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Heterogeneity in the Euro US Dollar Exchange Rate Expectations

Chartists, Fundamentalists, Noise Traders and News, a time series analysis

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Ad Antonio, Enrico, Sebastiano, e Simone, per esserci sempre stati,

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Introduction

The relevance of studying exchange rates and expectations

Exchange Rate Determination and prediction have been for a long time one of the unsolved puzzles in macroeconomics, as producing robust and reliable models remains challenging even today. Poor explanatory power, low out-of-sample robustness, and the difficulty of beating random walk processes in predicting exchange rates still constitute notable obstacles to the development of effective models (Engel, Mark, and West 2007; Engel and West 2005).

"Expectations of future exchange rate changes are a key determinant of asset demands and therefore of the current exchange rate", this is how Frankel and Rose wrote in 1995 in their "Empirical Research on Nominal Exchange Rates" (Frankel and Rose 1995).

A common observation in the literature is that exchange rate variation can be linked to market agents' expectations, thus studying expectations formation appears to me as a fundamental "piece of the puzzle" to be solved (Engel, Mark, and West 2007; Mankiw, Reis, and Wolfers 2003; Frankel and Rose 1995; Engel and West 2005). While during the previous decades many models have been developed over rational, homogeneous agents, a flourishing strand of literature advocated and pursued the development of agent-based models with heterogeneous agents. Heterogeneous agents can have heterogeneous expectation formation mechanisms, that might better represent reality. This is where my analysis starts from.

Whether heterogeneous expectations are due to different trading strategies or behavioral biases there is no unanimity in the literature. Some authors support the "Fundamentalists and Chartists hypothesis", where beliefs in mean reversion and trend following coexist in the market in presence of deviations from the fundamentals (Boswijk, Hommes, and Manzan 2007; Frankel and Froot 1990; Lux 1998; Alfarano and Lux 2007; Menkhoff, Rebitzky, and Schröder 2009; Frankel and Froot 1990; Westerhoff and Reitz 2003). Some others suggest that behavioral biases could be playing a role, with optimistic, boundedly rational investors becoming trend followers. Noise traders, and differentiated access and use of information could generate disagreement and heterogeneity (Boswijk, Hommes, and Manzan 2007; Capistrán and Timmermann 2009; Macdonald and Marsh 1996). Other determinants, such as the role of news have been studied: systematic misperception of new information depending on whether it is published in times of high or low volatility seemingly takes place (Manzan and Westerhoff 2005).

The distortions originated in the foreign exchange market can also be linked to behavioral biases, such as anchoring and adjustment heuristics, associated with mistakes' propagation and deviations in the perceived value of the fundamental (Westerhoff 2003).

If expectations could be affecting exchange rate movements (Engel, Mark, and West 2007; Frankel and Rose 1995), and individual expectations are inherently heterogeneous, the amount of Heterogeneity in exchange rate Expectations' could play a fundamental role in determining volatility. Which variables, models, and strategies can be linked to spikes in heterogeneity? Which explanations can be provided for what present day data show? Which differences can be found with previous papers covering this topic? In this thesis I attempt to answer all these questions.

Research question and purpose of the analysis

This work takes as its primary reference the 2009 paper of Menkoff et al. "Heterogeneity in Exchange Rate Expectations […]" and aims at replicating their econometric analysis using newly available, more recent data about exchange rate expectations made by major dealing banks. The purpose of this analysis is to replicate the original study, using analogous methodologies while exploring the robustness of their findings in the light of more recent data from the last decade (Menkhoff, Rebitzky, and Schröder 2009).

Menkoff et al. consider consensus expectations from about 300 forecasters over 15 years from December 1991 until August 2006, a period in which heterogeneity in exchange rate expectations shows remarkable time variation in dispersion. I consider data from Bloomberg of 144 financial institutions, and their quarterly prediction of the USD-Euro exchange rate, during the decade 2013-2022.

While this work does not aim at developing new hypotheses about the formation of heterogeneous expectations, I will verify and check the empirical validity of many econometric models of exchange rate determination, integrating where necessary the original paper analysis with other sources, and then formulate hypothesis and providing punctual comparisons with the original paper results. Given the scarcity of other published studies on expectations' heterogeneity, one of the main objectives of the thesis is to verify the generality and robustness of the results of Menkoff et al. The analysis will shed light on a different period, thus assessing if their findings maintain or lose some of their generality.

The analysis will identify significant determinants of heterogeneity throughout the decade and then provide evidence on the short and long-run effects of the variables defined on the level of heterogeneity in the pool of exchange rate expectations we study.

Chapter I - Literature and main references

1.1 Practical applications of the study of heterogeneity in exchange rate expectations

Aside from the pure academic interest and curiosity, studying exchange rate expectations and their heterogeneity can have multiple practical applications and be relevant for firms and policymakers. Providing a better understanding of the sources of heterogeneous expectations, and predicting periods of higher disagreement and uncertainty, can be crucial for reducing volatility and determining the timing and type of intervention of central authorities.

Exchange rate volatility can constitute a source of instability and uncertainty, therefore central authorities could adopt stabilizing interventions, once their dynamics are proven to not be fully endogenous (Wieland and Westerhoff 2005). The branch of literature on chaos control and central bank intervention covers also this aspect of exchange rates and suggests some different types of intervention. *"Delayed feedback control in the form of targeting long-run fundamentals"* is indicated as a way to stabilize the exchange rate. *"Beggar your neighbor"* policies could be adopted manipulating the exchange rate in the short run, intervening to promote domestic economies, and shifting it away from fundamentals (Wieland and Westerhoff 2005). In this context, extending the understanding of exchange rate expectations and their heterogeneity determinants could prove to be useful to design these types of policy instruments.

The role of news and access to information plays a role also in exchange rate volatility. Studying an artificial financial market, Manzan and Westerhoff confirm the evidence of a different perception of news by agents depending on the amount of current volatility (Manzan and Westerhoff 2005). In their paper of 2005, the authors describe agents as more receptive to new information during periods of high volatility, and careless of new information during periods of low volatility. The authors underline the relevance of news and their perception in the understanding of exchange rate dynamics. The relevance for policymakers can be found as they recommend central authorities to provide reliable information about fundamentals, reducing misperception and disagreement among market operators, and avoiding periods of instability (Manzan and Westerhoff 2005). Stabilizing exchange rates can be crucial also for exporter firms to avoid "exchange rate surprises" (Giovannini 1988).

Also Beckmann and Czudaj find evidence of the strong effect of announcements on expectations, but add to the argument also the degree of uncertainty in economic, fiscal, and monetary policy. In their paper they highlight the strong effect that an uncertain stance of economic policy has even on professionals' forecast errors and expectations (Beckmann and Czudaj 2017). Also in this case, the study of the factors causing disagreement, misperception, forecast errors and thus heterogeneity was revealed to be important.

Finally, some authors indicate the topic as relevant to policymakers when considering the choice of the exchange rate regime inside the "incompatible trinity" of Mundell (i.e. fixed exchange rates, monetary autonomy, and capital mobility). When also considering how to manage it, and the potential of finding new ways to reduce volatility in flexible exchange rate regimes, it can be said that the subject of this analysis is relevant (Jeanne and Rose 2002; Macdonald and Marsh 1996).

1.2 Models of Exchange Rate forecasting

Predicting exchange rates remains a challenging task for economic researchers, and while the best predictor, the random walk, still constitutes the almost unbeaten benchmark, we cannot define foreign exchange markets as efficient. In a comprehensive literature review, Rossi (2013) collects a large number of studied models and predictors economists adopted in the past when attempting to predict exchange rate changes. Rossi concludes that there exists a wide disagreement about exchange rate being forecastable, her review also presents the existing empirical findings. The models' performance can be summarized as follows: *"monetary and PPP fundamentals have no predictive ability on shorts horizons",* nonlinear models are the least successful, only Taylor Rule and Net Foreign Assets fundamentals appear promising on longer horizons (Rossi 2013).

A relevant example is Molodtsova and Papell (2009), they study out of sample exchange rate predictability using Taylor rule fundamentals, and find strong evidence of their predictive power. While pre-existing papers define exchange rates as unpredictable on any horizon, or predictable at longer horizons, the authors prove the symmetric Taylor rule model can predict exchange rates at one moth for a wide set of countries.

The relevance of having reliable and consistent forecasting methods and models cannot be discussed, as forecasts of inflation and of the state of the economy today are a major guide for economic and monetary policy.

While *"fully developed"* structural models i.e. not reduced form, or stylized models, generally don't fit exchange rate data, and are deemed as not suitable for forecasting, some models have made exchange rates apparently predictable: it is the case of the Taylor Rule fundamentals, and of net foreign assets fundamentals. Also in these cases, the random walk without drift remains undefeated. The literature is also sceptical about the use of PPP and monetary models when studying short horizons inferior to the 3 years (Rossi 2013).

In her literature review, Rossi draws some "general conclusions" that can help readers to better frame all the empirical econometric work done in this thesis.

First, the choice of predictors heavily affects the out of sample performance of the models, Taylor Rule fundamentals and net asset position appear promising in the short run, monetary models in the long run. Second, linear models are the most successful. Third, data transformations (detrending, seasonal adjustments) may lead to wide differences in results. Also the choice to use non-revised data can be problematic. Fourth: different methods, time horizons, and periods (this is particularly relevant in my analysis as Menkhoff et al. paper considers an "atypical" period in economic history, characterized by events leading to high instability) generate heterogeneous results (Rossi 2013).

Traditional predictors overall tend to lack of significance or out of sample forecasting ability, and generally are not able to outperform the random walk. More details about single papers and their results can be found in Rossi (2013).

Below, are outlined the most common predictors and their respective prediction models of origin.

Table 1. Main exchange rates' predictors and models. Source: Rossi (2013)

Interest rate differentials	*Uncovered interest rate parity models*	Expected return on purchased foreign assets must be comparable to the return of the home bonds.	Fisher (1896)
Price and Inflation differentials, PPP	*Purchasing Power Parity*	Nominal exchange rates should tend to the purchasing power parity of currencies in the long run, with small deviations.	Rogoff (1996)
Money and Output differentials	*Monetary Models of exchange rate determination*	Bilateral nominal exchange rates should fluctuate reflecting money movements, output, interest rates and prices.	Frenkel (1976) Mussa (1976)
Productivity differentials	*Extended Monetary Models*	As monetary models, but price differentials are expressed as productivity differentials.	Balassa (1964) Samuelson (1964)
Trade balances differentials, current account differentials (also cumulated), government debts	*Portfolio Balance models (Extended Monetary Models)*	Include a measure of stock balances comparing home and foreign assets held by home.	Frankel (1982) Hooper and Morton (1982)

Taylor Rule fundamentals generally provide better results and better predictions. By the Taylor Rule, monetary authorities set the interest rate based on the difference between the targeted level of inflation and its actual level, also considering the output gap and the difference between the nominal and real interest rate. Rossi refers to Molodtsova and Papell (2006), where the Taylor Rule is extended to an open economy setting and considers that central banks will also aim at keeping the real exchange rate near its PPP level (Rossi 2013). In my analysis, are both considered the inflation rate and the distance between the target and observed inflation. Are also considered output gaps and their variation. Euribor and Libor rates are included in order to test also the predictors indicated in the broader literature.

1.3 Exchange rate forecasting and the role of expectations

Within the vast literature developing models of prediction of exchange rates, I review here some papers I considered meaningful to introduce the specific aspects of exchange rates expectations and their heterogeneity.

Some hopes in predicting exchange rate changes come from Engel and West, who wrote about the link between the exchange rate and fundamentals such as interest rates, money supply and outputs. In their paper they suggest that *"exchange rates may incorporate information about future fundamentals",* and highlight this high correlation with news about the latter (Engel and West 2005).

Some stylized facts about bilateral exchange rates are the non-stationarity of the exchange rate and of its fundamentals and the correlation between the exchange rate and fundamentals, which appears to be stronger in the long run than in the short term (Engel and West 2005). These characteristics make particularly hard to predict exchange rates, and thus brought my analysis in the direction of studying expectations and disagreement or heterogeneity.

Also Williamson (2009), presenting the findings of De Grauwe and Grimaldi, reaffirmed that while exchange rate changes are disconnected from the fundamentals, the exchange rate is cointegrated with its fundamental value. It is also accepted that exchange rates have fat tails and are involved in *bubble-and-crash* dynamics.

1.3.1 Exchange rate expectations

"Expectations drive individual behaviours and individual behaviours determine the economic outcome, i.e. prices and trading. Therefore, a market, like other social environments, may be viewed as an expectations feedback system". G.Tedeschi et al. and Heemeijer et al.

Some of the main findings in the literature about exchange rate expectations are summarized in the following.

In the past decades, investors' expectations about exchange rates have been modelled using the forward exchange rates. Its *elusiveness and ubiquity* make the forward rate a weak predictor of exchange rate changes. This weakness leads to the definition of the "risk premium" (difference between the forward discount and the expected depreciation), as a fundamental variable to be considered when modelling rational expectations (Frankel and Froot, 2016). Both of these two variables have been included in the econometric analysis implemented in this thesis.

In 1995 Frankel and Rose wrote about the *microstructure of the foreign exchange market*, suggesting that not only macroeconomic variables should be considered to explain exchange rate changes: also different approaches should be adopted, for example studying heterogeneity in the forecasts of market participants (Frankel and Rose 1995).

Frankel and Froot find that exchange rate expectations appear to be not static, especially under a nonzero forward discount rate and a positive risk premium. They do not find evidence of *bandwagon effects*. The authors link inelastic expectations to the so-called *stabilizing speculation,* implying for example that current increases in the spot exchange rate lead to anticipations of future decreases. Testing a number of models, they conclude that the rationality of expectations depends on the sample period, and suggest already in 1987 the need to explore the hypothesis of coexisting heterogeneous expectation formation mechanisms in the market (Frankel and Froot 2016).

Unit roots in exchange rates, fat tails of returns and volatility clustering are three relevant characteristics of foreign exchange markets. Applying a chartists and fundamentalists model, Westerhoff et al. amongst others, suggests traders' interactions to be behind exchange rate dynamics. Behavioural biases such as anchoring and adjustment heuristics likely play a role. Other meaningful observations are the mistakes propagation during exchange rate adjustments, and the destabilizing and distorting effect of chartists' popularity in the agents' population (Westerhoff 2003; Manzan and Westerhoff 2007).

Frankel and Rose highlighted that expectations over future exchange rates affect assets' demand and therefore current exchange rates themselves. They state that expectations tend to be stabilizing, as appreciations may lead to expected future depreciations and steer the exchange rate back to its original path (Frankel and Rose 1995).

Given the relevance that expectations have in determining exchange rate dynamics, the next sections focuses specifically on their formation and effects.

Expectations play a fundamental role in macroeconomics, as agents are required to take forward-looking decisions based on the future expected value of assets such as equity, of interest rates, of the inflation rate and exchange rates. Mainly the literature adopts the view of Rational Expectations, where agents have their decision-making conditioned by their information set-making, and generate expectations able to shape the path of the economy and vice versa (Evans and Honkapohja 2012).

During the 70s and 80s, Rational-expectations efficient market models were the most relevant alternative in the literature. Under this hypothesis, exchange rates change only as a consequence of news about fundamentals (inflation, interest rates etc.), and remain in a stable relationship with fundamentals over time. Among these models it is worth mentioning the monetary model, the Dornbusch model, and the portfolio balance model. The empirical failure of these models and of the rational expectations hypothesis has led to a series of empirical puzzles, such as the *disconnection puzzle* of the exchange rate with its fundamentals, its *excess volatility*, and *fat tails in returns*. These evidences thus led to the adoption of models with heterogeneous agents and heterogeneous expectations, in some cases maintaining rationality in their formation as a framework. The main consequence of this type of setting is highlighted by De Grauwe and Grimaldi (2006): "*heterogeneous agents with rational expectations generate infinite regress, i.e. exchange rate depends on the expectations of other agents' expectations*".

This latter contribution reinforced my belief that studying heterogeneity in expectations may be relevant.

Some interesting results of their analysis were the importance of the history of fundamentals shocks, the role of interactions between disagreeing agents with different information sets, and the relevance of chartists' share in the market in determining volatility. They finally suggest that a high variance of changes in fundamentals should lead to "tighter links" between the exchange rate and fundamentals.

Using survey data, Ito (2020) finds evidence of the existence of individual effects in expectations' formation and of many violations of the rational expectations assumption. Other key findings about exchange rate expectations are: heterogeneity and *constant term biases* in agents' expectations formation process and wishful expectations (for example exporters tend to expect depreciation of their currency). He also finds evidence confuting the *unbiasedness of expectations* hypothesis. We can say that also for the rational expectations assumption empirical evidences are rather weak.

1.3.2 Expectations and the role of forecasting errors

As heterogeneous expectations may derive also from involuntary mistakes, I summarize the specific strand of literature which focuses on forecasting errors and biases.

Frankel and Froot (2016) confirm that understanding forecasting bias reveals to be tricky, as its origin could change over time, and as systematic forecasting errors are observed in data.

Some of the main causes on the presence of forecasting bias are announcements and policy uncertainty.

While announcements are well known for affecting expectations, a crucial finding of Beckmann and Czudaj (2017) is that the degree of uncertainty in economic, fiscal, and monetary policy also affect forecast errors in a heavier manner. An uncertain stance of future economic policy could have a strong influence on forecast errors in predicting exchange rate expectations too. In the context of financial markets and foreign exchange markets, announcements can be highly effective as proven by the historical "Whatever it takes" speech of the former ECB President Mario Draghi. In the summer of 2012, the announcement caused by itself an appreciation of the euro and thus a change in the forecasters' expectations.

Andersen et al. (2003) focus on announcement surprises or *new*s, as they can produce jumps in the mean and variance of an asset price. The authors underline how jumps in the mean have quick comebacks, while volatility and conditional variance adjustments tend to be gradual. They also highlight the relevance of timing and asymmetric effects depending on whether the news is good or bad.

Also Beckmann and Czudaj (2017) conclude that expectations are influenced by economic policy uncertainty, adding that high forecast errors are common, and that *considering disagreement measures among professional forecasters* can be an important topic of research.

1.3.3 Studying heterogeneity in exchange rate expectations

Heterogeneous agents with heterogeneous expectations constitute a fundamental part of contemporary exchange rate prediction models. Only Menkhoff, Rebitzky, and Schröder (2009), cover the specific topic of heterogeneity in exchange rate expectations. The lack of other, more recent empirical contributions heterogeneity (according to my knowledge) constitutes one of the reasons that motivated me to conduct this exercise. Aside from the former article, more reasons have been listed to justify the relevance of my analysis.

The literature on heterogeneous expectations is often associated with the role of news and announcements on financial markets, some relevant examples are Engel and Frankel (1984), Andersen (2003), Manzan and Westerhoff (2005), De Grauwe and Grimaldi (2006).

Frankel and Rose advocate for the study of heterogeneity already in 1995: they consider the evidence from survey data, proving the existence of heterogeneous market agents with different expectation formation mechanisms. They also highlight the relevant fluctuations in the number of market participants adopting each of the existing trading strategies over time. According to their analysis, heterogeneity in market participants' forecasts could be responsible for exchange rate volatility, exchange rate variation and for the high volume of trading on foreign exchange markets (Frankel and Rose 1995; Frankel and Froot 1990).

Papers studying heterogeneity in expectations historically have looked for differences in agents' expectation formation classifying them into groups. The main example is the chartists-fundamentalists hypothesis. Another relevant finding is the positive link between trade volumes and the "dispersion of opinion" in the market (Frankel and Rose 1995; Frankel and Froot 1990).

In the context of understanding the aforementioned link between heterogeneous expectations and trade volume, Frankel and Froot explanation comes handy. The proposed theory states that "the degree of dispersion has strong effects on the market", as "the existence of conflicting forecasts leads to noise-trading", increasing the volume of trading, and then bolstering volatility" (Frankel and Froot 1990). Therefore we can say that notable evidence exists of heterogeneity causing high trade volumes, and being able to explain excess volatility thorough the bandwagon effect of chartists. This dynamic, particularly strong in the short run, tends to weaken in the long run, when the stabilization to the Purchasing Power Parity is reached thanks to fundamentalists (Frankel and Froot 1990).

Understanding what causes heterogeneous expectations falls in the scope of my analysis, the theoretical and empirical literature propose some possible determinants are summarized below.

According to Bacchetta and Wincoop (2006), the use of the available information could be highly differentiated among market participants: *infrequent information processing* could verify even in the case of foreign exchange markets, and "partial information processing, the use of only a subset of the available information", could explain part of the heterogeneity we empirically observe. We may also add to the list the differences in available and accessible information sets, and the different use of available information by forecasters.

Another determinant of heterogeneous expectations is the frequency of action of market agents. The concept of infrequent acting derives from the observed low gains from active management of currency positions. The unpredictability of exchange rates, and delayed overshooting are indicated as causes of this behaviour (Bacchetta and Wincoop 2006). I speculate that a timing problem for agents' actions in the market could potentially arise and contribute again to heterogeneity, for example from delayed reactions to the new publicly available information.

As already mentioned in previous sections, also policy uncertainty can be accounted for among heterogeneity determinants, but evidence suggests it may decrease heterogeneity in some cases (Frankel and Kenneth A . Froot 2016).

Focusing now on the main reference for this work, Menkhoff, Rebitzky, and Schröder (2009), I outline the main existing hypotheses about exchange rate expectations determinants tested by the original authors.

The three main hypotheses considered, corresponding to the three relevant strands of literature consist of: models of heterogeneous agents such as the Chartists-Fundamentalists one, noise traders' models studying the link between risk and heterogeneity, and the role of information heterogeneity about macroeconomic fundamentals (Menkhoff, Rebitzky, and Schröder 2009). While in the original paper the second and third hypothesis were considered as a robustness check of the first, in my analysis (also given the different results obtained) any possible determinant is treated providing the same weight.

1.3.4 Survey data use in economic research about exchange rates

Survey data provide undoubted evidence of heterogeneous expectations, and while the reasons for their existence have already been discussed, also different forecasting abilities of market agents shall be accounted for (Macdonald and Marsh 1996). Even if economic research ignored survey data for a long time, today many pieces of literature make use of them, in this section I summarize some relevant examples.

Researchers have had contrasting opinions about the reliability and the use of survey data in studying exchange rate expectations. Skeptic positions are based on the fact that *economists generally distrust survey data* and the idea of *positive economics* given that the observation of agents' actions in the market can be more revealing than just surveys. Supportive positions argue that even alternative measures of expectations have their drawbacks. In the context of exchange rate expectations, we can argue that respondents have a more significant impact on the market, as they actually participate to the market, and have access to an extensive and up-to-date information set. In this case, given the professional nature of agents (forecasters), we can also deem their answers as reliable and truthful (Frankel and Froot 2016).

Menkhoff et al. use survey data from ZEW, not only having access to exchange rate forecasts of single market operators, but also to their self reported being chartists (trend followers) or fundamentalists (predicting convergence to the PPP). The group identification is obviously a necessary simplification as also the authors declare: *"we know that almost all market participants use fundamental as well as technical analysis simultaneously to some degree"*. In their sample they find a prevalence of self declared fundamentalists.

Coming from a qualitative survey, their dataset just distinguishes if the forecasters predict increases, decreases, or a stable trend in the exchange rate, without giving any "direct" information (they adopt a *quantification technique*) about the value of the exchange rate they expect (Menkhoff, Rebitzky, and Schröder 2009).

Frankel and Froot (2016) used survey data coming from independent surveys, and indicated it as potentially relevant for research purposes initiative, an example was the weekly Economist financial Report survey of expected future exchange rates of the 80s. Starting from survey data, they look for models able to represent agents' expectation formation. In their analysis, survey data provide evidence of expectations not being affected by bandwagon effects, and being *stabilizing* in the long run, predicting a convergence to fundamentals as happens in macro-based models.

Engel, Mark, and West (2007) also make extensive use of survey data to study exchange rate expectations and exchange rate models. The authors warn about some critical aspects of surveys: such as scarce availability of information about some variables, "answers" not coinciding with actual forecasts used in the market, or scarce accuracy. At the same time they describe the use of survey data as useful and relevant for research purposes. Prat and Uctum (2013) argue that survey data could or could not be based on a representative sample of the market population, or could being subject to measurement errors.

To conclude this section, we must recall that survey data provide the researcher with information otherwise not accessible, as agents form their expectations based on a set of nonmeasurable information or not possible to gather. They also let researchers to "avoid arbitrary hypotheses about expectation representation"(Frankel and Rose 1995; Prat and Uctum 2013). I conclude reminding that especially when studying reactions to announcements, or modelling the risk premium, surveys are fundamental, see Frankel & Rose (1995) and Prat & Uctum (2013).

Chapter II - Chartists and Fundamentalists models in financial markets

The Chartists and Fundamentalists model (C&F) consists of a model of heterogeneous agents with heterogeneous expectations' formation mechanisms. Its application to foreign exchange markets has its foundations in the relevance of technical analysis, used by chartists, as Menkhoff & Taylor (2007) show. Technical analysis in the foreign exchange market should provide forecasters with an intuitive understanding of market dynamics, even if they remain obscure (Taylor and Allen 1992).

In this framework, technical analysis and fundamental analysis are seen as alternative, but not binding for the individual forecaster, i.e. agents could be switching over time between being chartists and fundamentalists respectively. The C&F model agents can every period choose to behave as chartists, i.e. form expectations based on past realizations of the exchange rate (trend extrapolation from the recent past) or as fundamentalists, pivoting their expectations in the direction of the "fundamental" value of the exchange rate (usually coinciding with the PPP, seen as the long run equilibrium). Learning is admitted in this model, and agents over time change their strategy according to the performance of their strategy (Menkhoff, Rebitzky, and Schröder 2009).

An important passage in the original paper states that "the fundamental rule predicts higher expected returns and lower risks the farther exchange rates are from equilibria", where equilibria is represented by the PPP. As a direct consequence, we should expect a fall in expectation heterogeneity in times when fundamentalists represent a significant part of the market operators and the gap widens (Menkhoff, Rebitzky, and Schröder 2009). In my analysis given the lack of information about the type of forecaster, i.e. if they consider themselves Fundamentalists or Chartists, is it harder to gather evidence of chartists being predominant in specific periods, even if generally, spikes in heterogeneity should be associated with a higher presence of chartists and noise (Manzan and Westerhoff 2007).

The C&F model is applied to foreign exchange markets in the attempt to explain some of its features: large trading volumes, fat tails, and volatility clustering. (Frankel and Froot 1990). The failures of fundamental variables' use in structural models, and their scarce relevance in determining the exchange only reinforced the support for this kind of model, and today, the importance of considering heterogeneous expectations in this field is indisputable (Frankel and Froot 1990, Manzan and Westerhoff 2007).

The empirical evidences on heterogeneity in the expectation formation mechanisms of market agents provide wide support to models of this kind. Rational expectations' models are not empirically robust especially when studying forecasters' activity, as agents do not always have full rationality, do not have access to perfect information, use and interpret information differently, and generate heterogeneous expectations (Westerhoff 2004; Berardi 2011). The need to study exchange rate forecasters' behaviour finds its reason in the evidence that macroeconomic variables are perceived as relevant only in the long term, and in the fact that over short horizons, "non-fundamentals", the role of news, technical analysis and bandwagon effects are additional factors that matter (Manzan and Westerhoff 2007). In this analysis, the goal is to provide evidence supporting the C&F model, in the next paragraph are presented its characteristics.

2.1 The baseline C&F model from Frankel and Froot

The baseline Chartist and Fundamentalist model follows the structure outlined in Frankel and Froot (1990a, 1990b). Aside from the *current* spot rate, Frankel and Froot point out the relevance of other variables in the forecasters' expectations formation processes. They outline the extrapolative expectations of forecasters as dependent on the current change in the exchange rate. Supposing that forecasters assign a weight g to the lagged spot rate and a weight of $1-g$ to the current spot rate, their expectations take the form:

$$s_{t+1}^{m} = (1-g)s_t + gs_{t-1}$$

Where s_t is the logarithm of the current spot exchange rate, s_{t+1}^{m} the market's expected future exchange rate at time t. Therefore, the expected depreciation rate is so defined:

$$\Delta s_{t+1}^{m} = -g\Delta s_t$$

With this model of expectations, for a negative weight g, forecasters extrapolate the most recent trend in the variation of the exchange rate, and exhibit the so defined *bandwagon* expectations. Chartists would expect a continuation of the observed trend. In the opposite case, with positive g, forecasters could *behave as fundamentalists*, expecting depreciations to follow recent appreciations and vice-versa. The authors observe that extrapolative expectations to be destabilizing, and long-term expectations to be generally stabilizing.

Another proposed specification is based on "a weighted average of the current observed spot rate and the long run equilibrium spot rate". Considering the log long run equilibrium exchange rate s_t^{LR}, the market expectation takes the form:

$$s_{t+1}^{m} = (1-\theta)s_t + \theta s_t^{LR}$$

The expected depreciation:

$$\Delta s_{t+1}^{m} = \theta(s_t^{LR} - s_t)$$

This model specification let us define for θ positive the expectation that over time the spot exchange rate will converge to s_t^{LR}, therefore it is defined as regressive expectation. The behaviour described corresponds with the fundamentalists' approach, as there exists a set of fundamentals determining the long-term value of the exchange rate. In this setting, forecasters expect to observe a return to the fundamental value. The opposite case, in which $\theta<0$, would imply that agents expect the spot rate to diverge from its fundamental over time.

A third view, based on adaptive expectations, describes future expectations as derived from the "weighted average of the observed current exchange rate, and the lagged expected spot rate":

$$s_{t+1}^{m} = (1-y)s_t + ys_t^{m}$$

The expected depreciation rate takes the form:

$$\Delta s_{t+1}^{m} = y(s_t^{m} - s_t)$$

The authors conclude that generally market agents use a weighted average of the chartist and fundamentalist forecasting approaches, with different weights based on the time horizon. They indicate chartist expectations as destabilizing, and fundamentalist expectations as stabilizing. For long-term predictions the fundamentalist view prevails, in short term predictions the chartist one seems preferred. Different approaches could also stem from the degree of professionalism of forecasters, but given the common use of surveys in this field of study, also a discrepancy between the declared, economic reasoning based decision-making, and the true, instinctive behaviour adopted while on the market could be hypothesised.

2.2 Chartists' Behaviour and aggregated effects

The behaviour and expectation formation of chartists is extensively described in the literature. First, a definition to be used as a reference for the next sections of the thesis will be provided. Second, more insights about their behaviour and characteristics will be added.

Chartists are essentially described as trend followers which adopt extrapolative expectations, they consider past variations in exchange rates and the current observed trend to be a significant predictor of future exchange rate variations.

Chartists are also described as *"agents basing their trading decisions on the analysis of past price trends"* (Chiarella, Dieci, and Gardini 2002). They are deemed as *trend followers* as they trade based only on available information about the price process, whose they assume to incorporate all relevant information, suggesting they may also believe in the efficiency of the market (Baumann, Baumann, and Erler 2019; Chiarella, Dieci, and Gardini 2002).

About the way in which this trend extrapolation is performed some authors indicate the use of simple moving average procedures (De Grauwe & Dewachter, 1993).

A forecasting technique often associated with chartists is technical analysis, namely, the use of statistical tools, data from the past and the construction of statistical trends, today still constitute a commonly used tool for foreign exchange market operators (Menkhoff and Taylor 2007). This finding can only reinforce the argument in support of the use of C&F models.

Chartists usually operate on different markets, and enter and exit markets more frequently than fundamentalists. The use of extrapolative methods allow chartists to be more flexible, they tend to enter markets whose show clear trading signals and have not too misaligned price trends. A consequent finding is that chartists likely are less specialized than fundamentalists (Westerhoff 2004).

Other sources in the literature suggest chartists can become more aggressive in extrapolating current trends especially when wider deviations from the fundamental are observed (Manzan and Westerhoff 2007).

Many authors point at the complex dynamics emerging from the interactions of heterogeneous market agents. Westerhoff points out that the composition of the market between chartists and fundamentalists variations affects the stability of the market. The most common example is that the prevalence of chartists over a critical threshold may lead prices away from the fundamentals (Westerhoff 2004). In the case of exchange rates, empirical evidence shows that "prices" are highly unpredictable, excessively volatile, and subject to deviations from fundamentals: the model is therefore able to replicate some of these stylized facts. Discussing exchange rate volatility, C&F models repeatedly proven that chartists with bandwagon expectations exacerbate exchange rate swings (Frankel and Froot 1990).

When framing these considerations inside the chartists' expectation formation mechanism, which is based on the belief that past and current movements of the exchange rate provide an indication of the market sentiment, it can be more clear why by the shared view they fuel a positive feedback price dynamic (De Grauwe & Dewachter, 1993).

Chartists, acting with a *positive feedback reaction* therefore destabilize the equilibrium in which the price equals the fundamental, while fundamentalists reacting to the difference between the actual market price and the fundamental value, keep under control its fluctuations (Lux 1998).

More complex modelling of noisy fundamentalist and chartist agents can be found in Franke & Westerhoff (2012), where the trading strategy depends on differential wealth, herding behaviour, individual predispositions and the tendency to return to fundamentalist strategies once the gap with fundamentals has widened.

2.3 Fundamentalists' Behaviour and aggregated effects

Fundamentalists are commonly described as traders or forecaster who base their decisions and forecasts upon a fundamental value, and react to the "perceived over- or undervaluation" of the specific asset (Baumann, Baumann, and Erler 2019). They are described as fundamentalist as they believe in mean reversion of asset prices to their fundamental, and are said to have regressive expectations (Boswijk, Hommes, and Manzan 2007). They can also be seen as more professionalized forecasters, as they concentrate mainly on a single market, which requires a continuous observation to determine their subjective fundamentals (Westerhoff 2004).

The main fundamental used in the foreign exchange market indicated in the literature is the PPP. An interesting hypothesis studied by De Grauwe and Dewachter (1993) is that heterogeneity could also be present in the expectations of fundamentalists, as different estimates of the equilibrium or fundamental value of the exchange rate. The fundamentalists' behaviour is said to generate a "negative feedback" to exchange rate variations, as expression of their regressive expectations.

While trend followers or chartists aim at "magnifying the current trend" as they react strongly to sharper variations in asset prices, fundamentalists form expectations, buy and sell according to the price being above or below the fundamental, driving prices back to it (Baumann, Baumann, and Erler 2019). The literature does not express unanimously in favour of their strategy being dominant on *linear feedback trading strategies*, and suggests that their stabilizing effect on the market could be not significant or sufficient. In their paper, Baumann et al. hypothesize that only proper (still unknown) incentives could generate a similar type of trader or intervention able to avoid financial bubbles and stabilize the market. They also find the stabilizing effect of fundamentalists to be limited and not sufficient as a market stabilizing force.

Further arguments in favour of exchange rate evolution trends converging to fundamentals can be found in Levin (1997), in his model it is shown that monetary expansions can be associated with overshooting exchange rates as a consequence of fundamentalists' sales, followed by a convergent path which restores the interest parity. In this case, the joint activity of fundamentalists and chartists could be stabilizing.

2.4 Agents' switch between Chartist and Fundamentalist approaches: changing market composition and deriving effects

The group dynamic of market agents switching between the two forecasting methods, or entering and exiting the market constitutes a fundamental part of C&F models, as market composition is time varying. A relevant observation is that the forecasting horizon matters. Over short terms agents tend to behave like chartists, while on longer horizons they adopt regressive expectations. The prevalence of short-term traders or long term traders is relevant in determining the aggregate effects on the market (Frankel and Froot 1990). The difference in excess revenues also could contribute to strategy switching (Lux 1998). In our case, we could describe agents as boundedly rational, as the data generating process of exchange rate evolution is unknown, and the exchange rate remains hard to predict, they rationally change strategy over time.

The nature of foreign exchange markets is at the basis of these changes of strategy of market agents over time. Exchange rate changes tend to not be linked to changes in macroeconomic fundamentals, even though the exchange rate is cointegrated with its fundamental value. Exchange rate dynamics exhibit fat tails, and bubbles and crashes happen (Williamson 2009).

The dynamic proposed in Gaunersdorfer, Hommes, and Wagener (2008) suggests that low volatility periods corresponds to the ones in which fundamentalists dominate, whereas high volatility periods, are dominated by chartists. Temporary bubbles originate as a result of noise traders' activity. Agent based models with evolutionary strategies also suggest that based on past performances, agents over time switch between predictors and approaches, therefore are not of an "*inherited fixed type*" (Boswijk, Hommes, and Manzan 2007).

The relevance of exchange rate dynamics cannot be underestimated, as trend extrapolation will be stronger when sharp variations of asset prices are observed, and mean reversion in periods of gradual changes (Boswijk, Hommes, and Manzan 2007).

As many authors find, the share of chartists and fundamentalists varies over time, therefore the market composition constitutes a fundamental factor in determining prices, their dynamics, and the amount of noise. A dominance of chartists leads to destabilized prices and market bubbles, while a prevalence of fundamentalists guarantees stable prices (Franke and Westerhoff 2012; Brock and Hommes 1998). Researchers also link the noise level and the irregularly switches between volatile and calm periods with market's composition.

Another interesting result about Chartists and Fundamentalists' interactions comes from the study of chaos dynamics. Chaos arises because of chartists' trend extrapolation, which causes overshooting and volatility, and is reduced by fundamentalists who bring prices back to the fundamental. The authors suggest that the occurrence of intermittent chaos and stabilizing cycles depend on the strength of trend extrapolation and fundamental stabilization forces (Gaunersdorfer, Hommes, and Wagener 2008). Strong trend extrapolation forces may reinforce small price deviations over time, and as prices move far from fundamental, more and more of the chartists may abandon their forecasting rules and turn into fundamentalists (or just exit the market). Evolutionary interactions in this context are what causes intermittent chaos and irregular prices fluctuations. Weak trend extrapolation in opposition will generate more stable price levels. Also in this case, the destabilizing versus stabilizing effects find solid support (Gaunersdorfer, Hommes, and Wagener 2008).

While the presented sources describe fundamentalists' trading strategies as stabilizing, Lux (1998) suggests that "both groups act in a non-equilibrium manner" and that prices derive from a "non-market clearing interaction", in this case fundamentalists' activity aggregated effect can be seen as destabilizing too when combined with chartists' one.

While we could eventually expect chartists to be driven out from the market over time, some evidences that fundamentalists not necessarily outperform them exist. Phases of alternate dominance of fundamentalists and chartists happen, meaning that intrinsic heterogeneity never disappears in the market (Berardi 2011). Some authors indicate our specific case to be the opposite: in the context of exchange rates forecasting, as Chartism reveals to be a profitable strategy, chartists tend to often dominate foreign exchange markets (De Grauwe and Grimaldi 2006). In this thesis, I also attempt to frame the market composition in the recent past.

Chapter III - Two more strands of literature in studying heterogeneous expectations: Noise Traders, Consensus dynamics and the role of News

After presenting the C&F view, we consider a second strand of literature: the one studying noise and noise traders' activity effects on heterogeneity in expectations. I recall some fundamental results and present the literature.

By this hypothesis, heterogeneity is associated with the risk premium on the exchange rate, and derives from uncertainty and noise. As shown by Jeanne and Rose, unsophisticated noise traders generate "noise trading", heterogeneity in expectations, and leads to distortions and uncovered interest rate parity (Jeanne and Rose 2002). Regarding the increasing effect of noise trading on the dispersion of expectations and volatility, I suggest seeing Bacchetta & Wincoop (2006). If then we consider the exchange rate forward rate gap, again noise traders cause heterogeneous expectations: Flood and Rose rule out the peso problem, but find that not fully rational traders could be responsible for the forward discount bias (Flood and Rose 1996).

Finally, I present the third strand of literature, the one linking heterogeneous exchange rate expectations to uncertainty in fundamentals and macroeconomic variables' movements. Differentiated access to information, information dispersion and unobserved trades, lead to the agents' need to rely on fundamentals (Bacchetta and Wincoop 2006). It is also worth mentioning the infrequent decision and infrequent information processing effect on expectations as done in the previous sections, and the role of news and timing of arrival of news (more relevant in high volatility periods) in determining spikes in heterogeneity (Bacchetta and Van Wincoop 2006; Manzan and Westerhoff 2005).

3.1 A brief look to the forward discount puzzle and bias

Frankel and Froot (1989) write about the forward discount prediction error, they do not find evidence of prediction errors to capture a risk premium that changes in time. They conclude that that all the bias in the forward discount can't be attributed to either the risk premium or expectation errors entirely, only partially.

The forward premium bias consists in *"the empirical finding that the forward premium helps to predict the future percentage of currency depreciation but not with the sign implied by interest parity"*. Given that we could see the exchange rate as the domestic price of foreign currencies, the forward premium is defined as the difference between the nominal interest differential between domestic and foreign currency debt. In their paper, Mark and Wu (1998) advocate for quasi-rational noise traders' models, as they find weak evidence in favour of the chosen model of homogeneous rational agents.

In a paper studying the forward premium puzzle, Bacchetta and Wincoop (2006) consider the application of a model of incomplete information processing, and find evidence to support the "delayed overshooting" phenomena (exchange rates appreciation even after interest rate rise).

Some pieces of literature discussed in the previous sections are skeptical about the use of fundamentals as predictors of exchange rates: Kilian and Taylor (2003) study the exchange rate-PPP deviations and suggest it could be significant only in large samples and over long periods. In the empirical analysis I test also this hypothesis.

3.2 Consensus dynamics and herding behaviour

The notion of consensus in the exchange rate expectations' consists intuitively in the perceived *expectation of other market agents*. The role of this *market expectation* is deemed as fundamental in part of the literature, as the statement *the average of all forecasters' beliefs influences individual forecasts* is proven to be true. It is found that less experienced forecasters, especially if paid according to their performance or in the case of having a reputation to be defended, are more subject to this effect through herding dynamics (Rangvid, Schmeling, and Schrimpf 2013).

Herding dynamics, or "coordination in expectations" could be explained as the preference of agents for being *wrong together* with the large majority of other market participants, than taking the risk of *being right or wrong alone*. This mass behaviour in financial markets is particularly relevant, since market bubbles and market crashes are likely deriven by these dynamics. Herding dynamics do not coincide with consensus dynamics, but provide an example of mass behaviour in financial markets, and associated with positive feedback dynamics can justify the permanence of noise traders in the market (Tedeschi, Iori, and Gallegati 2012).

To justify consensus dynamics, which are present independently from reputational dynamics, Rangvid, Schmeling, and Schrimpf (2013) suggest that individual forecasters may believe the average expectation of the market to reveal "dispersed private information" and thus update individual expectations. Another reason suggests individual forecasters may believe that other market operators are taking as reference the consensus, this type of high-order expectation resembles the well-known Beauty Contest from Keynes (1936). Based on the assumption of other forecasters being rational, individuals may believe that the consensus expectation contains information about future returns and realizations. This possibility has been empirically ruled out in Rangvid, Schmeling, and Schrimpf (2013).

Furthermore, even if consensus dynamics could contribute to lower heterogeneity in expectations, it leads to overweighting of public signals and underweighting of private signals, thus providing a biased predictor of asset fundamentals. Rangvid, Schmeling, and Schrimpf (2013) find evidence of significant effects of lagged consensus expectations on individual expectations.

Another definition of Consensus in the literature consists of the average or median expectation in the sample of data considered. It is also the case for Menkhoff et al. paper and my analysis.

About the correctness of defining consensus measures as "the market expectation", Prat and Uctum (2013) suggest it being possible once accounted for a systematic and a random component representing white noise. In their paper, they also "speak" in favour of the consideration of consensus as a pure expectation (belief of forward rates being correct predictors).

Chapter IV - The original paper: models, methods, choices and main results

In the original paper, Menkoff et al. use a dataset covering 15 years of individual exchange rate expectations from ZEW's survey of around 300 market operators. The authors consider 6-months forecasts over the period December 1991 - August 2006.

The survey data, of qualitative nature, just identifies if individuals expect the exchange rate to increase, decrease or remain stable, and was transformed through a quantification technique to become treatable.

The main objective of the paper is to investigate heterogeneity determinants, and their short and long run effects on the level of disagreement among market agents.

They consider three exchange rates with the USD: the USD-Euro (before 1998, the German Mark), the Yen-Euro and GB-pound Exchange rate.

The main variable studied consists in the dispersion (i.e. standard deviation) of individual exchange rate expectations, defined as heterogeneity.

They test an extensive set of macroeconomic fundamental variables, and other indicators deemed as significant in the relevant literature about exchange rates forecasting.

Menkhoff et al. form a list of variables to be tested empirically in linear regressions, and then select the significant ones to be jointly considered in a Vector Error Correction Model framework.

For the C&F hypothesis, they characterize fundamentalists' expectations as regressive to the long run PPP equilibrium, and chartists' expectations as extrapolative and focused on the short-term variations of the exchange rate.

The chosen fundamental for the exchange rate is the Purchasing Power Parity index, the regressor for fundamentalists' influence in the market is constituted by the differential between the observed exchange rate and its PPP. To study chartists, they consider the most recent variation of the exchange rate (monthly in their case).

For the risk premium hypothesis, they compute the exchange rate risk premium as the difference between the expected exchange rate and its forward rate, following Froot and Frankel (1989) and Bams et al. (2004). A vast set of macroeconomic variables are also tested in their levels and rates of variation.

In this first stage of the analysis, they conclude that no macroeconomic fundamental is a significant regressor of heterogeneity. They find the regressive term (exchange rate-PPP differential), the extrapolation term (chartists' reference), and the risk premium to be significant, confirming the solidity of the C&F hypothesis in their sample and period.

The VECM is estimated considering the aforementioned variables, they study their persistent behaviour and their temporary or permanent effects.

The choice of the VECM is justified by the possibility that some of the selected regressors could be endogenous (they take as an example the potential impact of heterogeneity on the risk premium). Another reason to choose this approach is to make sense of the common stochastic trends that could be present among the variables, as they are persistent time series.

They also test some dummies for the intervention of central authorities, but this tool is not applied on the Euro-USD exchange that constitutes the focus of my analysis.

The authors conclude that:

"Findings are in accordance with underlying model assumptions and thus confirm the C&F approach. Moreover, the risk variable, which is unrelated to the C&F approach variables, has the sign as expected by the noise trading literature. Turning to dispersion's short-term relation, dispersion error-corrects significantly towards its long-term equilibrium. Moreover, in the short run, the extrapolation variable strongly pushes dispersion. This impact works in such a manner that the speed of the exchange rate change positively impacts dispersion, indicating the enormous relevance of extrapolation in the short run. An economic interpretation of this short-term effect may be that it indicates heterogeneity within the group of chartists as they react with different speed on the same strong signal" (Menkhoff, Rebitzky, and Schröder 2009).

Their results support the C&F hypothesis, the risk premium affects heterogeneity coherently with what expected in noise traders' models. They find trend extrapolation to increase heterogeneity on the short term, and the regressive term (PPP exchange rate differential) to reduce heterogeneity in the long run in response to wider gaps. The risk premium represents uncertainty and boosts heterogeneity, chartists over time decrease in number.

4.1 The Timing problem: information availability and the forecasting moment

One of the main issues when conducting this type of analysis is that some assumptions about the available information and timing of discover of new information need to be made. In order to study expectations, it is crucial to consider which information and data is available to forecasters in the moment of the expectation formation: while the availability of information about the past can be assumed with a high degree of confidence, the same does not hold about "present" information and "present" forecasts of the future.

The same holds for the timing of the forecast: the moment in which the forecast is made conditions the amount of available information and can make a future expectation made on the last day of the "current quarter" differ from one made at its beginning. Elias (2016) faces this same problem while modelling agents' expectation formation in a heterogeneous agent exchange rate model. The author assumes the value of the current period exchange rate to be unknown to forecasters, while current period fundamental variables to be available. In his model, provided the current exchange rate is indicated by S_t and current information by f_t, agents know S_{t-1} and f_{t-1}, observe the current realization of f_t, and then make their forecast of S_{t+1}. They do not have access to S_t.

In my analysis, it is assumed agents are aware of both current fundamentals and exchange rate movements, in an equivalent notation it is assumed that forecaster have access to both S_t , f_t, and their past realizations.

Chapter V - Methodology and Empirical Strategy

In this thesis, I perform an econometric analysis adopting the methodology known as the Vector Error Correction Model (VECM), to comprehend the reasons behind the use of this technique, I recommend seeing Zivot and Wang (2003) and Hamilton (2020). I also recall the reasons suggested by Menkhoff et al. cited in the previous section. In the next sections, are briefly presented the matter of cointegration and the main aspects econometric analysis applied to nonstationary time series.

5.1 Nonstationary series and cointegration

Studying equilibrium relationships among the levels of time series variables often leads to treat I(1) time series. The VECM model, used in this analysis derives from the VAR model, and is chosen to treat I(1) data. Its need cannot be understood without exploring the concept of cointegration (Zivot and Wang 2003).

While standard regression techniques require the use of covariance stationary series with finite and nonvarying mean and autocovariances, cointegration analysis lets us make estimates and inference on non-covariance stationary variables. In my case, as for many economic time series, variables can be nonstationary in levels, but stationary in their first differences. In this case we refer to them as I(1) “integrated of order 1 processes”. The most famous example of this type is the random walk, being not covariance stationary, but being first difference stationary (Stata Corp 2022). The application of conventional estimators when studying processes of this kind can be detrimental, as their asymptotic distributions become nonstandard. For these reasons, in section 7.2 I devote some time studying the presence of unit roots in the variables I use.

5.2 Spurious regressions and cointegration: VAR and VECM models

When applying econometric techniques to time series, we cannot guarantee all variables to be I(0) and stationary. In fact, if some or all the variables reveal to be I(1) the usual inferential results may not hold.

The main case of this phenomena is the one of spurious regressions where all regressors are I(1) and not cointegrated. For the technical considerations, see Hamilton (2020). For the purpose of introducing my analysis I recall that inference for regressions with I(1) data is nonstandard (Zivot and Wang 2003). Cointegration is a special case, in which stable relationships can be measured even in case of unstable or I(1) dynamics.

Let $Y_t = (y_{1t}, \dots, y_{nt})'$ denote an (nx1) vector of I(1) time series. Y_t is cointegrated if there exists an (nx1) vector $\beta = (\beta_1, \dots, \beta_n)'$ such that

$$\beta' Y_t = \beta_1 y_{1t} + \cdots + \beta_n y_{nt} \sim I(0)$$

Essentially the nonstationary time series in Y_t are cointegrated if there exists a linear combination of them which is stationary or I(0). If some of the elements of β are null, then only the subset of the time series in Y_t those have nonzero coefficient is cointegrated. The linear combination $\beta' Y_t$ is defined as long run equilibrium. In the case of cointegration, we expect that even if I(1) time series drift away from the equilibrium, the model will eventually return to it.

This long run relationship is more evident in the normalized form, useful to uniquely identify β. A typical normalization in the literature is $\beta = (1, -\beta_2, \dots, -\beta_n)'$ thus the cointegration relationship can be written as

$\beta' Y_t = y_{1t} - \beta_2 y_{2t} - \cdots - \beta_n y_{nt} \sim I(0)$ or as $y_{1t} = \beta_2 y_{2t} + \cdots + \beta_n y_{nt} + u_t$

Where u_t takes the name of cointegrating residual, taking value zero in the long run equilibrium (Zivot and Wang 2003).

5.2.1 Multiple cointegrating relationships and the error correction mechanism

In our case, we must also consider another degree of complexity, as we need to look for multiple cointegrating relationships.

If the (nx1) vector Y_t is cointegrated, there may be $0 < r < n$ linearly independent cointegrating vectors. We consider a simple example with n=3 and r=2 cointegrating vectors $\beta_1 = (\beta_{11}, \beta_{12}, \beta_{13})'$ and $\beta_2 = (\beta_{21}, \beta_{22}, \beta_{23})'$.

Then $\beta_1' Y_t = \beta_{11} y_{1t} + \beta_{12} y_{2t} + \beta_{13} y_{3t} \sim I(0)$ and $\beta_2' Yt = \beta_{21} y_{1t} + \beta_{22} y_{2t} + \beta_{23} y_{3t} \sim I(0)$. We can now define the matrix (3x2) **B**, basis of the space of cointegrating vectors.

$$\boldsymbol{B}' = \begin{pmatrix} \beta_1' \\ \beta_2' \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{33} \end{pmatrix}$$

5.2.2 Common stochastic trends

In our application case, we resort to the VEC model as the PPP implies the cointegration of the nominal exchange rate with foreign and domestic prices, while covered interest parity should imply forward and spot exchange rates to be cointegrated.

If the (nx1) vector time series Y_t is cointegrated with $0 < r < n$ cointegrating vectors then there are n-r common I(1) stochastic trends. We can show it considering $Y_t = (y_{1t}, y_{2t})' \sim I(1)$ and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})' \sim I(0)$ and supposing Y_t to be cointegrated with $\beta = (1, -\beta_2)'$ cointegrating vector. We can represent the cointegration relationship as below

$y_{1t} = \beta_2 \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{3t}$ $y_{2t} = \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{2t}$ where $\sum_{s=1}^{t} \varepsilon_{1s}$ is the stochastic trend.

Thorough the cointegration relationship $\beta = (1, -\beta_2)'$ we can annihilate the common stochastic trend:

$$\beta' Y_t = \beta_2 \sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{3t} - \beta_2 \left(\sum_{s=1}^{t} \varepsilon_{1s} + \varepsilon_{2t}\right) = \varepsilon_{3t} - \beta_2 \varepsilon_{2t} \sim I(0)$$

To explain the usefulness of the VEC model in the case of my analysis, I propose a bivariate example from Zivot and Wang (2003). We recall a well known result in the literature: cointegration implies the existence of an error correction model, the ECM describes the dynamic behavior of the two (or multiple) parts of the vector Y_t (Hamilton 2020; Zivot and Wang 2003).

Starting again from Y_t being a bivariate I(1) vector, cointegrated with $\beta = (1, -\beta_2)'$, such that $\beta' Y_t = y_{1t} - \beta_2 y_{2t}$ is I(0) the ECM describes the dynamic behaviour of y_{1t} and y_{2t} and takes the form of

$$\Delta y_{1t} = c_1 + \alpha_1 (y_{1t-1} - \beta_2 y_{2t-1}) + \sum_j \varphi_{11}^j \Delta y_{1t-j} + \sum_j \varphi_{12}^j \Delta y_{2t-j} + \varepsilon_{1t}$$

$$\Delta y_{2t} = c_2 + \alpha_2 (y_{1t-1} - \beta_2 y_{2t-1}) + \sum_j \varphi_{21}^j \Delta y_{1t-j} + \sum_j \varphi_{22}^j \Delta y_{2t-j} + \varepsilon_{2t}$$

The ECM essentially let us consider both the long-run equilibrium relationship that derives from the cointegration relationships, and the short-run dynamics of adjustment of the variables when moving away from the equilibrium (Zivot and Wang 2003).

5.2.3 Johansen procedure

The Johansen procedure for testing cointegration is based on residual-based tests for cointegration. Following the procedure, the researcher carries out the following steps:

a) Specify the VAR(p) model for Y_t ;
b) Conduct a Likelihood ratio test for the rank of P, determining the number of cointegrating vectors;
c) If necessary, identify restrictions and or normalize the model;
d) Estimate the VECM (Zivot and Wang 2003).

5.2.4 The cointegrated VAR

We start from a VAR(p) model for the (Kx1) vector of variables Y_t :

$$Y_t = v + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t \quad t = 1, \dots, T$$

Where v is a (Kx1) vector of parameters, which can also represent ΦD_t a deterministic trend D with coefficient Φ. The series of (KxK) matrices of parameters $A_1 \dots A_p$ represents the weights of the past realizations of Y_t. The vector ε_t is an i.i.d. vector of disturbances, with mean 0 and covariance matrix Σ. The stability condition of the VAR model consists in having all roots outside the unit circle. We require the existence of some linear combination of the variables in Y_t which is I(0).

We then transform the VAR into the VECM taking the difference by Y_{t-1} on both sides, we obtain a representation in differences of the former where Π is the long run impact matrix. One of the relevant changes is that ΔY_t and its lags become I(0).

Looking at the rank of Π, we can determine the number of cointegration relationships, in my case, Π has rank $0 < r < n$, implying Y_t being I(1) with r linearly independent cointegrating vectors and n-r common stochastic trends.

$$\Delta Y_t = \Phi D_t + \Pi Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \cdots + \Gamma_{p-1} \Delta Y_{t-p+1} + \varepsilon_t$$

$$or \qquad \Delta Y_t = v + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t$$

Where $\Pi = \sum_{j=1}^{j=p} A_j - I_k$ is known as long run impact matrix and $\Gamma_i = -\sum_{j=i+1}^{j=p} A_j \quad k = 1, \dots, p$ are the short run impact matrices. Given the rank of Π is 0 < r < K, we can then define $= \alpha\beta'$, two (rxK) matrices which dimensions depend on the number of cointegration relationships, and with rank r.

We obtain a new VECM substituting Π by $\alpha\beta'$, where α represents the speed of reaction of Y_t, i.e. it is the speed of adjustment coefficient.

$$\Delta Y_t = v + \alpha\beta' Y_{t-1} + \Gamma_1 \Delta Y_{t-1} + \cdots + \Gamma_{p-1} \Delta Y_{t-p+1} + \varepsilon_t$$

For in depth explanations and improve the understanding of this section see Hamilton (2020), and Zivot and Wang (2003) example 80 at page 456.

More can be said about the deterministic trends that can enter the cointegrating VECM. This component can derive from the mean of the cointegrating relationship or the mean of the differenced series. The general form we start from admits a constant and a linear trend:

$$\Delta Y_t = v + \delta t + \alpha\beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \varepsilon_t$$

In this formulation, δ is a (Kx1) vector of parameters, the constant v represents a linear time trend in the levels, δt a quadratic time trend. Given that is α a (Kxr) matrix, we could rewrite these two components to highlight 5 cases of trend: unrestricted trend, restricted trend, unrestricted constant, restricted constant and absence of a trend.

$$v = \alpha\mu + \gamma \quad \delta t = \alpha\rho t + \tau t$$

$$\Delta Y_t = \alpha(\beta' Y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \gamma + \tau t + \epsilon_t$$

In this analysis, it is considered a restricted trend ($\tau = 0$), therefore it is assumed the trends in the data levels to be linear, not quadratic, and the cointegrating equations to be trend stationary.

Chapter VI - Introduction to the empirical analysis

6.1 Expectations on the analysis results

Even following all the steps carried out by Menkhoff et al. I expect to find only partial confirmation of their results, as the two periods of analysis are different in length, and because of the occurrence of destabilizing events in the timespan they considered. Events such as the 2008 financial crisis or the 2011 sovereign debt crisis inside the Euro Area could have sparked periods of high uncertainty and heterogeneity, therefore I expect my results to not replicate entirely the same dynamics.

As evidences from Frankel and Froot (1990) show, another possible difference might depend on the forecasting horizon considered here, i.e. quarters, as on different forecasting horizons even the same agent could adopt different strategies .

At the same time, also some technical aspects must be considered, the choice of a smaller sample of market operators in my case could lead to different results: heterogeneity in exchange rate expectations may be linked to the cross sectional sample size. Other sources of concern affecting the comparability of the analysis with the original paper are the use of different data sources and calculation methods.

6.2 Data structure

The analysis starts by considering a dataset of quarterly exchange rate expectations of European and global financial institutions about the Euro-USD exchange obtained from Bloomberg. The final dataset of expectations takes the form of an unbalanced panel dataset, containing the expectations of 69 to 112 market operators at each period. The broad sample comprehends expectations from 144 different financial institutions, even though for some panellists an incomplete series of observations is obtained.

The individual time series of each forecaster spans over the last 9 years, starting from the first quarter of 2013 and lasting to the first quarter of 2022, for a total of 37 periods.

From this data, the main variable of interest is computed, that is, heterogeneity in expectations measured as the standard deviation of the individual expectations for each period. The average of the expectations, labelled as *consensus*, represents the market's expectation for each quarter.

The first difference with the original paper is in the data used and in the timespan considered here. Menkhoff et al. cover a period of 15 years, from December 1991 to August 2006, and use monthly data from the ZEW survey about fundamentals and expected exchange rate movements. In the case of the Euro-USD exchange rate, the period is particularly significant: until 1998 the data refers to the German Mark, and between 1999 and 2006 most of the enlargement of the Eurozone takes place. This work represents therefore a continuation of the original paper. Even if the period between 2006 and 2012 is left out, excluding the phase characterized by the 2008 financial crisis, the 2011 sovereign debt crisis, and the Euro instability that was observed in 2012, the analysis can shed light on the robustness of Menkhoff et al. results.
A second striking difference is in the type of data used. The original paper is based on survey data treated with a quantification technique (see Menkhoff, Rebitzky, and Schröder (2009)), while my analysis uses a dataset of individual expectations from Bloomberg. Finally, it is relevant to consider that the authors use monthly data, while I consider quarterly variables. In addition, their sample size

accounts for 300 market operators, while around 110 make mine. Finally, potential differences in how the dispersion of exchange rate expectations (i.e. heterogeneity) are present.

I present some descriptive statistics about the Consensus value of the spot exchange rate and its dispersion in expectations over the period of analysis. Dispersion corresponds to the standard deviation in agents' expectations i.e. heterogeneity.

Table 2. Descriptive statistics of Consensus and Expectations' Dispersion. Source: Menkhoff et al. (2009)

	Euro-USD	
	Consensus	Dispersion
Mean	1,133	0,0700
Std. Dev.	0,12	0,0170
25%-q.	1,049	0,0580
75%-q.	1,225	0,0780
Min.	0,881	0,0430
Max.	1,369	0,1320

Table 3. Descriptive statistics of Consensus and Expectations' Dispersion. Source: Data Elaboration.

	Euro-USD	
	Consensus	Dispersion
Mean	1,175	0,0420
Std. Dev.	0,083	0,0130
25%-q.	1,11	0,0333
75%-q.	1,218	0,0475
Min.	1,052	0,0233
Max.	1,342	0,0750

Above it is presented the table from Menkhoff et al. (2009) and an equivalent generated from my dataset.
Below are presented the detailed summaries tables obtained through Stata. It can be observed that in the period between 2013 and 2022 the consensus expectation about the Euro-USD exchange rate was lower than in 1991-2006. Also the amount of dispersion in expectations is different. In the original paper, the authors observed way higher levels of heterogeneity than I do in the last decade. If then we look at the consensus variability, I also observe that the market's expectation has been more volatile in the past than in the recent years. The general finding is that exchange rate expectations in the last decade have been less dispersed, as foreign exchange market's operators lived a phase of stability in which the exchange rate had a lower volatility.

Table 4. Extensive descriptive statistics of Consensus and Expectations' Dispersion. Source: Data elaboration.

Consensus				
	Percentiles	Smallest		
1%	1,0518	1,0518		
5%	1,0518	1,0518		
10%	1,0703	1,0594		
25%	1,1100	1,0703	Obs	37
50%	1,1654		Mean	1,175371
		Largest	Std. Dev.	0,083760
75%	1,2182	1,2994		
90%	1,2994	1,3201	Variance	0,007016
95%	1,3340	1,3340	Skewness	0,427670
99%	1,3418	1,3418	Kurtosis	2,175412

Dispersion				
	Percentiles	Smallest		
1%	0,0233	0,0233		
5%	0,0240	0,0240		
10%	0,0278	0,0252		
25%	0,0333	0,0278	Obs	37
50%	0,0389		Mean	0,042031
		Largest	Std. Dev.	0,012980
75%	0,0475	0,0614		
90%	0,0614	0,0637	Variance	0,000169
95%	0,0749	0,0749	Skewness	0,898569
99%	0,0750	0,0750	Kurtosis	3,370728

Below, the distribution density of the amount of heterogeneity in the last decade is presented, compared with the one observed by Menkhoff et al. The most striking feature of the graph is the lower amount of heterogeneity in expectations registered in the recent past of the Euro-USD exchange market.

Figure 1. Density Distribution of Expectations' Dispersion values over the period 1991-2006. Source: Menkhoff et al. (2009).

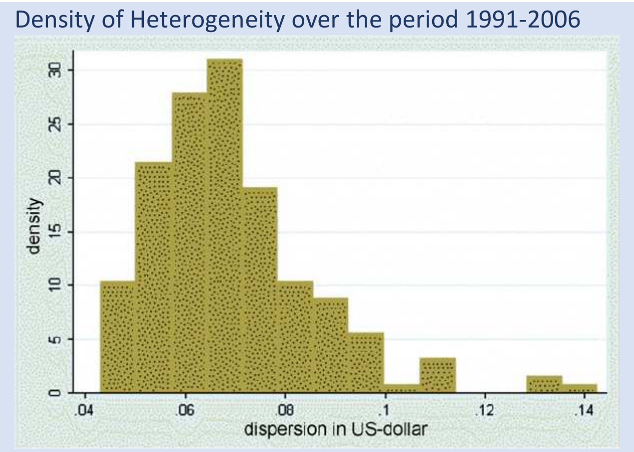
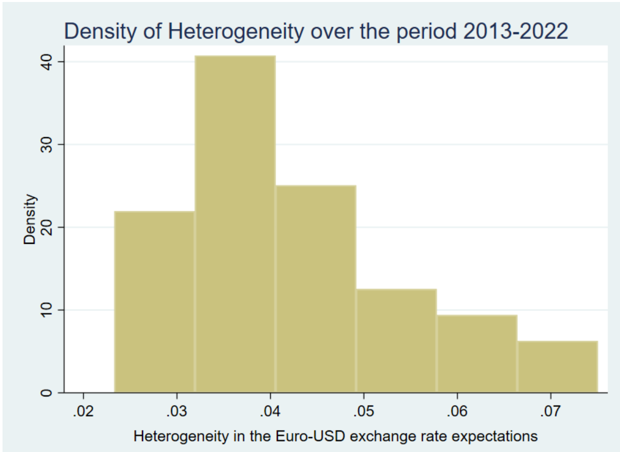
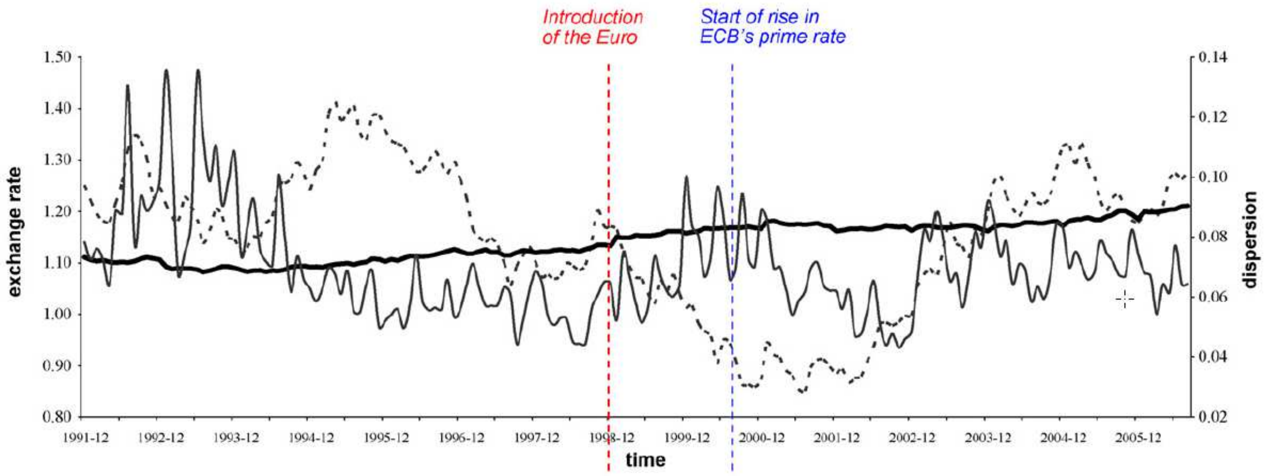


Figure 2. Density Distribution of Expectations' Dispersion values over the period 2013-2022. Source: Data elaboration from Stata.



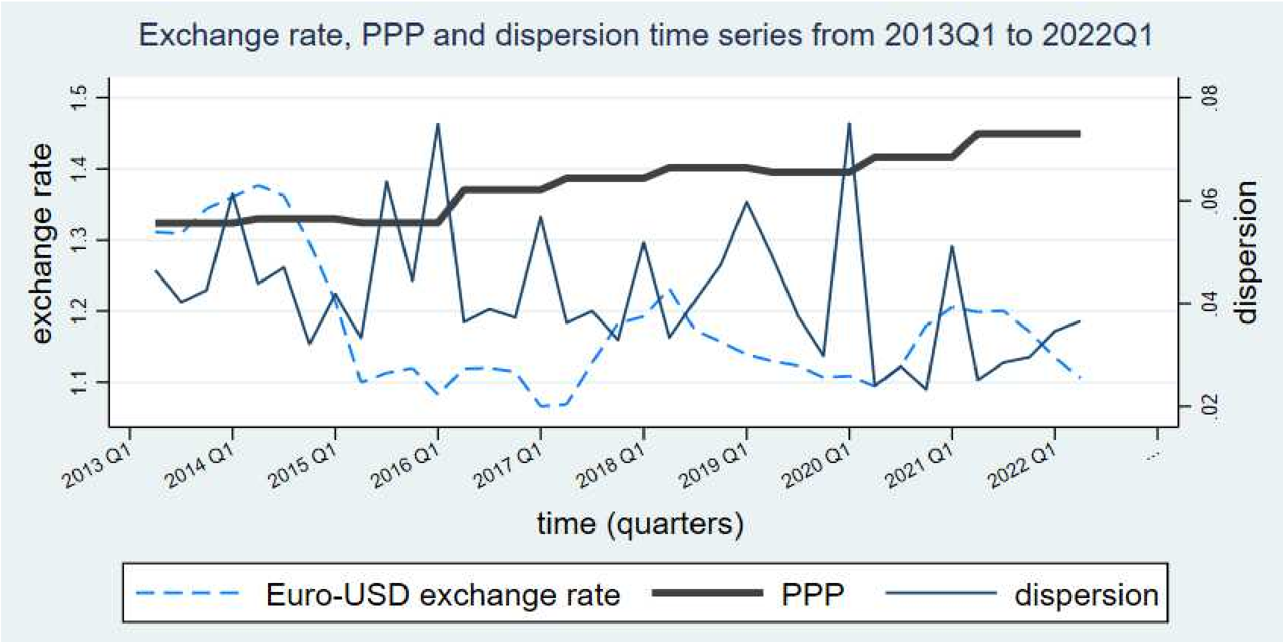
The comparison of the time evolution of the exchange rate, the amount of heterogeneity in expectations and PPP in the two period is presented in Figure 3. It can be argued that lower levels of dispersion in expectations are due to a more stable exchange rate, as also shown in the previous table. By graphical inspection, it can be noted that the Euro-USD exchange rate time series exhibits a more stable trend in the period 2013-2022 than it does in the period 1991-2006. Dispersion values are overall less volatile and lower in my period of analysis; the PPP follows in both cases an increasing trend.

Figure 3. Expectations' Dispersion, exchange rate and PPP values' evolution over the period 1991-2006. Source: Menkhoff et al. (2009).



The dispersion series is represented by the dotted line, the PPP by the bold line, the exchange rate by the thin line.

Figure 4. Expectations' Dispersion, exchange rate and PPP values' evolution over the period 2013-2022. Source: Data elaboration on Stata



6.3 Regressors and macroeconomic variables

Menkhoff et al. consider in their analysis some potential predictors of heterogeneity, considering the three main strands of literature they present: the Chartists & Fundamentalists Hypothesis, the Noise Traders' Hypothesis and the Macroeconomic Fundamentals' Information Heterogeneity Hypothesis.

For the C&F model, they consider two variables to represent the chartists' and fundamentalists' expectation formation. The first one is a measure of "extrapolation", chartists form their expectations by trend extrapolation of the recent past realizations of the exchange rate. The previous quarterly variation of the exchange rate is defined as follows

$$extrapolation\ term\ _t = \left(\frac{s_t}{s_{t-1}}\right) - 1$$

The second one is a measure of *regression*. As fundamentalists expect the exchange rate to converge to its fundamental in the long run, they expect to observe corrections towards the PPP. In this case the chosen variable is the PPP-exchange rate differential, relative to the "current" quarter.

$$regression\ term\ _t = s_{t\ (quarterly)} - PPP_{t\ (yearly)}$$

To study the Noise Traders' Hypothesis, Menkhoff et al. adopt a measure of the exchange risk premium that should derive from noise trading and heterogeneity, the risk premium constitutes the gap between the forward rate and the exchange rate expectation of the market (consensus).

$$risk\ premium_t = consensus_t - \ forward\ rate_t$$

For the third strand of literature, several macroeconomic variables are tested as regressors of heterogeneity, a table containing their description, source, and computation method is provided below. The macroeconomic variables' selection is inspired by the literature about models of exchange rate determination and by the collection of Menkhoff et al. paper. Additional variables are generated as absolute values of the following.

Table 5. List of variables considered as potential determinants of heterogeneity in exchange rate expectations. Data Source: Bloomberg, FRED, Data elaboration.

Variable	Description	Computation method
Consensus *(Ere)*	Market (Sample) exchange rate expectation of the next quarter (t+1)	Mean of the exchange rate expectation of the sample
Heterogeneity, dispersion in expectations (*Hetg*)	Standard deviation of the individual exchange rate expectations	Standard deviation of the exchange rate expectations in the sample
Quarterly exchange rate (*Exr*)	Quarterly average Euro-USD exchange rate	Mean of monthly exchange rates
Extrapolation Term: variation of the quarterly exchange rate (*Dexr*)	Percentage quarter by quarter variation of the Euro-USD exchange rate	$\left(\frac{Exr_t}{Exr_{t-1}}\right) - 1$
Regression Term: exchange rate – PPP differential (*Dppp_exr*)	Quarterly Euro-USD exchange rate – Power Purchasing Parity Index differential	$Exr_{t\,(quarterly)} - PPP_{t\,(yearly)}$
Forward Rate (*Forward*)	Quarterly forward rate relative to the quarterly expected exchange rate	$Exr_t * \left(\frac{1 + \text{ECB interest rate}}{1 + \text{FED interest rate}}\right)$
Exchange rate risk premium (*Riskp*)	Quarterly forward rate – Quarterly expected exchange rate (consensus)	$Forward_t - Ere_t$
M3 USD growth rate (*M3_usd*)	Quarterly percentage growth rate of M3 monetary aggregate - USD	$\left(\frac{M3\_usd_t}{M3\_usd_{t-1}}\right) - 1$
M3 Euro growth rate (*M3_eur*)	Quarterly percentage growth rate of M3 monetary aggregate - Euro	$\left(\frac{M3\_eur_t}{M3\_eur_{t-1}}\right) - 1$
Consumer Price Index in the U.S. (*Cpi_us*)	U.S. quarterly CPI with base Q1 2013	Rescaled CPI to 2013 Q1 as base year
Consumer Price Index in the Euro Area (*Cpi_eur*)	Euro Area quarterly CPI with base Q1 2013	Rescaled CPI to 2013 Q1 as base year
U.S. GDP growth rate (*Ggdp_us*)	Quarterly GDP growth rate - U.S.	$\left(\frac{GDP\_us_t}{GDP\_us_{t-1}}\right) - 1$
Euro Area GDP growth rate (*Ggdp_eur*)	Quarterly GDP growth rate - Euro area	$\left(\frac{GDP\_eur_t}{GDP\_eur_{t-1}}\right) - 1$
U.S. Inflation Rate (*Infl_us*)	Quarterly inflation rate - US	
Euro Area Inflation Rate (*Infl_eur*)	Quarterly inflation rate - Euro area	

U.S. Output Gap (*Ogap_us*)	Quarterly Output Gap – U.S.	
Euro Area Output Gap (*Ogap_eur*)	Quarterly Output Gap – Euro Area	
U.S. interest rate (*Int_us*)	FED Nominal interest rate	
Euro Area interest rate (*Int_eur*)	ECB Nominal interest rate	
Interest rates differential (*Diff_int*)	Quarterly Central Banks' Nominal Interest rates differential (Euro-USD)	$Int\_eur_t - Int\_us_t$
Inflation rate differential (*Diff_infl*)	Quarterly Inflation rates differential (Euro Area-US)	$Infl\_eur_t - Infl\_us_t$
Output Gap differential (*Diff_ogap*)	Quarterly Output gap differential (Euro Area-US)	$Ogap\_eur_t - Ogap\_us_t$
U.S. – E.U. trade balance variation (*Tr_bal*)	Percentage Quarterly variation of the US-EU trade balance	$\left(\frac{Trade\ balance_t}{Trade\ balance_{t-1}}\right) - 1$
Euribor Rate (*Ebor*)	3 months Euribor rate	
Libor rate (*Libor*)	3 months Libor rate	
Target inflation – actual inflation differential (D*Target_infl _us*)	Quarterly differential between the inflation rate and the quarterly inflation rate corresponding to the 2% target – U.S.	$Target\_infl - Infl\_us_t$ $Target\ infl = 0.00496293$
Target inflation – actual inflation differential (D*Target_infl _eur*)	Quarterly differential between the inflation rate and the quarterly inflation rate corresponding to the 2% target – Euro Area	$Target\_infl - Infl\_eur_t$ $Target\ infl = 0.00496293$
CPIs differential (*Diff_cpi*)	Quarterly CPIs differential between the Euro Area and the US.	$Cpi\_eur - Cpi\_us$
M3 Growth rate differential (*Diff_M3*)	Quarterly differential between the growth rates of M3 Euro and M3 USD aggregates	$M3\_eur - M3\_usd$

All data come from FRED - Federal Reserve Bank of St. Louis, except for Exchange rate Expectations, which have as their source Bloomberg. The majority of the variables are obtained as elaborations of FRED data. The target level of inflaton is set by the FED at a 2% yearly rate, the corresponding quarterly inflation rate is used. The target level of inflation is set by the ECB at a 2% yearly rate, the corresponding quarterly inflation rate is used.

Chapter VII - Empirical Analysis

7.1 OLS regression analysis of heterogeneity and methodological choices

To select the pool of regressors that will make part of the VAR and VECM, several OLS linear regressions of the heterogeneity in expectations are run. This allows to select the number of variables considered in the VEC model, thus making the econometric software ready to provide more reliable results. The significance of the variables that provided encouraging results in Menkhoff et al. paper is verified. Differences and similarities with the state of the art in the field are highlighted.

Menkhoff et al. test three main hypothesis behind heterogeneous exchange rate expectations, but focus mainly on the Chartists and Fundamentalists Hypothesis. The Noise Traders view is tested empirically and a list of macroeconomic variables from the literature on exchange rate expectations is considered to check the robustness of the previously mentioned hypotheses.

As in the original paper, variables in absolute value are used in the OLS regressions. As the purpose of this exercise is to study heterogeneity, dispersion regressors with negative values would provide results hard to be interpreted and likely, less relevant. Squared variables have been also considered, but were discarded to reduce the potential exposure of the analysis to outliers. The results are not reported here.

Considering once again the necessary assumptions made about the timing of information availability and the information set of the market's participants, 1-period-lagged variables are adopted as regressors. To clarify the reason of this choice, I provide an example. Agents in time t form their expectations about the exchange rate in time $t+1$, therefore the $expectation_t$ is referred to the exchange rate in $t+1$, and based on the information available at time t. As the expectation is time indexed to the period it predicts, regressing $heterogeneity_t$ on extrapolation requires to use $extrapolation_{t-1}$ as in $t-1$ the information about the fundamentals' value in t is unobservable (Elias 2016).

7.1.1 Testing the C&F and noise traders hypothesis

In the C&F hypothesis, heterogeneity in exchange rate expectations derives from the shift of market operators from the category and trading strategy of fundamentalists, who expect a long run convergence of the exchange rate to its PPP (regression to the fundamental), and the category of chartists, which base their projections on the current trend they observe (trend extrapolation).

The first variable empirically tested, represents the *regression to the fundamental* i.e. fundamentalists' expectation formation mechanism. The so defined *Regression Term* is calculated as the differential between the quarterly Euro-USD exchange rate and the annual Purchasing Power Parity index, namely.

$$regression\ term_t = Exr14_{t\ (quarterly)} - PPP_{t\ (yearly)}$$

The second variable tested is the extrapolation term representing the current trend extrapolation activity of chartists. As they extrapolate the current trend in the exchange rate, a possible dominance of chartists in the market should reduce the heterogeneity as a reaction to steep variations of the former.

The extrapolation term is computed as the quarterly percentage change of the exchange rate between the period $t-1$ and t.

$$extrapolation\ term_t = \left(\frac{Exr14_t}{Exr14_{t-1}}\right) - 1$$

The third variable tested is the Risk Premium. In fact, in Noise Trade Models, the existence of the exchange risk premium is linked to higher heterogeneity in expectations. Based on the idea of non-sophisticated traders who trade *"on noise"* and distort uncovered interest parity we expect to observe a gap between the consensus exchange rate expectation and the forward rate. The existence of this gap implies the existence of a larger exchange rate risk premium.

$$Risk\ Premium_t = Forward\ rate_t - Consensus\ expectation_t$$

The significance of the Forward Exchange rate as a regressor of heterogeneity is also tested. Given its relevance in exchange rates expectations' formation, and its link with both the current (spot) exchange rate and the level of interest rates in the two countries it was included in the regression. In models with covered interest rate parity, it reflects the incentive to invest in the foreign country for domestic investors, once accounted for the exchange rate.

$$Forward\ Exchange\ Rate_t = Spot\ Exchange\ Rate_t * \left(\frac{1+\ domestic\ interest\ rate\ (Euro\ area)}{1+foreign\ interest\ rate\ (US)}\right)$$

Once the variables found to be not significant to predict exchange rate expectations' heterogeneity are ruled out, the results below are obtained.

Table 6. Multivariate OLS regression of heterogeneity in exchange rate expectations.

```
. reg hetg  L.regression L.risk_premium L.forward_rate,robust

Linear regression                               Number of obs     =         36
                                                F(3, 32)          =       6.33
                                                Prob > F          =     0.0017
                                                R-squared         =     0.2614
                                                Root MSE          =     .01181
```

hetg	Coefficient	Robust std. err.	t	P>|t|	[95% conf. interval]	
regression						
L1.	-.1167895	.0430366	-2.71	0.011	-.2044521	-.0291269
risk_premium						
L1.	.178633	.057811	3.09	0.004	.0608758	.2963901
forward_rate						
L1.	-.105833	.0464081	-2.28	0.029	-.2003632	-.0113027
_cons	.1817513	.0638447	2.85	0.008	.0517038	.3117988

The main finding consists in the lack of significance of the extrapolation term; therefore the hypothesis of a dominance of chartists in the market is weakened, as no evidence is found for their presence. The data in this case suggests that the trend extrapolation approach may not contribute at all, neither positively or negatively, to the amount of heterogeneity in the agents' expectations. I hypothesize that chartists may be present in the market but representing a small minority.

The regression term is found to be significant, thus suggesting the presence of fundamentalists in the market in the decade I analysed. I find evidence in favour of the claim that a significant part of the market participants were behaving as Fundamentalists. The exchange rate – PPP differential is negatively correlated with the amount of heterogeneity in the expected future exchange rate. This finding suggests that agents may correct their expectations about the Euro-USD exchange rate in the direction of the fundamental value (PPP) after observing a "wider gap" between the two, implying that as the exchange rate moves further from the PPP, agents probably expect in a homogeneous way a return to its value.

While the C&F hypothesis is substantially confirmed, noise traders and uncertainty seemingly contribute to expectation heterogeneity. The risk premium constitutes a significant driver for heterogeneity, and appears to be positively associated with spikes in heterogeneity in exchange rate expectations. As in the literature, the existence of a risk premium may be a consequence of noise traders' activity. If we focus on the actual results, we can just conclude that the risk premium surges in periods of higher uncertainty or misalignment between agents expectations and the forward rate, leading to higher "disagreement" about the future.

Before presenting the results about the role of macroeconomic fundamentals, it can be highlighted the interesting finding of the significance of the forward rate in explaining reductions in the heterogeneity of expectations. While a link with heterogeneity is difficult to justify, we could still argue that its negative correlation with the amount of heterogeneity in exchange rate expectations may be due to symmetries in the intervention of the two central banks, the ECB and the FED, in the last decade. There are different possible reasons for that and here, two hypotheses can be made:

- Higher levels of the forward rates could derive from higher levels of exchange rates: given the correcting behaviour of Fundamentalists, once reached some sensible levels of the exchange rate, a broad agreement about a correction could reduce heterogeneity;
- Higher levels of the forward rate could also derive from a reduction of the "foreign" interest rate by the FED, or by raises of the domestic interest rate by the ECB: in both these cases, asymmetries could provide positive or negative signals about one of the two economies, thus giving more information to orientate expectations (and reducing heterogeneity).

Table 7. Multivariate OLS regression of heterogeneity in exchange rate expectations from Menkhoff et al.

Multivariate OLS-regression of dispersion		
	Euro to US-dollar	
const.	0,027	***
$Hetg_{-1}$	0,423	***
regression	-0,033	***
risk_premium	0,229	***
extrapol	0,249	***
vola.	0,024	
Adj. R^2	0,641	

*The variables are abbreviated as follows: constant (const.), lagged dispersion (Hetg.(−1)), regressive term – i.e. difference between the actual exchange rate and its fair value based upon relative ppp – (regression), risk premium – i.e. the expected (consensus) exchange rate change minus the relative bond rate (risk_premium), current exchange rate extrapolation (extrapol.) and exchange rate volatility – i.e. corresponding 1-month standard-deviation – (vola.). Please note that variables appear in absolute measure. Asterisks refer to the level of significance: *,**,*** to 10, 5 and 1%.*

When comparing the results of the multivariate analysis by Menkhoff et al. to mine, only some of them seem to hold. I find the same negative correlation of the regression term (PPP – exchange rate deviation) with heterogeneity, supporting the Fundamentalists' hypothesis. I also find coherent results about the risk premium: not only it is significant, but also positively correlated with dispersion in agents' expectations.

Although a partial correspondence in the results is obtained, some differences must be highlighted. While the extrapolation term is significant and positively correlated with heterogeneity in the original paper, no evidence of the trend extrapolation behaviour of chartists is present in my analysis: the variable is not significant. It can be argued that the activity of chartists is either not relevant in determining dispersion, or that it has a smaller magnitude in the period I study. Another striking feature of my results is the significance of the absolute value of the forward rate: likely not significant in the original paper, but significantly and negatively correlated with heterogeneity in my results. Nevertheless, it must be highlighted that the coefficient of this last regressor is the smallest. It must also be noted the lower value of the R^2 statistic.

7.1.2 Testing Macroeconomic Fundamentals

Aside from the mentioned differences between the obtained results and the original paper's ones, more arise when testing how several macroeconomic variables impact on heterogeneity.

The list of tested variables is reported in the previous sections.

While the vast majority of the tested macroeconomic variables turn out to be not significantly correlated with heterogeneity, some of them seemingly capture part of the variations of heterogeneity levels over time. I start testing their significance with univariate models, then moving to multivariate regressions. I do not consider variables *in levels,* but quarterly variations in order to check the robustness of the previous findings and the empirical relevance of the literature on news and announcements' effects. Quarterly variations in my analysis aim at mimicking present announcements, and in the case of sharp variations, the effect of news about macroeconomic fundamentals in the literature of exchange rate determination.

Once again, all variables are expressed in absolute value. This choice could be criticized as part of the information gets lost, i.e. whether the variations are negative or positive, but for the purpose of this analysis, they van be more useful if seen as "signals".

After a first screening, several of them result significant and all negatively correlated with heterogeneity when tested individually. An attempt at constructing an OLS linear regression of only macroeconomic variables is made before including them in the previous regression.

The vast majority of the chosen macroeconomic variables seem significant at a first sight. A second OLS regression of heterogeneity is constructed to clear out correlations among the regressors, and restrict the subset of variables that could enter the main regression.

Table 8. Multivariate OLS regression of heterogeneity in exchange rate expectations, only macroeconomic variables admitted

```
. reg hetg L.abs_diff_m3 L.abs_diff_ogap  L.abs_tr_bal L.abs_target_infl_dist_us,rob
> ust

Linear regression                               Number of obs     =         36
                                                F(4, 31)          =       8.70
                                                Prob > F          =     0.0001
                                                R-squared         =     0.4306
                                                Root MSE          =     .01054
```

hetg	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
abs_diff_m3 L1.	-.402863	.0781294	-5.16	0.000	-.5622089	-.2435171
abs_diff_ogap L1.	-.0149026	.0055467	-2.69	0.011	-.0262152	-.0035901
abs_tr_bal L1.	-.0696216	.0200305	-3.48	0.002	-.1104742	-.0287691
abs_target_infl~s L1.	-.0084009	.0035934	-2.34	0.026	-.0157297	-.0010721
_cons	.0659327	.0075612	8.72	0.000	.0505116	.0813537

The research for potential macroeconomic regressors of heterogeneity in exchange rate expectations is restricted to the following variables:

- Euro-USD quarterly differential of M3 growth rates;
- Quarterly Output gap variations' differential (Euro Area-US);
- Percentage Quarterly variation of the US-EU trade balance;
- Quarterly differential between the inflation rate and the target inflation – U.S.

The addition of each of the variables to the main regression is tested, the results are shown in *Table 9*. The measure of the quarterly variation of the trade balance (in absolute value) has a significant impact, and is compatible with the main regression. Both the quarterly differentials between the inflation rate and the target inflation of the US and the Euro-USD quarterly differential of M3 growth rates are suitable to enter the main regression.

The choice between the two variables, incompatible with each other, leads to the exclusion of the differential in the M3 expansion, for both a reason of better explanatory power and higher significance, but also given the possibility to justify the inflation-targeting variable by the Taylor Rule literature (Rossi 2013).
By the Taylor Rule, I argue that movements of the inflation rate away from the target could anticipate uncertainty about FEDs intervention, and thus lead to expectations that are more heterogeneous.
To keep the model as simple and robust as possible, and given the negligible coefficient obtained, this last regressor is also discarded.

Table 9. Extended multivariate OLS regression of heterogeneity in exchange rate expectations: all significant regressors admitted

```
. reg hetg L.regression L.risk_premium L.forward_rate L.abs_tr_bal L.target_infl_dis
> t_us,robust

Linear regression                               Number of obs     =         36
                                                F(5, 30)          =       4.55
                                                Prob > F          =     0.0033
                                                R-squared         =     0.3907
                                                Root MSE          =     .01108
```

hetg	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
regression L1.	-.1717543	.0555727	-3.09	0.004	-.285249	-.0582597
risk_premium L1.	.1910395	.0611642	3.12	0.004	.0661256	.3159534
forward_rate L1.	-.1597684	.0581412	-2.75	0.010	-.2785085	-.0410283
abs_tr_bal L1.	-.0486758	.0198203	-2.46	0.020	-.0891542	-.0081974
target_infl_dis~s L1.	-.0051457	.0025481	-2.02	0.052	-.0103496	.0000582
_cons	.2587064	.081172	3.19	0.003	.0929311	.4244816

A crucial difference with the original paper result can be observed. While Menkhoff et al. find no macroeconomic variable to be significant, I find both the measure of the quarterly variation of the US-EU trade balance and of the US target-observed inflation differential able to capture some of the variation of heterogeneity. In the next sections, I decide to exclude the inflation-target inflation gap as it is not significant at the 5% confidence level, and that its coefficient is negligible.

7.1.3 The OLS Model: the Main Regression

$$heterogeneity_t = \alpha * regression_{t-1} + \beta * risk\ premium_{t-1} + \gamma * forward\ exch.\ rate_{t-1} + \delta * trade\ balance\ variation_{t-1}$$

Table 10. Final multivariate OLS regression of heterogeneity in exchange rate expectations

```
. reg hetg  L.regression L.risk_premium L.forward_rate L.abs_tr_bal,robust

Linear regression                               Number of obs     =         36
                                                F(4, 31)          =       4.34
                                                Prob > F          =     0.0066
                                                R-squared         =     0.3591
                                                Root MSE          =     .01118
```

hetg	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
regression L1.	-.1235734	.0473867	-2.61	0.014	-.2202191	-.0269276
risk_premium L1.	.1814194	.0603455	3.01	0.005	.0583439	.3044948
forward_rate L1.	-.112898	.0492696	-2.29	0.029	-.2133841	-.0124119
abs_tr_bal L1.	-.0449386	.0199875	-2.25	0.032	-.0857033	-.0041739
_cons	.1966433	.0695956	2.83	0.008	.0547022	.3385845

The main regression therefore includes one macroeconomic variable, a term that differentiates my results from the ones of Menkhoff et al. I find the percentage change in the US-EU trade balance, measured quarterly and expressed in absolute terms, to be a significant regressor of heterogeneity in exchange rate expectations. The variable is negatively correlated with heterogeneity, suggesting that steep spikes or falls in the trade balance actually leads to small drops of heterogeneity. I suggest again, that under the predominance of fundamentalists in the sample, we may observe the reduction of disagreement among forecasters especially during times of uncertainty, given their expectation formation mechanism.

To summarize, the non-significance of the extrapolation variable, of the 1 lag heterogeneity, and the significance of the forward rate and trade balance change variable constitute the main differences in the preliminary results.

The pool of potential long term determinants of expectations heterogeneity has so been reduced to 5 quarterly variables, expressed in absolute terms, backed by OLS regressions:

- regression term (PPP – exchange rate differential);
- risk premium (exchange rate risk premium);
- forward rate (3 month forward exchange rate);
- trade balance percentage rate of variation;
- M3 quarterly percentage rate of variation;

Given its relevance in the specific strand of literature I study, I also consider the extrapolation term in the next stages of the analysis.

7.2 Johansen method, VAR and VECM specification

As a second step of the analysis, variables are modelled jointly in a Vector Autoregressive setup. A series of Dickey–Fuller tests are conducted on the heterogeneity variable and its OLS regressors to verify their suitability when used in a Vector Autoregressive Model. In fact, the application of inappropriate econometric techniques to I(1) time series could hamper the robustness and hold of the statistical results, especially in the case of non cointegrated time series. Just running OLS regressions on I(1) time series, could expose me to the risk of running spurious regressions (Zivot and Wang 2003; Hamilton 2020).

Then the Johansen procedure is followed, by checking for the existence of cointegration relationships. In presence of I(1) time series, it allows to obtain statistically meaningful results (Zivot and Wang 2003; Hamilton 2020).

The Johansen procedure for testing cointegration and formulate the VAR and VEC models requires first checking if the time series are stationary.

Dickey–Fuller tests are run for each variable used in the main regression. The test is computed both on levels and first differences variables. Using one and four lags in the auxiliary regression. To account for possible seasonality and given the relevance of yearly dynamics in studying quarterly data, 4 lags appear to be a suitable choice. The ADF test results are summarized as follows:

- Heterogeneity is not stationary, whereas its first differences are, since the null hypothesis is rejected even at the 1% level.
- The fundamentalists' regression term is not stationary, its first differences are, since the null hypothesis is rejected even at the 10% level. The evidence in favour of the time series being I(1) in this case is weaker.
- The exchange rate risk premium is not stationary, its first differences are, since the null hypothesis is rejected even at the 10% level. Also in this case, the evidence in favour of the time series being I(1) not indusputable.
- The forward exchange rate is not stationary, whereas its first differences are, since the null hypothesis is rejected even at the 5% level.
- The trade balance variations' series appears stationary, since the null hypothesis is rejected at the 10% level. Its first differences are indisputably stationary, since the null hypothesis is rejected even at the 1% level.

I generally do not obtain strong results to support the stationarity and absence of unit roots of all the time series I test. For this reason, and to remain adherent to Menkhoff, Rebitzky, and Schröder (2009), cointegration relationships are investigated, following the Johansen method.

It can be hypothesized that the use of variables expressed in absolute value may affect the results, as the transformation could make the time series more stable. I consider the results curious but not concerning.

7.2.1 VECM specification, identification of the number of lags and cointegrating relationships

The first step consists in the VAR(p) model specification, where *p* represents the number of lags considered in the right hand side components. Almost all the variables from the main OLS regression presented in the previous section enter the model; the forward rate, which reveals to be not significant in the VECM, is discarded.

In the formula below, *Het* stands for Heterogeneity, *Reg* for *the PPP-exchange rate differential*, *Risk* for risk premium, *Trbal* for trade balance variation, *Extr* for the extrapolation term, Mvar for the M3 variations' magnitude.

Returning to the notation used in the previous chapters, the VAR(p) can be represented as follows

$$\boldsymbol{Y}_t = v + A_1\boldsymbol{Y}_{t-1} + A_2\boldsymbol{Y}_{t-2} + \cdots + A_p\boldsymbol{Y}_{t-p} + \varepsilon_t \quad t = 1, \dots, T$$

Or alternatively, in extended form:

$$Het_t = v + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{1t}$$

$$Reg_t = \gamma + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{2t}$$

$$Risk_t = \delta + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{3t}$$

$$Extr_t = \vartheta + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{4t}$$

$$Trbal_t = \mu + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{5t}$$

$$Mvar_t = \pi + \sum_{i=1}^{k} c_i\, Het_{t-i} + \sum_{j=1}^{k} d_j\, Reg_{t-j} + \sum_{m=1}^{k} f_m\, Risk_{t-m} + \sum_{l=1}^{k} g_l\, Extr_{t-l} \sum_{p=1}^{k} r_p\, Trbal_{t-p} + \sum_{q=1}^{k} s_q\, Mvar_{t-q} + u_{6t}$$

It can be noted that in this form of the VAR(p), each variable is a function of its lagged values and the lagged values of the other variables. c, d, f, g, r and s represent the weight of the of the variables' past realizations in determining current levels.

Transforming the VAR(p) into the VECM requires taking the difference by Y_{t-1} on both sides, and obtaining a representation *in differences* of the former. While Y_t ($Heterogeneity_t$) is an I(1) process, ΔY_t ($\Delta Heterogeneity_t$) and its lags become I(0).

$$\Delta Het_t = \Phi D_t + \Pi Het_{t-1} + \Gamma_1 \Delta Het_{t-1} + \cdots + \Gamma_{p-1} \Delta Het_{t-p+1} + \varepsilon_t$$

$$or \qquad \Delta Het_t = v + \Pi Het_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Het_{t-i} + \varepsilon_t$$

Where $\Pi = \sum_{j=1}^{j=p} A_j - I_k$ is known as long run impact matrix and $\Gamma_i = -\sum_{j=i+1}^{j=p} A_j \quad k = 1, \ldots, p$ are the short run impact matrices. As it can be seen in *Table 11,* the rank of Π is $0 < r < K$, we can then define $\Pi = \alpha\beta'$, the two *(r x K)* matrices have rank r.

After substituting Π by αβ', the VECM takes the form:

$$\Delta Het_t = v + \alpha\beta' Het_{t-1} + \Gamma_1 \Delta Het_{t-1} + \cdots + \Gamma_{p-1} \Delta Het_{t-p+1} + \varepsilon_t$$

Once determined the optimal number of lags to be considered in the VAR model, and the number of cointegration relationships, the actual VECM specification is defined.

The optimal number of lags to be included in the VAR(p) and then in the VECM specification is found to be equal to two. The selection of the number of lags derives from the computation of the Schwarz's Bayesian information criterion (SBIC) lag order selection statistics, with the addition of one lag to autocorrelation in the VEC residuals (see *Appendix C*).

The second step of the Johansen procedure requires conducting a Likelihood ratio test for the rank of Π once estimated through maximum likelihood. The number of cointegrating equations is correctly identified as one.

Transforming the VAR(1) into the VECM, the number of lags reduces from K to K-1. The choice of using 2 lags is also reinforced by the short length of my data sample, as adding more lags increases the number of regressors in the VECM dramatically, and makes the ratio between the number of equations and the number of parameters to be estimated to shrink. While the choice of using 2 lags allows to estimate short run coefficients, this thesis focuses mainly on the estimation of long run equilibrium relationships.

In the compact form, the VECM specification is the following:

$$\Delta \boldsymbol{Y}_t = \boldsymbol{\sigma} + \sum_{i=1}^{k-1} c_i \, \Delta \boldsymbol{Het}_{t-i} + \sum_{j=1}^{k-1} d_j \, \Delta \boldsymbol{Reg}_{t-j} + \sum_{m=1}^{k-1} f_m \, \Delta \boldsymbol{Risk}_{t-m} + \sum_{l=1}^{k-1} g_l \, \Delta \boldsymbol{Extr}_{t-l} + \sum_{p=1}^{k-1} r_p \, \Delta \boldsymbol{Trbal}_{t-p} + \sum_{q=1}^{k-1} s_q \, \Delta \boldsymbol{Mvar}_{t-q} + \boldsymbol{\lambda} ECT_{t-1} + u_t$$

Where $\Delta \boldsymbol{Y}_t = [\Delta \boldsymbol{Het}_t \ \Delta \boldsymbol{Reg}_t \ \Delta \boldsymbol{Risk}_t \ \Delta \boldsymbol{Extr}_t \ \Delta \boldsymbol{Trbal}_t \ \Delta \boldsymbol{Mvar}_t]'$ is a (6x1) vector of the chosen variables. $\boldsymbol{\lambda}$**,** the coefficient of *ECT*, is the speed of adjustment at which $\boldsymbol{Y}$ returns back to the equilibrium as shocks in the regressors take place or as errors appear.
The term ECT_{t-1} constitutes the error correction term, corresponding to the lagged residuals of the cointegrating regression of the target variable on its regressors. ECT embeds the long run information from the long run cointegration relationship. u_t are the residuals, capturing the stochastic error term.

The restricted VECM can now be specified, with (p-1=k) lags and (r=1) rank. Short run coefficients are not included.

$$\Delta \boldsymbol{Het}_t = \sigma + \boldsymbol{\lambda} ECT_{t-1} + u_t$$

The ECT term again corresponds to the lagged residuals from the long run cointegrating equation:

$$\boldsymbol{Het}_t = \sigma + d_j \boldsymbol{Reg}_t + f_m \boldsymbol{Risk}_t + g_l \boldsymbol{Extr}_t + r_p \boldsymbol{Trbal}_t + s_q \boldsymbol{Mvar}_{t-1} + u_t$$

And takes the form:

$$\boldsymbol{ECT}_{t-1} = [\boldsymbol{Het}_{t-1} - d_j \boldsymbol{Reg}_{t-1} - f_m \boldsymbol{Risk}_{t-1} - g_l \boldsymbol{Extr}_{t-1} - r_p \boldsymbol{Trbal}_{t-1} - s_q \boldsymbol{Mvar}_{t-1}]$$

The model to be estimated is therefore:

$$\Delta \boldsymbol{Het}_t = \sigma + \boldsymbol{\lambda} [\boldsymbol{Het}_{t-1} - d_j \boldsymbol{Reg}_{t-1} - f_m \boldsymbol{Risk}_{t-1} - g_l \boldsymbol{Extr}_{t-1} - r_p \boldsymbol{Trbal}_{t-1} - s_q \boldsymbol{Mvar}_{t-1}] + u_t$$

The estimation of the new model, which now includes 2 lags, requires to follow the same steps described in the previous chapters. The assessment of the rank of the VAR(2) indicates again the existence of a single cointegration relationship, as shown in *Table 11.*

Table 11. Rank selection for the specification of the VEC model with two lags.

```
. vecrank hetg L.regression L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3,la
> g(2) trend(rtrend)

Johansen tests for cointegration
Trend: Restricted                              Number of obs  = 34
Sample: 4 thru 37                              Number of lags =  2
---------------------------------------------------------------------

                                                             Critical
Maximum                                            Trace        value
   rank   Params         LL   Eigenvalue       statistic           5%
      0       42   493.30134            .       125.8503       114.90
      1       54   514.02506      0.70449        84.4028*       87.31
      2       64   531.27881      0.63757        49.8953        62.99
      3       72   542.02835      0.46865        28.3963        42.44
      4       78   548.49404      0.31637        15.4649        25.32
      5       82   554.56883      0.30047         3.3153        12.25
      6       84   556.22647      0.09291
---------------------------------------------------------------------
* selected rank
```

The estimation of the long run coefficients and the adjustment coefficient is presented in the next chapter.

Chapter VIII – Results

8.1 VECM estimation

In this section, the parameters of the multivariate cointegrating VECM are estimated. The model is specified starting from the previously chosen time series. The parameters of interest generally are the ones in the cointegrating equation, the adjustment coefficient $\boldsymbol{\lambda}$, and the short-run coefficients. The model, whose estimates are presented in *Table 13* is characterized by a correctly specified single cointegration relationship and 2 lags. It reveals to be stable, to have stationary and non autocorrelated residuals, and provides long run coefficients comparable with the original results from Menkhoff et al.

Table 12. VECM model estimation of short run coefficients: 2 lags, one single cointegrating equation and a restricted trend included. Only significant coefficients are presented.

```
. vec hetg L.risk_premium L.extrapol L.regression L.abs_diff_m3 L.abs_tr_bal, lag(2)
>  rank(1) trend(rtrend)

Vector error-correction model

Sample: 4 thru 37                              Number of obs     =         34
                                               AIC               =   -27.0603
Log likelihood =  514.0251                     HQIC              =  -26.23357
Det(Sigma_ml)  =  2.98e-21                     SBIC              =  -24.63608

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_hetg               8     .014311   0.5917   36.23245   0.0000
LD_risk_premium      8     .033493   0.2380   7.810459   0.4522
LD_extrapol          8     .025469   0.3172   11.61567   0.1692
LD_regression        8     .033226   0.3599   14.05515   0.0803
LD_abs_diff_m3       8     .011177   0.3706   14.71821   0.0649
LD_abs_tr_bal        8     .089331   0.6450   45.41708   0.0000
----------------------------------------------------------------


-------------------------------------------------------------------------------
                 |  Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+-------------------------------------------------------------
D_hetg           |
            _ce1 |
             L1. |   .4809302   .1478358     3.25   0.001     .1911774     .770683
                 |
            hetg |
             LD. |  -.8979186   .2166373    -4.14   0.000     -1.32252   -.4733174
```

The estimation of the short run determinants of heterogeneity does not provide significant coefficients for the vast majority of the lagged variables, the lagged variation in heterogeneity has a short run negative impact on the present variation. The main implication of this result is that heterogeneity tends to be stationary, with sudden surges and higher levels that are not persistent and are corrected during the following period. The estimated value of the error correction coefficient λ is 0,4893.

Table 13. VECM model estimation of long run coefficients: 2 lags, one single cointegrating equation and a restricted trend.

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	5	88.07112	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
_ce1						
hetg	1	.	.	.	.	.
risk_premium						
L1.	-.7605772	.1417156	-5.37	0.000	-1.038335	-.4828196
extrapol						
L1.	.703059	.1636309	4.30	0.000	.3823483	1.02377
regression						
L1.	-.0770133	.0379493	-2.03	0.042	-.1513926	-.0026339
abs_diff_m3						
L1.	1.171258	.3091794	3.79	0.000	.5652778	1.777239
abs_tr_bal						
L1.	.4100307	.0468834	8.75	0.000	.318141	.5019205
_trend	.0008141	.0003909	2.08	0.037	.000048	.0015801
_cons	-.0838791	.	.	.	.	.

Now considering long run coefficients and the long run cointegrating equation, it can be noted that all the coefficients are significant at the 5% confidence level, therefore all of them impact the long term variations and evolution of the level of heterogeneity.

The resulting cointegrating equation, which enters in the VECM specification takes the form:

$$\boldsymbol{ECT}_{t-1} = [1{,}000\boldsymbol{Het}_{t-1} + 0{,}7605\boldsymbol{Risk}_{t-1} - 0{,}7030\boldsymbol{Extr}_{t-1} + 0{,}0770\boldsymbol{Reg}_{t-1} - 1.1712\boldsymbol{Mvar}_{t-1} - 0{,}4100\boldsymbol{Trbal}_{t-1} + 0.0838 - 0.0008t]$$

And the model:

$$\Delta\boldsymbol{Het}_t = -0{,}8979\Delta\boldsymbol{Het}_{t-1} + 0.4809\boldsymbol{ECT}_{t-1}$$

$$\lambda = 0{,}4809$$

In this formulation, λ represents the speed of error correction or convergence, i.e. the speed at which deviations from the equilibrium are corrected, in this case it's equal to 0,4809.

The main result that emerges is the significant and considerable impact of the extrapolation term on the long run equilibrium, which indicates unexpectedly that the activity of chartists actually should stabilize the market expectations and reduce heterogeneity. If we consider the evidence about the significance of the regression term, which should slightly increase heterogeneity over time, it can be said that this analysis finds encouraging evidence in support of the chartists and fundamentalists hypothesis.
Considering the other variables, a long term positive relation emerges between the risk premium and heterogeneity. As expected, as the risk premium increases, noise traders' activity increases driving up heterogeneity levels. Also the literature about noise trading appears to find supporting evidences in the results of my analysis.
The two selected macroeconomic variables, namely, the differential in the percentage quarterly variation of M3 in the Euro area and the US, and the quarterly percentage variation of the US-EU trade balance are also significant.
The M3 variable enters in the cointegrating equation with a negative coefficient, therefore it can be speculated that as more as the speed at which M3 increases in one of the two "countries" differs significantly from the other, a stabilizing effect emerges in the market's expectations. I speculate it could be due to the *emission* of a clear signal about the future evolution of the exchange rate, thus driving banking institutions' expectations in the same direction, and reducing the amount of dispersion in their expectations.
The same dynamic is observable for the measure of the trade balance variation. The quarterly change in the trade balance has a negative impact on the expectations' dispersion. I suggest that as forecasters observe a consistent improvement or worsening of the trade balance between the two countries, receive once again a signal that independently from its predictive power on the exchange rate, can lead market agents to form expectations that are more homogeneous. In this case, what matters could be the fact that agents tend to react homogeneously to trade balance variations' announcements, independently from their expectations being regressive, extrapolative or mixed.

To sum up, the chosen model specification indicates that based these data, in the interval 2013-2022, heterogeneity in exchange rate expectations among several banking institutions can be explained by several variables. The findings support some of the main strands of literature about heterogeneous exchange rates expectations. The presence of extrapolative expectations lead to decreases in dispersion, thus suggesting that a considerable share of the market to be constituted by trend followers chartists. Weaker evidence is found in favour of the effect of exchange rate deviations from the PPP on heterogeneity, with a slight tendency to increase it, arguably by the emergence of a minority of fundamentalists in a market made of chartists, even if other explanations could be proposed. It is observed that during the last decade, periods of higher uncertainty in which the exchange rate risk premium has been higher, it actually contributed to increase dispersion, maybe due to the entrance of noise traders in the market. I finally find evidence of the stabilizing effect that the asymmetric and sudden variation of macroeconomic variables such as the M3 monetary aggregate in the US and in the Euro Area, could have led to a convergence in the expected value or direction of variation of the exchange rate. A similar finding, but harder to explain, is obtained for the US-EU trade balance quarterly variation.
Further specifications have also been tested, their results are coherent with the results shown above, for a more complete overview, check *Appendix A and B*. For the preliminary analysis conducted to came up with the final specification, see *Appendix C.*

8.2 Some robustness checks

A *Lagrange multiplier (LM) test for autocorrelation in the residuals* of vector error-correction models is conducted. Autocorrelation is tested up to 5 lags. The null hypothesis of no autocorrelation cannot be rejected, as shown in *Table 14.* The results confirm that the addition of one lag actually let me overcome the autocorrelation issue that affected the previous specification.

Table 14. Veclmar testing of autocorrelation in residuals

```
. veclmar, mlag(5)

   Lagrange-multiplier test
```

lag	chi2	df	Prob > chi2
1	32.4296	36	0.63916
2	43.4123	36	0.18478
3	33.3845	36	0.59363
4	37.5782	36	0.39676
5	39.3873	36	0.32082

```
   H0: no autocorrelation at lag order
```

The second test performed checks for the distribution of the errors. A series of test statistics is computed in order to test the null hypothesis of normally distributed disturbances in the VECM. The test checks for normality, skewness and kurtosis jointly. As among the assumptions that are made applying the Johansen Method there are i.i.d. and normally distributed errors, this test reinforces the robustness of the previous estimates, and indicates the model is not misspecified.

Table 15. Vecnorm Jarque-Bera testing of errors' normality, skewness and kurtosis

```
. vecnorm, jbera

   Jarque-Bera test
```

Equation	chi2	df	Prob > chi2
D_hetg	1.294	2	0.52352
LD_risk_premium	0.443	2	0.80146
LD_extrapol	1.400	2	0.49648
LD_regression	4.300	2	0.11649
LD_abs_diff_m3	0.155	2	0.92559
LD_abs_tr_bal	0.876	2	0.64524
ALL	8.468	12	0.74756

The null hypothesis of normally distributed errors cannot be rejected, confirming that the model is correctly specified. Errors are not skewed, not kurtotic and are normally distributed for each variable and for the complete model.

The last robustness check conducted is a test for the stability of the model and the stationarity of the residuals. The command run in STATA™ is *vecstable*, the eigenvalues are plotted below . The test checks the eigenvalue stability condition in the VECM, necessary to ensure the stationarity of the cointegration relationship and the correct specification of the number of cointegrating relationships.

Following the STATA™ manual, the interpretation of the results must account for the fact that starting from K endogenous variables and r cointegrating vectors, the presence of K-r unit moduli in the companion matrix is expected. The graphical inspection should then check for the remaining moduli to not be *too close to one,* result that could imply a misspecification of the number of cointegrating equations, their nonstationarity or the presence of another common trend (Stata Corp 2022).

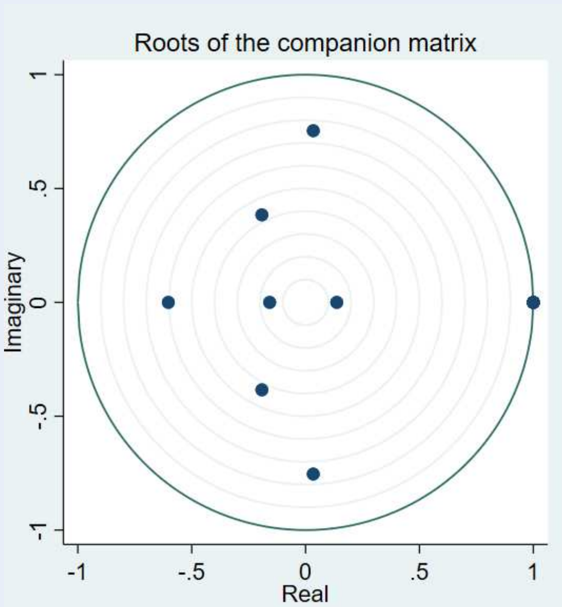
Table 16. Vecstable test for cointegrating equations' stationarity and model stability, graphical representation of the roots of the companion matrix.

```
vecstable, graph

 Eigenvalue stability condition
```

Eigenvalue	Modulus
1	1
1	1
1	1
1	1
1	1
.03367008 + .7539936*i*	.754745
.03367008 - .7539936*i*	.754745
-.60263	.60263
-.1920682 + .3842726*i*	.429599
-.1920682 - .3842726*i*	.429599
-.1575983	.157598
.1367685	.136768

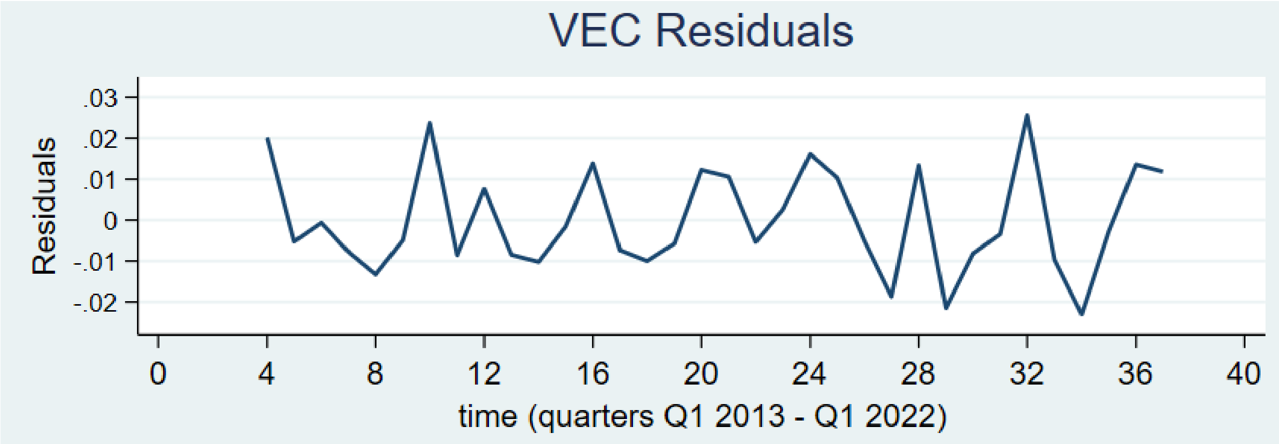
```
 The VECM specification imposes 5 unit modu
```



The results appear encouraging, and supporting the claim that the model is correctly specified, and the cointegrating equation is stationary.

Given the ambiguity of the conditions to reject the hypothesis of stationarity of the cointegration relationship and therefore of the stability of the model, a visual inspection of the residuals' is carried out (see *Figure 5*). Residuals appear stationary over time, thus confirming the stationarity also of the long term relationship identified in the previous sections. This result was also confirmed by a Dickey-Fuller test on the VEC residuals time series.

Figure 5. VEC residuals plot



8.3 Comparative analysis of the obtained results with the original paper

In this section are recalled the original results from Menkhoff, Rebitzky, and Schröder (2009). The main differences and similarities with my results are highlighted. I concentrate on the long run relationship given the scarcity of significant short run coefficients relative to the chosen variables. Considering the original paper's results *(Table 17)*, the cointegrating equation for heterogeneity takes the form:

$$\boldsymbol{ECT}_{t-1} = [1{,}000\ \boldsymbol{disp}_{t-1} + 0{,}298\boldsymbol{risk}_{t-1} - 0{,}066\boldsymbol{pppdev}_{t-1} + 0{,}710\boldsymbol{extrapol}_{t-1} + 0{,}048]$$

And λ, the speed of error correction or convergence, i.e. the speed at which deviations from the equilibrium are corrected, is equal to 0,421.

Just comparing those results with the ones obtained in this analysis, the most striking difference is in the number of variables considered, while the authors do not find any macroeconomic variable to be a significant determinant of expectations' heterogeneity, two of them are included in my model specification. Another considerable difference is observed in the sign of the long run coefficients for the three variables present in both the original and my model. The results obtained in this analysis are recalled below.

$$\boldsymbol{ECT}_{t-1} = [1{,}000\boldsymbol{Het}_{t-1} + 0{,}7605\boldsymbol{Risk}_{t-1} - 0{,}7030\boldsymbol{Extr}_{t-1} + 0{,}0770\boldsymbol{Reg}_{t-1} - 1.1712\boldsymbol{Mvar}_{t-1} - 0{,}4100\boldsymbol{Trbal}_{t-1} + 0.0838 - 0.0008t]$$

$$\lambda = 0{,}4809$$

The risk premium, the extrapolation term, and the PPP deviation term (i.e. the regressive term) are significant. In both results, the risk premium emerges as a positive determinant of heterogeneity, with an higher magnitude in my analysis. It is confirmed that the risk premium is a long run determinant of heterogeneity surges, and therefore that noise and noise traders may affect the dispersion of expectations.
The Noise Traders' hypothesis is supported by evidence in both analyses.

The first difference emerges in the regression term, as the original analysis finds it to be a variable decreasing heterogeneity, while mine finds it to actually generate slightly more heterogeneous expectations.

A potential explanation for this result may be found in a different market (and or sample) composition, while my findings could suggest that fundamentalists have likely constituted a minority in the last decade, this could not be the case for the 1991-2006 period. It can be argued that a different market composition could be derived from the different trend in the Euro-USD exchange rate observed during the two periods. In the previous decades, characterized by a more volatile exchange rate, fundamentalist expectations may have been more popular among forecasters who homogeneously expected to observe long run corrections to the PPP value (see *Figure 4 and 5*).

Another crucial difference can be observed in the effect that the extrapolation variable had in the two periods. As it emerges comparing the long run cointegrating equations, in Menkhoff, Rebitzky, and Schröder (2009), the extrapolation term emerged as a force fostering dispersion in expectations, indicating an increase in disagreement among forecasters, the opposite result emerged in my analysis. I speculate again that a different market composition could be responsible for this observed dynamics, as a fundamentalists' prevalence in the past could be associated with extrapolation pushing heterogeneity.

Overall, I confirm the relevance of the Chartists and Fundamentalists hypothesis as in both my analysis and Menkhoff et al., the regression term of fundamentalists' regressive expectations and the extrapolation term of chartists' trend-extrapolation expectations are significant.

Table 17. Original VECM model estimation and coefficients from Menkhoff et al. (2009).

The unrestricted VEC model for the US-dollar $\Delta \mathbf{x}_{1,t} = \boldsymbol{\theta}' \cdot \Delta x_{2,t} + \Gamma_{1.1} \cdot \Delta \mathbf{x}_{1,t-1} + \boldsymbol{\alpha} \cdot \boldsymbol{\beta}' \cdot x_{t-1} + \boldsymbol{\varepsilon}_t$ with $\{\mathbf{x}_t\} = \{\mathbf{x}_{1,t}, x_{2,t}\}$ and $\{\boldsymbol{\alpha}\} = \{\boldsymbol{\alpha}_1, 0\}$ with $\boldsymbol{\varepsilon}_t \sim N_p(0, \Sigma)$.

	$disp._{(-1)}$	$risk_{(-1)}$	$ppp\text{-}dev._{(-1)}$	$extrapol_{(-1)}$	const.
Cointegration equation					
$\beta'_{(1)}$	1.000***=	0.298***	−0.066**	0.710***	0.048***

	Δdisp.	Δrisk	Δppp-dev.
Error-correction equations			
$\alpha_{(1)}$	−0.421***	−0.114	−0.309**
[t-value]	[−5.310]	[−1.096]	[−2.041]
$\Delta disp._{(-1)}$	−0.123	−0.021	0.190
[t-value]	[−1.318]	[−0.175]	[1.061]
$\Delta risk_{(-1)}$	0.030	0.002	−0.411***
[t-value]	[0.388]	[0.022]	[−2.828]
$\Delta ppp\text{-}dev_{(-1)}$	0.053	0.041	0.264***
[t-value]	[1.372]	[0.818]	[3.582]
$\Delta extrapol._{(0)}$	0.309***	0.130**	0.015
[t-value]	[6.718]	[2.163]	[0.166]
$\Delta extrapol._{(-1)}$	−0.018	−0.078	−0.060
[t-value]	[−0.341]	[−1.129]	[−0.589]
Adj. R^2	0.289	0.030	0.131
Sum $resid^2$	0.021	0.037	0.078

*Notes: The variables are calculated in absolute values and are abbreviated as follows: Dispersion (disp.), risk premium (risk), regressive term (ppp-dev.) – i.e. current exchange rate minus fair value upon the relative ppp concept using CPI data – as well as 1-month exchange rate extrapolation (extrapol.). Based upon calculated t-values, corresponding cointegration parameters are highly significant. Nevertheless, since the test-statistics are not valid, they are limited to providing rough indications about the significances, which is why we do not represent them. Asterisks refer to the regressors' level of significance in the short-term relations: *,**,*** to 10, 5 and 1%.*

To sum up, this analysis confirms the relevance of the C&F and of the Noise Traders hypotheses when aiming at explaining the determinants of heterogeneity in exchange rates expectations. I generally confirm the robustness of Menkhoff et al. results, all the variables they selected are found to be significant when studying exchange rates over a different period and a different sample of market agents. I also find evidence of the relevance of macroeconomic variables in determining surges in heterogeneity as the announcements and role of news literature suggest.

For how much my results confirm the robustness of the original analysis results, it must be highlighted the considerable difference in the relation that some of the variables used in both models have with the amount of disagreement among market participants. In order to provide a critical and transparent analysis of my results, I devote some time considering which factors could have led to the difference in results in the following section.

8.4 Main Criticisms about the methods and the results

While this exercise was realized with the best intentions of replicating the original paper accurately, some methodological differences and some choices may hinder the robustness and credibility of the results.

First, the role of announcements has not been accurately studied, as this analysis does not consider the timing of news arrival. A determinant of heterogeneous expectations left out of the analysis could distort estimates and cause the very common omitted variable bias (Beckmann and Czudaj 2017) and deserve to be investigate in more depth.
The analysis of the seasonal effects of periodic announcements, especially from central banks, and the impact of specific events on exchange rates and the agents' expectations are two neglected aspects in this exercise which should be considered in future studies (Frankel 1979).

The use of the PPP yearly value as the exchange rate fundamental could generate distortions given that the analysis considers quarterly observations and data. Some criticism about the use of a yearly fundamental when studying quarterly values could be justified. The use of the PPP especially when making predictions over time horizons shorter than 6 months also lacks of significant supporting evidence in the literature (Kilian and Taylor 2003; Frankel and Froot 1990).
Another neglected variable that could be associated with higher dispersion in exchange rate expectations is the trade volume. Frankel and Froot present three hypotheses about trade volume and heterogeneous expectations: higher liquidity of the markets could reduce volatility and improve the efficiency of information processing; trade volume could be associated with the amount of noise in the market; foreign exchange markets would have to be more efficient to make it irrelevant (Frankel and Froot 1990). Given the suggested correlation with the risk premium, the latter could have partially captured the effect of trade volume on heterogeneity.

Further criticisms can be made given the inability to present the VECM short run coefficients, generally suggesting for the purpose of this kind of analysis the need to consider longer time series, or as in the original paper, monthly observations that provide a "denser" sampling of the chosen period.

The construction of the extrapolation variable, which should be associated with stronger chartists' trend extrapolation, can also be criticized. In the original paper, the variable is constructed as the last observed monthly variation, while in my analysis it is constituted by the variation of the exchange rate over the previous quarter with reference to the present. While the variable is constructed in a similar way, it cannot be denied that a monthly variation could have a different relevance in predicting short term variations of a variable such as the exchange rate, characterized by consistent monthly variations. To synthetize, quarters could be a too long period to consider to extrapolate current trends.

Concluding this section, a more in depth study of the potential seasonal dynamics present in the phenomena of exchange rate expectations' heterogeneity would have been recommendable.

Conclusions

Exchange Rate Determination and prediction have been for a long time one of the unsolved puzzles in macroeconomics, as forecasting exchange rates' movements remains a challenging task.

In this framework, expectations about future exchange rate changes are recognized as a key determinant of asset demands and therefore of the current exchange rate. The purpose of this thesis has been to shed some light about expectations' formation mechanisms in the foreign exchange market and identify the dynamics which influence the amount of disagreement among forecasters by using a new dataset.

While during the previous decades many models have been developed over rational, homogeneous agents, a flourishing strand of literature advocated and pursued the development of agent-based models with heterogeneous agents. Among them, the Chartists and Fundamentalists hypothesis is one of the most popular in the literature.

I take as reference the 2009 paper " Heterogeneity in exchange rate expectations: Evidence on the chartist–fundamentalist approach" by Menkhoff, Rebitzky, and Schröder, and replicate their analysis of the determinants of expectations' dispersion, studying the period 2013-2022, considering a sample of around 100 international banking institutions' exchange rate forecasts.

The three main hypotheses tested by the original paper are respectively the *Fundamentalists and Chartists hypothesis*, where beliefs in mean reversion and trend following coexist in the market in presence of deviations from the fundamentals; the *Noise traders' hypothesis* and the *Role of news and announcements* hypothesis.

The three aforementioned hypotheses have been tested through a wide set of predictors. The analysis was based on a battery of OLS regressions to first reduce the set of variables analysed to the significant regressors of heterogeneity in exchange rate expectations, and specify a Vector Error Correction Model to identify a long run relationship among expectations' dispersion and the set of selected variables.

I find that higher levels of disagreement among the Euro-USD exchange rate forecasters are associated with periods of higher risk premium, uncertainty, and arguably noise trading. Furthermore, with a smaller magnitude, with the widening of the gap between the exchange rate and the PPP fundamental. Three variables are associated with reductions in heterogeneity. The extrapolation term suggests the presence of a large proportion of chartists in the market, at least in the last decade. Two macroeconomic variables, the US-EU trade balance variations and the differential between the USD and the Euro M3 growth rates, also appear to reduce heterogeneity, indirectly likely providing signals to the market about future developments of the exchange rate.

This analysis finds evidence supporting all the three strands of literature tested, confirming the relevance of the Chartists and Fundamentalists model of heterogeneous agents, of Noise Traders models and of *the Role of news* concerning macroeconomic fundamentals for exchange rates' forecasters. Menkhoff et al. results only find partial confirmation, it can be argued that in different periods, the changing market composition and the different amount of exchange rate volatility observed, may justify the contrasting results obtained.

I conclude suggesting for future analyses to focus on longer periods and shorter forecasting horizons, to account for seasonal effects and specific, accurately chosen, events or historical announcements. Another suggestion is to back the empirical quantitative results with surveys that can provide an indication of the (declared) nature of the forecasting methodologies and relevant references used by market participants.

References

Alfarano, Simone, and Thomas Lux. 2007. "A Noise Trader Model as a Generator of Apparent Financial Power Laws and Long Term Memory." *Macroeconomic Dynamics* 11: 80–101. https://doi.org/10.1017/S1365100506060299.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Clara Vega. 2003. "Micro Effects of Macro Announcements: Real-Time Price Discovery in Foreign Exchange." *American Economic Review* 93 (1): 38–62. https://doi.org/10.1257/000282803321455151.

Bacchetta, Philippe, and Eric Van Wincoop. 2006. "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review* 96 (3): 552–76. https://doi.org/10.1257/aer.96.3.552.

Bacchetta, Philippe, and Wincoop. 2006. "Incomplete Information Processing: A Solution to the Forward Discount Puzzle." *Federal Reserve Bank of San Francisco, Working Paper Series*, 1.000-53.000. https://doi.org/10.24148/wp2006-35.

Baumann, Michael Heinrich, Michaela Baumann, and Alexander Erler. 2019. "Limitations of Stabilizing Effects of Fundamentalists: Facing Positive Feedback Traders." *Economics* 13: 1–25. https://doi.org/10.5018/economics-ejournal.ja.2019-44.

Beckmann, Joscha, and Robert Czudaj. 2017. "Exchange Rate Expectations and Economic Policy Uncertainty." *European Journal of Political Economy* 47: 148–62. https://doi.org/10.1016/j.ejpoleco.2016.06.003.

Berardi, Michele. 2011. "Fundamentalists vs. Chartists: Learning and Predictor Choice Dynamics." *Journal of Economic Dynamics and Control* 35 (5): 776–92. https://doi.org/10.1016/j.jedc.2011.01.010.

Boswijk, H. Peter, Cars H. Hommes, and Sebastiano Manzan. 2007. "Behavioral Heterogeneity in Stock Prices." *Journal of Economic Dynamics and Control* 31 (6): 1938–70. https://doi.org/10.1016/j.jedc.2007.01.001.

Brock, William A., and Cars H. Hommes. 1998. "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model." *Journal of Economic Dynamics and Control* 22 (8–9): 1235–74. https://doi.org/10.1016/s0165-1889(98)00011-6.

Capistràn, Carlos, and Allan Timmermann. 2009. "Disagreement and Biases in Inflation Expectations." *Journal of Money, Credit and Banking* 41 (2–3): 365–96. https://doi.org/10.1111/j.1538-4616.2009.00209.x.

Chiarella, Carl, Roberto Dieci, and Laura Gardini. 2002. "Speculative Behaviour and Complex Asset Price Dynamics: A Global Analysis." *Journal of Economic Behavior and Organization* 49 (2): 173–97. https://doi.org/10.1016/S0167-2681(02)00066-5.

De Grauwe, Paul, and Hans Dewachter. 1993. "A Chaotic Model of the Exchange Rate: The Role of Fundamentalists and Chartists." *Open Economies Review* 4 (4): 351–79. https://doi.org/10.1007/BF01011136.

De Grauwe, Paul, and Marianna Grimaldi. 2006. "Exchange Rate Puzzles: A Tale of Switching Attractors." *European Economic Review* 50 (1): 1–33. https://doi.org/10.1016/j.euroecorev.2004.07.005.

Elias, Christopher J. 2016. "A Heterogeneous Agent Exchange Rate Model with Speculators and Non-Speculators." *Journal of Macroeconomics* 49: 203–23. https://doi.org/10.1016/j.jmacro.2016.07.006.

Engel, Charles, Nelson C. Mark, and Kenneth D. West. 2007. "Exchange Rate Models Are Not as Bad as You Think." *NBER Macroeconomics Annual* 22: 381–442. https://doi.org/10.1086/ma.22.25554969.

Engel, Charles, and Kenneth D. West. 2005. "Exchange Rates and Fundamentals." *Journal of Political Economy* 113 (3): 485–517. https://doi.org/10.1086/429137.

Evans, George W., and Seppo Honkapohja. 2012. *Learning and Expectations in Macroeconomics*. *Learning and Expectations in Macroeconomics*. https://doi.org/10.1515/9781400824267.

Flood, Robert P., and Andrew K. Rose. 1996. "Fixes: Of the Forward Discount Puzzle." *The Review of Economics and Statistics* 78 (4): 748. https://doi.org/10.2307/2109962.

Franke, Reiner, and Frank Westerhoff. 2012. "Structural Stochastic Volatility in Asset Pricing Dynamics: Estimation and Model Contest." *Journal of Economic Dynamics and Control* 36 (8): 1193–1211. https://doi.org/10.1016/j.jedc.2011.10.004.

Frankel, J A, and K A Froot. 1990. "Chartists, Fundamentalists and the Demand for Dollars." *In: Courakis, A.S., Taylor, M.P. (Eds.), Private Behavior and Government Policy in Interdependent Economies*.

Frankel, Jeffrey A. 1979. "On the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differentials." *American Economic Review* 69 (4): 610–22.

Frankel, Jeffrey A., and Kenneth a. Froot. 1990. "Chartists, Fundamentalists, and Trading in the Foreign Exchange Market." *American Economic Review* 80 (2): 181–85.

Frankel, Jeffrey A., and Andrew K. Rose. 1995. "Empirical Research on Nominal Exchange Rates." *Handbook of International Economics* 3 (C): 1689–1729. https://doi.org/10.1016/S1573-4404(05)80013-9.

Froot, Kenneth A., and Jeffrey A. Frankel. 1989. "Forward Discount Bias: Is It an Exchange Risk Premium?" *Quarterly Journal of Economics* 104 (1): 139–61. https://doi.org/10.2307/2937838.

Gaunersdorfer, Andrea, Cars H. Hommes, and Florian O.O. Wagener. 2008. "Bifurcation Routes to Volatility Clustering under Evolutionary Learning." *Journal of Economic Behavior and Organization* 67 (1): 27–47. https://doi.org/10.1016/j.jebo.2007.07.004.

Giovannini, Alberto. 1988. "Exchange Rates and Traded Goods Prices." *Journal of International Economics* 24 (1–2): 45–68. https://doi.org/10.1016/0022-1996(88)90021-9.

Hamilton, James D. 2020. *Time Series Analysis*. Princeton University Press. https://doi.org/10.2307/j.ctv14jx6sm.

Ito, Takatoshi. 2020. "Foreign Exchange Rate Expectations: Micro Survey Data." *International Finance* 80 (3): 380–94. https://doi.org/10.4324/9780203168219-32.

Jeanne, Olivier, and Andrew K. Rose. 2002. "Noise Trading and Exchange Rate Regimes." *Quarterly Journal of Economics* 117 (2): 537–69. https://doi.org/10.1162/003355302753650328.

Jeffrey A . Frankel and Kenneth A . Froot. 2016. "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations." *The American Economic Review , Vol . 77 , No . 1 (Mar ., 1987),* 77 (1): 133–53.

Kilian, Lutz, and Mark P. Taylor. 2003. "Why Is It Difficult to Beat the Random Walk Forecast of Exchange Rates?" *Journal of International Economics* 60 (1): 85–107. https://doi.org/10.1016/S0022-1996(02)00060-0.

Levin, Jay H. 1997. "Chartists, Fundamentalists and Exchange Rate Dynamics." *International Journal of Finance and Economics* 2 (4): 281–90. https://doi.org/10.1002/(SICI)1099-1158(199710)2:4<281::AID-JFE56>3.0.CO;2-M.

Lux, Thomas. 1998. "The Socio-Economic Dynamics of Speculative Markets: Interacting Agents, Chaos, and the Fat Tails of Return Distributions." *Journal of Economic Behavior and Organization* 33 (2): 143–65. https://doi.org/10.1016/s0167-2681(97)00088-7.

Macdonald, Ronald, and Ian W. Marsh. 1996. "Currency Forecasters Are Heterogeneous: Confirmation and Consequences." *Journal of International Money and Finance* 15 (5): 665–85. https://doi.org/10.1016/0261-5606(96)00030-7.

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers. 2003. “Disagreement about Inflation Expectations.” *NBER Macroeconomics Annual* 18: 209–48. https://doi.org/10.1086/ma.18.3585256.

Manzan, Sebastiano, and Frank Westerhoff. 2005. “Representativeness of News and Exchange Rate Dynamics.” *Journal of Economic Dynamics and Control* 29 (4 SPEC. ISS.): 677–89. https://doi.org/10.1016/j.jedc.2003.08.008.

Manzan, Sebastiano, and Frank H. Westerhoff. 2007. “Heterogeneous Expectations, Exchange Rate Dynamics and Predictability.” *Journal of Economic Behavior and Organization* 64 (1): 111–28. https://doi.org/10.1016/j.jebo.2006.08.005.

Mark, Nelson C., and Yangru Wu. 1998. “Rethinking Deviations from Uncovered Interest Parity: The Role of Covariance Risk and Noise.” *Economic Journal* 108 (451): 1686–1706. https://doi.org/10.1111/1468-0297.00367.

Menkhoff, Lukas, Rafael R. Rebitzky, and Michael Schröder. 2009. “Heterogeneity in Exchange Rate Expectations: Evidence on the Chartist-Fundamentalist Approach.” *Journal of Economic Behavior and Organization* 70 (1–2): 241–52. https://doi.org/10.1016/j.jebo.2009.01.007.

Menkhoff, Lukas, and Mark P. Taylor. 2007. “The Obstinate Passion of Foreign Exchange Professionals: Technical Analysis.” *Journal of Economic Literature* 45 (4): 936–72. https://doi.org/10.1257/jel.45.4.936.

Molodtsova, Tanya, and David H. Papell. 2009. “Out-of-Sample Exchange Rate Predictability with Taylor Rule Fundamentals.” *Journal of International Economics* 77 (2): 167–80. https://doi.org/10.1016/j.jinteco.2008.11.001.

Prat, Georges, and Remzi Uctum. 2013. “Modeling the Horizon-Dependent Ex-Ante Risk Premium in the Foreign Exchange Market: Evidence from Survey Data.” *Journal of International Financial Markets, Institutions and Money* 23 (1): 33–54. https://doi.org/10.1016/j.intfin.2012.09.005.

Rangvid, Jesper, Maik Schmeling, and Andreas Schrimpf. 2013. “What Do Professional Forecasters’ Stock Market Expectations Tell Us about Herding, Information Extraction and Beauty Contests?” *Journal of Empirical Finance* 20 (1): 109–29. https://doi.org/10.1016/j.jempfin.2012.11.004.

Rossi, Barbara. 2013. “Exchange Rate Predictability.” *Journal of Economic Literature* 51 (4): 1063–1119. https://doi.org/10.1257/jel.51.4.1063.

Stata Corp. 2022. “Stata Manual.” *Stata Journal*, 1–4.

Taylor, Mark P., and Helen Allen. 1992. “The Use of Technical Analysis in the Foreign Exchange Market.” *Journal of International Money and Finance* 11 (3): 304–14. https://doi.org/10.1016/0261-5606(92)90048-3.

Tedeschi, Gabriele, Giulia Iori, and Mauro Gallegati. 2012. “Herding Effects in Order Driven Markets: The Rise and Fall of Gurus.” *Journal of Economic Behavior and Organization* 81 (1): 82–96. https://doi.org/10.1016/j.jebo.2011.09.006.

Westerhoff, Frank H. 2003. “Expectations Driven Distortions in the Foreign Exchange Market.” *Journal of Economic Behavior and Organization* 51 (3): 389–412. https://doi.org/10.1016/S0167-2681(02)00151-8.

Westerhoff, Frank H. 2004. “Multiasset Market Dynamics.” *Macroeconomic Dynamics* 8 (5): 596–616. https://doi.org/10.1017/s1365100504040040.

Westerhoff, Frank H., and Stefan Reitz. 2003. “Nonlinearities and Cyclical Behavior: The Role of Chartists and Fundamentalists.” *Studies in Nonlinear Dynamics and Econometrics* 7 (4). https://doi.org/10.2202/1558-3708.1125.

Wieland, Cristian, and Frank H. Westerhoff. 2005. “Exchange Rate Dynamics, Central Bank Interventions and Chaos Control Methods." *Journal of Economic Behavior and Organization* 58 (1): 117–32. https://doi.org/10.1016/j.jebo.2003.12.002.

Williamson, John. 2009. "Exchange Rate Economics." *Open Economies Review* 20 (1): 123–46. https://doi.org/10.1007/s11079-008-9091-7.

Zivot, Eric, and Jiahui Wang. 2003. "Cointegration." In *Modeling Financial Time Series with S-Plus®*, 415–60. https://doi.org/10.1007/978-0-387-21763-5_12.

Appendix A

Alternative VECM specification with four variables, the regression term has been excluded from the model as it had the worst significance level and a negligible coefficient. The signs of the long run coefficients in the long run cointegrating equations are coherent with the ones estimated in *Table 13*, thus confirming the robustness of the results.

This specification is characterized by the following results:

- Varsoc: 1 lag
- Vecrank: rank 4, updated to 2 after the addition of 1 lag
- Veclmar: autocorrelation in residuals at lag 2, then corrected with the addition of an additional lag
- Vecnorm: errors are normally distributed
- Vecstable: the model appears stable, but some roots are near the unit circle

Table 18. Cointegrating equation coefficients' estimation: alternative VECM specification, four variables, regression term excluded, 2 lags after the addition of 1 lag

```
. vec hetg L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3, lag(2) rank(1) tre
> nd(rtrend) noetable

Vector error-correction model

Sample: 4 thru 37                               Number of obs     =         34
                                                AIC               =  -23.22131
Log likelihood =  434.7623                      HQIC              =  -22.60892
Det(Sigma_ml)  =  5.38e-18                      SBIC              =   -21.4256

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  4    66.6458   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
_ce1         |
        hetg |          1          .        .       .            .           .
             |
    extrapol |
         L1. |   2.665158   1.102397     2.42   0.016     .5044998    4.825817
             |
risk_premium |
         L1. |  -3.157026   .9171365    -3.44   0.001    -4.954581   -1.359472
             |
  abs_tr_bal |
         L1. |   2.450935   .3045149     8.05   0.000     1.854097    3.047773
             |
 abs_diff_m3 |
         L1. |   7.602065   2.039283     3.73   0.000     3.605144    11.59899
             |
      _trend |  -.0008309   .0017009    -0.49   0.625    -.0041647    .0025029
       _cons |   -.323682          .        .       .            .           .
------------------------------------------------------------------------------
```

Table 19. Residuals autocorrelation testing: alternative VECM specification, four variables, regression term excluded, 2 lags after the addition of 1 lag

```
. veclmar
```

Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	30.0281	25	0.22324
2	35.6526	25	0.07704

H0: no autocorrelation at lag order

Table 20. Testing of normality of errors: alternative VECM specification, four variables, regression term excluded, 2 lags after the addition of 1 lag

```
. vecnorm, jbera
```

Jarque-Bera test

Equation	chi2	df	Prob > chi2
D_hetg	1.605	2	0.44832
LD_extrapol	0.672	2	0.71457
LD_risk_premium	2.021	2	0.36408
LD_abs_tr_bal	2.327	2	0.31236
LD_abs_diff_m3	0.806	2	0.66828
ALL	7.431	10	0.68425

Table 21. Testing of the stability of the model and stationarity of the cointegrating equation: alternative VECM specification, four variables, regression term excluded, 2 lags after the addition of 1 lag

```
. vecstable, graph
```

Eigenvalue stability condition

Eigenvalue	Modulus
1	1
1	1
1	1
1	1
-.02794233 + .7560818*i*	.756598
-.02794233 - .7560818*i*	.756598
-.4762863	.476286
-.45796	.45796
-.1353302 + .3694642*i*	.393469
-.1353302 - .3694642*i*	.393469

The VECM specification imposes 4 unit moduli.

Appendix B

Alternative VECM specification with three variables, aside from the regression term, excluded in the first alternative specification, also the quarterly variation of the US-EU trade balance does not enter the model. The signs of the long run coefficients in the long run cointegrating equations are coherent with the ones estimated in *Table 13*, thus confirming the robustness of the previous results.

This specification is characterized by the following:

- Varsoc: 1 lag
- Vecrank: rank 3
- Veclmar: no autocorrelation in the residuals
- Vecnorm: errors are normally distributed
- Vecstable: the model appears stable, roots are not near the unit circle

Table 22. Cointegrating equation coefficients' estimation: alternative VECM specification, three variables, regression term and trade balance variation excluded, 1 lag, imposed rank 1

```
. vec hetg L.extrapol L.risk_premium L.abs_diff_m3, lag(1) rank(1) trend(rtrend) noe
> table

Vector error-correction model

Sample: 3 thru 37                                Number of obs     =         35
                                                 AIC               =  -21.07773
Log likelihood =  380.8602                       HQIC              =  -20.89364
Det(Sigma_ml)  =  4.15e-15                       SBIC              =  -20.54446

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  3   18.37461   0.0004
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
_ce1         |
        hetg |          1          .        .       .            .           .
             |
    extrapol |
         L1. |   .2153633   .0745128     2.89   0.004     .0693208    .3614057
             |
risk_premium |
         L1. |  -.1680233   .0628955    -2.67   0.008    -.2912963   -.0447503
             |
 abs_diff_m3 |
         L1. |   .3143458   .1187296     2.65   0.008       .08164    .5470515
             |
      _trend |   .0003618   .0001387     2.61   0.009     .0000899    .0006337
       _cons |  -.0493306          .        .       .            .           .
------------------------------------------------------------------------------
```

Table 23. Residuals autocorrelation testing: alternative VECM specification, three variables, regression term and trade balance variation excluded, 1 lag, imposed rank 1

```
. veclmar

   Lagrange-multiplier test
```

lag	chi2	df	Prob > chi2
1	18.4994	16	0.29548
2	22.3407	16	0.13253

```
   H0: no autocorrelation at lag order
```

Table 24. Testing of normality of errors: alternative VECM specification, three variables, regression term and trade balance variation excluded, 1 lag, imposed rank 1

```
. vecnorm, jbera

   Jarque-Bera test
```

Equation	chi2	df	Prob > chi2
D_hetg	0.581	2	0.74784
LD_extrapol	3.056	2	0.21697
LD_risk_premium	0.872	2	0.64661
LD_abs_diff_m3	0.561	2	0.75554
ALL	5.070	8	0.75009

Table 25. Testing of the stability of the model and stationarity of the cointegrating equation: alternative VECM specification, three variables, regression term and trade balance variation excluded, 1 lag, imposed rank 1

```
. vecstable, graph

   Eigenvalue stability condition
```

Eigenvalue	Modulus
1	1
1	1
1	1
-.3492862	.349286

```
   The VECM specification imposes 3 unit moduli.
```

Appendix C

In this appendix the initial VECM specification is presented. First it is defined the number of p lags of the VAR(p) model behind the VECM. As the number of lags could also be chosen by an educated guess, I run the test in order to pick between 1 and 4 lags. The Schwarz's Bayesian information criterion (SBIC) lag order selection statistics is computed.

Another consideration that must be made is that the choice of 4 lags would have increased dramatically the number of regressors and equations, and heavily affected estimates' accuracy. The test selects 1 lag. The number of cointegration relationship in this specification is determined below.

Table 26. Selection of the optimal number of lags for the VAR model.

```
. varsoc hetg L.regression L.extrapol L.risk_premium L.abs_tr_bal  abs_diff_m3

Lag-order selection criteria

   Sample: 6 thru 37                                        Number of obs = 32
```

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	427.265				1.5e-19	-26.3291	-26.238	-26.0542
1	494.552	134.57	36	0.000	2.2e-20*	-28.2845	-27.6468	-26.3607*
2	524.553	60.003	36	0.007	4.1e-20	-27.9096	-26.7253	-24.3368
3	575.551	101.99	36	0.000	3.5e-20	-28.8469	-27.1161	-23.6252
4	647.939	144.78*	36	0.000	3.0e-20	-31.1212*	-28.8437*	-24.2505

```
   * optimal lag
   Endogenous: hetg L.regression L.extrapol L.risk_premium L.abs_tr_bal
                abs_diff_m3
    Exogenous: _cons
```

Table 27. Rank selection for the specification of the VEC model.

```
. vecrank hetg L.regression L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3,la
> g(1) trend(rtrend)

Johansen tests for cointegration
Trend: Restricted                           Number of obs  = 35
Sample: 3 thru 37                           Number of lags =  1
```

Maximum rank	Params	LL	Eigenvalue	Trace statistic	Critical value 5%
0	6	463.22363	.	169.0885	114.90
1	18	492.41836	0.81143	110.6990	87.31
2	28	515.04314	0.72551	65.4495	62.99
3	36	528.6312	0.53997	38.2734*	42.44
4	42	540.0565	0.47945	15.4228	25.32
5	46	546.00739	0.28827	3.5210	12.25
6	48	547.76789	0.09571		

```
* selected rank
```

The number of cointegration relationships is 3, as shown above. This implies Het_t is I(1) with 3 linearly independent cointegrating vectors and *n-3=3* common stochastic trends. Having several I(1) time series in the regression, having one or more linear combinations of the time series which is I(0) guarantees the robustness of the statistical results I obtain.

An issue emerges from the number of cointegrating relationships identified: checking multiple equations implies not being able to estimate all the long run coefficients for each of the heterogeneity determinants, as restrictions on at least one of them would be necessary.

I estimate the parameters of the multivariate cointegrating VECM with 1 lag, but imposing only a single cointegrating relationship to be estimated. The parameters of interest generally are the long-run coefficients and the adjustment coefficient $\boldsymbol{\lambda}$**.**

Table 28. VECM model estimation, long run coefficients' estimation with one single cointegrating equation imposed and one lag.

```
. vec hetg L.risk_premium L.extrapol L.regression L.abs_diff_m3 L.abs_tr_bal, lag(1)
>  rank(1) trend(rtrend) noetable

Vector error-correction model

Sample: 3 thru 37                                Number of obs     =         35
                                                 AIC               =  -27.10962
Log likelihood =  492.4184                       HQIC              =   -26.8335
Det(Sigma_ml)  =  2.43e-20                       SBIC              =  -26.30973

Cointegrating equations

Equation           Parms    chi2     P>chi2
-------------------------------------------
_ce1                  5   112.3913   0.0000
-------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed
------------------------------------------------------------------------------
        beta | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
_ce1         |
        hetg |          1          .        .       .            .           .
             |
risk_premium |
         L1. |  -.2913615    .038355    -7.60   0.000    -.3665359   -.2161871
             |
    extrapol |
         L1. |   .2104458    .046897     4.49   0.000     .1185292    .3023623
             |
  regression |
         L1. |  -.0301445   .0130467    -2.31   0.021    -.0557154   -.0045735
             |
 abs_diff_m3 |
         L1. |   .3696008   .0765731     4.83   0.000     .2195202    .5196813
             |
  abs_tr_bal |
         L1. |   .0663495   .0098789     6.72   0.000     .0469873    .0857117
             |
      _trend |   .0005906   .0001278     4.62   0.000     .0003401    .0008411
       _cons |  -.0526631          .        .       .            .           .
------------------------------------------------------------------------------
```

The resulting cointegrating equation, which enters in the VECM specification takes the form:

$$ECT_{t-1} = [1{,}000\boldsymbol{Het}_{t-1} + 0{,}0301\boldsymbol{Reg}_{t-1} + 0{,}2913\boldsymbol{Risk}_{t-1} - 0{,}2104\boldsymbol{Extr}_{t-1} - 0{,}0663\boldsymbol{Trbal}_{t-1} - 0.3696\boldsymbol{Mvar}_{t-1} + 0.0526 - 0.0005t]$$

And the model:

$$\Delta \boldsymbol{hetg}_t = -.0032328 + 1.096159\boldsymbol{ECT}_{t-1}$$
$$\lambda = 1{,}0961$$

In this formulation, λ represents the speed of error correction or convergence, i.e. the speed at which deviations from the equilibrium are corrected, in this case it's equal to 1,0961.

Robustness check

I check the robustness of the model specified with 1 lag and one cointegrating equation, I control for the autocorrelation in the residuals, I check the distribution of the errors and the stability of the model.

The first test is the test for residuals autocorrelation. A *Lagrange multiplier (LM) test for autocorrelation in the residuals* of vector error-correction models is implemented. Autocorrelation is tested up to 5 lags. The null hypothesis of no autocorrelation is rejected as shown below.

Table 29. Veclmar testing of autocorrelation in residuals

```
. veclmar, mlag(5)

   Lagrange-multiplier test
```

lag	chi2	df	Prob > chi2
1	28.4465	36	0.81091
2	53.1181	36	0.03284
3	34.5028	36	0.53983
4	60.2428	36	0.00687
5	36.4097	36	0.44959

```
   H0: no autocorrelation at lag order
```

The test for residuals' autocorrelation reveals that in this specification, exists some autocorrelation at lag 2 and lag 4. In order to deal with this negative result, I decide to add one lag and repeat the analysis from the beginning. The results are presented in chapter VII and VIII.

Appendix D


```
*HETEROGENEITY IN EXCHANGE RATE EXPECTATIONS

*Import Data
clear all
*Insert file location of the dataset
cd "C:\Users\initiald\Desktop"
use dataset_here

*Sets data as time series: based on 36 quarters (2013-2022) from q1_2013 to q1_2022
tsset time

*PPP stands for Purchasing Power Parity
*EXR stands for exchange rates
*ERE stands for exchange rate expectations

*VARIABLES NECESSARY TO THE ANALYSIS
*FUNDAMENTALISTS' EXPECTATIONS: dppp_exr = (ppp - actual exr)
                             ** used quarterly exr and yearly ppp values
*CHARTISTS EXPECTATIONS: (based on the most recent exr) the variation of the EXR during the previous quarter
*RISK PREMIUM: obtained as the difference between the expected EXR and the forward rate, riskp = (ERE - forward rate)
                             ** the forward rate is computed as (EXR*INTECB/INTFED)

*VARIABLES' LIST
*time: quarters from 1 to 37
*Ere: mean of the  Exchange rate expectations of the next quarter (time+1)
*Hetg: dispersion/heterogeneity of ERE
*Exr14: quarterly Exchange rate
*Dexr14: quarterly % variation of the Exr
*Dppp_exr14: differential between  quarterly exr and yearly ppp
*forward: forward rate = (EXR*INTECB/INTFED)
*Riskp = quarterly risk premium = difference between the expected exchange rate and the forward rate
*Dist_exr_forward = differential between the Exr and the forward rate
*M3_usd = quarterly growth rate of M3 monetari aggregate - USD dollar
*M3_eur = quarterly growth rate of M3 monetari aggregate - Euro
*ggdp_eur = quarterly gdp growth rate - Euro area
*Ggdp_us = quarterly gdp growth rate - US
*Cpi_us = Consumer price index in the US
*Cpi_eur = Consumer price index in the Euro Area
*Infl_us = quarterly variation of the inflation rate - US
*Infl_eur = quarterly variation of the inflation rate - Euro area
*Ogap_us = quarterly variation of the Output Gap - US
*Ogap_eur = quarterly variation of the Output Gap - US
*Int_eur = ECB interest rate
*Int_uw = FED interest rate
*Diff_int = Ecb int rate - Fed int rate differential
*Diff_infl = Euro area inflation rate - US inflation rate differential
*Diff_ogap = Euro area output Gap - US output Gap differential
*Tr_bal = quarterly rate of change of the US trade balance (deficit) with the EU
*Ebor = 3 months Euribor rate
*Libor = 3 months Libor rate
*target_infl_dist_us = differential of the quarterly inflation rate from the target 2% level (converted on quarterly basis = 0.0049629)
*target_infl_dist_eur = differential of the quarterly inflation rate from the target 2% level (converted on quarterly basis = 0.0049629)

*VARIABLES ARE RENAMED, GENERATED, LABELLED
*Mean of Exchange Rate Expectations for the quarter
rename mean ere
*Standard deviation/dispersion of ERE
rename std_dev hetg
*Quarterly variation of the ER on the previous quarter
rename delta_exr_Q14 dexr14
*Differential between PPP and Exchange rate
rename delta_ppp_exr14 dppp_exr14
*Extrapolation term, differential ERE - PPP
```

```
*cut
*Quarterly GDP growth - Euro area (19)
rename gdp_eur ggdp_eur
*Quarterly GDP growth - US
rename gdp_us ggdp_us
*Quarterly variation of CPI - US
rename CPI_USA cpi_us
*Quarterly variation of CPI - EUR area
rename CPI_EURO cpi_eur
*Quarterly Variation of the Output Gap - US
rename Out_gap_USA_ ogap_us
*Quarterly Variation of the Output Gap - EUR area
rename Out_gap_EURO ogap_eur
*Quarterly variation in the inflation rate - US
rename infl_USA infl_us
*Quarterly variation in the inflation rate - EUR area
rename infl_EURO infl_eur
*Quarterly interest rate - EUR area
*ECB interest rate
rename irate_EURO int_eur
*Quarterly interest rate - US
*Federal Reserve interest rate
rename irate_USA int_us
*Q. Differential between the Interest rate in EUR and US
rename diff_int_rate diff_int
*Q. Differential between EUR -US output Gap
rename diff_otpg diff_ogap
*Quarterly variation in the US trade balance with the EU
rename us_tr_bal_change tr_bal
*Euribor and Libor rates
rename euribor_3m ebor
rename libor_3m libor

*Generate the variabile "risk premium" defined as the difference between the expected exchange
rate and the forward rate
gen riskp = ere - forward
*Generate the variable "distance from target inflation" to then test the Taylor Rule
gen target_infl_dist_us = 0.00496293 - infl_us
gen target_infl_dist_eur = 0.00496293 - infl_eur
*Generate the variable "distance between the realized exr and the forward rate"
gen dist_exr_forward = exr14 - forward
*generate differentials of cpi, m3
gen diff_cpi = cpi_eur - cpi_us
gen diff_m3 = m3_eur - m3_usd

*GENERATE THE ABSOLUTE VALUES OF VARIABLES
gen abs_dexr14 = abs(dexr14 )
gen abs_dppp_exr14 = abs( dppp_exr14)
gen abs_riskp = abs( riskp)
gen abs_forward = abs(forward )
*RENAMED
gen regression = abs_dppp_exr14
gen risk_premium = abs_riskp
gen forward_rate = abs_forward
gen extrapol = abs_dexr14
*MACRO. VARIABLES ABSOLUTE VALUES
gen abs_m3_usd = abs(m3_usd )
gen abs_m3_eur = abs( m3_eur)
gen abs_ggdp_eur = abs(ggdp_eur)
gen abs_ggdp_us = abs(ggdp_us )
gen abs_infl_us = abs( infl_us)
gen abs_infl_eur = abs( infl_eur)
gen abs_ogap_us = abs(ogap_us )
gen abs_ogap_eur = abs( ogap_eur)
gen abs_diff_int = abs(diff_int )
gen abs_diff_infl  = abs(diff_infl  )
gen abs_diff_ogap = abs(diff_ogap )
gen abs_tr_bal = abs(tr_bal )
```

```
gen abs_target_infl_dist_us = abs(target_infl_dist_us )
gen abs_target_infl_dist_eur = abs(target_infl_dist_eur )
gen abs_diff_m3 = abs(diff_m3)
gen diff_ggdp = ggdp_eur - ggdp_us
gen abs_diff_ggdp = abs(diff_ggdp)


***************************************************************************
*summarize the descriptive statisics for the mean expected ERE and dispersion (heterogeneity)

*FIG 1
sum hetg, detail
sum ere, detail

*FIG 2
hist hetg
tab hetg
line hetg time
tsreport hetg
*summary of descriptive statistics about relevant variables of the original paper
sum ere, detail
sum exr14, detail
sum dexr14, detail
sum dppp_exr14, detail
sum riskp, detail

*FIG 3
gen PPP = exr14 + dppp_exr14
twoway line exr14 PPPP time, sort lpattern (dash solid) lwidth(medium thick) yaxis(1) || line hetg time, sort lpattern (solid) yaxis(2) yscale(alt) yscale(alt axis(1))


************************************************************
*LIST OF TESTED REGRESSIONS
*regression considering all variables
*regression considering only significant macro variables (not lagged)
*regression considering only significant macro variables (lagged)
*mostly differentials are used
*variables' selection is conducted to avoid collinearity and correlation among regressors

************************************************************
*SELECTED REGRESSIONS

* * * * * *  C&F AND NOISE TRADERS MODELS REGRESSION (ABSOLUTE VALUES) * * *
reg hetg  L.regression L.risk_premium L.forward_rate,robust
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* * * * * * MACRO VARIABLES REGRESSION (ABSOLUTE VALUES) * * * * * * * * * *
reg hetg L.abs_diff_m3 L.abs_diff_ogap  L.abs_tr_bal L.abs_target_infl_dist_us,robust
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* * * * * * * * * * * * * * JOINT REGRESSIONS * * * * * * * * * * * * * * *
reg hetg L.regression L.risk_premium L.forward_rate L.abs_tr_bal L.target_infl_dist_us,robust
*OR
reg hetg  L.regression L.risk_premium L.forward_rate L.abs_tr_bal L.abs_diff_m3,robust XXX

* * CHOSEN REGRESSION * *
reg hetg  L.regression L.risk_premium L.forward_rate L.abs_tr_bal,robust
*the M3 variation is chosen over the target inflation differential
*the use of the trade balance variation alone is preferred to the inclusion of also the M3 differential
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *


*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

*AUTOCORRELATION OF HETEROGENEITY
corrgram hetg
```

```
ac hetg
pac hetg, srv
dfuller hetg
reg hetg L.hetg L.L.hetg L.L.L.hetg L.L.L.L.hetg, robust
*"Applying OLS estimation to non-stationary variables can result in "spurious" results -> need a VAR model and tests for stationarity and unit roots' presence"


*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

**# VAR AND VEC MODELS

*VAR MODELS ARE SPECIFIED IN LEVELS: the dependent variable is a function of itself lagged and the other variables lagged
*To specify the VECM is necessary to identify the optimal number of lags, too many lags can cause the loss of degrees of freedom, too few can lead to specification errors
*the VAR gets differenced, we get a VECM, but we lose 1 lag (k-1)

*Specified Model from OLS
reg hetg  L.regression L.risk_premium L.forward_rate L.abs_tr_bal,robust

*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

**# Stationarity test for series (nonstat) and their first differences (stat) to verify they all are I(1) at 1 and 4 lags

** AT 1 LAG NO DIFF
dfuller hetg, lags(1)
*STAT 5%
dfuller regression, lags(1)
*NONSTAT
dfuller risk_premium, lags(1)
*STAT 1%
dfuller forward_rate, lags(1)
*NONSTAT
dfuller abs_tr_bal, lags(1)
*STAT 1%
dfuller abs_diff_m3, lags(1)
*STAT 5%

** AT 4 LAG NO DIFF
dfuller hetg, lags(4)
*NONSTAT
dfuller regression, lags(4)
*NONSTAT
dfuller risk_premium, lags(4)
*NONSTAT
dfuller forward_rate, lags(4)
*NONSTAT
dfuller abs_tr_bal, lags(4)
*NONSTAT (STAT 10%)
dfuller abs_diff_m3, lags(4)
*NONSTAT

** AT 1 LAG IN DIFF
dfuller d.hetg, lags(1)
*STAT 1%
dfuller d.regression, lags(1)
*STAT 1%
dfuller d.risk_premium, lags(1)
*STAT 1%
dfuller d.forward_rate, lags(1)
*STAT 5%
dfuller d.abs_tr_bal, lags(1)
*STAT 1%
dfuller d.abs_diff_m3, lags(1)
*STAT 1%
*broad weak results in favour of series being I(0) STAT IN THEIR 1ST DIFF
```

```

** AT 4 LAG IN DIFF
dfuller d.hetg, lags(4)
*STAT 1%
dfuller d.regression, lags(4)
*NONSTAT (STAT 10%)
dfuller d.risk_premium, lags(4)
*NONSTAT (STAT 10%)
dfuller d.forward_rate, lags(4)
*NONSTAT (STAT 10%)
dfuller d.abs_tr_bal, lags(4)
*STAT 1%
dfuller d.abs_diff_m3, lags(4)
*STAT 5%

***************************************************************************
*CHOICE OF THE TYPE OF TREND, CONSTANT
*trend(constant) include an unrestricted constant in model; the default
*trend(rconstant) include a restricted constant in model
*trend(trend) include a linear trend in the cointegrating equations and a quadratic trend in the undifferenced data
*trend(rtrend) include a restricted trend in model =defines a restricted trend model that excludes linear trends in the differenced data but allows for linear trends in the cointegrating equations. As in the previous case, a linear trend in a cointegrating equation implies that the cointegrating equation is trend stationary.
*trend(none) do not include a trend or a constant
***************************************************************************

*STATA COMMANDS FOR THE VECM ESTIMATION AND ROBUSTNESS CHECK
*Determine optimal lag lenght (p) for the model
*p opt lag number by SBIC, HQIC
varsoc
*Johansen cointegration test with p LAGS
*check for the number of cointegration relations
vecrank
*rank = so  cointegration relation exists
*Specify RESTRICTED VECM with (p) lags, keep in mind model estimates with (p-1) lags (MAX LAG=3-1=2, RANK=1)

vec
*1) we obtain short run coefficients for the target variable
            *ce1 is ADJ COEFF: SPEED OF ADJ ON SR
*2) we obtain a long run equation with hetg as a target

*THE ECT EQUATION: Long Run MODEL
*ECTt-1 = [LRcoeff*hetgt-1 - coeff*dpppt-1 - ... - constant]
*"the adj term ECT (...) is significant at X% level, suggesting that previous quarters' deviations from long run eq are corrected for within the current quarter at a convergence speed of (...%)"

*THE VECM MODEL EQUATION
*DELTAhetgt = const? - SRc*DELTAhetgt-1 +...-... - c*ECTt-1

*Diagnostic tests for VECMM
*1) Autocorrelation test:
veclmar
*H0 is lack of autocorrelation

*2) Normally distributed disturbancies test:
vecnorm, jbera

*3) Stability of the model test:
vecstable, graph
*We can use vecstable to check whether we have correctly specified the number of cointegrating equations. The companion matrix of a VECM with K endogenous variables and r cointegrating equations has K −r unit eigenvalues. If the process is stable, the moduli of the remaining r eigenvalues are strictly less than one. Because there is no general distribution theory for the moduli of the eigenvalues, ascertaining whether the moduli are too close to one can be difficult.
```

```
*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
**# VECM ESTIMATION

*(II): VECM SPECIFICATION WITH 2 LAGS, RANK 1, NOT AFFECTED BY AUTOCORRELATION, WITH NORMALLY DISTRIBUTED ERRORS, STABLE
vecrank hetg L.regression L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3,lag(2) trend(rtrend)
vec hetg L.risk_premium L.extrapol L.regression L.abs_diff_m3 L.abs_tr_bal, lag(2) rank(1) trend(rtrend)
veclmar, mlag(5)
vecnorm
vecnorm, jbera
vecstable, graph
*PLOT RESIDUALS AND GRAPHICAL INSPECT THEIR STATIONARITY
predict VECresiduals, residuals
line VECresiduals time
dfuller VECresiduals

*(III) SECONDARY VECM SPECIFICATION: Constructed by aggregation
varsoc hetg L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3
vecrank hetg L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3,lag(2) trend(rtrend)
vec hetg L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3, lag(1) rank(1) trend(rtrend) noetable
*+1 LAG
vec hetg L.extrapol L.risk_premium L.abs_tr_bal  L.abs_diff_m3, lag(2) rank(1) trend(rtrend) noetable
veclmar
vecnorm
vecnorm, jbera
vecstable, graph

*(IV) THIRD VECM SPECIFICATION: 3 variables
varsoc hetg L.extrapol L.risk_premium L.abs_diff_m3
vecrank hetg L.extrapol L.risk_premium L.abs_diff_m3, lag(1) trend(rtrend)
vec hetg L.extrapol L.risk_premium L.abs_diff_m3, lag(1) rank(1) trend(rtrend) noetable
vec hetg L.extrapol L.risk_premium L.abs_diff_m3, lag(1) rank(3) trend(rtrend)
veclmar
vecnorm
vecnorm, jbera
vecstable, graph

*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```