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Black Litterman and Carry: A new approach for asset allocation

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Ca' Foscari
Dorsoduro 3246
30123 Venezia

The Black-Litterman model (BL) enables investors to combine their views regarding the performance of various assets with the market equilibrium. Recently it has been shown that any security's expected return can be divided into its "carry" and its expected price appreciation, where carry can be observed in advance. The purpose of this paper is to exploit Carry as view in the Black Litterman model to make a diversified and customized portfolio

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Supervisor
Ch. Prof. Fulvio Corsi

Graduand
Iacopo Zecchi
845600

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Introduction

Most likely stock prices move together with the measures of fundamentals as dividends. The question of what moves stock prices is probably as old as the markets themselves, and has received much attention in financial literature[1]. Time variation in the price-dividend ratio is linked to time variation in expected returns and in expected dividend growth rates. However the question can be formulated in the opposite way as whether stock returns and dividend growth rates are predictable. The answer is fundamental to understanding the risk - return relationship. In the 1980s, the classical model assumed constant expected returns for stocks. Empirical evidence was uncovered showing that returns were predictable by financial ratios, as the price - dividend or price-earnings ratio. While other variables (spread between long and short term bond yields, consumption ratio, macro and micro variables, and corporate Managers) were also shown to have predictive ability. Studies in this field started to become interested in returns on other asset classes, such as government bonds, currencies, real estate, commodities, futures and others.

At the beginning predictability was interpreted as evidence against the efficient market[2]. Fama (1970) reviews the literature and tries to organize all principles. Fama describes increasingly fine information sets in a way that is useful when formulating the debate. Weak - form predictability uses the information in past stock prices. Semi - strong form predictability uses variables that are obviously publicly available, and strong - form uses anything else. Therefore the first studies revised by Fama in 1970, concluded that a martingale or random walk was an optimal model for stock prices and values. It is clear that the best forecast of the future price was the current one. However predicting price or rate return is very complicated and controversial. Financial studies reflect two views about predictability in stock returns. The first shows that any predictability represents exploitable inefficiencies in the way capital markets function. The second view argues that predictability is a natural outcome of an efficient capital market[3]. The useable inefficiencies view of return predictability is that in an efficient market traders would bid up the prices of stocks with predictably high returns, thus lowering their return and removing any predictability at the new price (see Friedman 1953 and Samuelson, 1965). Therefore market frictions are assumed to hinder such price - correcting (or arbitrage – trading). Predictable models can thus emerge when there are market imperfections such as: taxes, trading costs, information costs and human imperfections in processing or responding to information.

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[1] Koijen Ralph S.J. and Stijn Van Nieuwerburgh,2011, "Predictability of Returns and Cash Flows", *University of Chicago and Stern School of Business*

[2] The efficient market hypothesis (Fama, 1965, 1970; Jensen, 1978; Shiller, 1984; Summers, 1986; Fama, 1991). Testing the efficient market hypothesis requires a "market model" that specifies how information gets incorporated into asset prices

[3] Satchell S., 2007,"Forecasting Expected Returns in the Financial Markets", *Quantitative Finance Series,* Chapter one

These predictable patterns are thought to be exploitable, therefore an investor who could avoid the friction or cognitive imperfection could profit from the predictability at the expense of other traders.

Coming back to the efficient markets view of predictability described by Fama (1970). According to this theory returns may be predictable if they vary varies over time together with changing interest rates, risk or investors' risk-aversion. If required expected returns vary over time there may be no abnormal trading profits, and thus no incentive to exploit the predictability. Predictability may subsequently be expected in an efficient capital market.

Predictability in the 'efficient markets' view rests on systematic variation through time in the expected return. Modelling and testing for this variation is the focus of the conditional asset pricing literature (see Ferson, 1995; Cochrane, 2005). Not all of the predictability associated with stock prices involves predicting the levels of returns. A lot of literature models forecast second moments of returns (e.g. using ARCH and GARCH-type models as Engle in 2004) or other stochastic volatility models. Predictability studies have also examined the third moments (see Harvey and Siddique, 2001)[4].

For the financial economics and asset pricing allowing for predictability through time - variation in expected returns, risk measures and volatility have been some of the most significant developments of the past two decades. Research in asset pricing has proposed several equilibrium models with efficient markets that generate time variation in expected returns: models with time - varying risk aversion (Campbell and Cochrane, 1999), time - varying aggregate consumption risk (Bansal, 2004; Bansal, Kiku, and Yaron, 2009), time - varying consumption disasters (Gabaix, 2009), time-variation in risk-sharing opportunities among heterogeneous agents (Lustig and Van Nieuwerburgh, 2005), or time -variation in beliefs (Timmermann, 1993; Detemple and Murthy, 1994)[5].

The prevalent view today is that predictability of asset returns is no longer sure evidence of market inefficiency.

The majority of equilibrium literature takes time variation in expected returns as given and asks how it affects optimal asset allocation decisions (see Wachter, 2010). Return predictability is of considerable interest to practitioners who can develop market-timing portfolio strategies that exploit predictability to enhance profits. Despite the theoretical developments, return predictability is a subtle feature of the data.

Parallel studies developed in the 1990 question the strength of statistical evidence. This literature underlines problems such as biased regression coefficients, in - sample instability of estimates indicating periods with and without predictability, and poor out-of-sample performance.

Recently studies have come back to the question of whether or not measures of cash-flow growth, such as dividend-growth, are predictable as well. While interesting in its own

[4] We following the speech of Wayne Ferson, 2007, "Forecasting Expected Returns in the Financial Markets", *Quantitative Finance Series*

[5] Koijen Ralph S.J. and Stijn Van Nieuwerburgh, 2011, "Predictability of Returns and Cash Flows", University of Chicago and Stern School of Business

right and important for model design, dividend growth directly speaks to return predictability. Through the present-value relationship, which links asset prices today to future returns and future dividend growth, dividend growth and return predictability are two sides of the same coin (Lettau and Van Nieuwerburgh, 2008; Cochrane, 2008; Binsbergen and Koijen, 2010).
It is clear that in literature there are some different indicators for predicts return.

More recently Koijen, Pedersen, Moskowitz, and Vrugt (2013) define an asset's "carry" as its expected return assuming that market conditions including its price stay the same[6]. Based on this simple definition, any security return can be divided into its carry and expected and unexpected price appreciation:

$$\begin{aligned}\textbf{Return} = \textbf{Carry} + \textbf{E}(\textbf{price appreaciation}) \\ + \textbf{Unexpected price Shock}\end{aligned} \qquad \textit{Eq. 1}$$

$$\begin{aligned}&\textbf{Expected return} \\ &\qquad = \textbf{Carry} \\ &\qquad + \textbf{E}(\textbf{price appreciation})\end{aligned} \qquad \textit{Eq. 2}$$

Hence, an asset's expected return is its carry plus its expected price appreciation, where carry is a model-free characteristic that is directly observable ex ante.
Carry has been studied only for currency, however the above equation is a general definition that can be applied to any assets.

Koijen, Pedersen, Moskowitz, and Vrugt (2013) find that the predictability of carry is often stronger than that of these traditional predictors, indicating that carry not only provides a unified conceptual framework for these variables, but may also improve upon return predictability within each asset class.

Our goals are to exploit this new concept of Carry as the view in the Black Litterman (BL) model.

The BL was first published in 1990 by Fischer Black and Robert Litterman. In the last twenty years many authors have published research referring to their model, improving, enriching and customizing it. The classical model makes two significant contributions to the problem of asset allocation. First of all it provides an intuitive prior which is the equilibrium market portfolio, as a starting point for the estimation of asset returns. Secondly the Black-Litterman model provides a clear way to specify investors views on returns and to mix these views with prior information. Often investment managers have specific views regarding the expected return of some of the assets in a portfolio, which differ from the implied equilibrium one. The Black Litterman model allows such views to be expressed in either absolute or relative terms. Hence the BL model provides a

[6] For example in futures contract the carry is the expected return if the underlying spot price never changes. For assets with cash flows such as dividends, the carry is the expected return assuming the cash flow never changes

quantitative framework for specifying the investor's views, and a clear way to combine those views with an intuitive prior to arrive at new combined distributions.

Our Idea is to combine some predictor return indicators with an Asset allocation model that allows us to exploit predictability returns. In literature some of these indicators exist however the new concept of Carry developed by Koijen, Pedersen, Moskowitz, and Vrugt (2013) includes many of these. Carry is related to the typical known predictor returns as: the slope of the yield curve (for Bonds), the convenience yield (Commodities) and dividend yield (Equity)[7]. We believe that Carry is the best indicator of predictor returns. We want to exploit this forward – looking measure as views in Black Litterman Models. If we have a view regarding the asset's returns we can combine all this information with a model for asset allocation (BL).

The remainder of the paper is organized as follows. Chapter I: We present a literature review - Chapter II: We showed how calculated Carry in different asset classes and how carry is related to expected return. - Chapter III: We described the Black Litterman model - Chapter IV: Our approach is illustrated - Appedix I: Matlab Code.

1. Literature review

1.1 Literature about the Black Litterman Model

The Black-Litterman model, developed at Goldman Sachs in the early 1990s provides a framework for combining investor views with a global capital market equilibrium. Its aim is to help investment managers determine an optimal portfolio allocation for specific classes of assets in a way consistent with their market views. With this model, we can calculate optimal portfolio weights by using volatilities and correlations across asset classes.

<< Quantitative asset allocation models have not played the important role they should in global portfolio management. A major part of the problem is that such a model is difficult to use and tends to result in portfolios that are inconsistent. Consideration of the global CAPM equilibrium can significantly improve the usefulness of these models. In particular, equilibrium return for equities, bonds and currency provide natural starting points for estimating the set of expected excess return needed to derive the portfolio optimization process. This set of neutral weights can then be tilted in accordance with the investors' views.[8]>>.

As a result the benefits of this model are that one can blend a variety of views specified in different ways, absolute or relative, with a given prior (Market) estimate to generate a new and updated posterior estimate which includes all the views.

[7] See Chapter 2

[8] Black F.and Litterman R., 1992, "Global Portfolio Optimization", *Financial Journal, CFA Institute.*

This section will provide a survey of the literature and classify the model used by each of the various authors. We want to start by:

He and Litterman (1999)[9], which is the first paper by one of the original authors providing more detail on the workings of the model, however it does not include all formulas.

Bevan and Winkelmann (1998), for three years they have been publishing optimal global fixed income portfolios using the BL model to illustrate how it can be used to solve practical investment management problems. Their paper is a summary of what they have experienced while using the model for investment strategy[10].

Satchell and Scowcroft (2000)[11], attempted to demystify the BL model, but instead introduced a new non-Bayesian expression.

Their aim was to present details of Bayesian portfolio construction procedures which have become known in the asset management industry as BL models. They explain their construction, present some extensions and argue that these models are valuable tools for financial management.

Drobetz (2001), provides another description of the BL model which includes a debate of how to interpret the confidence in the estimates. This is one of the first papers on the canonical form not by an original author of the model. In his paper he speaks about asset allocation, in particular underlining that: "*It is well known that asset allocation policy is the major determinant of fund performance. However, there is substantial disagreement about the exact magnitude of the contribution of asset allocation. Following the approach in Ibbotson and Kaplan (2000), we use German and Swiss balanced mutual fund data to show that the correct answer depends on the specific question being asked. We find that more than 80 percent of the variability in returns of a typical fund over time is explained by asset allocation policy, roughly 60 percent of the variation among funds is explained by policy, and more than 130 percent of the return level is explained, on average, by the policy return level[12]".*

Fusai and Meucci (2003), providing the alternate formulation of the posterior variance and also a new measure for determining whether some views are extreme or not.

Meucci (2005), in his paper: "*Beyond Black-Litterman in Practice: A Five-Step Recipe to Input Views on Non-Normal Markets"*, introduced the *copula-opinion pooling* (COP) approach extends in principle the BL methodology to non-normally distributed markets and views. The implementations of the COP framework presented so far rely on restrictive quasi-normal assumptions. Here he present a general recipe to implement the COP approach in practice under all possible market and views specifications.

Meucci (2006), "*Beyond Black-Litterman: Views on Non-Normal Markets"*, he extends the BL to generic non-normal market distributions and non-normal views. He

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[9] I use this model for my Matlab application see Appendix 1.

[10] Bevan and Winkelmann, 1998, "Using the Black Litterman Global Asset Allocation Model: Three Years of Pratical Experience", *Goldman Sachs, Fixed Income Research.*

[11] Satchell S. and Scowcroft A., 2000, "*A demystification of the Black–Litterman model: Managing quantitative and traditional portfolio construction Received"*

[12] Drobetz W.and F. Köhler, 2003 ,"*The Contribution of Asset Allocation Policy to Portfolio Performance"*

draw on the copula and opinion pooling literature to express views directly on the market realizations, instead of the market parameters as in the BL case. He compare the two approaches and he show an application to a thick-tailed, skewed and highly dependent market, where the views are expressed as uncertainty ranges. In other word he illustrated a new method to use non-normal views in BL model.

Meucci (2010), "*The Black-Litterman Approach: Original Model and Extensions*", he showed how minor modifications of the original BL model greatly improve its range of applications. Moreover he discuss full generalizations of this and related models.

Meucci (2015), in his most recent paper: "*Dynamic Portfolio Management with Views at Multiple Horizons*", introduced *Dynamic Entropy Pooling* a quantitative technique to perform dynamic portfolio construction with discretionary, non-synchronous views.

Firoozye and Blamont (2003), demonstrated the reduction in variance of the posterior estimate. Moreover they concluded that tau needs to be between 0 and 1.

Herold (2003), describes an approach, in which the BL procedure can be employed with qualitative analysts and forecasts. He examines optimizing alpha generation, specifying that the sample distribution has zero average. The author also provides additional measures which can be used to validate that the views are correct.

Koch (2004), illustrates derivations of the *master formula* and the alternative form under 100% certainty. He does not mention posterior variance, or show the alternative form of the *master formula* under uncertainty (general case). He includes a formulation on the sensitivity of the posterior estimate on τ using the alternative reference model.

Idzorek (2005), in his famous paper: "*A step by Step guide to the Black Litterman model*[13]", introduced a new method for controlling the tilts and the final portfolio weights caused by views. The new method asserts that the magnitude of the tilts should be controlled by the user-specified confidence level based on an intuitive 0% to 100% confidence level.

Krishnan and Mains (2005), take they recession factor [14]into the original BL model and set up a new one named the two-factor BL model. The authors show how the recession factor has an impact on the expected returns computed from the model[15]

Mankert (2006), gives an interesting analysis of the model and provides a detailed transformation between the two specifications of the BL *master formula* for the estimated asset returns. She provides a new approach regarding the value τ.

Beach and Orlov (2006), in their paper: *An Application of the Black-Litterman Model with EGARCH-M-Derived Views for International Portfolio Management*[16], show an application of the Black and Litterman methodology to portfolio management in a global setting. The novel feature of this paper is that they use GARCH derived views as an input

[13] Where the focus is incorporating user-specified confidence levels.

[14] Where the first factor is the market and the second factor is an orthogonalized recession risk factor. They show how to generate equilibrium returns and how to then revise them according to investor views. They incorporate a downside risk factor, but in a linear framework.

[15] Krishnan H. and Mains N., 2005, "The two-factor Black Litterman model", *Financial Journal.* Here they rewrited the BL equation in a two-factor framework that takes long-term recession risk into account

[16] Use *EGARCH-M* model to set the investors' view and set up new BL model based on *EGARCH-M* model.

into the BL model. The returns on their portfolio surpass those of ones that rely on market equilibrium weights or Markowitz's optimal allocations.

Braga and Natale (2007), propose a new measure for the marginal contribution of each view to the ex - ante tracking error volatility (TEV)[17]. They also provide sensitivities for the posterior estimates to the various views.

Martellini and Ziemann (2007), extending BL beyond the MV framework. They describe an approach to active management of hedge funds.
Their results suggest that the systematic implementation of active style allocation decisions can add significant value to a hedge fund portfolio.

Giacometti, Bertocchi, Rachev and Fabozzi (2007)[18], improve the classical BL model by applying more realistic ones for asset returns (the normal, the t - student, and the stable distributions) and by using alternative risk measures (dispersion-based risk measures, value at risk, conditional value at risk). At the end they find that incorporation of the views of investors in the model provides information as to how the different distributional hypotheses can effect the optimal composition of the portfolio.

Bertsimas, Gupta and Paschalidis (2013), in their famous papers: "*Inverse optimization, a new prospective for asset allocation"*, propose a richer formulation that is flexible enough to incorporate investor information on volatility and market dynamics. Their approach allows movement beyond the traditional MV paradigm of the original model and construct *BL - type* estimators for more general notions of risk such as coherent risk measures. They introduce two new *BL - type* estimators and their corresponding portfolios: a mean variance inverse optimization (MV-IO) portfolio and a robust mean variance inverse optimization (RMV-IO) portfolio. These two new approaches are based on ideas from arbitrage pricing theory and volatility uncertainty. They show that both methods often demonstrate a better risk-reward trade-off than their BL counterparts and are more robust to incorrect investor views.

Michaud, et al (2013)[19], provides a critique of the *Alternative Reference Model*. In their opinion the BL portfolio is often uninvestable in applications due to large leveraged short allocations. BL use an input tuning process for computing acceptable sign constrained solutions. They compare constrained BL with MV and Michaud (1998) optimization for a simple dataset. They show that constrained BL is identical to Markowitz and that Michaud portfolios are better diversified under identical inputs and optimality criteria.

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[17] *TEV Sensitivity to Views in Black-Litterman* Model. The question of the TEV sensitivity to the views is important because: provide the asset managers with a method for revising the portfolio consistently with a given TEV constraint; make the specialists responsible for the generation process of the views and set a mechanism to connect the incentive fees not only to the excess return but also to the marginal contribution of each view to the TEV.

[18]Giacometti, Bertocchi, Rachev and Fabozzi, 2007, "Stable distributions in the BL approach to asset allocation", *Financial Journal*

[19] Michaud et al, 2013, "Deconstructing black–litterman: how to get the portfolio you already knew you wanted", *Journal Of Investment Management*,Vol. 11, No. 1, 6–20.

These are just some of the many studies about BL, which have been developed and improved.

The focus of this thesis is not the alternative methods for BL, but rather easier ways to specify the views. During this work we are following He and Litterman approach in particular for my Matlab implementation.

1.2 Literature about Carry

The concept of carry has been studied in literature almost exclusively for currencies, where it represents the interest rate differential between two countries. The currency literature looks at the uncovered interest rate parity (UIP)[20] and explains the empirical deviations from UIP.

The *carry trade strategy* consists in borrowing low-interest-rate currencies and lending high interest-rate currencies. This strategy is motivated by the failure of uncovered interest parity (UIP) documented by Bilson (1981) and Fama (1984).

The literature about Carry goes back at least to Meese and Rogoff (1983) who discovered that exchange rates follow a "near random walk" allowing investors to take advantage of the interest differential without suffering an exchange rate depreciation. Moreover currency spot rates are nearly unpredictable out of sample.

Surveys are presented by Froot and Thaler (1990), Lewis (1995), and Engel (1996) regarding the literature on uncovered interest parity.

Explanations of the UIP failure include: liquidity risk, crash risk , volatility risk and Peso problems:

1.Liquidity risk: Carry performs worse when there are liquidity squeezes (Brunnermeier, Nagel, and Pedersen (2008)) and increases in foreign exchange volatility (Menkhoff, Sarno, Schmeling, and Schrimpf (2011)). Its risk exposures are also time-varying, increasing in times of greater uncertainty (Christiansen, Ranaldo, and Söderlind (2010)).

2.Crash risk : Farhi and Gabaix (2008) develop a model in which the forward premium arises because certain countries are more exposed to rare global fundamental disaster events. Their model is calibrated to also match skewness patterns obtained from FX option prices. Instead of focusing on exogenous extreme productivity shocks, evidence is provided with a theory that currency crashes are often the result of endogenous unwinding of carry trade activity caused by liquidity spirals.

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[20] A parity condition beginning from the difference in interest rates between two countries is equal to the expected changes in exchange rates between the countries' currencies. If this parity does not exist, there is an opportunity to make a profit.

3.Volatility risk: Bhansali (2007) argues that carry trades are essentially short volatility and documents that option based carry trades yield excess returns. Lustig and Verdelhan (2007) focus on the cross-sectional variation between the returns of high and low interest rate currencies and make the case that the return on currencies with high interest rates have higher loading on consumption growth risk. Jurek (2007) finds that the return to the carry over period 1999-2007 with downside protection from put options of various moneyness is positive. Furthermore he finds that the more protection one buys on the carry trade, the smaller is the average return and Sharpe ratio. Ranaldo and Söderlind's (2007) findings that safe-haven currencies appreciate when stock market volatility increases, can be related to our third set of findings where the unwinding of carry trades is correlated with the volatility index, VIX. Lustig, Roussanov, and Verdelhan (2011) however show that the risk of carry trades across currency pairs is not completely diversified, so there is a systematic risk component.[21]

4.Peso problems: Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011)

Burnside (2007) argues, however, that their model leaves a highly significant excess zero-beta rate unexplained. Furthermore Eichenbaum, Kleshchelski, and Rebelo (2006 - 2007) find that the return of the carry trade portfolio is uncorrelated to standard risk factors, attributing instead the forward premium to market frictions (bid-ask spreads, price pressure, and time-varying adverse selection). Suominen (2008) argues that inflation risk is higher in high interest rate currencies and shows a positive relationship between carry trade returns and hedge fund indices. Burnside, Eichenbaum, and Rebelo (2008) also show that a well-diversied carry trade attains a Sharpe ratio that is more than double that of the US stock market itself a famous puzzle (Mehra and Prescott (1985)). Bacchetta and van Wincoop (2007) again attribute the failure of UIP to infrequent revisions of investor portfolio decisions.

Finally, there are many papers that study crash risk and skewness in the stock market for example Chen, Hong, and Stein (2001)

Therefore, this strategy has received a great deal of attention in academic literature as researchers struggle to explain its apparent profitability. Consequently while the carry trade crashed, a diversified currency strategy fared quite well in this turbulent period. At the same time literature on alternative currency investments have seen major developments since 2008. Menkhoff, Sarno, Schmeling and Schrimpf (2011) document

[21] They form an empirically motivated risk factor the return of high-yielding currencies minus low-yielding currencies (HMLFX) close in spirit to the stock market factors of Fama and French (1992) and show that it explains the carry premium. But the HMLFX is itself a currency strategy, so linking its returns to more fundamental risk sources is an important challenge for research in the currency market. Some risks of the carry trade are well known. High yielding currencies are known to go up by the stairs and down by the elevator implying that the carry trade has substantial crash risk. Carry performs worse when there are liquidity squeezes

the properties of currency momentum, Burnside (2011) examines a combination of carry and momentum, Asness, Moskowitz, and Pederson (2009) study a combination of value and momentum in currencies (and other asset classes), and Jord and Taylor (2009) combine carry, momentum and the real exchange rate. Most of the studies on alternative currency strategies focus on simple, equal weighted portfolios. At the end, Burnside (2011) also examines a combination of carry and momentum. Melvin and Taylor (2009) also provide a vivid narrative of the major events in the currency market during the crisis. Our goal is to combine Carry's idea with a model (BL) that can optimize the weight for different assets.

2. Carry

2.1 The new concept of Carry

In the analysis below we follow Koijen, Pedersen, Moskowitz, and Vrugt (2013) and compute the carry of each asset from Futures (F) and Spot prices (S). They define the carry as the return an investor would earn if market conditions stay constant[22]. If we consider this assumption, any asset's return can be divided into Carry and both its expected and unexpected price appreciation:

$$\begin{aligned}\mathbf{Return} = \mathbf{Carry} + \mathbf{E(price\ appreaciation)} \\ + \mathbf{Unexpected\ price\ Shock}\end{aligned} \qquad \textit{Eq. 3}$$

$$\begin{aligned}&\mathbf{Expected\ return} \\ &\qquad = \mathbf{Carry} \\ &\qquad + \mathbf{E(price\ appreciation)}\end{aligned} \qquad \textit{Eq. 4}$$

Therefore, an asset's expected return is the Carry plus expected price appreciation, where Carry is model-free and is observable in advance. This concept as shown below has received a great deal of attention in academic literature as researchers struggle to explain its apparent profitability.

[22] For example, for a futures the carry is the expected return if the underlying (spot price) never changes. For assets with cash flows such as dividends, the carry is the expected return assuming the cash flow never changes. However for asset with dividend (or cash flows) the carry is defined as the expected return assuming the cash flow never changes.

If we consider eq.1 and 2 from Koijen, Pedersen, Moskowitz, and Vrugt (2013), this relation can be enforced regarding any securities.

These authors show that Carry is a predictor of return in all asset classes with different magnitudes. In particular they conclude that if the carry's coefficient is greater than 1 (see the regression in chapter 2.3. Carry and Expected Return) it predicts a positive future price change that adds to return that is over and beyond itself.

The expectedness of carry lends support to the models of varying expected return, but what is the source of this change? Researchers and theory advances consider that expected returns can vary due to macroeconomic risk[23], limited arbitrage[24], market liquidity risk[25], funding liquidity[26], volatility risk[27] or exposure to other and different global risk factors.

Consequently if the carry is less than one, this implies that the market takes back part of the Carry[28].

Koijen, Pedersen, Moskowitz, and Vrugt (2013) consider how much of the returns to Carry strategies can be unfolded by other famous global return factors such as value, momentum and time series momentum in each asset class. They find that Carry is not explained by the other factors and that it is a unique return predictor in each asset class.

However carry strategies have little exposure to traditional macro indicators. On the other hand carry returns tend to be lower during a global crisis. If we consider both the worst and the best carry return episodes and call these Drawdowns and Expansion, we find that during carry drawdowns all strategies do poorly however they perform significantly worse than the passive exposures to the same market.

Another problem for the Carry strategies are skewness and kurtosis, but the cross all asset classes diversified carry factor have a skewness close to zero and thinner tails. This idea that crash risk theories about currency carry premium is unlikely to explain the general one.

Next they consider another problematic aspect for carry strategies: liquidity and volatility risk[29]. They find that Carry strategies are more frequently exposed to liquidity shocks and negatively exposed to volatility risk. Therefore its strategies tend to incur losses in this scenario with low liquidity and high volatility.

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[23] Campbell and Cochrane (1999) and Bansall et al (2004)
[24] Shleifer and Vishny (1997)
[25] Pàstor and Stambaugh (2003), Acharya and Pedersen (2005)
[26] Brunnermeier and Pedersen (2009), Garleanu and Pedersen (2011)
[27] Bansal, Kiku, Shaliastovich, and Yaron (2013) and Campbell, Giglio, Polk, and Turley (2012)
[28] Although not all, as implied by UIP/EH
[29] Amihud Y., Mendelson H. and Pedersen L.H., 2013, "Market Liquidity, asset pricing, Risk and Crises", *Cambridge Uviversity Press*

If we had a global vision about carry return premium, we can divide the return to carry strategy in each asset class into a passive and dynamic component. The passive component derived from being on average long (short) assets with high (low) average return, with the dynamic component consider how variations in carry around its average predicts future returns.

Koijen, Pedersen, Moskowitz, and Vrugt (2013) show as the dynamic component of carry strategies that characterize the return to the equity, fixed income and option, and contributes about half of the return to the US Treasury, currency, credit and commodities. Their studies also related to the literature on return predictability see above.

However Koijen, Pedersen, Moskowitz, and Vrugt (2013) show that Carry is also related to other classical predictor returns. For example, they demonstrate how the carry for bond is connected to the slope of the yield curve studies. Commodities Carry is related to the convenience yield and equity carry is a forward looking measure of dividend yield[30]. Therefore there are some indices that can predict return in different asset classes, but the power of the Carry is that it unifies these measures and allows us to investigate return predictability across these asset classes. Carry is related to the typical know predictors return however there is a difference because the predictability of Carry is often stronger than traditional predictors. This is the confirmation that carry not only unifies the different frameworks, and may also improve upon return predictability in each asset class.

2.2 Carry in different Asset Classes

Following Koijen, Moskowitz, Pedersen and Vrugt (2013) we can divide the return to any securities into two different components: Carry and Price Appreciation.

As we state above this concept is true if we assume that the price (or market condition) stay the same. In this case Carry can be observed in advance. For example, if we consider Future contracts [31]that expire in period $t+1$ with a current future price F_t and Spot

[30] See again Cochrane(2011) and Ilmanen (2011)

[31] **Bloomberg Definition**: "*A financial contract obliging the buyer to purchase an asset (or the seller to sell one), such as a physical commodity or a financial instrument, at a predetermined future date and price. Futures contracts detail the quality and quantity of the underlying asset; they are standardized to facilitate trading on a futures exchange. Some futures contracts may call for physical delivery of the asset, while others are settled in cash. The futures markets are characterized by the ability to use very high leverage relative to stock markets. Futures can be used either to hedge or to speculate on the price movement of the underlying asset. For example a producer of corn could use futures to lock in a certain price and reduce*

price of the underlying securities S_t we start to define the return of the futures. Assuming that an investor allocates X_t dollars today of capital to finance each futures contract, in the next period the price of the marginal capital and the future contracts is equal to $X_t(1 + r_t^f) + F_{t+1} - F_t$, where r_t^f is the risk free interest rate today that is earned on the Marginal Capital. Hence, the return for the capital over one period is

$$r_{1t+1} = \frac{X_t(1 + r_t^f) + F_{t+1} - F_t - X_t}{X_t} = \frac{F_{t+1} - F_t}{X_t} + r_t^f \qquad \textit{Eq. 5}$$

Moreover, the return in excess of the risk-free is:

$$r_{t+1} = \frac{F_{t+1} - F_t}{X_t} \qquad \textit{Eq. 6}$$

The Carry (C_t) is calculated as the Future Excess return under the assumption that the Spot price stays the same from t to $t+1$. We know that the future price expires at the future spot price ($F_t = S_{t+1}$), assuming that the spot price stays the same ($S_{t+1} = S_t$), we have that ($F_{t+1} = S_t$). As a consequence Carry can be defined as:

$$C_t = \frac{S_t - F_t}{X_t} \qquad \textit{Eq. 7}$$

We can compute returns and carry based on a fully collateralized position[32]. That is the amount of capital allocated to the position is equal to the futures price $X_t = F_t$

Therefore, the carry of a fully - collateralized position is

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risk (hedge). On the other hand, anybody could speculate on the price movement of corn by buying and selling long or short using futures".

[32] A fully collateralized position has two components of return: the change in the value of the derivative plus any return on the collateral. If we define R as the percentage change in the value of the derivate based on notional value and using continuous compounding the return on a fully collateralized position R_{fcoll}, can be expressed as: $R_{fcoll} In(1 + R) + R_f$ where R is the change in the derivatives price divided by its previous price or notional value. The first term is the continuously compounded percentage change in the fully collateralized position due to change in the value of the derivative. While the second term is the percentage change in the fully collateralized position from interest on the collateral, the sum represents the total return on the fully collateralized position.

$$C_t = \frac{S_t - F_t}{F_t} \qquad \textit{Eq. 8}$$

As described above, we can divide the return into its expected one plus unexpected price appreciation to know how carry relates to expected returns. Following the Carry definition the decomposition of excess return on the futures such as:

$$r_{t+1} = C_t + E_t\left(\frac{\Delta S_{t+1}}{X_t}\right) + u_{t+1} \qquad \textit{Eq. 9}$$

Where $\Delta S_{t+1} = S_{t+1} - S_t$, however the unexpected price shock is $u_{t+1} = \left(S_{t+1} - E_t(S_{t+1})\right) / X_t$.

The Equation above shows how Carry is related to the expected return $E_t(r_{t+1})$ even if the two are not the same. The expected return is comprised of both the carry and expected price appreciation which depend on the specific asset pricing model used to create expectations and its risk premium. However the component Carry can be measured ex - ante without the need to specify the pricing model and consequently Carry is a component of the expected return on an asset. Moreover Carry can be relevant for predicting expected price changes on an asset which also contribute to its expected return.

Below we can see how carry is related to different asset classes and how it can be calculated.

2.2.1 Currency Carry

For a currency the carry is the local interest rate in the corresponding country. Investing in a currency by putting cash into a country's money market increases the interest rate if the exchange rate (price of the currency) stays the same. To calculate the carry of a currency from forward rates we have to remember that the no - arbitrage price of a currency forward contract with spot exchange rate S_t (measured in number of local currency units per unit of foreign currency), local interest rate r^f, and foreign interest rate r^{f*} is

$$F_t = \left[\frac{S_t\left(1 + r_t^f\right)}{\left(1 + r_t^{f*}\right)}\right] \qquad \textit{Eq. 10}$$

However the carry of the currency is

$$C_t = \frac{S_t - F_t}{F_t} = \left(r_t^{f*} - r_t^f\right) - \frac{1}{1 + r_t^f} \qquad \textit{Eq. 11}$$

The carry is the interest-rate spread $\left(r_t^{f*} - r_t^f\right)$, adjusted by a scaling factor that is close to one $1/1 + r_t^f$.

Hence the carry is the foreign interest rate in excess of the local risk-free rate r_t^f because the forward contract is a zero-cost instrument whose return is an excess one.[33]

2.2.2. Global Equity Carry

Koijen, Moskowitz, Pedersen and Vrugt (2013) implement a global equity carry strategy via futures. While we do not always have an equity futures contract with exactly one month before expiring. We interpolate between the two nearest – to - maturity futures prices to compute a consistent series of synthetic one - month equity futures prices[34]. The no-arbitrage price conditions in a Futures contracts is $F_t = S_t\left(1 + r_t^f\right) - E_t^Q(D_{t+1})$, where D is the expected dividend payment under the risk-neutral measure Q and r_t^f which is the risk free in time t in a specific country (Binsbergen, Brandt, and Koijen (2012) and Binsbergen, Hueskes, Koijen, and Vrugt (2013). As a result we can rewrite a carry for the equity future as:

$$C_t = \frac{S_t - F_t}{F_t} = \left(\frac{E_t^Q(D_{t+1})}{S_t} - r_t^f\right)\frac{S_t}{F_t} \qquad \textit{Eq. 12}$$

Where $E_t^Q(D_{t+1})/S_t$ is the expected dividend yield, r_t^f being the local risk-free rate and S_t/F_t is a scaling factor that is close to one.

[33] The scaling factor reflects currency exposure using a forward/future contract correspond to buying one unit of foreign currency in the future.

[34] More details are discused in Appendix 1.

In other word Carry is the expected dividend yield minus local risk-free rate adjusted by a scaling factor that is close to one. Therefore, if a stock price stays the same, the stock return comes only from the dividend or put simply carry is the dividend yield.

If expected returns stay constant, the dividend growth will be high when the dividend yield will be low so that the two parts of E(R) would offset each other.

While expected returns do vary, it is natural to expect carry to be positively related to expected returns: "*If a stock's expected return increases while dividends stay the same, then its price drops and its dividend yield increases*[35]".

A high expected return leads to a high carry and the carry predicts returns more than one-for-one. Carry is a forward-looking measure while the classic dividend yield used in literature is backward - looking.

2.2.3 Commodity Carry

Here the carry is the convenience yield or net benefits of the use of the commodity in excess of storage costs. While the actual convenience yield is hard to measure (and may depend on the specific investor), the carry of a commodity futures can be easily computed and represents the expected convenience yield of the commodity.

Similar to the dividend yield on equities, where the actual dividend yield may be hard to measure since future dividends are unknown in advance, thus the expected dividend yield comes from futures prices.

Consequently one of the reasons we employ futures contracts is to easily and consistently compute the carry across asset classes.

The no - arbitrage price of a commodity futures contract is $F_t = S_t\left(1 + r_t^f - \delta_t\right)$, where δ_t is the convenience yield in excess of storage costs. As a result the carry for a commodity futures contract is:

$$C_t = \frac{S_t - F_t}{F_t} = (\delta_t - r^f)\,\frac{1}{1 + r_t^f - \delta_t} \qquad \textit{Eq. 13}$$

[35] Campbell and Shiller, 1988, "Stock prices, earnings and expected dividends", *Scholars at Harvard*

where the commodity carry is the expected convenience yield of the one in excess of the risk free rate (adjusted for a scaling factor that is close to one).

To calculate the Carry from the above equation, we need data on the current spot price S_t. and current futures price F_t. Commodities spot markets are often illiquid and clean spot price data on commodities are frequently unavailable. To oppose this problem, instead of examining the slope between the spot and futures prices we can consider the slope between two futures prices of different maturity. In particular we examine the price of the nearest-to-maturity commodity futures contract with the price of the next-nearest futures contract on the same commodity.

For example, if the nearest to maturity futures price is F_t^1 with T_1 months to maturity and the second futures price is F_t^2 with T_2months to maturity, where $T_2 > T_1$. When we rewrite the no-arbitrage futures price as $F_t^{Ti} = S_t(1 + (r^f - \delta_t)T_i)$ the carry of holding the second contract can be computed by assuming that its price will converge to F_t^1 after $T_2 - T_1$ months, that is considering that the price of a T_1-month futures stays the same:

$$C_t = \frac{F_t^1 - F_t^2}{F_t^2\ (T_2 - T_1)} = \left(\delta_t - r_t^f\right) \frac{S_t}{F_t^2} \qquad \textit{Eq. 14}$$

we can simply use data from the futures market - specifically, the slope of the futures curve—to obtain a measure of carry that captures the convenience yield.

Hence carry provides an interpretation of some of the predictors of commodity returns examined by Gorton, Hayashi, and Rouwenhorst (2007), Hong and Yogo (2010), Yang (2011) and is linked to the convenience yield on commodities.

2.2.4 Global Bond Carry

Koijen, Moskowitz, Pedersen and Vrugt (2013) continue with their representation of Carry and now focus their attentions on the Global Bond. There is still the problem of liquidity because liquid bond futures contracts are only traded in some countries and, if they exist, they are very few.

Moreover there is another intricacy because different bonds have different coupon rates and the futures price is subject to cheapest - to - deliver option.

Koijen, Moskowitz, Pedersen and Vrugt (2013) derive synthetic future price based on data regarding zero coupon rates as follows.

They consider a futures contract that gives the obligation to buy a 9-year-and-11-months zero-coupon bond in one month from now.

The price of this one-month future is $F_t = \left(1 + r_t^f\right) / (1 + y_t^{10Y})^{10}$, where y_t^{10Y} is the current yield on a 10-year zero-coupon bond. At the same time, the "spot price" is $S_t = 1/(1 + y_t^{9Y11M})^{9+11/12}$.

As a consequence if we follow the first equation the carry is given by :

$$C_t = \frac{S_t}{F_t} - 1 = \frac{\frac{1}{(1 + Y^{9Y11M})^{9+\frac{11}{12}}}}{\frac{\left(1 + r_t^f\right)}{(1 + y_t^{10Y})^{10}}} - 1 \qquad \textit{Eq. 15}$$

We can approximate carry using the modified duration D^{mod} as follows :

$$C_t \cong y_t^{10y} - r_t^f - D^{mod}\,(y_t^{9Y11M} - y_t^{10Y}) \qquad \textit{Eq. 16}$$

Therefore it is intuitive that the Carry's bond is the Bond's yield spread to the risk-free rate (also called the slope of the term structure) plus the roll down which explains the price increase due to the fact that the bond rolls down the yield curve.

2.2.5 Option Carry

Finally, carry is applied to U.S. equity index options. They define the price of a call option at time t with maturity T[36], strike price K[37], implied volatility σ_t, and underlying spot price S_{it} as $F_t^{Call}\,(S_{it}, K, T, \sigma_t)$. The equivalent put price is denoted by $F_t^{put}(S_{it}, K, T, \sigma_t)$.

[36] The expiration date of an option contract is the last date on which the holder of the option may exercise it according to its terms.
[37] The strike price for an option is the price at which the underlying asset is bought or sold if the option is exercised.

We apply the same concept of carry as before, regarding the return on a security if market conditions do not change. In the context of options this implies the definition of carry (j = Call, Put)

$$C_{it}^{j}\,(K,T,\sigma_T) = \frac{F_t^{j}\,(S_{it},K,T-1,\sigma_{T-1})}{F_t^{j}(S_{it},K,T,\sigma_T)} - 1 \qquad \textit{Eq. 17}$$

which depends on the maturity, the strike, and the type of option traded. We could subtract the risk-free rate from this expression, however all options are traded in US markets and therefore this will not change the rank of the signals in our cross-sectional strategies.

While we compute option carry using the exact expression Eq.17 throughout the paper, we can have some ideas through an approximation based on the derivative of the option price with respect to time (θ[38]) and implied volatility (v[39]):

$$\begin{aligned} F_t^{j} = (S_{it},K,T-1,\sigma_{T-1}) \\ &\cong \left(F_t^{j}(S_{it},K,T,\sigma_T)\right) \\ &- \theta_t^{j}(S_{it},K,T,\sigma_T) \\ &- \nu_t^{j}(S_{it},K,T,\sigma_T)\,(\sigma_T - \sigma_{T-1}) \end{aligned} \qquad \textit{Eq. 18}$$

This allows us to write the option carry as:

$$C_{it}^{j}\,(K,T,\sigma_T) \cong \frac{-\,\theta_t^{j}(S_{it},K,T,\sigma_T) - \nu_t^{j}(S_{it},K,T,\sigma_T)\,(\sigma_T - \sigma_{T-1})}{F_t^{j}(S_{it},K,T,\sigma_T)} \qquad \textit{Eq. 19}$$

[38] Theta of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time. The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remains the same, the value of a long call or put option declines. Or more compacts Theta is the change in price of the option with time thus a measure of the decay of time value.

[39] Vega is the rate of change of the value of a derivatives portfolio with respect to volatility.

The dimension of the carry is subsequently driven by the time decay (via θ) and the roll down on the implied volatility curve (via ν). The option contracts that we consider differ in terms of their moneyness, maturity, and put/call characteristic as we describe below.

Therefore this general concept of carry provides a unifying framework that synthesizes much of the literature about the return predictability in different asset classes. Hence as we show above the carry[40] in equities is related to the dividend yield. Then one in fixed income is related to the yield spread, and in commodities one is related to the convenience yield. These predictors are often treated as separate and unrelated in each asset class. However this new concept of carry provides a common theme that links these predictors across asset classes. Carry is also different from these standard predictors and adds to the predictability literature.

2.3 Carry and Expected Return

The significant returns to the carry trade indicate that it is indeed a signal of expected returns.

To understand the relation between carry and expected returns consider the first equation, which divides expected returns into carry and expected price appreciation. To estimate this relationship we run the following panel regression for each asset class:

$$r_{t+1} = c + b_t + \beta C_t + \varepsilon_{t+1} \qquad \textit{Eq. 20}$$

Where c is an intercept or fixed effects, b_t are the time fixed effects, C_t is the Carry at the time t and β is the coefficient that measures how well carry predicts returns. We can consider different cases[41]:

1. $\beta = 0$ means that carry does not predict returns
2. $\beta = 1$ means that the expected return moves one-for-one with carry
3. $\beta > 0$ means that a positive carry is associated with a positive expected price appreciation so that an investor has the carry and price appreciation too - thus, carry predicts further price increases.
4. $\beta < 0$ would imply that there is an inverse relationships between Carry and expected returns

[40] Koijen, Moskowitz, Pedersen and Vrugt

[41] Koijen Ralph S.J, Moskowitz T. J., Pedersen L.H., Vrugt E. B., 2013, "Carry", *AQR Capital.*

Without fixed effects and time fixed effects, β rapresents the total predictability of returns from carry from both its passive and dynamic components. If we include the time fixed effects they remove the time-series predictable return component coming from general exposure to assets at a given point in time. If we include the asset-specific fixed effects they remove the predictable return component of carry coming from passive exposure to assets with different unconditional average returns. If we include both asset and time fixed effects, the coefficient represents the predictability of returns coming simply from variation in carry.

3. The Black Litterman Model

3.1 The Equilibrium

The BL model uses a Bayesian approach to combine the investor's views regarding the expected returns of assets with the market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns. The result is a new vector of returns (the posterior distribution) leading to intuitive portfolios with sensible portfolio weights. Consequently the Black Litterman model uses equilibrium return as a neutral starting point.[42]

If we employ the Quadratic Utility function and risk free rate the equilibrium model is the Capital asset pricing Model (CAPM)[43]. One results that there is a linear relationship between risk and return:

$$E(r) = r_f + \beta r_m + \alpha \qquad \textit{Eq. 21}$$

Where r_f is the risk free rate, r_m is the excess return of the market portfolio, β is the regression coefficient computed as $\beta = \rho\, \sigma_p / \sigma_m$ and α is the idiosyncratic excess return. Under the assumption of CAPM all investors hold the same portfolio called CAPM Market portfolio.

[42] Jay W.(Initial 2007 and Revised 2014), "The Black- Litterman Model In Detail Initial", <u>*SSRN Electronic Journal*</u> *07/2011; DOI: 10.2139/ssrn.1314585*

[43] In equilibrium the risk premium of an asset is the coefficient of the projection of its return on the market return times the market risk premium.

We consider a market with *n* assets and *one* risk-free asset in which the investor chooses asset weights in order to maximize the value of a utility function that rewards higher expected returns and penalizes portfolio risk. Assume the utility function has this simple quadratic form:

$$U = w'\Pi - \frac{\delta}{2} w'\Sigma w \qquad \textit{Eq. 22}$$

Examining in more detail: U Investors utility (convex function), which is the objective function during Mean-Variance Optimization, w Vector of weights invested in each asset, Π Vector of equilibrium excess returns for each asset, δ Risk aversion parameter and Σ Covariance matrix of the excess returns for the assets.

The quadratic form of the utility function (U) represents the assumption that as risk grows there is an increasing aversion (in the form of willingness to forgo expected return) to additional increases in risk.

The utility function has its maximum if:

$$\frac{dU}{dw} = \Pi - \delta\Sigma w = 0 \qquad \textit{Eq. 23}$$

And solving for Vector of implied equilibrium excess returns for each asset we can derive the reverse optimization as:

$$\Pi = \delta\Sigma w_{mkt} \qquad \textit{Eq. 24}$$

Missing only a components: the risk aversion coefficient.

This one characterizes the expected risk-return trade-off. It's the rate at which a investor will forego expected returns for less variance. In the reverse optimization the risk-aversion coefficient acts as a scaling factor for the reverse optimization estimate of excess returns; the weighted reverse optimized excess returns equal the specified market risk premium. "*More excess return per unit of risk (a big lambda) increases the estimated excess returns*[44]". Therefore implied risk aversion δ for a portfolio can be estimated following Grinold and Kahn (1999):

[44] Jay W.(Initial 2007 and Revised 2014), "The Black- Litterman Model In Detail Initial", <u>*SSRN Electronic Journal*</u> *07/2011; DOI: 10.2139/ssrn.1314585*

$$\lambda = \frac{E(r_m) - r_f}{\sigma^2} \qquad \textit{Eq. 25}$$

Where $E(r_m)$ is the expected market total return, r_f is the risk free rate and σ^2 is the variance of the market excess return[45].

Then summarizing: *in equilibrium the excess return is equal to the risk aversion parameter multiplied by the variance of the portfolio*.

If we modify eq.24. and replacing μ that is any vector of excess return for Π (vector of implied excess equilibrium returns) this leads to eq.22, the results to the unconstrained maximization problem : $\max_w w'\mu - \delta w'\Sigma/2$ is:

$$w = (\delta\Sigma)^{-1}\mu \qquad \textit{Eq. 26}$$

And if μ is different from Π , w will not the same w_{mkt}. The implied equilibrium return Π is the market starting point for BL model.

A Introduction to Bayesian approach[46]

$$P(A|B) = \frac{P(B|A)\, P\,(A)}{P(B)} \qquad \textit{Eq. 27}$$

$P(A|B)$: The conditional probability of A, given B also known as the posterior distribution.

$P(B|A)$: The conditional probability of B given A. Also known as the sampling distribution.

$P\,(A)$: The probability of A. Also known as the prior distribution.

$P(B)$: The probability of B. Also known as the normalizing constant.

One of the assumptions of the BL model is that asset returns are normally distributed. Another assumption in BL is that the variance of the prior and the conditional distributions about the actual average are known.

[45] $\sigma^2 = \omega'_{mkt}\Sigma\omega_{mkt}$

[46] Jay W.(Initial 2007 and Revised 2014), "The Black- Litterman Model In Detail Initial", *SSRN Electronic Journal* *07/2011; DOI: 10.2139/ssrn.1314585*

One assumptions in the Black-Litterman model is that the covariance of the prior estimate is proportional to the covariance of the actual returns, however the two quantities are independent. The parameter τ will serve like the constant of proportionality.

We can define[47]:

$E(R)$	the new (or posterior) combined return vector $(Nx1)$.
τ	a scalar[48]
Σ	the covariance matrix of excess return (NxN)
P	the matrix that identifies the assets involved in the views. If there k views regards N asset is $(K\ x\ N)$.
Ω	diagonal covariance matrix of errors terms from the expressed views. $(K\ x\ K)$
Π	the implied equilibrium return vector $(Nx1)$
Q	the view vector $(K\ x\ 1)$

The prior distribution for the BL model is

$$P(A) \sim N(\Pi, \tau\Sigma) \sim N(P(A), \Sigma) \qquad \textit{Eq. 28}$$

Instead the conditional distribution is based on the investor's views. These views are specified as returns to portfolios of assets, and each view has an uncertainty which will impact the overall mixing process. The conditional distribution from the investor's views is:

$$P(B/A) \sim N\ (P^{-1}Q,\ (P'\Omega^{-1}P)^{-1}) \qquad \textit{Eq. 29}$$

Given equations above for our prior and conditional distribution respectively we can compute the following formula for the posterior distribution of asset returns

$$P(A|B) \sim N([(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}\ [(\tau\Sigma)^{-1}\ \Pi + \mathrm{P}'\Omega^{-1}Q]\ , [(\tau\Sigma)^{-1} + \mathrm{P}'\Omega^{-1}\mathrm{P}]^{-1}\ ) \qquad \textit{Eq. 30}$$

[47] If we assumed that: K is the number of views, N is the number of assets

[48] Idzorek (2005) and other do not calculate a new posterior variance, instead use the known variance of the returns regard the mean. In different approach the value of τ which is closer to 0 than to 1. Black and Litterman (1992), He and Litterman (1999) and Idzorek (2005) used a values of τ from 0.025 to 0.050.

Therefore the Black-Litterman formula is:

$$E(R) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}\,[(\tau\Sigma)^{-1}\,\Pi + \mathrm{P}'\Omega^{-1}Q] \qquad \textit{Eq. 31}$$

3.2 The view

The BL model does not require that investors specify views regards all assets. The uncertainty of the views results in a random, independent, unknown and normally-distributed error term vector (ε) with a zero mean and $\boldsymbol{\Omega}$ covariance matrix. Hence a view as a following form[49]:

$$Q + \varepsilon = \begin{pmatrix} Q_1 \\ \vdots \\ \vdots \\ Q_k \end{pmatrix} + \begin{pmatrix} \varepsilon \\ \vdots \\ \vdots \\ \varepsilon_t \end{pmatrix} \qquad \textit{Eq. 32}$$

In the general the error term ε is a negative or positive value[50].

The error (ε) does not enter straight in BL formula while variance of each error term ω [51] does enter in the formula.

The variance of the error terms describe the $\boldsymbol{\Omega}$, one which is the diagonal covariance matrix with zeros in all of the off-diagonal positions[52].

$$\Omega = \begin{bmatrix} \omega_{11} & 0 \\ 0 & \omega_{22} \end{bmatrix} \qquad \textit{Eq. 33}$$

The variances of the error terms (ω) represent the uncertainty of the view. Therefore high value of ω is equal to a great uncertainty of views.

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[49] Idzorek T.*, 2005, "*A Step-By-Step guide to the Black-Litterman Model, Incorporating UserSpecified Confidence Levels*", faculty.fuqua.duke.edu*.
[50] Except in the case in which an investor has 100% confidence in his views.
[51] That is the absolute difference from the error term's ε expected value of 0.
[52] Because the model assumes that the views are independent of the other.

Unfortunately determining the individual variances of the error term ω is one of the most complicated aspects of the BL model.

The expressed views in column vector Q are matched to specific assets by matrix P:

$$P = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \quad \textit{Eq. 34}$$

$$Q = \begin{bmatrix} 5\% \\ 10\% \end{bmatrix} \quad \textit{Eq. 35}$$

In this example there are three assets and two views. The first is a relative view (first row of P) in which the investor believes that Asset 1 will outperform Asset 3 by 5% with ω_{11} confidence. While the second is an absolute view in which the investor believes that Asset 2 will return 10% with ω_{22} confidence.

In other world an investors can have two different type of views absolute or relative. In the first case each row of matrix P sums to 1. In the second case each row of matrix P sums 0[53].

Given this concept of the views we can compute the conditional distribution mean and variance in view as:

$$P(B/A) \sim N\ (Q, \Omega) \quad \textit{Eq. 36}$$

If we did express the views in asset space the formula is:

$$P(B/A) \sim N\ (P^{-1}Q,\ (P'\Omega^{-1}P)^{-1}) \quad \textit{Eq. 37}$$

Incomplete views and relative ones make the variance non-invertible, and relative views also impact the mean term.

[53] Different authors compute the differents weights scheme, He and Litterman (1999) and Idzorek (2005) use a market capitalization weighed scheme, whereas Satchell and Scowcroft (2000) use an equal weighted scheme in their papers.

3.3 Matrix of the covariance of the views Ω

Ω is a matrix of the covariance of the views. One is diagonal because the views are independent and uncorrelated. Ω^{-1} is known as the confidence in the investor's views. The i-th diagonal element of Ω is represented as ωi. Ω and is symmetric and zero on all non-diagonal elements, but may also be zero on the diagonal if the investor is certain of a view.

Therefore the variance of the views (Ω) is inversely related to investor confidence in their views. The original BL does not provide an intuitive way to compute this relationship. Therefore there are different methods to compute Ω[54].

1. Proportional to the variance of the prior
2. The variance of residuals in a factor model
3. Idzorek's method

Below the different methods are shown.

1. Proportional to the variance of the prior

We can consider that the variance of the views will be proportional to the one of the returns. This method is followed by He and Litterman (1999), and Meucci (2006) even if with slight variations.

He and Litterman (1999) compute the variance of views as:

$$w_{ij} = p(\tau\Sigma)p' \qquad \forall i = j \qquad \textit{Eq. 38}$$

$$w_{ij} = 0 \qquad \forall i \neq j \qquad \textit{Eq. 39}$$

Or

$$\Omega = diag\ (P(\tau\Sigma)P') \qquad \textit{Eq. 40}$$

This approach of the variance for views equally weights the investor's views and the market equilibrium has the same weights. On our approach we follow this method.

Instead, Meucci (2006) does not consider the diagonal to compute the matrix Ω as:

[54] Jay W., Initial 2007 and Revised 2014, "The Black- Litterman Model In Detail", *SSRN Electronic Journal 07/2011; DOI: 10.2139/ssrn.1314585*

$$\Omega = \frac{1}{c} P\Sigma P' \qquad \textit{Eq. 41}$$

2. The variance of residuals in a factor model

In this case the investors use factor model to make their views. Consequently they must use the variance of the residuals from the model to drive the variance of the return estimates. For example, Beach and Orlov in 2006 use GARCH models to calculate their views for use with the BL. They compute the precision of the views using the GARCH models[55]

3.Idzorek's method

Idzorek presents a new method for determining the implied confidence levels in the views.

The implied confidence level can be computed with an intuitive 0% to 100% user-specified confidence level in each view to determine the values of Ω. At the same time this one removes the difficulty of specifying a value for the scalar τ. The individual variances of the error term w that form the diagonal elements of the covariance matrix of the error term Ω were based on the variances of the view portfolios $p_k \Sigma p'_k$ multiplied by the scalar τ.

In the opinion of Idzorek there is other information in addition to the variance of the view portfolio that affect an investor's confidence. Therefore a intuitive level of confidence (0% to 100%) can be assigned to each view

Idzorek argues that: *"If we consider the diagonal of Ω equal to zero it is obvious that the investor has 100% confidence in their view. With 100% confidence the Black Litterman formula becomes"*[56]:

$$E(R_{100\%}) = \Pi + \tau\Sigma P'(P\tau\Sigma P')^{-1}(Q - P\Pi) \qquad \textit{Eq. 44}$$

[55] GARCH model can be represented by two equations: one for the conditional mean and the other for the conditional variance: $y_t = x_t'\gamma + \epsilon_t$ $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ where y is the dependent variable, x_t is a vector of exogenous variables, $\epsilon_t \sim N(0, \sigma^2)$ is an error term, and $\alpha, \beta, and\ \gamma$ are the coefficients to be estimated. The one-period ahead forecast variance σ_t^2 (conditional variance) depends on the mean (ω), news about volatility from the previous period (ϵ_{t-1}^2, the ARCH term), and last period's forecast variance (σ_{t-1}^2, the GARCH term). GARCH (1,1) refers to the presence of a first-order GARCH term (the first term in parentheses) and a first-order ARCH term (the second term in parentheses).

[56] Idzorek T., *2005,* "A Step-By-Step guide to the Black-Litterman Model, Incorporating UserSpecified Confidence Levels", *faculty.fuqua.duke.edu*.

Replacing $E(R_{100\%})$for μ in equation $w = (\delta\Sigma)^{-1}\mu$ we can calculate $w_{100\%}$ that is weight vector based on 100% confidence in the views.

Idzorek argues that: "*If an asset is only named in one view, the vector of recommended portfolio weights based on 100% confidence ($w_{100\%}$) enables the calculation of an intuitive 0% to 100% level of confidence for each view*"[57].

Therefore the new unconstrained maximization problem can be solved in two different ways: first using $E(R)$ and secondly using $E(R_{100\%})$.

Idzorek showed that :"*The new return vector $E(R)$ resulting from the covariance matrix of the error term Π leads to vector $\widehat{w}$, while the new return vector $E(R_{100\%})$ based on 100% confidence leads to vector $w_{100\%}$*". [58]

The starting point of these new weight vectors is the market capitalization weights w_{mkt} hence $\widehat{w} - w_{mkt}$ and $w_{mkt} - w_{100\%}$, respectively.

After we can compute the implied level of confidence in the views by dividing each weight difference ($\widehat{w} - w_{mkt}$) from the corresponding weight difference $w_{mkt} - w_{100\%}$.

Given the difference between the declared confidence levels and the implied confidence ones, we can show this difference as w_s, and recalculate the new combined return vector $E(R)$ as well as the new set of recommended portfolio weights.

Therefore the implied confidence level is

$$confidence = (\widehat{w} - w_{mkt})/(w_{100} - w_{mkt}) \qquad \textit{Eq. 45}$$

Where $w_{100}$is the weight of the asset under 100% certainty in the view, w_{mkt} is the weight of the asset under no views and $\widehat{w}$ is the weight of the asset under the specified view.

[57] Idzorek T., *2005, "*A Step-By-Step guide to the Black-Litterman Model, Incorporating UserSpecified Confidence Levels*", faculty.fuqua.duke.edu*.
[58] Idzorek T*., 2005, "*A Step-By-Step guide to the Black-Litterman Model, Incorporating UserSpecified Confidence Levels*", faculty.fuqua.duke.edu*.

4. Our Approach

4.1 Compute Carry

We take the equity index futures data from 11 countries: the U.S. (S&P 500), Australia (S&P ASX 200). AS51, Germany (DAX), France (CAC), Spain (IBEX), Hong Kong (Hang Seng), Italy (FTSE MIB), the UK (FTSE 100), Switzerland (SMI) and Sweden (OMX). We collect data on spot, nearest-(Generic 1)[59], and second-nearest (Generic 2)-to-expiration contracts to calculate the carry as:

$$C_t = \frac{S_t - F_t}{F_t} \qquad \textit{Eq. 46}$$

Bloomberg tickers are reported in the table below.

Table 1 Bloomber Tickers

Country	Index	Generic 1	Generic 2
US	SPX	SP1	SP2
Australia	AS51	XP1	XP2
Germany	DAX	GX1	GX2
France	CAC	CF1	CF2
Spain	IBEX	IB1	IB2
Hong Kong	HSI	HI1	HI2
Italy	FTSEMIB	ST1	ST2
UK	UKX	Z1	Z2
Switzerland	SMI	SM1	SM2
Sweden	OMX	QC1	QC2

[59] For example we consider SP1: CME S&P 500 Index Futures, Launch date: April 21, 1982, Exchange Symbol: SP. Trading Hours: 1. Open Outcry: MON-FRI: 8:30 a.m.-3:15 p.m and 2. CME Globex: Effective 11/18/2012, MON-FRI: 5:00 p.m. previous day - 4:15 p.m. CT, trading halt from 8:15 a.m. - 3:30 p.m. Last Trade Date/Time: 1. Open Outcry: 3:15 p.m. on Thursday prior to 3rd Friday of the contract month and 2. CME Globex: On the rollover date (typically eight days prior to last trade date for open outcry) when the lead month goes off the screen and the deferred month becomes the new lead month. Contract Months: 1. Open Outcry: Eight months in the March Quarterly Cycle (Mar, Jun, Sep, Dec) plus three additional Dec contracts and 2. CME Globex: One month in the March Quarterly Cycle (Mar, Jun, Sep, Dec)

I calculate daily return for the most active (high volume) equity future contracts (which is the front-month contract) switching from one to another when the trading volume was greater and then aggregate the daily returns to monthly ones. This procedure is slightly different from Koijen, Moskowitz, Pedersen and Vrugt (2013), which allows me to use the most liquid contracts. For more details see appendix one

The starting date for all my indexes are:

Table 2 This table lists all the instruments that we use in our analysis, and reports the starting date for which the returns and carry are available for each instrument.

Index	**Synthetic Future**	**Begin sample**
SPX	SPX	04/29/82
AS51	XPX	01/31/01
DAX	GXX	11/29/90
CAC	CFX	01/28/93
IBEX	IBX	08/30/93
HSI	HIX	04/29/92
FTSEMIB	STX	08/28/04
UKZ	ZX	06/29/93
SMI	SMX	10/29/98
OMX	QCX	02/28/05

To understand the relation between carry and expected returns consider the first equation decomposing expected returns into carry and expected price appreciation. To estimate this relationship we run the following regression for each index:

$$r_{t+1} = c + \beta C_t + \varepsilon_t \qquad \textit{Eq. 47}$$

This formulation is different from the classic relationship because we have chosen only one asset class: Equity. Without fixed effects and time fixed effects, β rapresents the total predictability of returns from carry from both its passive and dynamic components. Following this regression I calculate the coefficients β for all indexes considering all the sample:

Table 3 The table reports the results from the regression of equation 47 for each index without asset and time fixed effects

Variable	Coefficient	t-Statistic	Prob.
SPX	1.73	1.87	0.0634
AS51	2.35	3.71	0.0002
DAX	2.46	1.89	0.061
CAC	1.53	1.5	0.1354
IBEX	-1.9	-1.85	0.066
HSI	2.1	3.03	0.0027
FTSEMIB	1.22	1.35	0.1488
UK	1.25	2.01	0.046
SMI	1.23	2.17	0.0317
OMX	1.78	1.73	0.0862

Beta (β) is the coefficient of interest that measures how well carry predicts returns. Our regression is different from the original relationships of Koijen, Moskowitz, Pedersen and Vrugt (2013). We don't consider asset and time fixed effects, because our applications regard only Equity. As a consequence we don't use the panel regression.

The results in the Table 3. indicate that carry is a strong predictor of expected returns apart from Ftsemib and Cac Indexes, however we have decided to include these in our analysis.

There are positive coefficients in all indexes except for IBEX where the coefficient is negative. In one there is an inverse relationship between expected returns and Carry.

Looking at the dimension of the predictive coefficient. Table 3. shows that the point estimate of β is greater than one for the: U.S. (S&P 500), Australia (S&P ASX 200). AS51, Germany (DAX), France (CAC),Hong Kong (Hang Seng), Italy (FTSE MIB), the UK (FTSE 100), Switzerland (SMI), Sweden (OMX), and smaller than one in Spain (IBEX).

These results imply that when the dividend yield is high regarding equities, not only is an investor rewarded by directly receiving large dividends (relative to the price), but also equity prices tend to appreciate more than usual, consistent with the discount-rate mechanism.

Our idea is that each month we could change the weights in all indexes. Therefore all the dates are monthly. This procedure allows me to buy and sell just once a month (only one rebalancing for each month).

4.2 Build the input about Black Litterman

During this paper we have been following the He - Litterman approach.

Therefore we have been considering the Risk tolerance from the equilibrium portfolio (Delta) equal to 2.5 and the Coefficient of uncertainty in the prior estimate of the mean (Tau) equal to 0.05 for each period t. Following Appedix 1 we can compute the different inputs for our model.

First we computed the Σ Covariance matrix of the excess returns for all the assets.

Table 4 Matrix of Variance and Covariance

Var/Cov	s&p	as51	dax	cac	ibex	hsi	ftse	uk	smi	omx
s&p	0.00180	0.00132	0.00185	0.00174	0.00175	0.00187	0.00203	0.00145	0.00117	0.00148
as51	0.00132	0.00162	0.00150	0.00153	0.00149	0.00180	0.00175	0.00129	0.00102	0.00135
dax	0.00185	0.00150	0.00284	0.00237	0.00219	0.00224	0.00277	0.00175	0.00140	0.00204
cac	0.00174	0.00153	0.00237	0.00240	0.00232	0.00201	0.00276	0.00173	0.00141	0.00188
ibex	0.00175	0.00149	0.00219	0.00232	0.00347	0.00221	0.00314	0.00171	0.00137	0.00185
hsi	0.00187	0.00180	0.00224	0.00201	0.00221	0.00391	0.00250	0.00179	0.00125	0.00207
ftsemib	0.00203	0.00175	0.00277	0.00276	0.00314	0.00250	0.00412	0.00197	0.00163	0.00212
uk	0.00145	0.00129	0.00175	0.00173	0.00171	0.00179	0.00197	0.00158	0.00107	0.00147
smi	0.00117	0.00102	0.00140	0.00141	0.00137	0.00125	0.00163	0.00107	0.00139	0.00116
omx	0.00148	0.00135	0.00204	0.00188	0.00185	0.00207	0.00212	0.00147	0.00116	0.00236

Furthermore we also showed the correlation matrix:

Table 5 Correlation Matrix

Corr	s&p	as51	Dax	Cac	ibex	hsi	ftse	Uk	smi	omx
s&p	1	0.773305	0.819488	0.83471	0.702427	0.706641	0.744785	0.857862	0.742348	0.715837
as51		1	0.699702	0.773752	0.627789	0.713485	0.676671	0.801845	0.682156	0.689149
dax			1	0.907956	0.69892	0.673592	0.809999	0.82377	0.7052	0.786568
cac				1	0.80451	0.654242	0.876947	0.886731	0.770679	0.787503
ibex					1	0.601144	0.832174	0.73004	0.626587	0.645056
hsi						1	0.623441	0.719917	0.535887	0.679829
ftsemib							1	0.771393	0.683297	0.68061
uk								1	0.721353	0.762341
smi									1	0.643824
omx										1

Then we consider the different Market Value for each Country w_{mkt} . I add each Market Capitalization [60]value to discover the total market value. Dividing the total estimated by the value of each country we are able to calculate the real percentage of Market capitalization of each nation. This method ensures that the sum of all Market capitalization is equal to 1 (100%). This procedure is repeated monthly.

Table 6 Bloomberg tickers and market attributions are reported. An Example of market distributions is also shown. This procedure is repeated every month. All data are in Mil. USD.

Country	Ticker Market Cap	Begin sample	Market Cap	Market Attributions
US	WCAUUS Index	09/22/03	12358098	63.22%
Australia	WCAUAUST Index	09/22/03	469804.9063	2.40%
Germany	WCAUGERM Index	09/22/03	861137.75	4.41%
France	WCAUFRAN Index	09/22/03	1198522.75	6.13%
Spain	WCAUSPAI Index	09/22/03	408350.4063	2.09%
Hong Kong	WCAUHK Index	09/22/03	791610.3125	4.05%
Italy	WCAUITAL Index	09/22/03	525161.75	2.69%
UK	WCAUUK Index	09/22/03	2084361	10.66%
Switzerland	WCAUSWIT Index	09/30/03	594990.1875	3.04%
Sweden	WCAUSWED Index	09/22/03	254770.75	1.30%
		Tot	19546807.81	1

The matrix P identifies the assets involved in the views. As described above an investor can have two different types of views absolute or relative. In this case we have only absolute views regarding all indexes in each period. We consider ten different indexes and ten different views that regard the indexes as a consequence the P matrix is a 10 x 10.

Different authors compute the various weights within the view in different ways (see chapter 3), here we chose to make the matrix P as an Identity matrix (10x10):

[60] The company's worth calculated by multiplying the shares outstanding by the price per share. For companies with multiple shares, the market capitalizations pf all common stock classes. For indices, this equals the sum of the current Market Value of the securities used to compute the index.

Table 7 Matrix P as an Identity matrix (10x10). We have only absolute views regarding ten differents indexes

P	s&p	as51	dax	cac	ibex	hsi	ftsemib	uk	smi	omx
View1	1	0	0	0	0	0	0	0	0	0
View2	0	1	0	0	0	0	0	0	0	0
View3	0	0	1	0	0	0	0	0	0	0
View4	0	0	0	1	0	0	0	0	0	0
View5	0	0	0	0	1	0	0	0	0	0
View6	0	0	0	0	0	1	0	0	0	0
View7	0	0	0	0	0	0	1	0	0	0
View8	0	0	0	0	0	0	0	1	0	0
View9	0	0	0	0	0	0	0	0	1	0
View10	0	0	0	0	0	0	0	0	0	1

The Q matrix is a (10 × 1) vector of the excess returns for each view. I compute one for each month multiplying Carry for Beta as :

$$E(return_{t+1}) = c + \hat{\beta}\ Carry \qquad \textit{Eq. 48}$$

I repeat this for each month. Our calculus starts at 05/01/2005. For more details see the Appendix 1.

After the matrix of variance of the views Omega (10x10) is computed as:

$$\begin{pmatrix} diag\left(P(\tau\Sigma)P'\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & diag\left(P(\tau\Sigma)P'\right) \end{pmatrix} \qquad \textit{Eq. 49}$$

It is clear that it is diagonal matrix. Also this matrix changes with time (once a month). Hence it is calculated every month from 05/31/2005 to 03/31/205

Now we can calculate the output of our models.

First compute the Reverse optimizations and take out the equilibrium returns

$$\Pi = \delta \Sigma w_{mkt} \qquad \textit{Eq. 50}$$

We can compute the Matrix return as :

$$\bar{\mu} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} [(\tau\Sigma)^{-1} \Pi + P'\Omega^{-1}Q] \qquad \textit{Eq. 51}$$

Consequently we can obtain different portfolios with diverse features. To do this the maximization problem is estimated as:

$$\max \ \omega'\bar{\mu} - \frac{\delta}{2} \omega'\bar{\bar{\Sigma}}\omega^{*} \qquad \textit{Eq. 52}$$

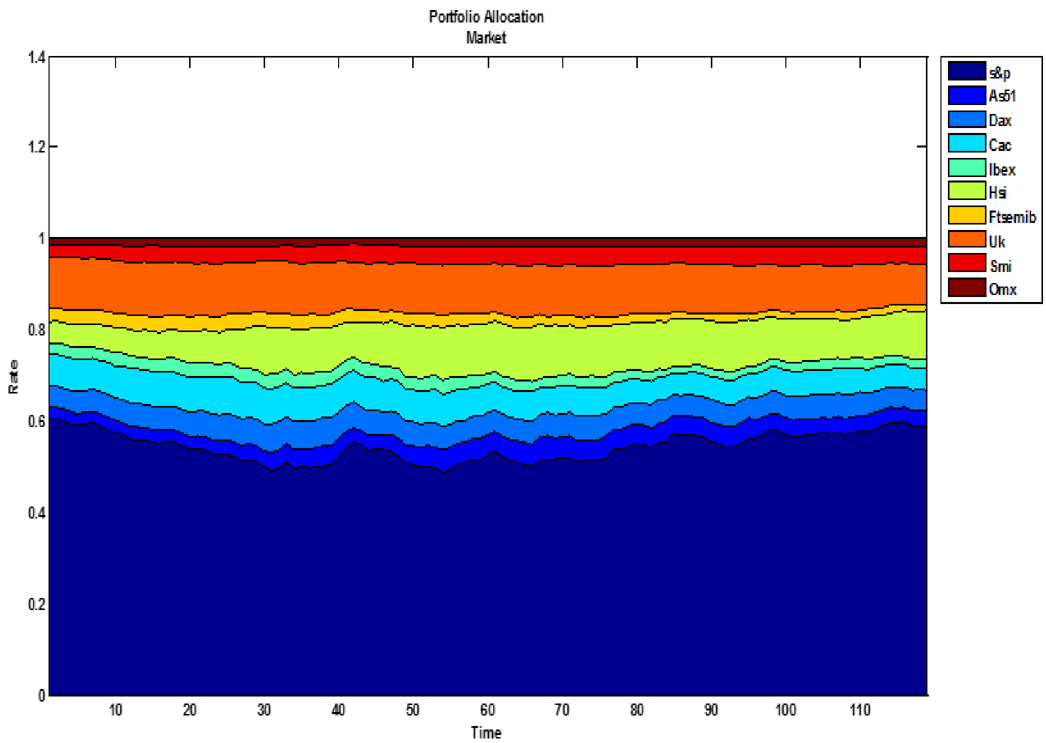
And make different constraints as we want to obtain different portfolios with diverse features.

We consider how the weights change and then we look at the different results regarding the cumulate returns in each portfolio.

4.3 Results and conclusion

In this sections we have considered four different portfolios with diverse features. In all of them there is the constrain that all indexes considered have to be bought.

The first portfolio is our Benchmark because it represented the different Market Capitalization of our indexes, in other words the weights are equal to those of the Market (the weights in this one are the same as the matrix w_{mkt}) [61]. In this portfolio we haven't introduced the views regarding Carry.

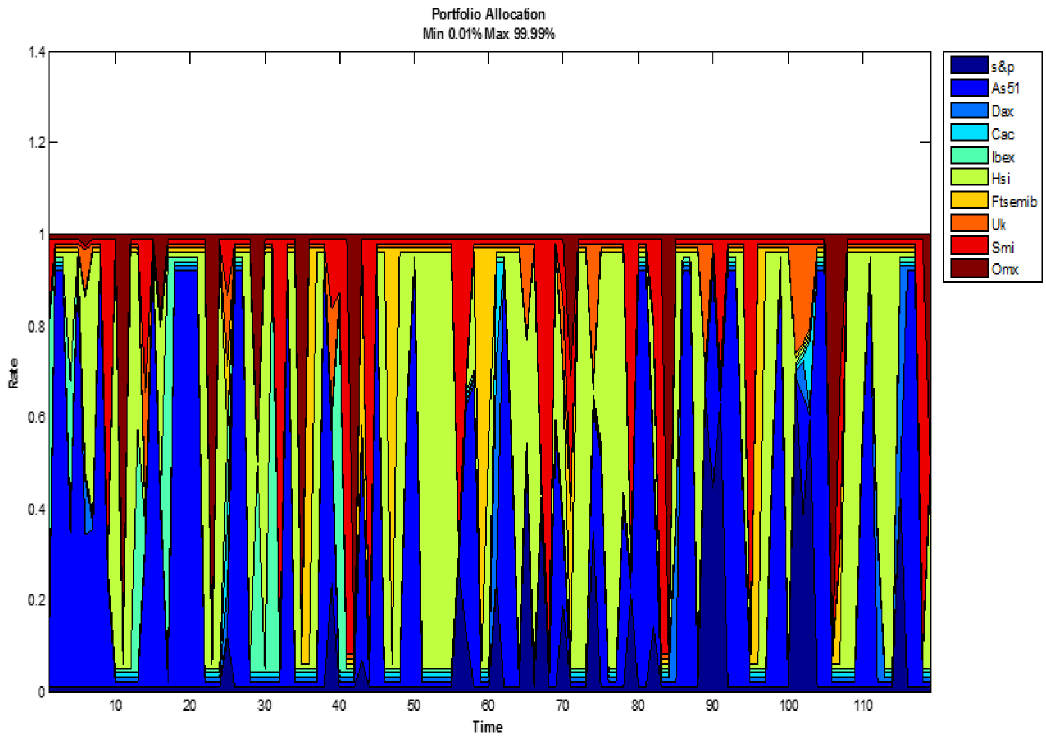


As shown above our analysis started from 05/31/2005 to 03/31/2015 hence we have been considering 119 (one per month) rebalancing.

In the second portfolio we exploited Carry as views in the Black Litterman Model and our strategy regards all our indexes. In this one the weight in each asset can have a maximum

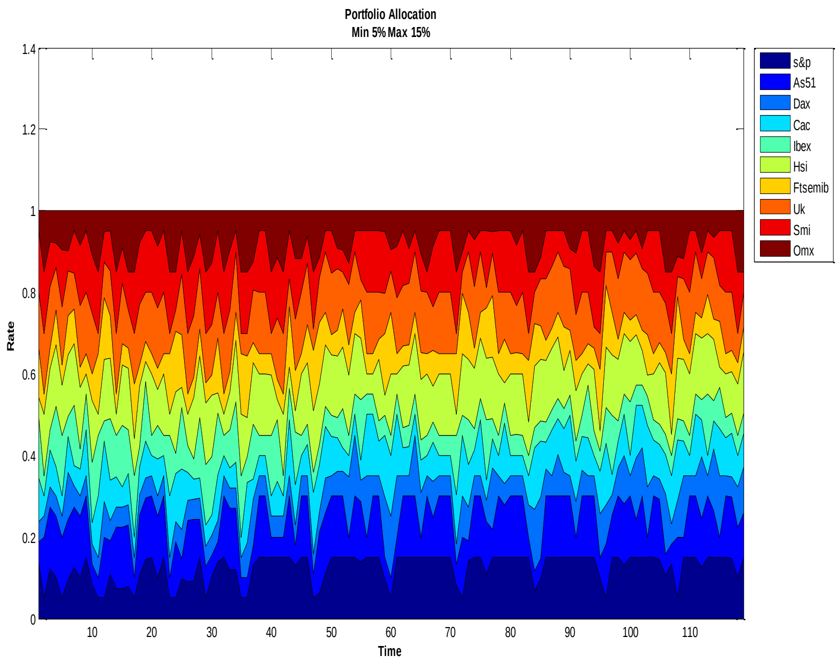
[61] See weight attributions scheme in Table 6

of 99.99% or a minimum of 0.01%. We haven't considered short selling and leverage. The weights change as we can see on the chart below:

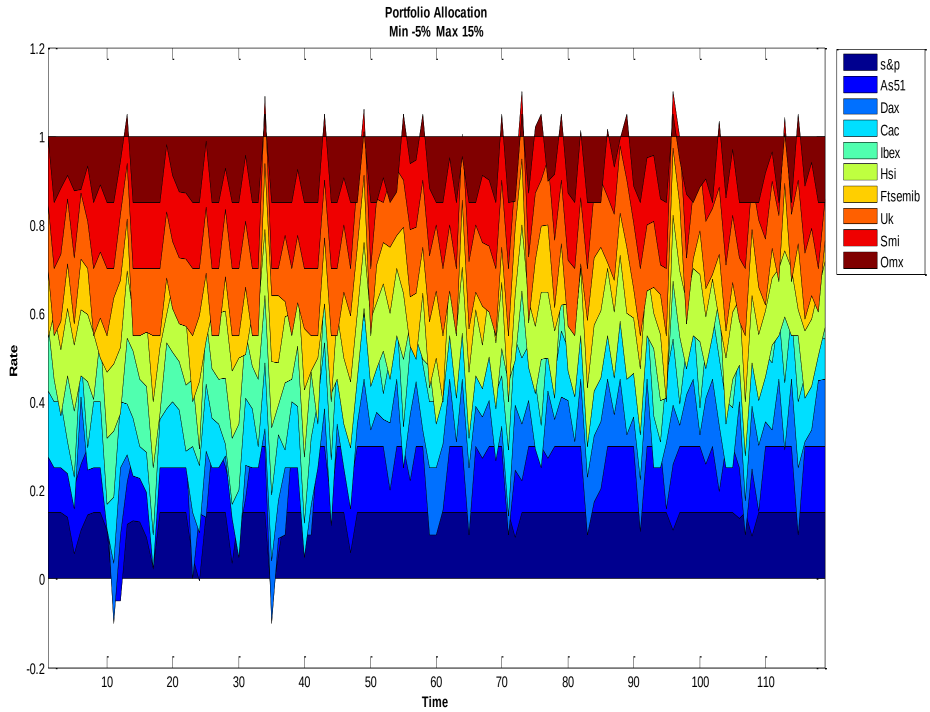


For more details about this one see Appendix 1.

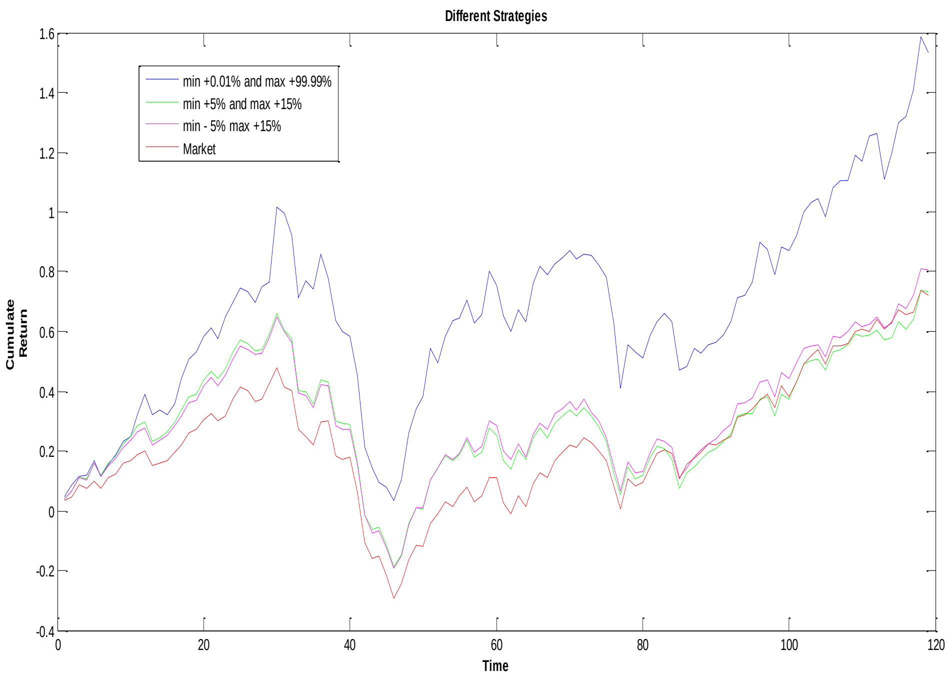
The third is a Portfolio without short selling with a maximum 15% weights in each asset. We showed the weights:



The fourth is a portfolio that allows short (max 5% in each position) and long (max 15% in each position).



Now we can consider the different results regarding cumulate return from the different portfolios:



Our goals are to exploit this new concept of Carry as the view in the Black Litterman (BL) model.

If we examine the graph above we can come to different conclusions.
If we consider the portfolio 2 (Blue line) where the weights in each index can change from 99.99% to 0.01% the results of our strategy are very encouraging. In others portfolios (3 and 4) the results are very similar to the Benchmark (portfolio 1).
This is predicted because in these portfolios the weights are almost equal weights, as we preferred the diversification among the different indexes. Furthermore we must underline the fact that in our strategy there are two indexes Ftsemib and Cac where the statistical significance of Carry is low (see table 3) however we have chosen to include this in our portfolios. Our portfolios however are limited if we consider only Equity index instead of Bond, Currency and Options. If we had realised a portfolio across all asset classes the results would have been better. Another aspect also has to be taken into consideration: this approach is not very optimal during the period where there is liquidity squeeze or more volatility.
If we consider the graph above from the period 2007 -2009 our trading strategy suffers from a significant loss even if the cumulate returns from the first portfolio remain positive. Also this result was predicted previously.

It would be interesting in the future to analyse this model during a period of recession using the same concepts. It could be possible to find a relationship between Carry and expected returns during crises. Or calculate an indicator that allows us to compare the expected returns when there is high volatility and low liquidity. It may be interesting to realise a model made up of two components:
The first exploiting the strategy of Carry combining with Black Litterman when the Market shows significant liquidity and the volatility is low.
And the second part that is able to give a view regarding the expected returns where there is either high volatility or low liquidity. And switch from one to another when market conditions change.

Or another possibility could be to consider the model of Bertsimas D., Vishal and Ioannis Ch. Paschalidis with robust mean variance inverse optimization (RMV-IO) portfolio during an economic crisis. "*The main advantage of this model is the increased flexibility for specifying views and the capacity to consider more general notions of risk than the traditional mean-variance approach. The authors have used this flexibility to present two new BL-type estimators and their corresponding portfolios, a mean-variance inverse optimization (MV-IO) technique and a robust mean - variance inverse optimization (RMV-IO) approach. The most important distinction between these approaches is that the first allows investors to capitalize upon any private information they could have on volatility, whereas the second tries to insulate investors from volatility uncertainty when they have no such information. The results indicate that these approaches provide certain benefits* *beyond the traditional BL model, especially in scenarios where views are not absolutely certain*[62]*”*.

When the market demonstrates high volatility and low liquidity our expectations regarding returns using Carry (our views) are not very precise.
As a consequence using the model of Bertsimas D. Vishal and Ioannis Ch. Paschalidis could be a practical solution.

[62] Bertsimas D., Gupta V., Paschalidis Ch. I., 2012, “Inverse optimization: A new perspective on the Black Litterman Model”, *Operations Research*, Vol.60,No.6

Appendix 1

%%

MATLAB CODE

%%

```
%%%% CARRY CODE

%% The purpose of this code is to create a synthetic Future using the
return for the most liquidity future contracts.

Input:

Spx         spot price
Sp1         nearest future price or Generic_1
Sp1_v       Generic 1 daily volume
Sp2         second nearest future price or Generic_2
Sp2_v       Generic 2 daily volume

Outputs
Carry


I calculate daily return for the most active (high volume) equity future
contracts (which is the front-month contract) switching from one to
another when the trading volume was greater and then aggregate the daily
returns to monthly ones. This procedure is a bit different from Koijen,
Moskowitz, Pedersen and Vrugt (2013). But allows me to use the most
liquid contracts.


%% Import data

date=matrix(:,1);
spx=matrix(:,2);
sp1=matrix(:,3);
sp1_v=matrix(:,4);
sp2=matrix(:,5);
sp2_v=matrix(:,6);


%% I calculated daily return for the Generic_1 (Sp1) and
Generic_2(Sp2)
```

```
[ret1,int1]=price2ret(sp1,date);
[ret2,int2]=price2ret(sp2,date);


%%I created vector all return that is composed of the return of the
nearest and second nearest future.
If the Volume of Generic1 is greater than Generic 2, we choose the
return of Generic 1, otherwise we choose the return of Generic 2.

b=sp1_vCopy>sp2_vCopy;
all_return=b.*ret1 + (1-b).*ret2;


%% From return to price for creating a synthetic future

[price_new,int_new]=ret2price(all_return,[start price],int1,[start
time]);


%% Plot the difference between the sp1 sp2 and new future series

diff1=price_new -sp1;
plot(diff1)
diff2=price_new -sp2;
plot(diff1)


%% Create time series with new synthetic future

ts_price=fints(date,price_new);


% Converts price from daily to monthly

month_price=tomonthly(ts_price);


% Ft2mat, takes the data series in the financial time series object
(month_pice,1) and puts them into the matrix m_p as columns.

m_p=fts2mat(month_price,1);


%% Create a time series with spot price (Spx)

spxts=fints(date,spx);
spxmonth=tomonthly(spxts);
```

%% calculate Carry as: $C_t = \frac{Spot\ price - Synthetic\ future\ price}{Synthetic\ future\ price}$

```
carry1=spxmonth - month_price;
CARRY=carry1/ month_price;
CARRY_TS=fts2mat(CARRY);
```

```
%% Export Carry in excel

xlswrite('carry_ufficiale',CARRY_TS);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Black Litterman Code

This code is divided into two parts. The fist shows the classical approach (He and Litterman 2009) and the second shows our results.

First part:

```
%% Inputs

%   delta  - Risk tolerance from the equilibrium portfolio
%   weq    - Weights of the assets in the equilibrium portfolio
%   sigma  - Prior covariance matrix
%   tau    - Coefficiet of uncertainty in the prior estimate of the
             mean(pi)
%   P      - Pick matrix for the view(s)
%   Q      - Vector of view returns
%   Omega  - Matrix of variance of the views (diagonal)

% Outputs
%   Er     - Posterior estimate of the mean returns
%   w      - Unconstrained weights computed given the Posterior
             estimates
%            of the mean and covariance of returns.
%   lambda - A measure of the impact of each view on the posterior
             estimates.
%   theta  - A measure of the share of the prior and sample
             information in the posterior precision.


%% Import file

load('import.mat')


%% Asset’s name

assets=
{'S&P','AS51','DAX','CAC','IBEX','HSI','FTSEMIB','UK','SMI','OMX'};
Assets=(assets)'
```

```
%% With Reverse Optimization and back out the equilibrium returns

pi1 = weq(1,:) * sigma * delta;


%% We use tau * sigma in many places so just compute it once

ts = tau * sigma;


%% Compute posterior estimate of the mean. This is a simplified
version of the original formula.

er = pi1' + ts * P' * inv(P * ts * P' + omega) * (Q(1,:)' - P * pi1');


%%  We can also do it the long way to illustrate that d1 + d2 = I

d = inv(inv(ts) + P' * inv(omega) * P);
d1 = d * inv(ts);
d2 = d * P' * inv(omega) * P;
er2 = d1 * pi1' + d2 * pinv(P) * Q(1,:)';


%% er = er 2 are two different way for the same result


%% Compute posterior estimate of the uncertainty in the mean

ps = ts - ts * P' * inv(P * ts * P' + omega) * P * ts;
posteriorSigma = sigma + ps;


%% Compute the share of the posterior precision from prior and
views,then for each individual view so we can compare it with lambda

theta=zeros(1,2+size(P,1));
theta(1,1) = (trace(inv(ts) * ps) / size(ts,1));
theta(1,2) = (trace(P'*inv(omega)*P* ps) / size(ts,1));
for i=1:size(P,1)
    theta(1,2+i) = (trace(P(i,:)'*inv(omega(i,i))*P(i,:)* ps) /
size(ts,1));
end


%% Compute posterior weights based solely on changed covariance

w = (er' * inv(delta * posteriorSigma))';


% Compute posterior weights based on uncertainty in mean and
covariance

pw = (pi * inv(delta * posteriorSigma))';
```

```
% Compute lambda value

lambda = pinv(P)' * (w'*(1+tau) - weq(1,:))';
```

Second part

```
%% Inputs

%   delta  - Risk tolerance from the equilibrium portfolio
%   weq    - Weights of the assets in the equilibrium portfolio
%   sigma  - Prior covariance matrix
%   tau    - Coefficiet of uncertainty in the prior estimate of the
             mean (pi)
%   P      - Pick matrix for the view(s)
%   Q      - Vector of view returns
%   Omega  - Matrix of variance of the views (diagonal)

% Outputs
%   Er     - Posterior estimate of the mean returns
%   w      - Unconstrained weights computed given the Posterior
             estimates
%            of the mean and covariance of returns.
%   lambda - A measure of the impact of each view on the posterior
             estimates.
%   theta  - A measure of the share of the prior and sample
             information in the posterior precision.

% Portfolio weights outputs

%1.w_noshort_noleva     Portfolio without short selling and leverage
%2.w_max_5noshort15long Portfolio without short selling and maximum 15%

                        weights in each asset

%3.w_max_5short15long'  This portfolio allows short( max 5% in each

                        position) and long (max 15% in each position)



%% Reverse optimize and back out the equilibrium returns. This allows
Matrix (PI) to be used for different market capitalizations.

for i=1:119;
pi(i,:)=weq(i,:) * sigma * delta;
end;


%% Matrix return, Compute posterior estimate of the mean.

all_r(:,1)= pi(1,:)' + ts * P' * inv(P * ts * P' + omega) * (Q(1,:)' -
P * pi(1,:)');
```

```
all_r(:,2)= pi(2,:)' + ts * P' * inv(P * ts * P' + omega) * (Q(2,:)' -
P * pi(2,:)');
for i=1:119;
all_r(:,i)= pi(i,:)' + ts * P' * inv(P * ts * P' + omega) * (Q(i,:)' -
P * pi(i,:)');
end;


%% Matrix of weight unconstrained

w = (er' * inv(delta * posteriorSigma))';
all_w=ones(10,119);
for i=1:119;
all_w(:,i)=(((all_r(:,i))' * inv(delta * posteriorSigma)))';
end;


%% Lambda matrix

lambda = pinv(P)' * (w'*(1+tau) - weq(1,:))';
all_lamba=ones(10,119);
for i=1:119;
all_lamba(:,i)= pinv(P)' * ((all_w(:,i))'*(1+tau) - weq(i,:))';
end;


%% Create function for weights.

My functions can introduce lower bounds( minimum weight in one position)
and upper bounds ( maximum weight in one position), but the sum of the
weight is 1.

function eu = mvopt(w,mu,delta,Sigma)
eu = - (w'*mu-delta/2*w'*Sigma*w);


options = optimoptions('fmincon','Algorithm','active-set');
w =
fmincon(@mvopt,repmat(eps,10,1),[],[],ones(1,10),1,[],[],[],options,er
,2.5,ps);

% funz,x0, a, b, aeq, beq, lb, ub, nonlcon, options)


%% Portfolio without short selling and leverage

options = optimoptions('fmincon','Algorithm','active-set');
for i= 1:119;
    w_noshort_noleva(:,i) =
fmincon(@mvopt,repmat(eps,10,1),[],[],ones(1,10),1,0.01*ones(1,10),[],
[],options,all_r(:,i),2.5,ps);
end;
```

```
%% Portfolio without short selling and maximum 15% weights in each
asset

options = optimoptions('fmincon','Algorithm','active-set');
for i= 1:119;
    w_max_5noshort15long(:,i) =
fmincon(@mvopt,repmat(eps,10,1),[],[],ones(1,10),1,0.05*ones(1,10),0.1
5*ones(1,10),[],options,all_r(:,i),2.5,ps);
end;


%% This portfolio allows short( max 5% in each positions) and long
(max 15% in each positions)

options = optimoptions('fmincon','Algorithm','active-set');
for i= 1:119;
    w_max_5short15long(:,i) =
fmincon(@mvopt,repmat(eps,10,1),[],[],ones(1,10),1,-
0.05*ones(1,10),0.15*ones(1,10),[],options,all_r(:,i),2.5,ps);
end;


%% Export results

xlswrite('w_noshort_noleva',w_noshort_noleva);
xlswrite('w_max_5noshort15long',w_max_5noshort15long);
xlswrite('w_max_5short15long',w_max_5short15long);
```

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