlation Plot
1980-1999: 10 countries

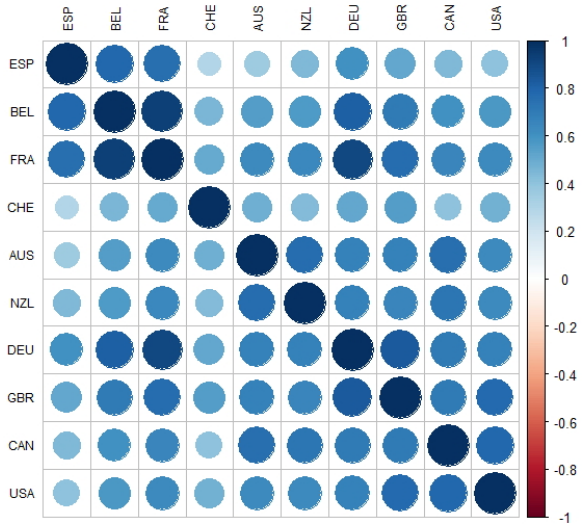


Figure 10: Correlation Plot
2002-2017: 10 countries

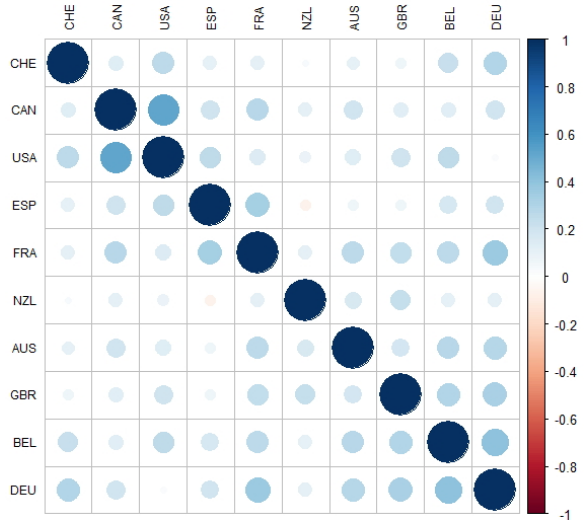


Figure 11: Correlation Plot
1980-2017: 10 countries

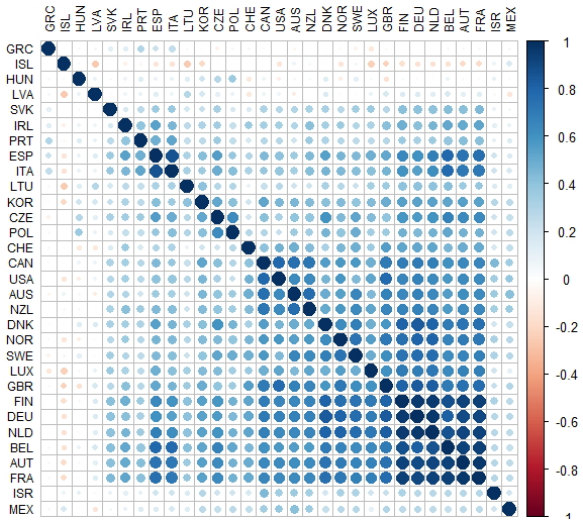


Figure 12: Correlation Plot
2002-2017: All available countries

8 Conclusion

It is not redundant to mention the importance given by Central Banks to forecast future potential recessions. Even when a vast collection of key economic indicators is readily available, a model where information is attainable in real–time is presented. The term spread is employed as a considerably accurate proxy for the term structure of yield curves. This indicator is constructed from short and long term interest rates with monthly observations, despite recognizing that those could also be composed in a daily frequency. This same variable is later employed as the input for a Granger causality network with analogous time frequency. As a consequence, a first advantage of the proposed model is the flexibility of daily calibration and not monthly or quarterly revisions that constrain many of the current key indexes.

A Granger causality network was constructed from a universe of 28 economies on its widest side and 10 countries in its most narrow end. A benchmark model by Estrella and Trubin (2006) was applied to the 28 countries, were 18 economies attained a coefficient with the expected negative sign and a significance level of 1%: AUS, CAN, CHE, DEU, ESP, FRA, GBR, USA, NZL, NOR, AUT, FIN, SWE, ISL, ITA, ISR, LTU, LVA and MEX. Five countries: ESP, IRL, PRT, GRC, HUN, attained a significance level of 5% but the coefficient value had a positive sign. This finding is somewhat counterintuitive.

Notwithstanding, the predictive power of the term structure depends on the monetary parameters that respond to output gap, inflation and interest rates; therefore we cannot claim that the effect of the yield curve on the policy rule is structural even when it is robust. Five countries differed from the general finding: BEL, DNK, NLD, CZE, LUX. A plausible explanation may suggest that in the case of small open economies, the influence of the term spread over the monetary policy of the nations is diluted. Moreover, it is possible for countries whose financial market is not as developed to account for structurally different parameters in comparison with more developed economies.

The network parameter W_i had a favorable outcome: 16 out of 28 economies included the network parameter as a statistically significant covariate at a 5% level (using White standard errors). The value of the coefficient was larger than zero for 15 countries and negative for one. Empirically, a possible conjecture about the effect of the in-degree network parameter could be that the more connected a country is, the more prone or vulnerable it is to suffer a recession in the short run if. If the parameter has a coefficient larger than zero, then whenever the number of in-degree connections rises, the probability of a future downturn in the output will also increase.

More investigation is required regarding the optimal calibration of the networks employed as covariates. There is a potential bi-causality as a consequence of country coordination during widespread crisis periods. Moreover, whenever implementing Newey-West standard errors to test for Granger causalities, the number of economies with a 5% level of statistical significance

reduces from 16 to 15 cases, but remains constant in 16 countries when applying a 10% significance level. Further exploration of the network dynamics is advised; given the fact that the Newey-West and White standard errors played the most significant difference in the model without foreign spreads, there is a potential omitted variable bias (OVB). If such bias is present then the admittance of foreign spreads into the model would be reinforced.

The generalized adjusted pseudo–R^2 criteria, followed by most authors in the literature surrounding the probit model fitting, offers a clear preference of the newly proposed general model with a network component over the classical examples of Estrella and Trubin (2006) and Bernard and Gerlach (1998). The advantage is consistently maintained for different model diagnosis, including Youden's J–statistic, residual deviance, Pearson residuals and in–sample AUC of ROC. There is a drawback, however, in the out–of–sample predictive power of the fitted models. Similar issues have been addressed since the Lucas critique (1976) in the sense that even if historically the model seems to be a good fit in–sample, there is no guarantee that the model will perform equally properly in an out–of–sample environment. Whenever the number of observations was reduced, the out–of–sample efficiency of the newly proposed general model lost power considerably.

A final remark about the networks and correlation plots depicts the undeniable evidence that the term spreads between economies are each time more linked between each other. Curiously, evidence on a mechanism through which smaller economies may affect their larger counterparts is presented. The yield spreads for the countries inside the Economic and Monetary Union (EMU) of the European Union have dramatically increased their correlations, particularly given the Euro convergence criteria implemented by the EMU. Moreover, the economies that result elected as a reference, according to the Maastricht criterion, tend to motion into the center of the Granger causality network.

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Appendix A Key Economic Indicators

Table A.1: NBER Key Economic Indicators as seen in Stock and Watson (1989)

Transformation	**Description**
Growth rates	Industrial production
Growth rates	Personal income
Growth rates	Manufacturing and trade sales
Growth rates	Employee-hours (non-agricultural)
Levels	New building permits
Growth rates	Manufacturer's unfilled orders
Growth rates	Trade-weighted nominal exchange rate
Growth rates	Part-time work in non-agriculture industries
Differences	Yield on 10-year US Tresaury bonds
Levels	Spread between corporate paper and US T-Bills (6-months)
Levels	***Spread between 10-year and 1-year Treasury securities**

Source: Stock and Watson (1989)

***Spread between 10-year and 1-year Treasury securities**: Stock and Watson define this indicator as the spread between the yield on constant-maturity portfolio of 10-year US Treasury bonds and the yield on 1-year US Treasury Bills; using as a source of yield rates the Federal Reserve Board.

Table A.2: Index components by the Department of Commerce

Key Economic Indicators	**Note**
Personal income and outlays	
GDP	
Advance Monthly Sales for Retail and Food Services	
U.S. International Trade in Goods and Services	
Monthly Wholesale Trade	
Services Survey	Q
Retail Trade	Q
Manufacturing, Mining, Wholesale Trade, and Selected Service Industries	Q
Manufacturers' Shipments, Inventories, and Orders	
Construction Spending	
Durable Goods Manufacturers' Shipments, Inventories, and Orders	A
Monthly Retail Inventories	A
U.S. International Trade in Goods	A
Monthly Wholesale Inventories	A
New Residential Sales	
New Residential Construction	
Manufacturing and Trade Inventories and Sales	
Homeownership Rate	
Rental Vacancy Rate	

Q: Quarterly
A: Advanced

Table A.3: Index components: The Conference Board

Leading Index (April 2008)
Average weekly hours, manufacturing
Average weekly initial claims for unemployment insurance
Manufacturers' new orders, consumer goods and materials
ISM new order index
Manufacturers' new orders, non-defense capital goods excl. aircraft
Building permits, new private housing units
Stock prices, 500 common stocks
Leading Credit Index
***Interest rate spread, 10-year Treasury bonds less federal funds**
Avg. consumer expectations for business and economic conditions
Coincident Index
Employees on nonagricultural payrolls
Personal income less transfer payments
Industrial production
Manufacturing and trade sales
Lagging Index
Average duration of unemployment
Inventories to sales ratio, manufacturing and trade
Labor cost per unit of output, manufacturing
Average prime rate
Commercial and industrial loans
Consumer installment credit to personal income ratio
Consumer price index for services

***Interest rate spread, 10-year Treasury bonds less federal funds**: The Conference Board defines this indicator as the spread or difference between long and short rates is often called the yield curve. This series is constructed using the 10-year Treasury bond rate and the federal funds rate, an overnight interbank borrowing rate. It is felt to be an indicator of the stance of monetary policy and general financial conditions because it rises (falls) when short rates are relatively low (high). When it becomes negative (i.e., short rates are higher than long rates and the yield curve inverts) its record as an indicator of recessions is particularly strong.

Appendix B Granger Causality between Yield Spreads and Inflation

In order to assess the existence or non-existence of causality between the yield spread (denoted by s_t) and the expectation of inflation (denoted by $E_t\pi_{t+1}$), a Granger Causality test was performed for the following countries:

Table B.4: Granger causality: Yield spreads and Inflation

Granger Causality	Country	Lags
s_t GC $E_t\pi_{t+1}$	Australia	1, 2, 3, 4
s_t GC $E_t\pi_{t+1}$	Austria	1, 2, 3, 4
s_t GC $E_t\pi_{t+1}$	Belgium	2
s_t GC $E_t\pi_{t+1}$	Finland	1, 2, 3, 4
s_t GC $E_t\pi_{t+1}$	Germany	2, 3
s_t GC $E_t\pi_{t+1}$	Luxembourg	1
s_t GC $E_t\pi_{t+1}$	Portugal	1
s_t GC $E_t\pi_{t+1}$	Spain	1, 3, 4
s_t GC $E_t\pi_{t+1}$	Sweden	1, 2, 3, 4
s_t GC $E_t\pi_{t+1}$	United Kingdom	1, 2, 3, 4
s_t GC $E_t\pi_{t+1}$	United States	1, 2, 4
$E_t\pi_{t+1}$ GC s_t	Australia	4
$E_t\pi_{t+1}$ GC s_t	Czech Rep.	1
$E_t\pi_{t+1}$ GC s_t	Denmark	1
$E_t\pi_{t+1}$ GC s_t	Finland	1
$E_t\pi_{t+1}$ GC s_t	France	2
$E_t\pi_{t+1}$ GC s_t	Slovakia	2
$E_t\pi_{t+1}$ GC s_t	Slovenia	3, 4

There seems to be a clearer Granger Causality from the yield spread to the expectation of inflation for the majority of lags and a larger amount of countries. There is however, a Granger Causality from the expectation of inflation in the yield spreads, specially in the shorter lags.