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Counterparty Credit Risk in OTC derivative products and Credit Value Adjustment

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COUNTERPARTY CREDIT RISK in OTC DERIVATIVE PRODUCTS AND CREDIT VALUE ADJUSTMENT

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ACRONYMS

OTC Over The Counter

SFT Security Financing Transactions

IRS Interest Rate Swap

CDS Credit Default Swap

FX Forward Foreign Exchange Forward

R Repo

RR Reverse Repo

SL Security Lending

SB Security Borrowing

MCR Minimum Capital Requirement

EAD Exposure at default

WWR Wrong Way Risk

CVA Credit Value Adjustment

MPR Margin Period of Risk

C Recovery Rate

LGD Loss Given Default

MtM Mark to Market

CPE Current Positive Exposure

CNE Current Negative Exposure

PFE Potential Future Exposure

EE Expected Exposure

NPV Net Present Value

NS Netting Set

EFV Expected Future Value

Effective EE Effective Expected Exposure

ABSTRACT

This thesis has the object to give a general overview of the Credit and Counterparty Risk and to evaluate how the factors linked to this risk influence the price of OTC products. Counterparty Credit Risk comes from the probability at default of an institute that could potentially cause losses to its own creditors. The fundamental concept that is linked to this value of risk is called Exposure, concept that will be carefully presented and computed to a set of OTC products. There exist several measures that allow to quantify the amount of credit exposure during the life of a product. Measures as Expected Exposure, Potential Future Exposure or Expected Positive Exposure are auxiliary to the definition of the fundamental measure Credit Value Adjustment (CVA). This metric define the true value of a risky product, as it is computed as the difference between the risk free value of a contract and its real value taking into consideration the probability at default of the counterparty. The last part of the thesis is dedicated to the computation of the Exposure and the CVA for European Options.

INTRODUCTION

Counterparty risk has become the key financial risk since the global financial crisis and the awareness of the importance of Over The Counter (OTC) derivatives products in defining crises. The myth of the creditworthiness of the “too big to fails” institutions have for many years obscured the importance of the size and the scale of Counterparty Risk.

As it has been said the relatively recent financial crisis that bring to bankruptcy some of the biggest bank institutions, had the consequence to increase the attention to the Counterparty Credit risk. It becomes clear that no counterparty could ever be regarded as risk-free. Spurious credit rating, legal assumptions or collaterals have for many years hidden the riskiness of the counterparty. Moreover, a relatively new risk measure, rarely used before the crisis, become a buzzword constantly associated with financial markets: Credit Value Adjustment (CVA), which defined the price of counterparty risk. Pricing counterparty risk into trades (via CVA charge) is now becoming the rule and not the exception.

As a matter of fact, global financial markets became aware of the potential devastating effects that a high exposure versus a counterparty could have on the Over the Counter derivative products. In fact, recent studies proved that the increasing losses in the crisis period were mainly due to a collective risk underestimation of the investors of the traded instruments. Moreover, the attention is focused on the OTC market because whilst it allow counterparty risk transfer, as other market traded product (credit derivatives such as single-name and index credit default swaps), being OTC instruments also introduce their own form of counterparty risk. Despite the riskiness associated to such products, in recent years the OTC market has increased very fast and has created a tight interconnection between banks. The object of this elaborate is to quantify the risk within the OTC contracts.

Because of the above, it is necessary to consider how to define and quantify counterparty risk. Counterparty risk mitigation methods need to be understood, and their side-effect and any residual risk need to be defined. It is important to define how CVA can be quantified and managed. Wrong Way Risk must be understood and mitigated.

The first chapter will give a general overview of what is meant by Counterparty Credit Risk and how the Regulation of Basel formally treat this field. The second chapter focus on the technical issues regarding the quantification of the Counterparty Risk trough the computation of the CVA. It will be analyzed and shown the calculation techniques useful in order to quantify this measure and how the Probability at Default enter in the computations. Moreover, some hints will be given of what is meant by Wrong Way Risk (the risk of unfavorable correlation) and its effect on the Exposure and CVA.

The third chapter define the concept of Credit Exposure, a general definition is given and some examples will be illustrated based on real values. The mathematical methods described in order to quantify the Exposure is the one used by UniCredit Group in its internal computations. The last theoretical chapter focus on the main techniques of Counterparty risk mitigation and the effect that these methods have on the Exposure and CVA.

Will follow a computational part implemented in MATLAB. In this chapter simple option instruments will be priced and the related Credit Value Adjustment will be defined with the purpose of compute the final price of such instruments that take into account the riskiness of the defaulting counterparty.

CHAPTER I.

DEFINING COUNTERPARTY CREDIT RISK

Over The Counter Derivatives Market

Derivatives contracts represent agreement either to make payments or to buy or sell an underlying contract at a time or times in the future. They are based on one or more underlying assets and their value will change according to the movement of this and to the decisions made by the parties to the contract. Derivatives are no different from the underlying cash instrument, they simply allow one to take a similar position in a synthetic way, therefore the most important feature is Leverage. In fact, derivatives are executed with only a small (with respect to the notional value of the contract) or no upfront payment. Because of this, derivatives have been shown to be capable of creating major market disturbances: the fact is that as with any other invention that bring significant advantages, derivatives can be extremely dangerous.[1]

Many of the simplest products within the derivatives markets are traded trough exchanges. Exchange trade market has the benefit of facilitating liquidity and making trading of positions easy. Moreover, an exchange mitigates all counterparty risk since the default of a member of the exchange would be absorbed by the market.

Compared to exchange-traded derivatives, OTC derivatives tend to be less standard structures and are typically traded bilaterally. They are structured as private contracts and not protected by any government insurance or customer protection programme. Basically, each party takes counterparty risk with respect to the other party, moreover, many players in the OTC derivatives market do not have high credit quality nor are they able to mitigate their risk with, for example, collaterals. This counterparty risk is therefore an unavoidable consequence of the OTC derivatives

[1] Gregory Jon, Counterparty credit risk and credit value adjustment : a continuing challenge for global financial markets – 2012.

market, that additionally tends to create highly connected counterparties such as in interbank trading.

However, thanks to its customers-oriented feature the OTC derivatives marked has been subject to a great growth over the last fifteen years, as shown in the figure below.

Figure 1. Global over-the-counter (OTC) Derivatives outstanding (as of 2017)[2]

As it can be seen the diffusion of these type of instruments has been driven by Interest Rate Products followed by Foreign Exchange and Credit Derivatives Products. The underlying of the first type of products are the interest rates, in fact, these derivatives are often used as hedges by institutional investors, namely banks, to protect themselves against changes in market interest rate. They can also be used to increase or reduce holder's risk profile depending on the type of position they have. Among the most common instruments of these type there are Swaps (allow the parties of the contract to swap their underlying interest rate; from floating to fixed rate payment or vice versa) , Caps and Floors (that allow the owner to have an additional protection on the movements of the underlying). Credit derivatives products are also of much importance, given the fact that they can be potentially very dangerous if not managed carefully. They can be represented as Forward Contracts (credit spread forward, collateralized debt obligation), Swap (Credit Default Swap, total return swap) and Options (Credit Default Swap option). Their price is driven by the credit risk of economic agents, such as private investors or governments. With

[2] Source Bank for International settlement semiannual OTC survey.
Note: amount outstanding are reported in gross notional terms.

these type of instruments, mostly banks, have the possibility to transfer the risk of a loan given to its customers (that has to purchase the Credit derivative as condition of the loan) to other institutions (called third party in the transaction), under the payment of a fee. The third party act as guarantor of the borrower as it will honors its payment in case of default. Therefore, the value of the credit derivative is dependent on both the credit quality of the borrower and the one of the third party, that is of much importance.[3]

Important is then quantifying the risk involved in these type of transaction, not only as a matter of pricing need but also in order to have an overview of the overall riskiness of the credit exposure.

Essential is then identify which type of risk we are dealing with. It is known that the financial market presents numerous risks that are classified differently depending on their source: one of them is known as Counterparty Risk. Counterparty risk is arguably the most complex financial risk to deal with since it is driven by the intersection of different risk types (market and credit) and is highly sensitive to systemic traits, such as the failure of large institutions.[4] Moreover, counterparty risk involves the most complex financial instruments, derivatives.

Counterparty Credit Risk

Every day new financial instruments become tradable and this increase the complexity within the financial market, complexity that is reflected by the increasing in volatility and uncertainty. The highest portion of these instruments is composed by the already mentioned Over The Counter products that refer to all such instruments that are not exchanged in the regulatory market. The exchange is done under an agreement between the two parties, both parties players are subjected to the risk at default of its own counterparty.

Counterparty risk is a peculiar aspect of credit risk and it is strictly linked to the market factors fluctuations; financial institutions can't disregard this component. Counterparty risk is traditionally thought of as credit risk between OTC derivatives counterparties; is the risk that the entity with whom one has entered into a financial contract will fail to fulfill their side of the contractual agreement (they default). As it

[3] http://www.investopedia.com: definition of credit derivative.
[4] Financial Times, 2009, "Which risks have been underestimated?" article by West Hedrich.

can be note, this risk is a pillar component of all bank activities that are linked to loans, and so it influences all the investment decisions made by banks, financial intermediaries and investors.

Counterparty risk represents a combination of Credit Risk, which defines the counterparty credit quality and Market Risk, which defines the exposure. Moreover, it is mainly associated to Pre-settlement risk, which is the risk of default of a counterparty prior to expiration (settlement) of the contract.[5]

The major transactions that are exposed to counterparty risk are the one referred to the Over The Counter (OTC) derivatives financial instruments such as: Interest Rate Swap (IRS), Credit Default Swap (CDS), Foreign Exchange forward (FX forward); and the Security Financial Transaction (SFT) operations such as: Repo (R), Reverse Repo (RR), Security Lending (SL) and Security Borrowing (SB). The former category (OTC) is the most significant due to its size and diversity and the fact that a significant amount of risk is not collateralized. We already gave a brief introduction of the products composing it, let's now focus the attention on the latest category: the one of SFT. For example, in a Repo transaction, one party exchanges securities in return for cash with an agreement to repurchase the securities at a specified future date, the securities essentially act as collateral for a cash loan (vice versa for RR). Security lending (borrowing) transactions are similar to repo (reverse repo) except that securities are exchanged for collateral rather than cash for collateral, in fact one party borrow securities from another and provide collateral of comparable value. For these features SFT products are less sensible to counterparty risk.

Regulatory framework: Basel III[6]

Basel agreements generated a series of rules that have the aim to define a prudential regulatory framework in order to manage the risk involved in the credit sector. These agreements have been firstly settled in 1988 by the Basel Committee on Banking Supervision, that is an international organization constituted by the central banks of

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[5] Jon Gregory, 2015, the xVa Challenge: counterparty Credit Risk, Funding, Collateral and Capital.

[6] Official journal of the European Union, 2016, commission delegated Regulation, supplementing Regulation (EU) No 648/2012 of the European Parliament and of the Council on OTC derivatives, central counterparties and trade repositories with regard to regulatory technical standards for risk-mitigation techniques for OTC derivative contracts not cleared by a central counterparty.

the top 10 countries in term of industrialization. The aim of the Basel organization is to promote the cooperation within the nations in order to establish a financial and monetary stability. The last step of the treaty is to establish the minimum capital requirement (MCR) that every bank should follow with regards its financial and credit activities.

In light of the recent financial crisis of 2007-2008, the Basel Committee, trough the publication of a new regulatory document, Basel III (December 2010), attempted to reform some points of the latter treaty that presented some regulatory lack. What became clear after the publication of the new treaty was the particular attention of the computation and management of counterparty risk, previously ignored. In order to reach a better risk management and more transparent OTC market, Basel III promote a reorganization of credit risk definition, introducing some measures that will be briefly described:

- Wrong Way Risk management and governance. The negative correlation among probability at default and exposure has now to be introduced in the computation of the Exposure at Default (EAD) in order to have a complete overview of the risk that the institution really incur in to. Stress Test activities have to be conducted by all institutions in order to identify the micro and macro scenarios potentially bearers of WWR.
- Stressed scenario in the computation of Effective EPE (Expected Positive Exposure). Stress Testing activities allows institutions to foreseen potential devastating scenarios and value the effect on their financial stability.
- CVA in pricing activities. CVA is defined with the aim to capture and compute the losses incurring in OTC due to the volatility of Credit Spread. Basel II were accounting for the counterparty risk at default but not for the CVA.

  "under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults." (BCBS, June 2011)

- Interbank correlation coefficient. During the financial crisis the correlation within the activities of the financial institutions exceeded the 25% with respect a stationary market situation. The new regulation oblige to introduce this type of correlation in the regulatory capital. A coefficient of 1.25 is applied to every portfolio exposure exceeding $100 billion.
- Margining period of risk increment. MPR stand for the time period from the most recent exchange of collateral covering a netting set of financial instruments with a defaulting counterparty until the financial instruments are closed out and the resulting market risk is re-hedged. It allow to evaluate the liquidity of the OTC market. The threshold increased from 10 to 20 days. In this way also the riskiness of the transaction results to be higher .[7]

Components and Terminology

Credit Exposure

Credit exposure defines the loss in the event a counterparty defaulting.

Exposure can take positive and negative values: the positive value of a financial instrument corresponds to a claim on a defaulted counterparty; in case of negative value an institution is still obliged to honor its contractual payments. The institution will incur in a loss whenever it owed money and its counterparty default, whereas it cannot gain from the default by being released from its liability.

Default probability, credit migration and credit spreads

Important when defining counterparty risk is to consider the credit quality of the counterparty over the entire lifetime of the relevant transactions.

Crucial in influencing the term structure of default probability are the discrete changes in credit quality, such as due to rating changes and credit migrations.

Important is also underlying the alternative methods used in order to define the default probability: real-world or risk-neutral. The former detect the actual default probability which it is estimated via historical data, it aims to reflect the true value of some financial underlying. The latter calculate the default probability from market credit spreads, it reflects parameters derived from market prices. The method used is relevant when computing the CVA; real-world measures are used for risk

[7] The directive analyzed above are all referred to the new field introduced by Basel II: internal regulatory Framework for Banks, June 2011.

management purposes, whereas for pricing purposes risk-neutral measure should be preferred.[8]

Loss given default

Important in the valuation of CVA are: Recovery rates (C) and Loss given default (LGD). The former represent the percentage of the outstanding claim recovered in case a counterparty defaults; the latter is linked to recovery rate by the relationship, $LGD = 1 - C$.

As mentioned recovery rates play a critical role in quantifying counterparty risk via Credit Value Adjustment even if credit exposure is traditionally measured gross of any recovery.

Mitigating Counterparty risk

Diversification is surely the first step in mitigating the risk in any investment activity, whereas there are other techniques massively used in order to bring the exposure to its lowest amount.

Netting agreement. Refer to the legal practice that the counterparty of a transaction agree in order to offset positive and negative contract value within their transaction with the aim to reduces the exposure at counterparty level.

Collateral agreement. The most commune practice is to hold cash or securities against an exposure, the institution positively exposed over a counterparty could require collateral posting activities.

Note that in discussing exposure and CVA below, it will be used the term "netting set": which correspond to a set of trades that can be legally netted together in the event of a default. A netting set may be composed by a single trade and there may be more than one enforceable for a given counterparty. Across netting sets, exposure will always be additive, whereas within a netting set MtM values can be added together.

Mark to Market

As said, within a netting set positive value of exposure are netted against negative value, the result is the so called Mark to Market that represent what could be

[8] The term risk-neutral refers to option pricing: the option pricing is based on the cost of a hedging strategy which ideally replicates the option without any risk. In a risk-neutral valuation, the free simulation parameters like volatility are estimated in a way that the theoretical price and the traded price match; the mean equals the risk-free interest rate. This way, one obtains market consistent option prices for similar options, which are not traded.

potentially lost today (if positive). The current MtM presents the value of all the payments an institution is expecting to receive, less those it is obliged to make.

$$MtM = Positive\ Current\ Exposure\ (PCE) - \ Negative\ Current\ Exposure\ (NCE).$$

Credit Value Adjustment

CVA is a risk measure that imply the pricing of counterparty risk and can generally be defined in one of two ways:

- Actuarial price that represents the expected value of future cash flows, incorporating some adjustment for risk being taken;
- Market implied price representing the price as the cost of an associated hedging strategy.

Basel III regulatory capital rules advocate a risk neutral approach to CVA.

Pricing counterparty risk is fundamental in the decision-making process: the question of whether to do a transaction becomes whether or not it is profitable once the counterparty risk component has been "priced in".

$$Instrumet\ final\ Price = Riskfree\ price - \ CVA$$

Where the Risk-free price of a derivatives is the price assuming no counterparty risk.

As it has been mentioned, Credit Value Adjustment and the related definition of the Exposure is fundamental for any institution in assessing its operational implied risk. Understanding the meaning of the variables composing CVA is then important in order to be capable of make the right decision when entering in any financial transaction. In the following chapter, it will be deeply define the computational path of CVA and of the components defining it: default probability of the counterparty; default probability of the institution; transaction in question; netting of existing transactions with the same counterparty and collateralization.

CHAPTER II.

CREDIT VALUE ADJUSTMENT

Introduction to Credit Value Adjustment

For many years, derivatives pricing techniques gave any value to the counterparty creditworthiness.[9] All cash flows where discounted at the LIBOR curve and the final value of any instrument where view as risk free. However , recent studies proved that more than two third of the losses occurred in the crisis period were due to the erroneous valuation of the derivatives Mark-to-Market and only the residual part could be addressed to default events.[10]

The introduction of counterparty risk lead to the definition of a new risk measure able to quantify it: the Credit Value Adjustment (CVA).

CVA is the difference between the risk free value of the portfolio of derivatives instruments and its real value taking into consideration the counterparty probability at default. In other words CVA is the market value of the counterparty credit risk.

In practice an institution holding a portfolio of instruments within a counterparty has the object to identify if the gains from the transactions are in accordance with the new regulatory demands and if the total exposure do not exceed the limit set unit. In order to accomplish to this task it is necessary to compute which is the risk that could rise from each transaction and the related CVA of the portfolio.

Risk neutral parameters in the computation of CVA

In general, CVA is computed using risk neutral (market implied) parameters[11]. Such an approach is in line with the pricing theory based on the non arbitrage principle;

[9] Gregory Jon, Counterparty credit risk and credit value adjustment : a continuing challenge for global financial markets – 2012.
[10] Basel III: internal regulatory frame work for Bank, June 2011.

computing the expected value of future payoff through this measure, allow to define the fair price of a portfolio. Of course certain parameters cannot be risk neutral since they are not observed in the market (ex. Correlation) or may require interpolation or extrapolation assumptions (volatility). Moreover, risk neutral parameters such as volatilities may generally by higher than their real world equivalents (historical estimates) with the consequence that both Positive Future Exposure (PFE) and Expected Exposure (EE) result to be higher. However, it is important to consider that higher value of risk could be a proxy that the market is pricing a higher level of volatility than it has been registered in the past. In this case risk neutral parameters may be a superior measure, with respect real world one, since they represent the expectations of the market. Finally, other problems could arise when approaching risk neutral measure, for example, the default probability could be negatively affected[12], however, this can be avoid by using Credit spreads[13] (identify as a proxy of the riskiness of a counterparty) in reporting CVA.

Unilateral CVA

Credit Value Adjustment is a fundamental measure in defining the overall risk faced by an institution. What is important is to understand for which scope the CVA is being computed (pricing or hedging purposes) and the metrics composing it.

In this chapter the attention is focused on the definition of the CVA computational path and on the derivation of one of its metrics: Probability at default.

For derivation purposes choosing among Unilateral or Bilateral CVA is not of much importance as it changes only the riskiness take into account. In the unilateral CVA

[11] In mathematical finance, a risk-neutral measure, is a probability measure such that each share price is exactly equal to the discounted expectation of the share price under this measure. This can be possible only assuming the completeness of the market and the absence of arbitrage opportunity with the consequence that investors are view as risk-neutral.

[12] It has been proved that the mentioned assumptions of risk-neutral probability measures (investors are risk-neutral and desire no risk premia) bias the measurement of probability at default. In particular, the fact that risk-neutral default probability is linked to the actual default probability (estimated via historical data) with an adjustment made for risk premia seems not to be satisfied: the ration between the two (RNPD/ADP) result to be higher for high rated bond than for lower one (not consistent with the assumption that risk-adverse investors require risk premia) .

[13] Coming from Rating Agencies that base their computation on Actual Default Probabilities (ADP).

only one counterparty is considered to be exposed to default risk, inversely, in the Bilateral CVA methods, both counterparty are assumed to be at risk.

The following computational method refers to Unilateral CVA, therefore, it is important to set in which side of the transaction we are located, in this case the view is from the side of the institution free of default risk that is dealing with a potentially defaulting counterparty.

Before formally derive the general CVA formula, it is necessary to give an example in order to have a preliminary understanding of its properties.

Given a derivative set, with respect one counterparty, we assume that the contract with the highest expiring date is in *T*. The first step is to compute the Net Present Value (NPV) of the residuals payoff within the time set (t, T).

The NPV is the expected value of the risky discounted payoff $E_t[V(t,T)]$at any time *t*.

Let's assume that if the default occur this will happen at time τ , then the institution can face two possible scenarios:

1. The counterparty do not default before T, then the payoff will be given by:

$$\Phi = I(\tau > T) * V(t, T)$$

2. The counterparty fail before T, the payoff will be given by the sum of the payments done before time T plus the recovery rate (supposes to be constant):

$$\Phi_1 = I(\tau \leq T) * V(t, \tau),$$
$$\Phi_2 = I(\tau \leq T) * [\delta * V(\tau, T)^+ + V(\tau, T)^-],$$

Where:

- $I(\tau \leq T)$ represent the Indicator function that take value 1 if the default time τ occure before the maturity T and 0 otherwise;
- $V(\tau, T)$ represent the Net Discounted Cash Flows risk-free at time t.

The payoff profile in case of default results to be highly asymmetric: if the portfolio presents a positive Net Present Value then the institution will received only a partial amount of what owed (recovery amount). Whilst, if the NPV is negative the institution is still obliged to honor its obligations.

Finally, defining the NPV in the instant of default τ as :

$$E_\tau[V(\tau,T)]$$

the risky portfolio value could be defined as:

$$\tilde{V}(t,T) = I(\tau > T) * V(t,T) + I(\tau \leq T) * V(t,\tau) + I(\tau \leq T) * [\delta * D(t,\tau) * V(\tau,T)^+ + V(\tau,T)^-],$$

Where $D(t,\tau)$ is the discount factor within the time interval (t,τ).

Proposition. (General unilateral counterparty risk pricing formula)

At time t, assuming no default happened, the price of a portfolio taking into consideration the riskiness of the counterparty is given by:

$$E_t[\tilde{V}(t,T)] = E_t[V(t,T)] - E_t\{(1-\delta) * I(\tau \leq T) * D(t,\tau) * NPV(\tau)^+\}.$$

(See Appendix A)

Finally we can define the Credit Value Adjustment as the following amount:

$$CVA(t,T) = E_t\{(1-\delta) * I(\tau \leq T) * D(t,\tau) * NPV(\tau)^+\}.$$

The expected value of the risky portfolio will be the difference within the risk-free and the CVA:

$$E_t[\tilde{V}(t,T)] = E_t[V(t,T)] - CVA(t,T)$$

This derivation result to be useful as the problem of valuing a transaction and computing its counterparty risk can be separated. It allows also the division of responsibility within a financial institution. Transactions and their associated counterparty risk may be, then, priced and risk managed separately.

However, the linearity of the equation is only apparent. As it will be analyzed later, risk relievers such as netting and collateral agreements, vanish the additive properties of CVA. This means that the risky value of a given transaction can't be calculated individually, but only at netting set level.

From the above example, it is clear that the starting point of all CVA valuations is the computation of the probability of default and the exposure linked to the counterparty.

As a result, when formally deriving the CVA formula, two initial hypothesis are needed:

1. Recovery value is independent with respect to the default event and the expected exposure;
2. Absence of WWR: there is no correlation between the exposure and the default event of the counterparty.

Let's then derive the Unilateral CVA.

Be $V(u,T)^{*} = V(u,T)[_{\tau=u}$ the value of the portfolio in τ (the default timing event), integrating each default event we obtain:

$$CVA(t,T) = E\{(1-\delta) * I(\tau \leq T) * D(t,\tau) * NPV(\tau)^{+}\}$$
$$= -\left(1-\bar{\delta}\right) * E\left[\int_{t}^{T} D(t,u) * V^{*}(u,T)^{+} * dS(t,u)\right]$$
$$= -\left(1-\bar{\delta}\right) * E\left[\int_{t}^{T} D(t,u) * EE(u,T) * dS(t,u)\right].$$

Where $D(t,u)$ represents the risk free discount factor and $S(t,u)$ is the survival probability of the counterparty. Moreover, all the measures are assumed to follow the risk neutral assumption method.

Finally, the discrete Unilateral Credit Value Adjustment formula con be represented as follow:

$$CVA(t,T) \approx -\left(1-\bar{\delta}\right) * \sum_{i=1}^{m} D(t,t_i) * EE(t,t_i) * [S(t,t_{i-1}) - S(t,t_i)]$$

Where:

[1] i represent the indicator of time instants on which the default event could occurs;

[2] $1-\bar{\delta}$ represents the Loss Given Default (LGD), the expected value of the loss;

[3] $D(t,t_i)$ represents the risk-free discount factor in i, alternatively we can assume that the Exposure is already discounted but care must be taken if explicit discount factors are requires (this could happen in order to vanish the convexity effects that some risk discount factor could have, for example with interest rate products where high rates will imply a smaller discount factor and vice versa);

[4] $EE(t,t_i)$ represents the Expected Exposure;

[5] $S(t, t_{i-1}) - S(t, t_i)$ define the marginal probability at default within the time interval *(i,i-1)*, the marginal credit spreads.

Impact of default probability and recovery rate

There are several aspects to consider in computing CVA: level of credit spread (proxy of default probabilities), the overall shape of the credit curve[14], the impact of recovery rates and the basis risk arising from recovery rate assumptions.

The increase in credit spread clearly increase the CVA, however this effect is not linear since default probabilities are bounded at 100%. As the credit quality of the counterparty deteriorates, the CVA will obviously increase but at some point , when the counterparty is near to default, it will decrease again.

Interesting is analyze the impact of recovery rates. Changing recovery rate assumptions has a reasonably small impact on the CVA since there is a cancellation effect: increasing recovery rates increase the implied default probability but reduce the resulting loss.

Probability at Default

As we mentioned so far, the Probability at default is one of the parameters composing CVA amount, and with the Exposure is the measure that most influence the risk related to a counterparty. What follow is a brief description of the models used in order to quantify it. These computational methods can be divided within: Intensity Models and Structural Models. Following, the attention will be focused on the previous as it is the mostly used one.

Poisson Process[15]

Credit risk models based on intensity consider the risk at default as an exogenous event: what is modeled is the way on which the default will occur. The probability at default within a time interval is computed proportionally to the length of the interval considered, in particular the attention is focused on the Hazard rate of default, that is

[14] The shape of the credit curve is strongly linked to economical expectation (expansion or recession) and so to future default probabilities. CVA will be higher in case the Credit curve is decreasing, vice versa, it will be lower with increasing credit curve. This because it comes from expectation of future exposure and default probabilities.

[15] The Poisson process is one of the most widely-used counting process: it is used in scenarios where we are counting the occurrences of certain events that appear to happen at a certain rate/intensity (Λ), but completely random.

the instantaneous probability of default on payment in a short period of time assuming no earlier default event occurred. Intensity models make use of Poisson process in modeling the probability at default, that means to consider default events as random. The risk is modeled trough a counting process $\mathrm{N(t)}$ that increase proportionally by unit jumps when the default event occurs. In particular, assuming that each jump happen in $\tau_1, \tau_2, \tau_3, \tau_4$ ... we have that $\mathrm{N(t)} = \sum_{i=1}^{n} 1_{\tau_1 \leq t}$ where $1_{\tau_1 \leq t} = 1$ if the default event occurs otherwise is 0. The frequency on which these events occur in a time interval $(t, t + \Delta(t))$ is described trough a function called Intensity. This function is denoted by λ(t), that could be stochastic or deterministic16. Moreover, the probability that two events occur simultaneously in such models is zero.

Depending on which type of intensity model is chosen, two process can be used: Homogeneous Poisson process (deterministic intensity function) or Inhomogeneous Poisson process (stochastic intensity function).

Homogenous Poisson process

In such process the intensity of default is constant over time, then the probability that will occur N jumps in a time interval $(t, t + \Delta(t))$ is proportional to $\Delta(t)$.

$$P[N(t + \Delta(t)) - N(t) = 1] = \lambda\Delta(t)$$

As a consequence the probability that there will be no jumps is 1- $\lambda\Delta(t)$. As we are interested in only one jump, presumably the first, we can divide the interval $\big(t, t + \Delta(t)\big)$in n subinterval of length $\Delta(t) = (T - t)/n$, every interval can be seen as an independent binomial experiment[17] on which the jumps occur with probability $\lambda\Delta(t)$. Moreover, as events are independent the probability of survival (no default) in a period [t,T] for n → ∞ is equal to:

$$P[N(T) - N(t) = 0] = \big(1 - \lambda\Delta(t)\big)^n \rightarrow \exp\big(-\lambda(T - t)\big)$$

The probability at default will be equal to:

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[16] In deterministic models, the output of the model is fully determined by the parameter values and the initial conditions. Stochastic models, instead, possess some inherent randomness: the same set of parameter values and initial conditions will lead to an ensemble of different outputs.

[17] A binomial experiment is a statistical experiment that has the following properties: n repeated trials; each trial can result in just two possible outcomes (success or failure); the probability of success is the same on every trial; the trials are independent.

$$PD = 1 - \exp\left(-\lambda(T - t)\right)$$

The time at default is then distributed as an exponential random variable having parameter λ.

Inhomogeneous Poisson process

In such process the intensity is defined trough a function dependent on time.

The survival probability will be:

$$P[N(T) - N(t) = 0] = \exp\left(-\int_t^T \lambda(\tau) d_\tau\right)$$

Moreover, let's define the cumulate intensity function (hazard function) as $\int_0^T \lambda(\tau) d_\tau = \Gamma_T$.

If N(t) is a standard Poisson process, then the inhomogeneous one will be denoted as M(t) and will be defined by:

$$M(t) = N(\Gamma_t)$$

The inhomogeneous Poisson process is defined as a Poisson standard that take into account the changes in the time factor.

As a consequence, if in M(t) the first jump will occur in τ, in N(t) it will occur in Γ_τ, then the probability of survival will be equal to:

$$P(\tau > t) = P(\Gamma_\tau > \Gamma_t) = \exp(-\Gamma_t)$$

Alternatively to Intensity Models, one can choose to apply Structural Models, that take into account fundamental economics variable of the institution (ex. Debt/Equity rate) in modeling the probability at default. Then, the risk factor is considered as endogenous. Within these models a fundamental role is given to rating agencies, that for each rating class associate a default probability that is increasing with respect time. Typical structural process are: Merton Model, Black Cox model and Hull&White model.

Effects of Netting and Collateral in the computation of CVA

The structure and the logics behind netting and collateralizations agreement will be carefully discussed in the last chapter. Here it will be given some hints on the effects that such techniques have when computing Credit Value Adjustment.

The introduction of a new product within a portfolio could lead to a reduction of the exposure (and so the CVA) due to the exploit of netting and collateral techniques. However, as it has been mentioned above, this lead to a major complexity in the computation of CVA, due to the fact that the riskiness of a transaction is no longer computable separately to the others within the same netting set or collateral agreement.

Therefore, it is known that for a set of netted trades (NS):

$\mathrm{CVA(NS)} \leq \mathrm{sum(stand\ alone\ CVA)}$.

The above reduction can be substantial and the question becomes how to allocate the netting benefits to each individual transactions. The most obvious way to do this is to use the concept of Incremental CVA. Here the CVA of a transaction *(i)* is computed based on the incremental effect that the trade has on the netting set:

$\mathrm{CVA}_i^{\mathrm{incremental}} = \mathrm{CVA}_{\mathrm{NS}+i} - \mathrm{CVA}_{\mathrm{NS}}$.

Therefore, CVA depends on the order in which trades are executed.

It can be derived the following formula for Incremental CVA:

$$\mathrm{CVA}_i^{\mathrm{incremental}} = (1 - \mathrm{Rec}) \sum_{j=1}^{m} \mathrm{DF}(t_j) \mathrm{E}_i^{\mathrm{incremental}}(t_{j-1}, t_j) \mathrm{PD}(t_{j-1}, t_j).$$

Now EE is the incremental one and not the stand alone as shown in the general equation. This is obvious since netting agreements change only the exposure and have no impact on recovery values, discount factors or default probabilities. Moreover, the fact that the incremental CVA in presence of netting agreements is never higher than the stand alone one, lead to the opportunity of an institution to offer more favorable conditions to a counterparty with respect a new trade.

As it will be discussed in the last chapter, for real values computational purpose, when incorporating netting activities a general Monte Carlo simulation for EE quantification is required.

Similar to the effect of netting activities, the influence of collateral on the standard CVA formula is straightforward. Collateral only changes the EE and hence the same formula may be used based on collateralization assumptions.

Defining a real scenario: the Wrong Way Risk

In the analysis conducted so far we assumed independency between the probability at default of the counterparty and the annex credit exposure. It is obvious that such scenarios could be hardly realistic, even if not including correlations assumptions it is a preliminary activity in order to fully understand the magnitude and the meaning of CVA. When talking about correlations among probability at default and exposure, several scenarios have to be taken into account:

- There could be a positive dependency among these two variables , meaning that the probability at default of the counterparty tend to increase (decrease) whenever the exposure versus the same counterparty tend to increase (decrease): this situation is defined as Right Way Risk
- Whereas, if we assume that there is a negative dependency among the two figures, the probability at default tend to increase (decrease) when the exposure tend to decrease (increase), we talk about Wrong Way Risk.

Among the scenarios of negative correlation (WWR) another important distinction has to be taken into account: the one between Specific WWR and General WWR. In particular, Specific Wrong Way risk arises due to counterparty specific factors like rating downgrade, poor earning or litigation: for example, it could happen that within a collateral agreement coming from a trade activity, the counterparty post one of its bond as collateral, then in covering the exposure the institution is highly dependent on the credit situation of the same counterparty. Whereas, General Wrong-way risk occurs when the trade position is effected by macroeconomic factors like interest rates, inflation or political tension. In such cases, the likelihood of the counterparty default is negatively correlated with general market risk factors. The analysis below focus the attention on the latest scenario as is the one that most influence the risk associated to a counterparty.[18]

It is obvious that the presence of WWR will increase CVA. In fact, we can still use the same CVA expression as long as we calculate the exposure conditional upon default of the counterparty. Computing conditional exposure is not easy but broadly speaking there are two ways to go about computing it: consider the exposure and

[18] Gregory Jon, Counterparty credit risk and credit value adjustment : a continuing challenge for global financial markets – 2012.

default of the counterparty together and quantify the economic relationship between them (in other words the correlation), but this may be extremely hard to define; or incorporate WWR via simple conservative assumptions “rules of thumb” or simple generic models.

The latest method is also mentioned in the Basel II Accord and the Capital Requirements Regulation on IMM banks that pay particular attention to the treatment of General Wrong Way Risk. In fact, as a relevant tool for monitoring and managing GWWR, Stress Testing on exposure has been identified and required. Currently GWWR is only monitored via Current Exposure Stress Testing. Moreover, in trying to facilitate the introduction of such metric in the computation of CVA, Basel II identify a constant term (set at 1.4) to be added in the base model in order to increase the credit exposure. This means that at minimum CVA will be increased by 40% with respect to the CVA computed assuming independence.[19] However, as it is stated above, a deeper analysis could be conducted through the estimation of the correlation within default probability and exposure. In order to do that, the probability at default has to be model so that it results correlated with a risk factor linked to the underlying instrument of the transaction. One should model a stochastic process both for the intensity of default and for the risk factor , in fact in order to take into consideration the correlation it is necessary to assume joint stochasticity (both have to be modeled trough stochastic process).

An example is given in order to better understand the computational process.

Such example is constructed in order to compute the CVA for a Bond instrument taking into account WWR: as stated above, it is necessary to model the correlation between the default probability and the risk factor linked to such instrument (namely interest rates). In particular, assume that interest rate r follow a Vasicek process[20] for which:

$$dr_t = k(\theta - r_t)dt + \sigma dW_r(t)$$

[19] UniCredit Holding, September 2013, Counterparty Credit Risk Internal Model Methodology: General Wrong-Way Risk.

[20] Vasicek model is a mathematical model describing the evolution of interest rates. It is a typo of one-factor short rate model as it describes interest rate movements as driven by only one source of market risk ($W_r(t)$).

Where $W_r(t)$ represent a Brownian motion process (modeling the random market risk factor); σ is the standard deviation, determining the interest rate volatility and the parameters $k(\theta - r_t)$ characterize completely the dynamic. Moreover, the process is of type "mean reverting" meaning that the value of r will reach its long period value represented by θ after a defined time interval.

As stated above, the probability at default is determined by means of the Intensity Model, meaning that in presence of WWR it is asked to compute it by correlating its random intensity to the interest rate. Therefore, it is necessary to model the random intensity of the probability at default by means of $CIRR^{++}$(extension of CIR model for short rate): $\lambda_t = y_t + \psi(t)$ [short rate function], where λ is the intensity at default of the counterparty, y_t follow a Cox Ingersoll Ross (CIR)[21] process and ψ is a positive deterministic function used in order to adapt exactly the model to the implicit probability at default of Bond instruments. Assuming that y_t follow a dynamic model (CIR) under risk neutral probability measures, it can be rewrite as: $dy_t = k(\mu - y_t)dt + \upsilon\sqrt{y_t}dW_y(t)$, where k, μ and υ are positive and deterministic constants; W_y is a Brownian Motion process and represents the stochastic shock in the dynamic model.

The correlation within intensity and interest rate is determined assuming that the two Brownian Motion $W_r$and W_y are instantaneously correlated (Hazard rate), following the relation: $d(W_r, W_y)(t)=\rho dt$. Therefore, this create a linkage within r and λ dependent on ρ. If ρ is negative, as intensity increase (default probability increase), r will decrease due to the negative correlation. It is straightforward that whenever r decrease, the value of the Bond increase and so the CVA will be amplified by the correlation effect. Than it can be state that in case of Bond instruments, Wrong Way Risk exist whenever $\rho<0$.

[21] The CIR model is an extension of the Vasicek model, on which the element $\upsilon\sqrt{y_t}$ avoids the possibility of negative interest rates for all positive values of k and μ: this in order to avoid possible disturbing consequence in case of negative interest rates (crisis periods).

CHAPTER III.

CREDIT EXPOSURE

This chapter will be focused on the definition of credit exposure (also known just as exposure) in more detail, along with the explanation of the main metrics use in its quantification. The computational path will be carefully analyzed trough the presentation of the internal method model (IMM) used by UniCredit.

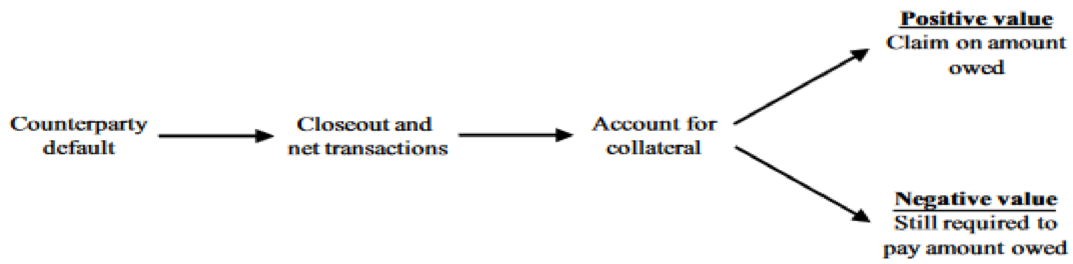
Having in mind what said so far, it is clear that understanding exposure is relevant for the following reasons:

- Pricing and hedging counterparty risk (CVA).
- Calculating economic and regulatory capital.
- Trade approval by comparing against credit limits.

Definition of Credit Exposure

An important feature of counterparty risk arise from the asymmetry of potential losses with respect to the value of the underlying transactions. In the event that a counterparty has defaulted, an institution may close out the relevant contracts and cease any future contractual payments. Following this, they may determine the net amount owing between them and their counterparty and take into account any collateral posted. Once the above steps have been followed, there is a question of whether the net amount is positive or negative. As it has been mentioned in previous chapters, positive values in transaction activities are in some sense in favor of the institution as they represent a right to receive from the counterparty, what it is registered. However, in case of counterparty default, only a partial amount of what owed will be received: that is the recovery rate, the institution will incur in a loss equal to the difference between the MtM value of the exposure and the Recovery rate. A reverse situation is defined whenever the counterparty default and the Mark-to-Market value of the transaction is negative, in such cases the institution is still obliged to pay the amount owed: no gain could be registered.

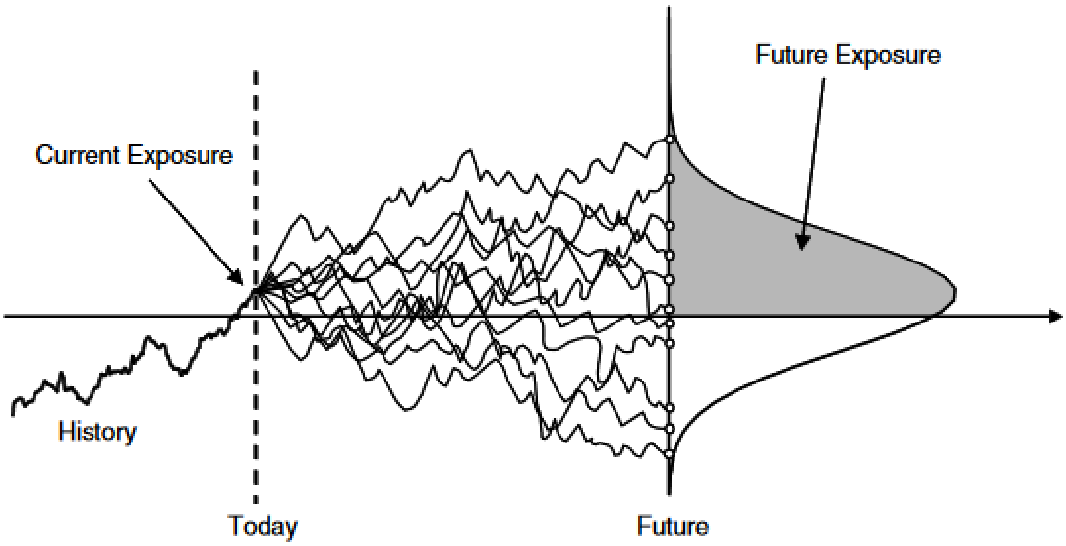
Figure 2. Illustration of the impact of a positive or negative value in the event of the default of a counterparty



The above feature – an institution loses if the value is positive and does not gain if it is negative- is a defining characteristic of counterparty risk.[22] Therefore, the exposure can be simply define as: $\text{Exposure} = \text{Max}(\text{value}, 0)$. As result, defining exposure at a given time is rather easy. However, when approaching exposure for risk management or pricing purposes, institutions are not interested only in the current situation, what is of much importance is the future trend. In fact, exposure is a very time-sensitive measure, since a counterparty can default at any time in the future and so it has to be considered the impact of such events many years from the current situation. It is clear that future exposure is a rather complicate measure that has to be defined probabilistically and should encompass the risk arising from actual claims, potential claims as well as contingent liabilities.

[22] Jon Gregory, 2015, the xVa Challenge: counterparty Credit Risk, Funding, Collateral and Capital.

Figure 3. Illustration of future exposure with the grey area representing exposure (positive future value) and the white area representing negative exposure



Linked to the above feature, is the bilateralism of credit risk: meaning that both parties to a transaction can default and therefore both can experience losses. As a matter of fact also the exposure presents this feature. So far the attention has been focused on the counterparty point of view, only this were subjected to default and the institution was the only party suffering a loss. However, from an institution's point of view, its default can potentially cause a loss to any counterparty, whenever a negative mark-to-market is registered. This is defined in term of negative exposure as follow: $\text{Negative Exposure} = \text{Min}(\text{value}, 0)$. Therefore, in case an institution defaults a negative exposure leads to a gain, which is relevant since the counterparty is making a loss. In such cases, it is important to highlight the effects that Recovery values have on Exposure (which identify positive MtM amount) and Negative Exposure. This is a reverse situation of what explained above, in such cases, an institution will pay only the recovery fraction of the positive amount, but will receive the entire value of the transaction in case of negative exposure: it will gain.

Figure 4. Illustration of payoffs taking into account the bilateralism of Counterparty Risk

		Impact	Payoff
Counterparty defaults	Exposure	Loss	$Rec_C * Max(value, 0)$
Institution defaults	Negative exposure	Gain	$-Rec_I * Min(value, 0)$

Moreover, the fact that Counterparty risk creates such asymmetric risk profile allow to link the definition of Exposure to the one of a short option position[23] (as exposure identify a loss). It follows that in order to quantify the exposure one should take into consideration the volatility of the value of the relevant contracts and collateral. Moreover other important elements have to be considered:

Time horizon. Unlike some other risk measures (like VaR), in order to understand fully the impact of time and specifics of the underlying contracts, exposure needs to be defined over multiple time horizons. This lead to another important futures of exposure: when looking at longer time horizons, the trend (drift) of market variable is relevant, therefore volatility is not the only relevant metric.

Risk mitigants. Exposure is typically reduced by risk relievers such as netting and collateral agreements and the impact of these figures must be considered in order to properly estimate the future exposure.

Relevant Metrics

In this section, it will be defined the measures commonly used to quantify exposure, focusing the attention on the figures important in Credit Value adjustment application. The nomenclature followed is the one defined by the Basel Committee on Banking Supervision (2005). Moreover, note that in discussing exposure below, the total number of relevant trades netted and collateralized is still used, and so all the measures are view under the definition of a netting set.

[23] This comes from the fact that the payoff of a short option position can be defined as follow: SHORT CALL Payoff=min(K-S(t),0) and SHORT PUT Payoff=min(S(t)-K,0) – where in this case K-S(t) (or vice versa) represent the value of the exposure.

The metrics that will be analyzed are referring to the future position that the parties of a transaction face during the life of the contract, therefore, it is important to mention some characteristics of expected value. In fact, Expected future value (EFV) may vary significantly form current value for a number of reasons:

- Cash flow differential. Dealing with derivatives products means to consider transactions that may be rather asymmetric. For example, in an interest rate swap, early in its life time typically fixed cash flows exceed the floating one (assuming that the underlying yield curve is upwards sloping), this situation could reverse (and often it does) as time goes on. This lead to a situation in which an institution expects a transaction in the future to have a value significantly above or below the current one.
- Forward rates. Linked to the example above, it can happen that forward rates differ significantly from current spot variables. This difference introduces an implied drift (trend) in the future evolution of the underlying variables. Drifts in market variables will lead to a higher or lower future value for a given netting set even before the impact of volatility.
- Asymmetric collateral agreements. Favorable or unfavorable collateral terms can obviously effect future exposure expectations.

Let's now detect some important metrics when approaching credit exposure:

Potential Future Exposure (PFE)

Is the worse exposure that an institution could have at a certain time in the future taking into consideration a confidence level. The PFE at a confidence level of 99% will define an exposure that would be exceeded with a probability of no more than 1%. It is clear, that this measure of exposure is very similar to the one of VaR (Value at Risk), however there are two important differences: PFE needs to be defined for the whole life of the contract and it represents gains (exposure) rather than losses. Moreover, being the worst scenario prediction of the current exposure lead to use this measure for risk management purposes. In fact, PFE is used in order to quantify the sensibility of exposure to future changes of stock prices of market rates, at counterparty level. The shape of the PFE curve is sensitive on time and represent the future exposure profile until the contract expires. This curve is also known as "line charge" and it is computed using stochastic models that simulate the payoff of the portfolio in different time instant (called bucket) in relation to a confidence interval. The pick of the exposure is defined as the Maximum Potential Future Exposure and it is used when a comparison with exposure limits, imposed by regulators, is needed.

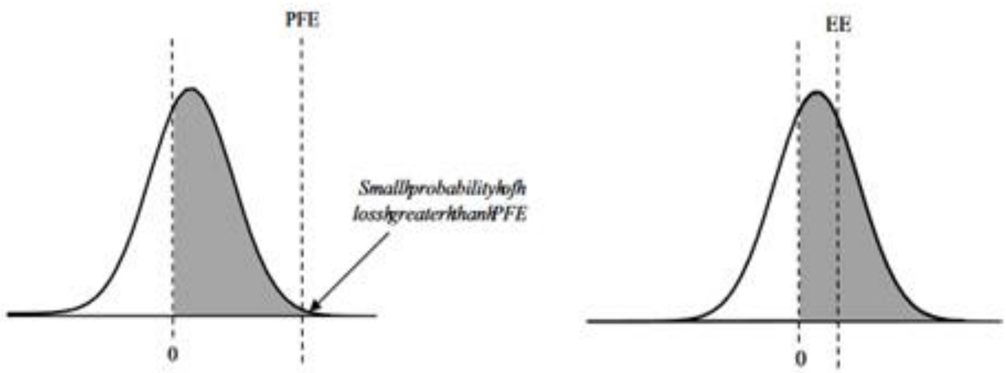
Expected Exposure (EE) and Effective Expected Exposure (Effective EE)

Is the average of all exposure values needed in pricing counterparty risk. As it can be seen in the figure above (Figure 3), only positive values give rise to exposures (other value give zero contribution), it result that the expected exposure will be above the expected future value (this is similar to the concept of an option being more valuable than the underlying forward contract).

Linked to this measure is the Effective Expected Exposure that define the maximum value of EE in a restricted period of time. This measure has been introduced for short-dated transaction in order to capture "rollover risk" that can be underestimated by EE.

In the figure below a normal distribution in shaping EE and PFE has been considered, however, in general the probability distribution function is a results of market factors and the characteristics of the contract considered.

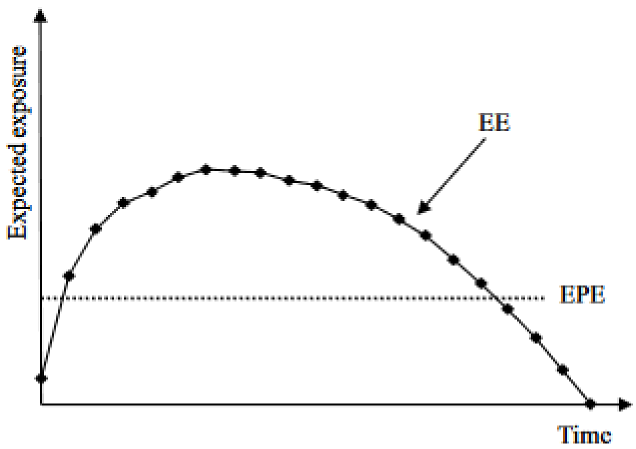
Figure 5. Illustration of Potential Future Exposure (PFE) and Expected Exposure (EE). The grey area represents positive values which are exposures



Expected Positive Exposure (EPE)

It is defined as the average exposure across all time horizons. This value is able to give an unique measure for the exposure (both positive or negative) that is linked to the specific contract considered.

Figure 6. Illustration of maximum PFE



It is straightforward to understand that all the figures defined so far are strictly related one to another; all of them can be derived from the probability distribution of the exposure in future dates, identifying, in this way, important information for the parties of the transactions.

Quantification of Credit Exposure: the Internal Method Model

Once the relevant measures of Credit Exposure have been defined, it is necessary to analyze the computational techniques in order to quantify it.

First of all, before deciding the metrics to use and start the calculations it is important to structure a precise path for the computational process in order to avoid errors and time consuming activities.

The starting point is to define the simulation model: risk factors have to be identify and their evolution have to be modeled. In this phase it is important to pay attention on balancing reality and parsimony, for example involving two or three factors in the model may represent the right compromise. Moreover, correlation analysis have to be implemented in order to capture the correct multidimensional behavior of the netting sets to be simulated.

The second step is to generate scenario, under which computing the exposure, via simulation of the risk factors chosen: each scenario is a joint realization of risk factors at various points in time. It is then necessary to choose a grid of simulation that have to present a number of grid points reasonably large in order to capture all the details of the exposure (the final simulation date has to be grater or equal to the longest maturity instrument under consideration).

Revaluation activities are also needed. Once the scenarios have been generated, it is necessary to revalue the individual positions at each point in future times. So, first the risk factor linked to the instrument has to be calculated and then it has to be implemented the standard pricing function for that instrument as a function of the risk factor.

Finally all the results have to be aggregated in order to get the total amount of Expected Exposure. The future value of the Exposure is then a function of three components: trade, simulation and time step.

IMM for UniCredit Group[24]

Basel III capital regulation ratified by the capital requirements directive IV (European Commission) has introduced new standards in the calculation of the regulatory requirement specific to Counterparty Credit Risk. Exposures to any counterparty are calculated on a position by position basis, unless a legally enforceable netting agreement is in place.

UniCredit Group developed an Internal Method Model (IMM) to simulate the exposure profiles of its Over the Counter derivatives business that will gradually cover the vast majority of the positions across all its Legal Entities. As a matter of fact the model is an extension of the Basel II compliant scenario generator and has been approved by the regulators.

The final purpose of the Internal Model is the accurate calculation of Expected Positive Exposure (EPE) – relevant for the determination of the Regulatory Capital Requirement (via CVA computations); and of the Potential Future Exposure (PFE) , also known as Peak Exposure, relevant for lines management. The process followed is the one described above (simulation of scenarios, valuation of all trades on each scenario and aggregation of results).

General Settings

Regarding the definition of the simulation model, the risk factors that are commonly taken into consideration are: equity and commodity prices, foreign exchange and

[24] Group Risk Methodologies and Model (UniCredit Group), December 2016, Counterparty Credit Risk, Internal Model Method: Scenario Generation.

interest rates, credit spreads. The evolution of such risk factors future value is implicitly based on a dynamic specified by the following generic Stochastic Differential Equation (SDE):

$$dX_t^i = \mu^i(t)dt + \sigma^i(t)dW_t^i,\ \ X_0^i = x_0^i$$

Where i indicates the i-th risk factor, $i = 1, \dots d$, μ^i and σ^i are bounded measurable functions and the Brownian motions $W^1, \dots, W^d$ that are correlated, with correlation coefficient $\rho_{ij}, |\rho_{ij}| \leq 1, i, j = 1, \dots, d: E(W_t^i W_t^j) = \rho_{ij} * t$. The annualized variance-covariance matrix $\sum$ is $\sum = (c_{ij})$ for every $i, j = 1, \dots, d$. As it will be stated below, the variance-covariance structure is computed from historical returns, while the drifts are set so that the average of future risk factors matches the current forward values. In such way, the stochastic process X models:

- The logarithm of foreign exchange (FX) rates, equity and commodity prices, implied volatilities and inflation indexes;
- The square roots of interest rate (IR), credit spreads (from CDS) and inflation rates.

In the current setup of the variance-covariance matrix the elements c_{ij} with $i, j = 1, \dots, d$, are estimated from time series containing three years of history, considering equally-weighted five-day lags. This lag is meant to provide a more stable estimation of the correlation structure, smoothing possible daily noisy and idiosyncratic spikes of the time series.

Choosing the measures

As it has been state above, when defining the calibration philosophy for CVA parameters, risk-neutral measures were chosen for pricing purposes and real-world one for risk-management-oriented calibrations. The division can’t be considered linear as some measures are difficult to estimate in both cases: some assumptions have to be implemented. For example when approaching to risk-neutral measures (as the one used for CVA computational purposes) there is the problem of how to estimate the volatility: in the case of the Internal Model this is calibrated from an historical time series; instead, the drift (mean) of risk factors is estimated trough the assumption that its value coincide to its currently observed forward value, via no arbitrage consideration (in such case the risk factor is modeled as a martingale).

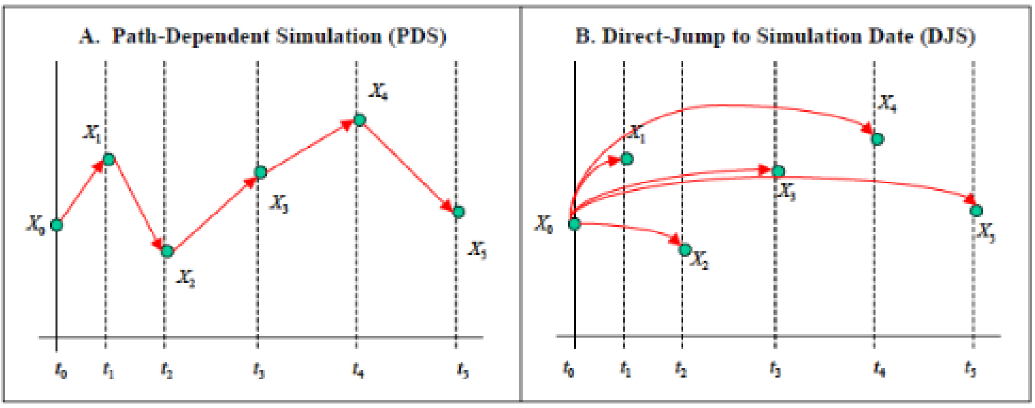
Scenario Generation Philosophy

When simulating a process defined by a SDE two approaches can be chosen:

Path dependent simulation: the value of a market risk factor at any date depends on its values at all the previous dates;

Simulation that evolves (jumps) directly to the desired simulation date: the risk factors value X_t^i at time *t* only requires the knowledge of its initial value x_0^i and of *t*.

Figure 7. Path-dependent versus direct-Jump Simulation



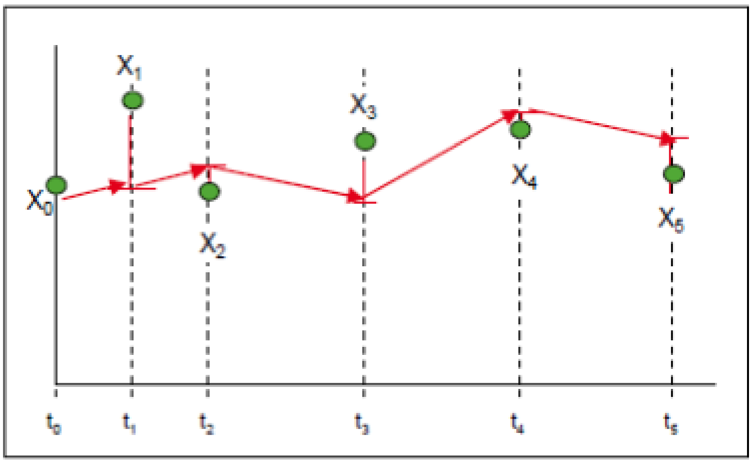
The simulation algorithm implemented in the Counterparty Credit Risk engine is closer in logic to the first of the two schemes. In particular the evolution is carried out with a number of finite steps equal to the number of grid points in the simulation. All the scenarios for the risk factors are eventually driven from a Wiener process, that is evolved according to the following scheme:

A path from a drift-less Brownian process (but variance obtained from the calibration)evolved through the grid points;

For each grid point the relevant (for the grid pint) forward value of the risk factor being evolved is added to the Brownian realization.

This guarantees that the underlying Wiener process is consistently evolved in time along each path, while the risk-factor matches the desired marginal distributions at each relevant time (defined by the grid).

Figure 8. Simulation path in IMM



The CCR simulation is then performed by:

[1] Drawing correlated scenarios;

[2] At different time horizons;

[3] From known risk factors distributions: whose mean is set according to the assumption stated before for the drift; and whose volatility is calibrated from a several years of market data;

[4] Finally, in order to identify the EPE or PFE value a Montecarlo simulation approach has been chosen due to the complexity of the portfolio of OTC derivatives.

CHAPTER IV.

MITIGATION METHODS: Collateral and Netting agreements

So far, the attention has been focused on the concept of Counterparty Risk, the elements determining it and the methods in order to have a preliminary estimation of its magnitude. However, once the risk in a transaction has been identify, the real challenge is to find techniques in order to decrease it. In this chapter the most common mitigation techniques will be briefly described, along with the presentation of a real case related to UniCredit Group.

Introduction

The International Swaps and Derivatives Association (ISDA) is a trade organization for OTC derivatives practitioners. In 2002 the ISDA Master Agreement has been designed in order to eliminate legal uncertainties and provide mechanisms for mitigating counterparty risk. In particular, it specifies the general terms of the agreement between parties with respect to general questions such as netting, collateral, definition of default and other termination events. The real achievement of this agreement is that multiple transactions can be subsumed under it, forming a single legal contract of indefinite term, covering many or all of the transactions traded.

As mentioned in previous chapters, there are many ways in order to mitigate counterparty risk: these include netting and collateralization (margining). It is clear that even after the effects of such techniques, counterparty risk can’t be completely eliminated but only reduced. Moreover, any mitigation of counterparty risk is a double edged sword since while partially reducing the overall risks, it potentially allows financial markets to develop too quickly or to reach dangerous sizes, due to the exploit of the benefits of such techniques. As a matter of fact, netting and collateralization have facilitated counterparty risk management to the extent that they have allowed a massive expansion of OTC derivatives market, with major dealers having massive notional risks.

It has been said that the reduction of exposure is only partially true as mitigation methods only change the nature of the underlying market risk components: counterparty risk, in fact, is reduced only by transformation into other financial risks such as:

Market risk. Taking collateral to minimize counterparty risk creates market risk since exposure exists in the time taken to receive the relevant collateral amount (margin period of risk) and collateral itself may have price and FX volatility;

Operational risk. Relating to organizational activities that are required whenever a new netting or collateral agreement occur;

Liquidity risk. This type of risk arise whenever the collateral needs to be sold at some point due to a default.

Netting

Substantially, netting techniques consist in an aggregation of different derivatives products that are being traded among two counterparties: the goal of such agreements is to create folders that present MtM nearly null, so to cut down losses in case of default of one of the two counterparties.

Netting gives preferential benefit to derivatives counterparties at the expense of other creditors (bondholders and shareholders). Shareholders and bondholders, in fact, could argue that such agreements adversely influences their position due to the increase in default probability and reduction of recovery potentially caused by sizable derivatives exposure. Moreover, whilst netting reduces exposure dramatically, the resulting exposure could result to be highly volatile, making the control of it more complex.

The real challenge when approaching such techniques is to be able to create the best netting with the available instruments traded with the counterparty. In order to do this it is important to remember that exposure is an additive measure so when netting agreements are not in place the total exposure within a counterparty is the sum of all the trade with positive MtM value. Therefore, in order to create the best netting set it is necessary to aggregate trades with opposite MtM sign that present a certain grade of correlation.

Collateralization

Another very popular technique, mostly after 2008, is the one defied as collateralization (margining): this process is used whenever the netting agreements between two parties is not possible due to the high amount of positive MtM contracts, or whenever the exposure is not completely cut down within a netting set.
As netting agreement, collateralization is used in order to creates additional business opportunity: by reducing the total exposure the institution could dispose the residual capital in other investments.
The underlying idea is to construct a collateral (namely an asset) which allow to have a legal protection in case the counterparty default. ISDA Master Agreement establish the necessary documentation related to this techniques, and in particular specifies the characteristics that the asset (identify as collateral) has to presents. Due to the bilateralism of such agreements both counterparties have to mark periodically all positions to market and check the net value: depending on the MtM value it will be decide who has to post the collateral. Obviously, these posting activities rise operational costs, so often the counterparties agree that posting of collateral is not continuous and can occur in blocks according to predefined rules: however this could make liquidity and market risk to rise.
The motivation for collateral agreements is clearly to reduce counterparty risk and can be summarized in more details as follow:

- Reduce credit exposure so as to be able to expand the business.
- To maintain exposure within credit lines not being forced to cease trading with certain counterparties.
- To enable one to trade with a particular counterparty: for example, rating restrictions may not allow uncollateralized credit lines to certain counterparties.
- To reduce capital requirements: Basel regulatory capital rules give capital relief for collateralized exposures
- To fix more competitive pricing of counterparty credit risk.

Moreover, while uncollateralized credit exposure should be considered over the full time horizon of the transaction, collaterals change the above picture by transforming this “long-term” risk into one that need only be considered over a much shorter period: the margin period of risk.

Taking into consideration the advantages mentioned above of such methods it is straightforward to understand why the OTC market has growth so fast in recent years. Moreover, market participants are likely to overestimate the benefit of risk relievers: since counterparty risk acts to reduce profits on transactions, it would not be surprising that the reduction in risk offered by a risk mitigant would be overstated in order to maximize the profitability of such transactions.[25] As a matter of fact, without netting agreements, the current size and liquidity of the OTC derivatives market would be unlikely to exist: netting means that the overall credit exposure in the market grows at a lower rate than the notional growth of the market itself.

Netting and Collateral in real financial transactions

At this point it is necessary to give an example of how such techniques are exploit in real world transactions. What follow is an extract of the current financial activities among UniCredit Group and Goldman Sachs International. The view is from UniCredit Group, meaning that whenever the exposure show positive amounts (CPE) the Group is gaining from the transaction, vice versa, whenever negative amounts (CNE) occur.

Firstly let’s analyze the total amount of trades in place within the two parties as of 30th June 2017:

[25] Jon Gregory, 2015, the xVa Challenge: counterparty Credit Risk, Funding, Collateral and Capital.

Table 1.Trades in place among UniCredit Group and Goldman Sachs international (as of 30 June 2017): total CPE and CNE

Product Type	Notional amount Euro	Current Positive Exp.	Current Negative Ex	Mark to Market
Cap Floor	1,784,050,512	8,800,949	- 55,125	8,745,823
Commodity OTC	2,607,694		- 355,453	- 355,453
Credit Default Swap	2,549,756,522		- 2,976,115	- 2,976,115
Equity Derivatives (linear)	190,201,232		- 40,331	10,285,669
Forex Derivatives	5,085,470,876	34,662,775	- 2,260,352	32,402,423
Forex Option	2,762,712,302	976,707	- 7,779,736	- 6,803,029
IRS	10,806,339,650	467,152,930	- 119,496,270	347,656,661
Repo	14,457,912,420	216,583,664	- 70,371,513	146,212,152
Sec. Lending/Borrowing	1,428,223	7,911,101		7,911,101
Swap Options	1,741,625,000		- 26,493,046	- 26,493,046
Total	**39,874,490,698**	**736,088,125**	**- 229,827,940**	**506,260,185**

The Notional amount has been added in order to show the potential magnitude of the exposure if the underlying, and not the derivatives, would have been traded: almost €40 Billion of notional against a bit more than €500 Milion of MtM.

As it can be seen, the MtM is then split among the two measure of exposure: CPE and CNE. In this extract the Current Positive Exposure account for €736 Million and the Current Negative, around €230 Million. It is clear that in the current situation, UniCredit is exposed to the default of Goldman Sachs, therefore, in such case only a partial amount of it will be actually received (depending on the agreement in place for each trade).

Having determined the total exposure let's see how the netting agreements in place within the counterparties change the current situation:

Table 2. Netting set agreements and the effect on UniCredit Group Current Exposure

Product Type	Current Positive Exp.	Current Negative Exp.	Mark to Market
No nettign Agreements			
Equity Derivatives (linear)		- 19,918	- 19,918
No Netting Agreements			
Equity Derivatives (linear)		- 12,672	- 12,672
No Netting Agreements			
Equity Derivatives (linear)		- 7,741	- 7,741
Netting Agreements n.17			
Cap Floor	- 55,125		- 55,125
Forex Option	976,707		976,707
IRS	- 119,496,270		- 119,496,270
Real Collateral OTC	119,064,000		119,064,000
Netting Agreements n.122			
Real Collateral SFT		700,000	700,000
Repo		- 879,684	- 879,684
Netting Agreement n.559357			
Real Collateral SFT		353,837	353,837
Repo		- 69,491,828	- 69,491,828
Netting Agreement n.560394			
Cap Floor		8,800,949	8,800,949
Commodity OTC		- 355,453	- 355,453
Credit Default Swap		- 2,976,115	- 2,976,115
Equity Derivatives (linear)		10,326,000	10,326,000
Forex Derivatives		34,662,775	34,662,775
Forex Option		- 7,779,736	- 7,779,736
IRS		452,229,951	452,229,951
Real Collateral OTC		- 477,358,000	- 477,358,000
Swap Options		- 26,493,046	- 26,493,046
Netting Agreement n.620291			
Real Collateral SFT		- 8,260,710	- 8,260,710
Sec. Lending/Borrowing		7,911,101	7,911,101
Netting Agreement n.147069			
Real Collateral SFT	- 10,162,375		- 10,162,375
Repo	216,583,664		216,583,664
Netting Agreement n.372033			
Forex Derivatives	-	- 2,260,352	- 2,260,352
IRS	-	14,922,979	14,922,979
Real Collateral OTC	-	- 12,676,996	- 12,676,996
Total amount	**206,910,602**	**- 78,664,661**	**128,245,941**

As we can see there are 7 netting agreements in place, both CPE and CNE have dramatically decreased, the MtM is almost one fifth of the original amount. From this overview it is clear why netting and collateral agreements have became so popular in daily trading activities. If being in an OTC derivative contract already permit to exploit the leverage of such instruments, agreed to organize the future trades within such mitigation activities bring even more benefits.

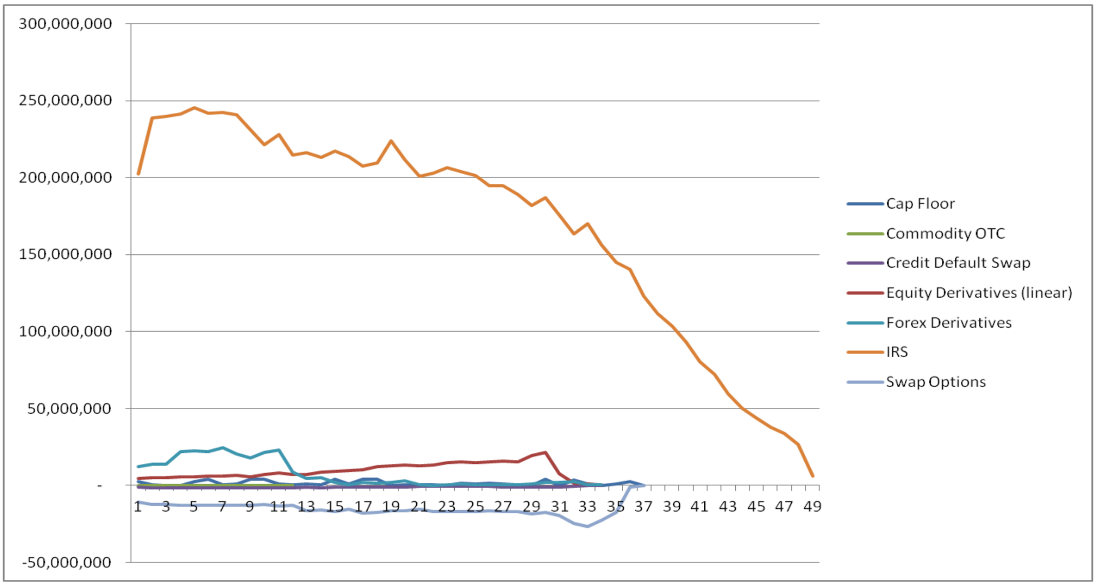
The attention is now focused on one of this netting set, in particular the Netting Agreement n.560394. Firstly the analysis will be centered only on the effect of the netting activity, secondly the collateral will be introduced and the final exposure will be analyzed.

Table 3. Example of a real Netting Agreement

Netting agreement number: 560394	
Product Type	**Current exposure**
Cap Floor	8,800,949
Commodity OTC	- 355,453
Credit Default Swap	- 2,976,115
Equity Derivatives (linear)	10,326,000
Forex Derivatives	34,662,775
Forex Option	- 7,779,736
IRS	452,229,951
Swap Options	- 26,493,046
Total final exposure	**468,415,324**

The above can be graphically shown as follow:

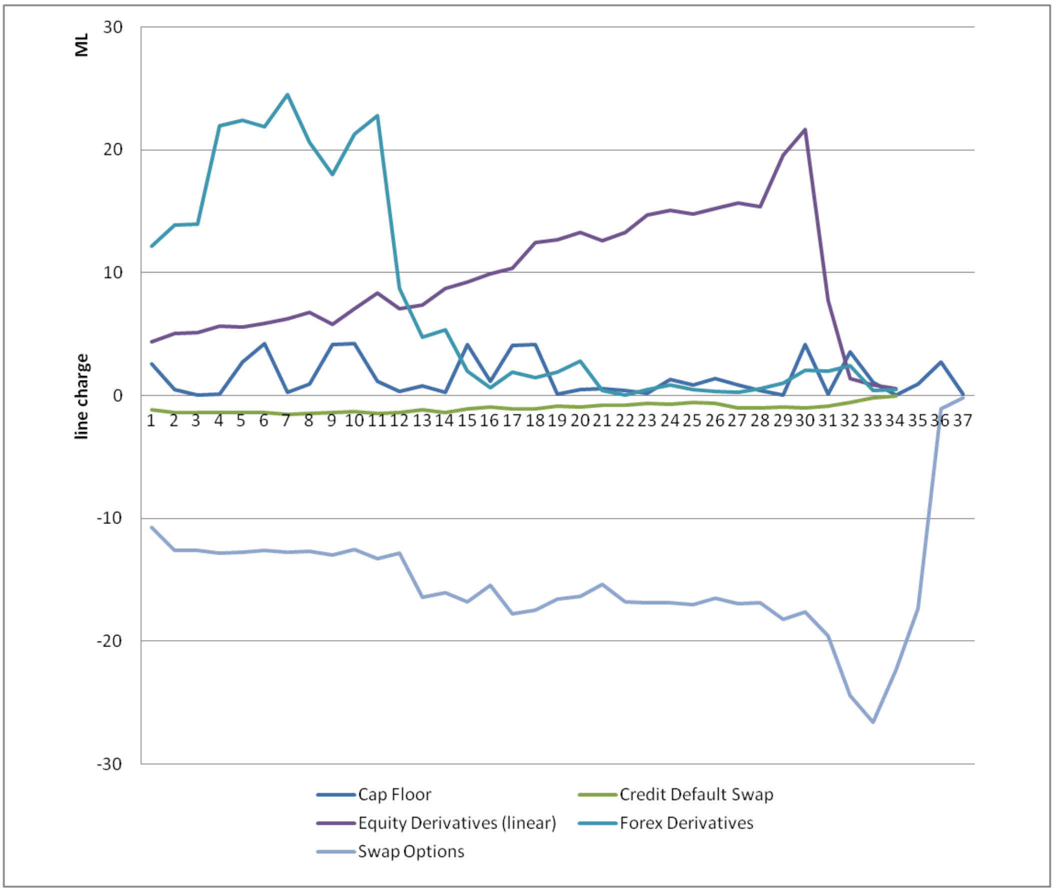
Figure 9. Graphical representation of a netting set



It is clear that the product driving the sign of the exposure is the Interest Rate Swap: even within the Netting set the current exposure seems quite high and not really well managed.

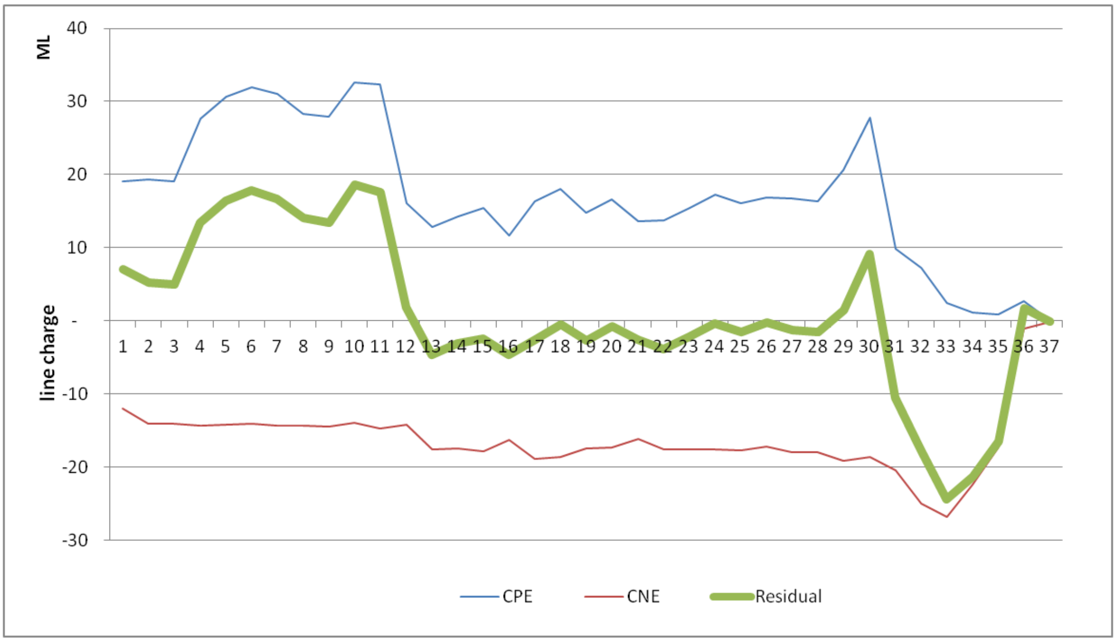
However, the other traded products seem to offset one to another, the following graph better highlight this behavior:

Figure 10. Graphical representation of a netting set: understanding the internal construction



From the above figure a certain grade or correlation among the products presented can be detected, especially of what concern to Equity Derivatives and Swap Options: whenever one of the two increase the other follow the same behavior causing a natural decrease of the total exposure. Overall, it can be state that, in the construction of such netting set, the attention has been focused on the length of the exposure, all the products detected, in fact, have a duration that do not exceed 37 years. From the figure below it can be seen how these products together manage to reduce the overall exposure at list in the long-run:

Figure 11. Behavior of the exposure for a set of products within a netting agreement



Where the CPE is defining as the sum of all the position presenting a positive MtM (IRS excluded) and CNE is the sum of all the position presenting a negative MtM. The green line is the Final Exposure from netting of such MtM amounts.

Let's now introduce another element constituting the original netting set: the collateral.

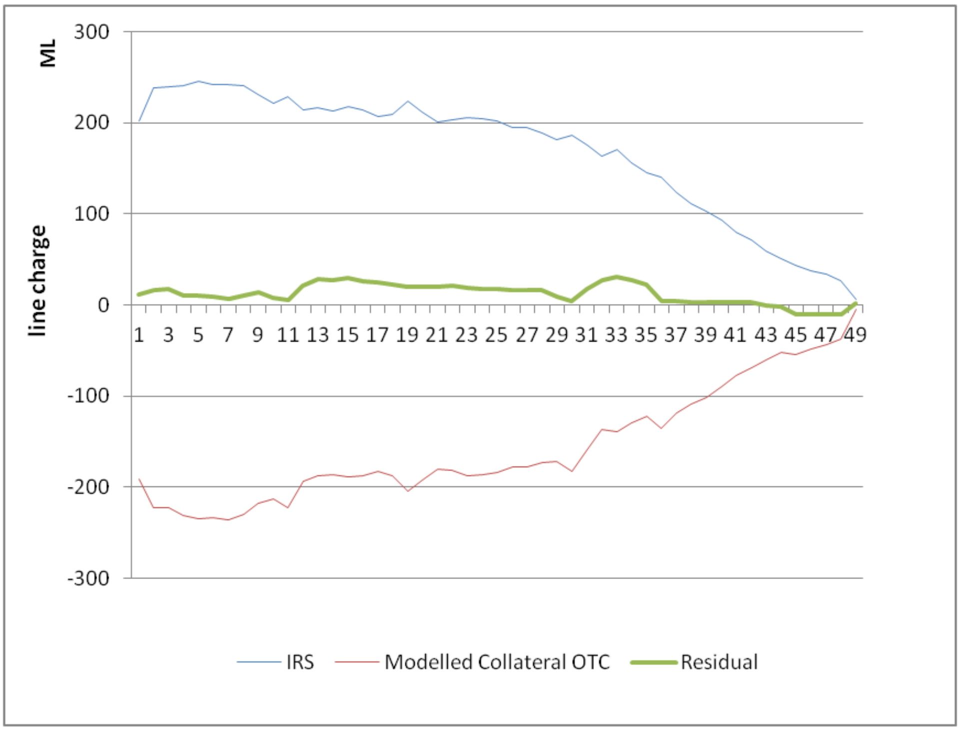
Table 4. ***Netting Set and Collateral***

Netting agreement number: 560394	
Product Type	**Current negative exposure**
Cap Floor	8,800,949
Commodity OTC	- 355,453
Credit Default Swap	- 2,976,115
Equity Derivatives (linear)	10,326,000
Forex Derivatives	34,662,775
Forex Option	- 7,779,736
IRS	452,229,951
Real Collateral OTC	- 477,358,000
Swap Options	- 26,493,046
Total final exposure	**- 8,942,676**

Finally with the introduction of the Real Collateral OTC (defining an OTC asset that act as collateral) we can note how the final exposure is drastically cat down: from a positive exposure we now end with a negative amount. In such situation, UniCredit is no longer negatively exposed to default probability of its counterparty, the scenario has been inverted.

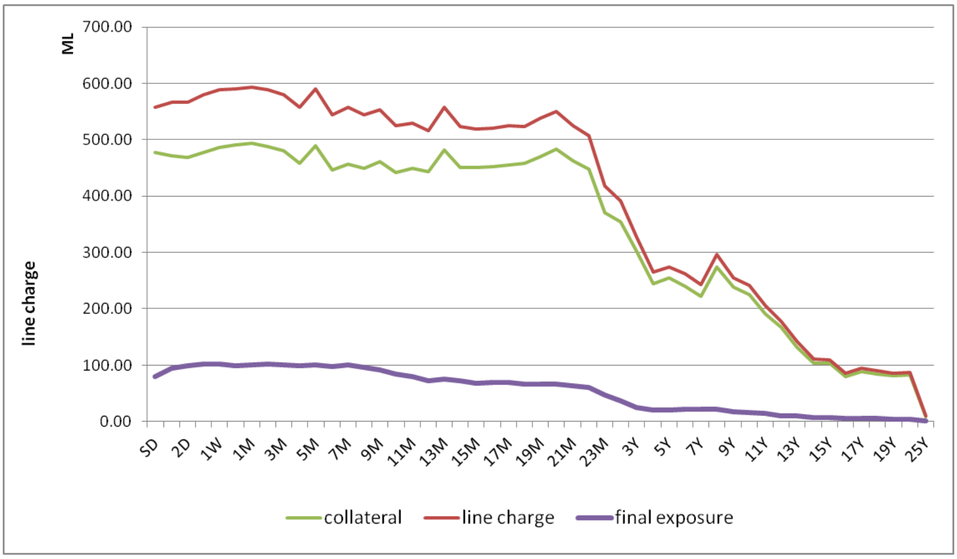
It is clear that the collateral has been posted (from Goldman Sachs) to offset the exposure coming from the Interest Rate Swap. In fact, this can be seen from the below figure: the Modeled Collateral perfectly follow the behavior and the length of such product reducing the related PFE almost to zero (green line).

Figure 12. IRS and Modeled Collateral: the residual exposure



The introduction of the collateral bring benefits to the overall netting set, as seen in the table above. This feature can be shown graphically by adding together the lines charge of each products and netting the Modeled collateral. It ends that the behavior of the netting set in the long run can be represented as follow:

Figure 13. Final Exposure after Netting and Collateral for a set of products



As mentioned in previous chapters the benefit of netting and collateral agreement indirectly decrease the related CVA of the transaction by means of Exposure: this is a clear example of how such agreements potentially null the risk related to a counterparty and so the CVA.

CHAPTER V.

CVA IMPLEMENTATION USING MATLAB

In the following chapter the theory presented so far will be implemented in order to compute the CVA for some simple European option instruments. We will try to capture the behavior of the CVA to variation of the parameters influencing probability at default and Exposure. Finally the Wrong Way Risk will be included in the computational process in order to detect its effect on option pricing activities. The entire analysis will be implemented on Matlab.

CVA for an European Option

Before starting the computational process it is useful to give a preliminary theoretical explanation of the steps to follow in order to determine the Credit Value Adjustment for such products.

First of all, Options are derivatives financial instruments characterize by the fact that they can potentially benefit from favorable price movements, while not being hit by unfavorable movements. Formally, an option is a contract on which a party (buyer) buy the right to oblige its counterparty (seller) to purchase, at a certain future date, the underlying instrument of the contract. There are two type of options: Put Option, on which the owner of the instruments has the right to sell, at maturity, the underlying to its counterparty; Call Option, on which to the owner is reserved the right to buy. We talk about European Option when the right incorporated in the instrument can be exercise only at maturity of the contract.

For our computation on European Call Option, let’s consider the following specification.

Maturity set at T=1 year;

Strike Price K=25€ as defined in the contract;

Underlying initial price S_0=35€, so the option is “in the money”.

Moreover, it is assumed that S (the price of the underlying) follows a geometrical Brownian Motion process and it is computed under risk-neutral measures. The parameters defining the process are, in particular, set as follow: interest rate $r = 1\%$ and volatility $\sigma = 35\%$.

Let's assume also that the option instrument is own at time *t* from a counterparty A, and if the option is exercised at maturity, the counterparty B has to sell the underlying (S) at the Strike Price K. Moreover, it is assumed that the probability at default of the counterparty B is defined under a Homogeneous Poisson intensity process on which λ is constant over time and uniformly distributed within the interval (0,L). In this first part of the project it is assumed also independence within the Default Probability and the price of the underlying (S).

In order to determine the CVA, let's begin with its definition formula (as in Chapter 2.3):

$$CVA(t,T) = E\{(1-\delta) * I(\tau \leq T) * D(0,\tau) * NPV(\tau)^{+}\}$$

In case of European Call Option the residual NPV at time τ is equal to the expected value (in τ) of the discounted option payoff: $NPV(\tau) = E_{\tau}[D(\tau,T)(S_T - K)^{+}]$. Therefore, we can re-write the CVA as follow:

$$CVA(t,T) = E_0\{(1-\delta) * I(\tau \leq T) * D(0,\tau) * E_{\tau}[D(\tau,T)(S_T - K)^{+}]^{+}\}$$

$$CVA(t,T) = (1-\delta) * E_0\left[I(\tau \leq T) * D(0,\tau) * E_{\tau}[D(\tau,T)(S_T - K)^{+}]\right]$$

$$CVA(t,T) = (1-\delta) * E_0[I(\tau \leq T) * E_{\tau}(D(0,\tau) * D(\tau,T)(S_T - K)^{+})]$$

$$CVA(t,T) = (1-\delta) * E_0[E_{\tau}(I(\tau \leq T) * D(0,T)(S_T - K)^{+})]$$

$$CVA(t,T) = (1-\delta) * E_0[I(\tau \leq T)D(0,T)(S_T - K)^{+}]$$

$$CVA(t,T) = (1-\delta) * E_0[I(\tau \leq T)]E_0[D(0,T)(S_T - K)^{+}]$$

$$CVA(t,T) = (1-\delta) * P(\tau \leq T)E_0[D(0,T)(S_T - K)^{+}]$$

$$CVA(t,T) = (1-\delta) * P(\tau \leq T)CallPrice_0$$

On which $P(\tau \leq T)$ is the Default Probability.

Therefore, in order to compute the CVA it is necessary to define the Call Price at time t=0 and the probability at default.

Let's begin with the later, knowing that Probability of survival in homogeneous Poisson process can be modeled as:

$$P(\tau \geq T) = E[\exp(-\int_0^{\mathrm{T}} \lambda(\mathrm{t})\mathrm{d}_{\mathrm{t}})] = \mathrm{E}[\exp{(\lambda \mathrm{T})}]$$

Knowing that λ is constant over time and uniformly distributed with density equal to $\frac{1}{L}$, we can proceed as follow:

$$= \int_0^L \exp(-xT)\, p_\lambda(x) dx = \int_0^L \exp(-xT) \frac{1}{L} dx = \frac{1}{LT}(1 - \exp(-LT))$$

So if $P(\tau \geq T)$ is the probability of survival, $P(\tau < T) = 1 - \frac{1}{LT}(1 - \exp(-LT))$ is the probability of default.

Within the data framework chosen the mean of the Probability of default results to be equal to 0.0938.

```
%default probability estimation

lambda=0.01:0.01:0.19;
pd=zeros(size(lambda));%define the interval of variation for Lambda (the intensity homogeneous function)
for i=1:numel(lambda)
    pd(i)=1-(exp(-lambda(i))); %vector of PD for different value of lambda
end
pd1=mean(pd); %computing the mean of PD to insert in CVA formula
```

Having determined the probability at default let's now derive the price of the Call Option by mean of the Black&Scholes Model as follow:

$$CallPrice_0 = S_0 N(d1) - Kexp(-rT)N(d2)$$

Where $d1 = \frac{\log\left(\frac{S_0}{K}\right) + (r + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $d2 = d1 - \sigma\sqrt{T}$.

Within the data framework chosen the Price of the Call Option is equal to 11.1152.

```
%pricing the Option with Black&Scholes Model

StockSpec=stockspec(Sigma,AssetPrice); %create stock structure
PriceBLS=optstockbybls(RateSpec,StockSpec,settled,maturity,OptSpec,Strike);
```

Finally, the CVA formula for an European Call Option can be constructed as follow:

$$CVA(t,T) = (1 - \delta) * (1 - \frac{1}{LT}(1 - \exp(-LT)))\big(S_0 N(d1) - Kexp(-rT)N(d2)\big)$$

Having determined both PD and the Call Price, with a Recovery value of 49% (that represent the everage registered after the crisis 2007-2008), the CVA result to be 0.5109.

```
%CVA estimation with crescent Recovery Rates and lambda variation

rec=0.40:0.01:0.58; %specify different value for the Recovery rate
lgd=1-rec; %loss given default
for j=1:numel(rec)
    lgd(j)=1-rec(j); %LGD for different value of recovery rates
end
ExpperProb=pd*PriceBLS; %correspond to(1-1/LT(1-exp(-LT))(S0 N(d1)-Kexp(-rT)N(d2))
CVA=lgd'*ExpperProb; %CVA definition
```

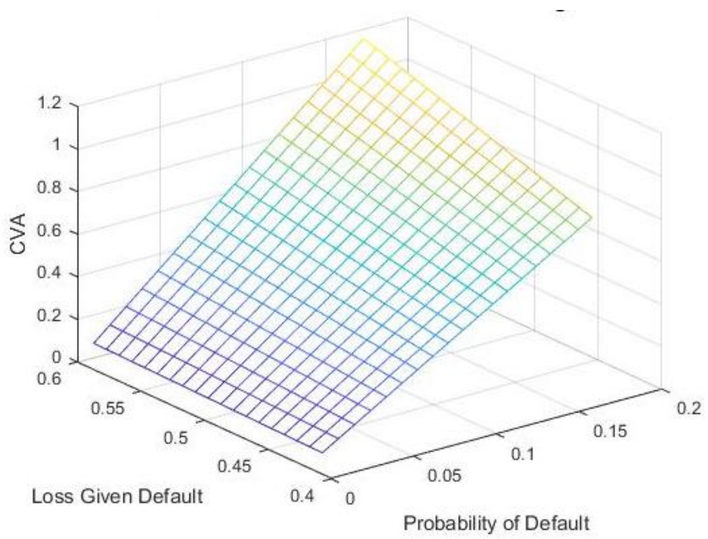
Knowing that the final Price of an instrument, taking into consideration the riskyness of the counterparty is defined as follow:

$$Instrumet\ final\ Price = Riskfree\ price - \ CVA$$

We can and saying that the final price of the European Call Option is equal to 10.6043.

In the following figure it is shown the behaviour of the Credit Value adjustment as result lambda variations and increasing Recovery Rates:

Figure 14. CVA values as function of λ and C



As we could have expected, the Credit Value Adjustment increase as result of increases in the Probability at default and Loss Given Default, that represents the amount of exposure that is loss in case the counterparty defaults.

In the following table are reported the results from the computation of CVA and the Final Price of the Call Option when λ and LGD change in value:

Table 5. CVA for a Call Option as function of Lambda and LGD

LGD/Lambda	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190
0.600	0.066	0.132	0.197	0.261	0.325	0.388	0.451	0.513	0.574	0.635	0.695	0.754	0.813	0.871	0.929	0.986	0.104	0.110	0.115
0.590	0.065	0.130	0.194	0.257	0.320	0.382	0.443	0.504	0.564	0.624	0.683	0.742	0.799	0.857	0.913	0.970	0.103	0.108	0.113
0.580	0.064	0.128	0.191	0.253	0.314	0.375	0.436	0.496	0.555	0.613	0.672	0.729	0.786	0.842	0.898	0.953	0.101	0.106	0.112
0.570	0.063	0.125	0.187	0.248	0.309	0.369	0.428	0.487	0.545	0.603	0.660	0.716	0.772	0.828	0.883	0.937	0.990	0.104	0.110
0.560	0.062	0.123	0.184	0.244	0.304	0.362	0.421	0.479	0.536	0.592	0.648	0.704	0.759	0.813	0.867	0.920	0.973	0.103	0.108
0.550	0.061	0.121	0.181	0.240	0.298	0.356	0.413	0.470	0.526	0.582	0.637	0.691	0.745	0.799	0.852	0.904	0.956	0.101	0.106
0.540	0.060	0.119	0.177	0.235	0.293	0.350	0.406	0.461	0.517	0.571	0.625	0.679	0.732	0.784	0.836	0.887	0.938	0.989	0.104
0.530	0.059	0.117	0.174	0.231	0.287	0.343	0.398	0.453	0.507	0.561	0.614	0.666	0.718	0.770	0.821	0.871	0.921	0.970	0.102
0.520	0.058	0.114	0.171	0.227	0.282	0.337	0.391	0.444	0.497	0.550	0.602	0.654	0.705	0.755	0.805	0.855	0.904	0.952	0.100
0.510	0.056	0.112	0.168	0.222	0.276	0.330	0.383	0.436	0.488	0.539	0.590	0.641	0.691	0.741	0.790	0.838	0.886	0.934	0.981
0.500	0.055	0.110	0.164	0.218	0.271	0.324	0.376	0.427	0.478	0.529	0.579	0.628	0.677	0.726	0.774	0.822	0.869	0.916	0.962
0.490	0.054	0.108	0.161	0.214	0.266	0.317	0.368	0.419	0.469	0.518	0.567	0.616	0.664	0.712	0.759	0.805	0.851	0.897	0.942
0.480	0.053	0.106	0.158	0.209	0.260	0.311	0.361	0.410	0.459	0.508	0.556	0.603	0.650	0.697	0.743	0.789	0.834	0.879	0.923
0.470	0.052	0.103	0.154	0.205	0.255	0.304	0.353	0.402	0.450	0.497	0.544	0.591	0.637	0.682	0.728	0.772	0.817	0.861	0.904
0.460	0.051	0.101	0.151	0.200	0.249	0.298	0.346	0.393	0.440	0.487	0.533	0.578	0.623	0.668	0.712	0.756	0.799	0.842	0.885
0.450	0.050	0.099	0.148	0.196	0.244	0.291	0.338	0.385	0.431	0.476	0.521	0.566	0.610	0.653	0.697	0.740	0.782	0.824	0.866
0.440	0.049	0.097	0.145	0.192	0.239	0.285	0.331	0.376	0.421	0.465	0.509	0.553	0.596	0.639	0.681	0.723	0.765	0.806	0.846
0.430	0.048	0.095	0.141	0.187	0.233	0.278	0.323	0.367	0.411	0.455	0.498	0.540	0.583	0.624	0.666	0.707	0.747	0.787	0.827
0.420	0.046	0.092	0.138	0.183	0.228	0.272	0.316	0.359	0.402	0.444	0.486	0.528	0.569	0.610	0.650	0.690	0.730	0.769	0.808

Table 6. Final Price of a Call Option taking into account CVA

LGD/Lambda	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090	0.100	0.110	0.120	0.130	0.140	0.150	0.160	0.170	0.180	0.190
0.600	11.049	10.983	10.918	10.854	10.790	10.727	10.664	10.602	10.541	10.481	10.421	10.361	10.302	10.244	10.186	10.129	10.073	10.017	99.612
0.590	11.050	10.985	10.921	10.858	10.795	10.733	10.672	10.611	10.551	10.491	10.432	10.374	10.316	10.258	10.202	10.146	10.090	10.035	99.804
0.580	11.051	10.988	10.925	10.862	10.801	10.740	10.679	10.620	10.560	10.502	10.444	10.386	10.329	10.273	10.217	10.162	10.107	10.053	99.996
0.570	11.052	10.990	10.928	10.867	10.806	10.746	10.687	10.628	10.570	10.512	10.455	10.399	10.343	10.287	10.233	10.178	10.125	10.072	10.019
0.560	11.053	10.992	10.931	10.871	10.812	10.753	10.694	10.637	10.579	10.523	10.467	10.411	10.356	10.302	10.248	10.195	10.142	10.090	10.038
0.550	11.054	10.994	10.935	10.875	10.817	10.759	10.702	10.645	10.589	10.533	10.478	10.424	10.370	10.317	10.264	10.211	10.159	10.108	10.057
0.540	11.055	10.996	10.938	10.880	10.822	10.766	10.709	10.654	10.599	10.544	10.490	10.436	10.384	10.331	10.279	10.228	10.177	10.126	10.077
0.530	11.057	10.999	10.941	10.884	10.828	10.772	10.717	10.662	10.608	10.555	10.502	10.449	10.397	10.346	10.295	10.244	10.194	10.145	10.096
0.520	11.058	11.001	10.944	10.889	10.833	10.779	10.724	10.671	10.618	10.565	10.513	10.462	10.411	10.360	10.310	10.261	10.212	10.163	10.115
0.510	11.059	11.003	10.948	10.893	10.839	10.785	10.732	10.679	10.627	10.576	10.525	10.474	10.424	10.375	10.326	10.277	10.229	10.181	10.134
0.500	11.060	11.005	10.951	10.897	10.844	10.792	10.739	10.688	10.637	10.586	10.536	10.487	10.438	10.389	10.341	10.293	10.246	10.200	10.154
0.490	11.061	11.007	10.954	10.902	10.850	10.798	10.747	10.696	10.646	10.597	10.548	10.499	10.451	10.404	10.357	10.310	10.264	10.218	10.173
0.480	11.062	11.010	10.958	10.906	10.855	10.804	10.755	10.705	10.656	10.607	10.559	10.512	10.465	10.418	10.372	10.326	10.281	10.236	10.192
0.470	11.063	11.012	10.961	10.910	10.860	10.811	10.762	10.714	10.666	10.618	10.571	10.524	10.478	10.433	10.388	10.343	10.298	10.255	10.211
0.460	11.064	11.014	10.964	10.915	10.866	10.817	10.770	10.722	10.675	10.629	10.583	10.537	10.492	10.447	10.403	10.359	10.316	10.273	10.230
0.450	11.065	11.016	10.967	10.919	10.871	10.824	10.777	10.731	10.685	10.639	10.594	10.550	10.505	10.462	10.418	10.376	10.333	10.291	10.250
0.440	11.067	11.018	10.971	10.923	10.877	10.830	10.785	10.739	10.694	10.650	10.606	10.562	10.519	10.476	10.434	10.392	10.351	10.310	10.269
0.430	11.068	11.021	10.974	10.928	10.882	10.837	10.792	10.748	10.704	10.660	10.617	10.575	10.533	10.491	10.449	10.409	10.368	10.328	10.288
0.420	11.069	11.023	10.977	10.932	10.888	10.843	10.800	10.756	10.713	10.671	10.629	10.587	10.546	10.505	10.465	10.425	10.385	10.346	10.307

CVA for European Options taking into account General Wrong Way Risk: Monte Carlo approach

In this paragraph it will be analyzed the Credit Value Adjustment for two European Options taking also into consideration the presence of General Wrong Way Risk (that define the negative correlation among Credit Exposure and Default Probability). Differently of what defined in the previous example, in this simulation a Monte Carlo model will be used in order to compute the Price at maturity of the two instruments. Such model permit to simulate the different path of the underlying price under risk-neutral assumptions and to obtain the final value of the options as the mean of the discounted simulated payoff.

Therefore, the first step is to generate the time paths of the stock price until maturity by means of a General Stochastic Process with solution: $\mathrm{S(t) = S(0)e^{y(t)}}$ where y(t) is an Itô Process ruled by: $\mathrm{dy(t)} = \left(\mathrm{r} - \frac{\sigma^2}{2}\right)(\mathrm{T}) + \sigma\sqrt{\mathrm{T}}\mathrm{Z}$ and Z is a random variable distributed as a standard Normal (0,1).

The behavior of the stock price is then modeled by the following SDE:

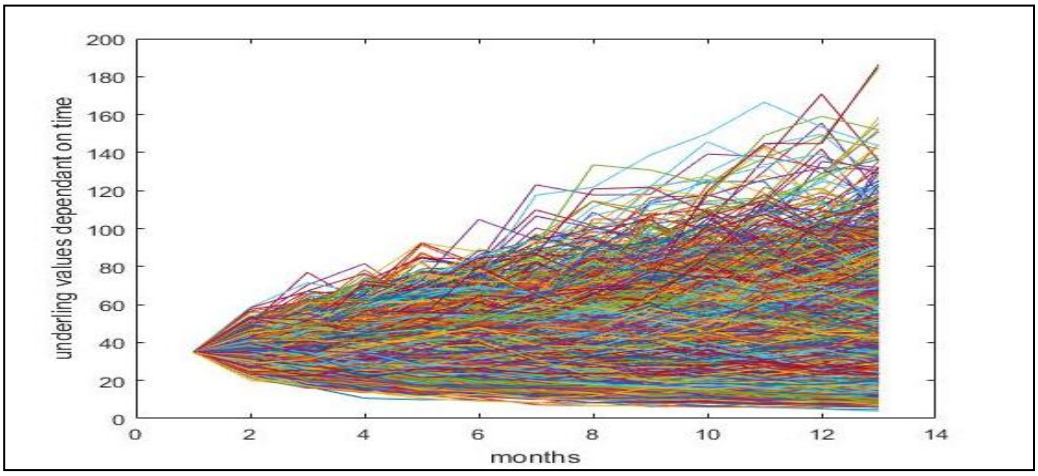
$$S_{t+1} = S_t \exp\left(\left(r - \frac{\sigma^2}{2}\right)(T) + \sigma\sqrt{T}Z\right).$$

```
%simulation of the underlying using a log-normal model

dW=sigma*sqrt(h)*randn(n,M);%defining σ√TZ
S=zeros(n+1,M);
S(1,:)=s0;
for i=1:n
    S(i+1,:)=S(i,:).*exp((r-sigma^2/2)*h+dW(i,:)); %representing the behaviour
of the underlying trough a SDE
end
```

The following figure show the values of the underlying stock simulated 10.000 times, under the same data framework as the example above.

Figure 15. Simulation of the underlying at different time interval



The second step foreseen the computation of the instruments payoff (representing the Credit Exposure) at maturity for ach path: knowing that CallPayoff= max[S-K,0] and

PutPayoff= max[K-S,0]. Therefore, the price of the option will be defined as the discounted mean of all such measures.

Within the data set considered the Price of the Call from the MonteCarlo simulation is equal to 12.2364.

```
%Call option payoff computations for each grid

payoffcall=max(S(n+1,:)-K,0);%payoff for each path
Vcall=exp(-r*T)*payoffcall;%Expected present value of the future payoff
```

Having computed the Price of the option that will be insert in the CVA formula, it is time to model the probability at default. Let's assume that PD is defined under an Inhomogeneous Poisson process (stochastically modeled intensity) for which:

$$P(\tau < T) = 1 - exp\,(-\int_0^T \lambda(S_t)d_\tau)$$

On which we can approximate the integral as:

$$\int_0^T \lambda(S_t)d_\tau \approx \sum_{n=1}^{N} \lambda(S_t)\Delta t_n$$

In such way the intensity parameter λ is inversely dependant on the underlying value, through the following relation:

$$\lambda(S) = aS^{-1}$$

Where $a = 0.9$. By means of this model, the intensity at default will increase whenever the value of the underlying decrease. The probability at default will be equal to:

$$P(\tau < T) = 1 - \exp\,(-\sum_{n=1}^{N} \lambda(S_t)\Delta t_n)$$

Within the data framework analyzed the expected value of PD is equal to 0.0310.

```
%simulating probability at default

a=0.9;
lambda=a./S; %definig the relation within default probability and underlying
lambdai=sum(lambda)*h; %probability of survival taking into consideration GWWR
pd=1-exp(-lambdai); %probability of default
pd1=mean(pd); %mean value of PD
```

As we are computing the value of the underlying trough a MonteCarlo process with 10000 simulations, we will end with a vector of 10000 values of the underlying that will give a vector of payoff of equal length. Moreover the same vector will result on the computation of the default probability. Assuming a Recovery rate of 49% the Credit Value Adjustment computed for the European Call Option will be equal to 0.1428.

As above, knowing that the final Price of an instrument, taking into consideration the riskyness of the counterparty is defined as follow:

$$\text{Instrumet final Price} = \text{Riskfree price} - \text{CVA}$$

Where the CVA is computed also taking into consideration the General WWR effects.

<u>We can state that the Final Price of the European call option is equal to 12.0936.</u>

The following Tables show how CVA, Default Probability and Option Prices vary for different value of σ:

Table 7. CVA, PD and Call Price for different value of σ.

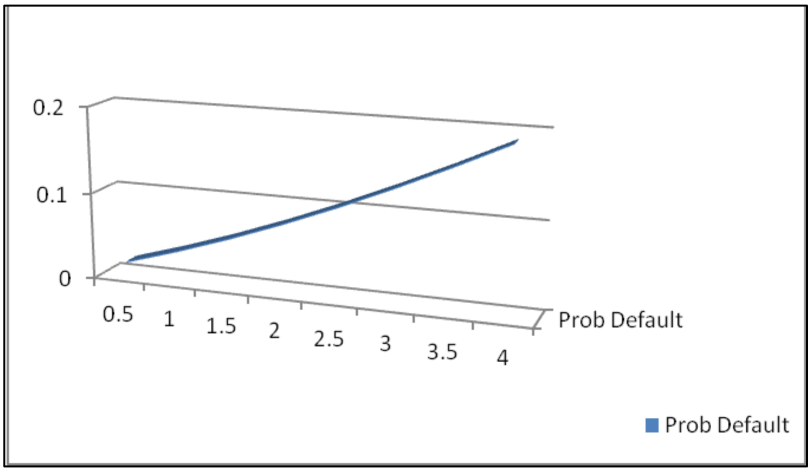
Sigma	0.01	0.03	0.10	0.25	0.50
CVA call	0.1428	0.1426	0.1408	0.1362	0.1428
Price call	10.2457	10.2396	10.2190	10.4547	12.2364
Prob Default	0.0273	0.0274	0.0275	0.0282	0.0310
Final Price	10.1029	10.0970	10.0782	10.3185	12.0936

Table 8. CVA, PD and Put Price for different value of σ.

Sigma	0.01	0.03	0.10	0.25	0.50
CVA put	0.1568	0.1572	0.1609	0.1924	0.3039
Price put	9.5553	9.5614	9.5909	10.4881	13.6208
Prob Default	0.0273	0.0274	0.0275	0.0282	0.0310
Final Price	9.3985	9.4042	9.4300	10.2957	13.3169

Interesting is also analyze the behavior of the probability at default at different time to maturity:

Figure 16. Default probability at different time to maturity



It is evident that the default probability of the counterparty increase as the time interval considered become wider: it is surely lower in the first stage of the contract, becoming more consistent as time goes on. It can be also noted how such probability presents an increasing behavior that is not fully linear: in other words it is not possible to establish a direct proportionality between the time interval considered and the related solvency rate.

In light of what presented so far, we can conclude that the Monte Carlo approach, that considers the underlying price as path dependent in the definition of the default probability, is able to capture the generic risks linked to the derivative financial instrument (General Wrong Way Risk). Considering simulated scenario, even for these simple instruments, allow to anticipate the market future behavior and to better manage the counterparty risk. On the other hand, assuming the presence of interconnection among probability at default and the market risk factors, requires a higher computational effort compared to the one under independence assumption: the final result, however, can be considered more effective and significant.

APPENDIX A

Unilateral CVA

Being a risk-free portfolio value defined as:

$$V(t,T) = I(\tau > T) * V(t,T) + I(\tau \leq T) * V(t,T);$$

To each side of the equation it is added the same quantity, resulting:

$$V(t,T) + (\delta - 1)I(\tau \leq T) * [NPV(\tau)]^{+} =$$
$$I(\tau > T) * V(t,T) + I(\tau \leq T) * V(t,T) +$$
$$-I(\tau \leq T) * [NPV(\tau)]^{+} + \delta * I(\tau \leq T) * [NPV(\tau)]^{+},$$

Where $[X]^{+} = (X,0)$ e $[X]^{-} = \min\,(X,0)$.

Analyzing the tow central terms, the expected value conditional to the information until time τ, it is as follow:

$$E_{\tau}[I(\tau \leq T) * V(t,T) - I(\tau \leq T) * [NPV(\tau)]^{+}]$$
$$= E_{\tau}[I(\tau \leq T) * \{V(t,T) + D(t,\tau)V(\tau,T) - D(t,\tau)E_{\tau}[V(\tau,T)]^{+}\}].$$

Knowing that $V(\tau,T) = V(\tau,T)^{+} + V(\tau,T)^{-}$ and that the expected value is deterministic in the time interval [t, τ], the equation above could be view as:

$$= I(\tau \leq T) * \{V(t,\tau) + D(t,\tau) * E_{\tau}[V(\tau,T) - (E_{\tau}[V(\tau,T)])^{+}]\} =$$
$$= I(\tau \leq T) * V(t,\tau) - D(t,\tau) * (E_{\tau}[V(\tau,T)])^{-}.$$

Exploiting the properties of $[.]^{+}$ and $[.]^{-}$ the equation above could be re-write as:

$$= I(\tau \leq T) * V(t,\tau) - D(t,\tau) * (E_{\tau}[-V(\tau,T)])^{+}$$
$$= I(\tau \leq T) * V(t,\tau) - D(t,\tau) - D(t,\tau) * (-E_{\tau}[V(\tau,T)])^{+}$$
$$= I(\tau \leq T) * V(t,\tau) - D(t,\tau) - D(t,\tau) * (-NPV(\tau))^{+}.$$

Re-writing the two terms of the above equation $[I(\tau > T) * V(t,T) + +\delta * I(\tau \leq T) * [NPV(\tau)]^{+}]$ with what just defined, it results the expression of the risky portfolio:

$$\tilde{V}(t,T) = I(\tau > T) * V(t,T) + I(\tau \leq T) * V(t,\tau) + I(\tau \leq T)$$
$$* [\delta * D(t,\tau) * V(\tau,T)^{+} + V(\tau,T)^{-}].$$

It follow that, making use of the properties of the conditional expected value $E_{t}\big[E_{\tau}[.]\big] = E_{t}[.]$:

$$E_{t}[\tilde{V}(t,T)] = E_{t}[V(t,T)] - E_{t}\{(1-\delta) * I(\tau \leq T) * D(t,\tau) * NPV(\tau)^{+}\}.$$

APPENDIX B.

SCRIPT MATLAB: CVA FOR AN EUROPEAN CALL OPTION

```
%Option pricing using Black&Scholes Model
%defining the data set
settled='Oct-01-2016'; %starting date
maturity='Oct-01-2017'; %maturity of the contract
Strike=25; %strike price settled at the beggining of the contract
OptSpec='call'; %specifying the type of the Option: call
AssetPrice=35; %underlying price at t=0
Sigma=0.35; %volatility
StartDates='01 Oct 2016';
EndDates='01 Oct 2017';
Rates=0.01; %interest rate
ValuationDate='01 Oct 2016';
Compounding= -1; %Scalar value representing the rate at which the
input zero rates were compounded when annualized.Compounding = -1
                 %Disc = exp(-T*Z), where T is time in years.
RateSpec=intenvset('Compounding', Compounding, 'StartDates',
StartDates, 'EndDates', EndDates,'Rates',Rates,
'ValuationDate',ValuationDate);%set the properties of interest rate
structure
StockSpec=stockspec(Sigma,AssetPrice); %create stock structure
PriceBLS=optstockbybls(RateSpec,StockSpec,settled,maturity,OptSpec,S
trike); %pricing the stock with Black&Scholes Model
%default probability estimation
lambda=0.01:0.01:0.19;
pd=zeros(size(lambda));%define the interval of variation for Lambda
(the intensity homogeneous function)
for i=1:numel(lambda)
    pd(i)=1-(exp(-lambda(i))); %getting the vector of probability at
defaults for different value of lambda
end
pd1=mean(pd); %computing the mean to insert in CVA formula
%CVA estimation with crescent Recovery Rates and lambda variaiton
rec=0.40:0.01:0.58; %specify different value for the Recovery rate
lgd=1-rec; %loss given default
for j=1:numel(rec)
    lgd(j)=1-rec(j); %LGD for different value of recovery rates
end
ExpperProb=pd*PriceBLS; %correspond to(1-1/LT(1-exp(-LT))(S0 N(d1)-
Kexp(-rT)N(d2))
CVA=lgd'*ExpperProb; %CVA definition

%representation of CVA behaviour
figure(2)
mesh(pd,lgd,CVA);
xlabel('Probability of Default')
ylabel('Loss Given Default')
zlabel('CVA')
title('CVA with lambda variations and increasing C')
%Final price of the Option taking into account the riskiness of the
%counterparty
FianlPrice=PriceBLS-CVA;
```

APPENDIX C

SCRIPT MATLAB: CVA FOR EUROPEAN OPTION in presence of GENERAL WRONG WAY RISK

```
%Data for the Call Option
randn('state',3)
K=25;%Stike Price
r=0.01;%interest rate
sigma=0.5;%volatility
T=1;%maturity set at 1 year
s0=35;%underlying Price at t=0
n=12;%number of month
h=T/n;%sub-interval
M=10000;%number of times of which the underlying will be simulated by
Monte Carlo model
%Data for the Put Option
randn('state',3)
K=45;
r=0.01;
sigma=0.5;
T=1;
s0=35;
n=1;
h=T/n;
M=10000;
%simulation of the underlying using a log-normal model
dW=sigma*sqrt(h)*randn(n,M);%defining σTZ
S=zeros(n+1,M);
S(1,:)=s0;
for i=1:n
    S(i+1,:)=S(i,:).*exp((r-sigma^2/2)*h+dW(i,:)); %representing the
behaviour of the underlying trough a SDE
end
plot(S);
xlabel('months');
ylabel('underling values dependant on time')
title('simulation of the underlying at different time interval')
%Call option payoff computations for each grid
payoffcall=max(S(n+1,:)-K,0);%payoff for each path
Vcall=exp(-r*T)*payoffcall;%Expected present value of the future
payoff
%Pricing using MonteCarlo
PriceCall=exp(-r*T)*mean(payoffcall);%MonteCarlo simulation with
M=10000
call=blsprice(s0,K,r,T,sigma);
%simulating probability at default
a=0.9;
lambda=a./S; %definig the relation within default probability and
underlying
lambdai=sum(lambda)*h; %probability of survival taking into
consideration GWWR
pd=1-exp(-lambdai); %probability of default
pd1=mean(pd); %mean value of PD
```

```
%CVA definition
Recovery=0.49; %Recovery value
CVAcall=(1-Recovery)*mean(Vcall.*pd);
%Put Option Payoff for each grid
payoffput=max((K-S(n+1,:)),0);
%discounted MtM Put value
Vput=exp(-r*T)*(payoffput);
%CVA simulation for Put option
Recovery=0.49;
CVAput=(1-Recovery)*mean(Vput.*pd);
%Put Derivative price with Monte Carlo Simulation
PricePut=exp(-r*T)*mean(payoffput);
```

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