Università
Ca'Foscari
Venezia

Ca' Foscari
Dorsoduro 3246
30123 Venezia

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Final Thesis

On the application of Filtering Historical Simulation to the HAR-RV for VaR forecasting

We propose a Filtered Historical Simulation method, applied to the Heterogeneous AutoRegressive - Realized Volatility (measured on tick-by-tick data) model, to predict one-day-ahead Value at Risk. A great number of ticks are available during the trading day, but they are not present overnight. On a similar research, Realized Volatility has been re-scaled to take into account overnight returns and related volatility. We want to apply a different procedure, in order to test whether the specification of a dynamic model to the overnight returns, in particular, a GARCH model, could improve the Volatility forecasts to the aim of VaR computations. We find that the two approaches lead to similar results, but the model for 1% VaR with the GARCH specification on overnight returns allows for lower failures and performs better on the Dynamic Quantile test and Conditional Coverage test. We tested the versions of FHS-HAR-RV model on Standard and Poor's 500 Index Futures, from January 2, 1996, to December 14, 2011 of tick-by-tick data and daily returns.

Keywords: VaR, FHS, HAR-RV, GARCH, Forecast

Supervisor
Ch. Prof. Fulvio Corsi

Graduand
Simone Bortolato
Matriculation Number 850673

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Contents

Introduction

The estimation and forecasting of Value at Risk is one of the most studied fields in risk management and financial econometrics. The idea behind this concept is very simple, but there is not a unique solution to this problem. There is a wide literature made of different models able to predict VaR and practitioners are faced with the non-trivial issue of choosing between the alternatives. Different models applied to the same portfolio of securities, often lead to different estimates of VaR and to significant errors. Empirical evidences tell us that, to perform a good estimate of VaR, a dynamic volatility model is necessary.

Probably, the most influential work on this argument, that is a column of modern risk management, is the Autoregressive Conditional Heteroscedasticity ARCH model for time-varying volatility proposed by Robert Engle (1982), employed to model the conditional heteroscedasticity of financial returns series. A lot of variants of this model are present in the literature and the great majority of them have been used to VaR estimation purposes.

A different approach has raised and become a very useful tool in volatility estimation, prediction and, consequently, in risk management: this is the Realized Volatility, defined as the square root of the sum of intraday squared returns. The most influential works on the argument are made by Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold, and Labys (2003), Andersen, Bollerslev, Diebold and Ebens (2001), and Barndorff, Nielsen and Shephard (2002, 2004), where they describe the Realized Volatility as a non-parametric method to measure the volatility and suggested that standard time series procedures can be used to model it. In this way, the volatility is simply observed and is model-free (instead, GARCH models are quite complex and computationally expensive). Moreover, it has been shown by Martens (2002), Koopman et. al. (2005) and others, that Realized Volatility models are superior to ARCH-Type models in volatility forecasting.

Particularly, the Heterogeneus AutoRegressive Realized Volatility model of Corsi (2009) (HAR, from now on) has become a standard to model the long memory features of the volatility process: its simple form (as an additive cascade process made of three different aggregated components, to capture the daily, weekly and monthly contribution to the volatility process) and its consequent parsimony and computational efficency are uncommon qualities to find in other volatility-related models; all this features are coupled with a consistent estimation of the volatility. We propose a Filtered Historical Simulation (FHS) method, applied to the HAR-RV model, with Realized Volatility measured on tick-by-tick data, to predict one-day-ahead Value at Risk. Particularly, on this research we have applied the standard HAR-RV model, the Asymmetric HAR-RV (AHAR) and these models with a GARCH specifications for their residuals.

A great number of ticks are available during the trading day, but they are not present overnight. On a similar research by Louzis, Xanthopoulos-Sisinis and Refenes (2011), Realized Volatility has been re-scaled to take into account overnight returns and related volatility. We want to apply a different procedure, in order to test whether the specification of a dynamic model to the overnight returns, in particular, a GARCH model, could improve the Volatility forecasts to the aim of VaR computations. Specifically, we applied to the overnight returns a standard GARCH(1,1) model and the GJR-GARCH(1,1) model, to capture the asymmetries in the returns. The main result is that the use of a dynamic model able to promptly adapt to the modifications of the market conditions, is a significant improvement with respect to the re-scaling of the entire dataset based on a unique coefficient. This becomes evident when the backtests are compared on pre and post 2008-2009 crisis periods, where will show how the dinamical structure is superior the the re-scaled one. As an additional benchmark, we have used the FHS approach as proposed by Barone-Adesi, Giannopoulos and Vosper (1999), that is the milestone of this empirical procedure. The first chapter of the present thesis is dedicated to the volatility models, how they are constructed and how they are able to estimate and predict volatility. The second chapter will be devoted to the functioning of the FHS, as an improvement of the more basic Historical Simulation. The third chapter is about the Value at Risk estimation and the comparison methods employed. The fourth and final chapter will present the data and the empirical results.

Chapter 1

Volatility Models

1.1 An introduction to volatility for VaR purposes

Historically, the volatility of a financial time series was simply estimated as the square root of the variance of the logarithmic returns. Assuming the returns are normally distributed, the estimated volatility was coupled with the expected rate of return to compute the Value at Risk at the desired coverage level:

$$VaR(\alpha) = \hat{\mu} + \hat{\sigma} * \Phi^{-1}(\alpha),$$

where $\hat{\mu}$ is the expected return, $\hat{\sigma}$ is the estimated volatility, Φ^-1 is the quantile function of a standard normal and α is the desired coverage level.
The assumption of normality and the use of historical volatility is the simplest way, but often leads to mistakes, since financial time series are often non-normal.
Moreover, the employment of historical volatility can lead to mistakes itself, since it do not take into account the current market conditions, but it is just an "average" of the past conditions.
Accordingly, empirical evidences suggest the use of a dynamic model to estimate the volatility, on the current market conditions. One of the most influential work on

the subject, is surely that of Engle(1982), who published a paper about its ARCH model (which will be discussed later). Such a model and subsequent improvements, are able to take into account the changes in the volatility behavior, providing a dynamic estimation (and forecast) of it.

Concurrently, a different way to measure volatility has arised. Realized volatility is measured as the square root of the sum of squared intraday returns. The higher the frequency of the intraday returns, the higher the precision of this measure and, at the same time, the higher the bias due to the microstructure of the financial markets. Variuos techniques are available to correct the microstructure noise: here, we have used the Two-Scale correction.

1.2 The models

The volatility models here analyzed are the the GJR-GARCH(1,1), HAR, HAR with a GARCH(1,1) specification for the residuals, the AHAR, the AHAR with a GARCH(1,1) specification for the residuals.

1.2.1 ARCH-type models

The first Autoregressive Conditional Heteroscedasticity model has been developed by Engle (1982). An ARCH(q) is specified as:

$$\begin{aligned} X_t =& \sigma_t \epsilon_t, \\ \sigma_t^2 =& \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2, \end{aligned} \tag{1.1}$$

where ϵ_t represents a random standard normal innovation. The parameters can be estimated via maximum likelihood.

A significant improvement of the ARCH model (the Generalised ARCH) has been realised by Bollerslev (1986) and has become a standard in time series volatility analysis. a GARCH(p,q) model is defined as:

$$
\begin{aligned}
X_t =& \sigma_t \epsilon_t, \\
\sigma_t^2 = \omega + & \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2,
\end{aligned}
\tag{1.2}
$$

with ϵ_t a random standard normal innovation. The maximum likelihood approach is applied here, too. The GARCH model is a more parsimonious way to describe the conditional volatility process, since an equivalent ARCH model would have much more parameters.

The GJR-GARCH model by Glosten, Jagannathan and Runkle (1993) permits to capture the asymmetries in the innovations, adding a term when (for example) the last innovation is negative. Therefore, the variance equation of a GJR-GARCH(1,1) can be set as:

$$
\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \gamma \epsilon_{t-1}^2 I(\epsilon_{t-1} < 0) + \beta \sigma_{t-1}^2 \tag{1.3}
$$

We have employed the GJR model as the asymmetric GARCH model used in Barone-Adesi et al.(1999) on the application of the FHS (that will be explained in the next chapter).

1.2.2 Realized Volatility

The starting point for the estimation of realized volatility is the availability of intra-day data. Assume $S(t)$ is the price of a financial asset, and define $p(t)$ as $log(S(t))$. Then, assume that $p(t)$ is ruled by the following diffusion process:

$$
dp(t) = \mu(t)dt + \sigma(t)dW(t) \tag{1.4}
$$

where $\mu(t)$ is a cadlàg finite variation process, $W(t)$ is a standard brownian motion and $\sigma(t)$ is a stochastic process independent of $W(t)$ (Corsi, 2009). The integrated variance associated with one day (t), is defined as:

$$IV_t = \int_{t-1}^{t} \sigma^2(\omega) d\mathbb{P}(\omega) \tag{1.5}$$

Accordingly, the integrated volatility is:

$$\sigma_t = \sqrt{IV_t} \tag{1.6}$$

As shown by Andersen et al. (2001, 2002a, 2002b), the integrated variance is approximable as the sum of squared intraday returns. Andersen et al. (2003) suggested that the use of the usual time series modeling outperforms the popular GARCH and stochastic volatility models. The realized volatility (over a one-day period) can be defined as:

$$RV_t^{(d)} = \sqrt{\sum_{i=0}^{M-1} r_{t-i\cdot\Delta}^2} \tag{1.7}$$

where $\Delta = 1\text{-}day/M$, M is the number of intraday returns (Δ can be viewed as the arbitrary precision of the realized volatility measurement), $r_{t-i\cdot\Delta} = p(t-i\cdot\Delta) - p(t-(i+1)\cdot\Delta)$ represent intraday returns sampled at Δ frequency (Corsi's notation, 2009).

1.2.3 HAR-RV models

The intuition of this model comes from the Heterogeneous Market Hypothesis by Müller et al. (1993). The theory states that market participants with different investing horizons and trading frequency, differently contribute to the market movements. As a consequence, the HAR-RV model is composed of three parts: the daily (just explained), weekly and monthly realized volatility components, to capture the heterogeneity of the views of the different market participants. The weekly and monthly pieces are just simple averages of the daily quantities:

$$
\begin{aligned}
RV_t^{(w)} =& \frac{1}{5}\left(RV_t^{(d)} + RV_{t-1}^{(d)} + \cdots + RV_{t-4}^{(d)}\right) \\
RV_t^{(m)} =& \frac{1}{22}\left(RV_t^{(d)} + RV_{t-1}^{(d)} + \cdots + RV_{t-21}^{(d)}\right)^{1}
\end{aligned}
\tag{1.8}
$$

The components are then combined to obtain a simple model, equivalent to an AR(22) with constrained parameters:

$$
RV_{t+1}^{(d)} = c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + \omega_{t+1} \tag{1.9}
$$

where ω_{t+1} is an independent, zero-mean noise, with a truncated left tail to ensure the positivity of the volatility. Equation (1.9) represents the basic HAR-RV model. Corsi et al.(2008) show that the HAR model presents heteroscedastic residuals. They suggested the use of a GARCH(1,1) specification for the residuals to solve the problem:

$$
\begin{aligned}
RV_{t+1}^{(d)} &= c + \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} + u_{t+1} \\
u_{t+1} &= h_{t+1}\epsilon_{t+1} \\
h_{t+1}^2 &= \omega + \alpha\epsilon_t^2 + \beta h_t^2
\end{aligned}
\tag{1.10}
$$

The other realized volatility model employed in this research is the Asymmetric HAR-RV (AHAR) model, as designed by Louzis, Xanthopoulos-Sisinis and Refenes (2012). The additional terms that are present in this model are thought to take into account asymmetries in the HAR model, through the use of standardized returns and absolute-standardized returns. The model writes as:

$$
\begin{aligned}
RV_{t+1}^{(d)} = c &+ \beta^{(d)} RV_t^{(d)} + \beta^{(w)} RV_t^{(w)} + \beta^{(m)} RV_t^{(m)} \\
&+ \vartheta^{(d)} z_t^{(d)} + \vartheta^{(w)} z_t^{(w)} + \vartheta^{(m)} z_t^{(m)} \\
&+ \gamma^{(d)} \left|z_t^{(d)}\right| + \gamma^{(w)} \left|z_t^{(w)}\right| + \gamma^{(m)} \left|z_t^{(m)}\right| + u_{t+1}
\end{aligned}
\tag{1.11}
$$

[1] We assume a month made of 22 trading days

where $z_t^{(h)} = \sum_{i=1}^{h} r_{t-i+1} \Big/ \sqrt{\sum_{i=1}^{h} \sigma^2_{\mathrm{RV},t-i+1}}$ are the standardized innovations, for $h = 1, 5, 22$. It is possible to add a GARCH specification for the residuals, as in equation (1.10).

Chapter 2

Filtered Historical Simulation

Filtered Historical Simulation is an alternative to the variance-covariance method in the estimation of the Value at Risk. It is an improvement of the simple Historical Simulation and presents advantages with respect to both methodologies. It is distribution free: as it is for the historical simulation, no assumption about the distribution of the returns is needed; using the historical (empirical) distribution allows to take into account the fat tails and the volatility clusters that financial data usually present. As Barone-Adesi and Giannopoulos (1996) pointed out, the drawback of the historical simulation is that it is based on past historical returns and relies on them without taking into account the changes in the market conditions. The same kind of problem applies to the variance-covariance method, where the var-cov matrix is estimated on historical data and is not able to capture current market conditions.

The intuition behind the Filtered method consists in the standardization of the return series, dividing it by the conditional standard deviations series estimated by a GARCH model. In this way, an almost i.i.d. innovation sample is obtained. Then, a forecast of the day-ahead volatility is provided, and a number of the innovations obtained before is randomly picked and multiplied by the volatility forecast to obtain an empirical distribution that is responsive to the changes in the market conditions (Barone-Adesi et al.,1999).

2.1 The original approach as benchmark

One of the main applications of FHS comes from Barone-Adesi et al. (1999). The first step of their approach is to remove serial correlation and volatility clusters from the returns series, by applying an ARMA-GARCH specification. For example, an ARMA(1,1)-AGARCH(1,1) should remove any serial correlation and volatility clusters from the series, obtaining an i.i.d. residuals series. The choice of an Asymmetric GARCH model is made in order to take into account any asymmetry present in the process.

The model specification wuold be similar to:

$$\begin{aligned} r_t &= \mu r_{t-1} + \theta \epsilon_{t-1} + \epsilon_t \\ \epsilon_t &\sim N(0, h_t) \\ {h_t}^1 &= \omega + \alpha(\epsilon_{t-1} - \gamma)^2 + \beta h_{t-1} \end{aligned} \tag{2.1}$$

where μ is the autoregressive coefficient, θ is the moving average one, ω is a constant, α and γ are the influence and the asymmetry of the last innovation and β is the coefficient of the volatility of the previous day.

To obtain i.i.d. standardized residuals, the estimated residuals are divided by the corresponding daily volatility estimated by the GARCH equation:

$$e_t = \frac{\epsilon_t}{\sqrt{h_t}} \tag{2.2}$$

where e_t corresponds to the standardized residuals.

The procedure applies as follows:

- Randomly draw a standardized residual return, from the vector of standardized residuals, calling it e^*;
- Multiply the picked standardized residual by the day-ahead volatility forecast, to obtain an innovation forecast:

[1] We have employed the GJR-GARCH(1,1), with a conditional variance equation stated as: $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 \cdot I(\epsilon_{t-1} > 0) + \beta\sigma_{t-1}^2$

$$z^*_{t+1} = e^* \cdot \sqrt{h_{t+1}} \tag{2.3}$$

This passage is crucial, because the standardized innovation is here scaled in order to take into account the current market conditions (represented by the volatility estimate);

- The simulation of the return is then computed as:

$$r^*_{t+1} = \mu r_t + \theta z^*_t + z^*_{t+1} \tag{2.4}$$

where z^*_t is estimated as in (2.3)

Then, the simulated empirical distribution of the returns for the day-ahead time period, is constructed by repeating the procedure a high number of times (let's say 10,000).

2.2 FHS with realized volatility

There is a very limited literature about the application of FHS to Realized Volatility measures and where it is present, it is on non-ufficially-published working papers. In particular, in an interesting research by Louzis, Xanthopoulos-Sisinis and Refenes (2011), where a comparison between various VaR estimation techniques is provided, FHS method is applied to Realized Volatility.

Since Realized Volatility is measured on intraday returns and it does not take into account nightly volatility, usually, it is re-scaled to take into account the overnight volatility:

$$RV_t = \Phi \cdot \sum_{j=1}^{M} r^2_{t,j}, \quad \Phi = \frac{\sigma^2_{oc} + \sigma^2_{co}}{\sigma^2_{oc}} \tag{2.5}$$

where σ^2_{oc} and σ^2_{co} are, respectively, the open-to-close and close-to-open sample variances. The realized volatility is the square root of the above formula.

One-day-ahead volatility forecasts are estimated applying the HAR, HAR-GARCH, AHAR and AHAR-GARCH models to the re-scaled RV, on a moving window of

1,000 observations (i.e., 1,000 obs. are used to estimate the parameters of the models and to estimate the $1,001^{\text{st}}$).

Then, the daily returns are standardized, dividing them by the daily re-scaled realized volatilities:

$$e_t = \frac{r_t}{RV_t}. \tag{2.6}$$

To obtain a simulated return, it is sufficient to randomly pick a standardized return (namely, e^*) and multiply it by the volatility forecast provided by the HAR models:

$$r^*_{t+1} = har_{t+1} \cdot e^* \tag{2.7}$$

The simulated empirical distribution of the one-day-ahead return is obtained, again, repeating the procedure 10,000 times.

We have applied a slightly different procedure, without re-scaling the realized volatility to take into account the overnight volatility. Instead, we have applied a GARCH specification to the overnight returns, in order to have a dynamic response, instead of a fixed one.

The main difference of this approach with the previous one is the non-re-scaling of the Realized Volatility: we keep it as it is calculated and we just apply a dynamic model to the overnight returns.

We will use two innovations vector: one for the realized volatility (that is measured on the daily movements) and the other for the overnight returns. The former, is obtained as in (2.6), with the difference that RV_t is not rescaled as in (2.5). The standardized innovations will be obtained as:

$$e_t^{(oc)} = \frac{r_t^{oc}}{RV_t} \tag{2.8}$$

where r_t^{oc} represents the returns realized during the trading days (open-to-close).

For the latter, we need some volatility estimates. These volatilities are estimated with the use of a GARCH(1,1) model with an exogenous variable that appears in the equation: the exogenous variable used is the lagged Realized Volatility:

$$\begin{aligned} r_t^{on} &= \sigma_t \epsilon_t, \qquad \epsilon_t \sim N(0,1) \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta RV_{t-1}(^2) \end{aligned} \tag{2.9}$$

The lagged RV appears in the conditional variance equation since it is plausible that the level of volatility realized during the day affects the following overnight return. The GJR-GARCH(1,1) model with an exogenous variable has been applied to the overnight returns, too: the exogenous variable is still the lagged RV. The related conditional variance equation is:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \gamma \epsilon_{t-1}^2 \cdot I(\epsilon_{t-1} > 0) + \beta \sigma_{t-1}^2 + \delta RV_{t-1} \tag{2.10}$$

The same conditional variance equations are used to have volatility predictions. Given the overnight volatilities estimates, in accordance with equation (2.8), we create a vector of standardized innovations, that will be used to simulate overnight returns:

$$e_t^{(co)} = \frac{r_t^{(co)}}{\sigma_t^{(co)}} \tag{2.11}$$

where $r_t^{(co)}$ represents the close-to-open returns and $\sigma_t^{(co)}$ represents the volatility estimates from the GARCH or GJR-GARCH models.

We have now all the ingredients to simulate the one-day-ahead return. Let us denote with $e^{*(oc)}$ the randomly picked open-to-close standardized innovation and with $e^{*(co)}$ the randomly selected close-to-open standardized innovation. The simulation will be:

$$r_{t+1}^* = har_{t+1} \cdot e^{*(oc)} + \sigma_{t+1}^{(co)} e^{*(co)} \tag{2.12}$$

where har_{t+1} represents the RV forecast obtained by one of the previously introduced HAR models and $\sigma_{t+1}^{(co)}$ the forecast provided by one of the GARCH models described. Repeating the procedure, let's say, 10,000 times, we obtain the empirical distribution

[2]Realized Volatility is always a non-negative measure, since it is measured as the sum of squared numbers: therefore, it is not necessary to raise it to the square in the conditional variance equation

of the one-day-ahead estimate.

Chapter 3

VaR Estimation and Comparison Methods

The Value at Risk has become one of the most used risk management tools. The intuition is simple: the VaR represents the within a given confidence level and a given time horizon. For example: a VaR($\alpha = 0.05$) $= -10\%$ means that, with a confidence of $1 - \alpha$ ($= 95\%$), the maximum loss is -10%. Despite its simplicity, there is not a unique way to estimate it. Instead there are a lot of parametric and non-parametric ways. A common approach is to estimate the volatility and to have a parametric VaR estimation assuming the return process is normally distributed:

$$VaR(\alpha) = \hat{\mu} + \hat{\sigma} \cdot \Phi^{-1}(\alpha)$$

where $\hat{\mu}$ is the sample mean, $\hat{\sigma}$ is the volatility estimated with any kind of volatility model, and $\Phi^{-1}()$ is the quantile function of a standard normal distribution.
To keep our VaR estimation model-free, we estimate it simply as the α quantile of the empirical distribution, simulated as in the previous chapter:

$$VaR(\alpha) = \Omega^{-1}(\alpha) \tag{3.1}$$

where $\Omega^{-1}()$ is the quantile function of the one-day-ahead empirical distribution.

3.1 Failure rate

One of the most common validation methods for the VaR valuation is the Failure Rate. Firstly, the VaR forecast is compared with the effectively realized returns: when the return exceeds VaR, there is a so-called Hit_t:

$$Hit_t = \begin{cases} 1, & \text{if } r_t < VaR_t^{\alpha} \\ 0 & \text{if } r_t \geq VaR_t^{\alpha} \end{cases} \tag{3.2}$$

Then the number of $Hits$ is divided by the number of observations, to get the Failure Rate: a well constructed VaR estimation model, should give a failure rate that is near the desired coverage level.

3.2 POF test

The POF(proportion of failure) test, designed by Kupiec (1995), is a Likelihood Ratio test. On this kind of test, the ratio is defined as the maximum probability of the observed result under the null hypothesis over the maximum probability of the observed result under the alternative hypothesis. The null hypothesis of the POF test is $H_0 : \alpha = \hat{\alpha}$, that is, the observed coverage level (the failure rate) is equal to the desired one.

The test is:

$$LR_{POF} = -2 \ln \left(\frac{(1-\alpha)^{T-x} p^x}{\left[1 - \left(\frac{x}{T}\right)\right]^{T-x} \left(\frac{x}{T}\right)^x} \right) \tag{3.3}$$

where T is the number of observations and x is the number of failures of the model. The test is asymptoticaly $\chi^2(1)$ distributed. The null hypothesis is rejected if the test gives a number greater than the χ^2 critical value, i.e., if the failure rate is far from the desired coverage level.

3.3 Coverage tests

Christoffersen (1998) figured out a test to check whether the failure rate is statistically equal to the desired coverage level, without taking into account the history of the "hit process" Hit_t. Accordingly, the null hypothesis of correct unconditional coverage is $E(Hit_t) = \alpha$. On this test, the independence is considered an assumption. The Likelihood Ratio test to verify if the failure rate ($\hat{\alpha}$) is equal to the coverage level (α) is:

$$LR^{uc} = 2[\log\left((1-\hat{\alpha})^{n_0}\hat{\alpha}^{n_1}\right) - \log\left((1-\alpha)^{n_0}\alpha^{n_1}\right)] \tag{3.4}$$

where n_1 is the number of failures (number of hits in the Hit process), $n_0 = n - n_1$ and n is the number of observations.

LR^{uc} is asymptotically distributed as a $\chi^2(1)$. The null hypothesis is rejected if there are too many or too few exceptions.

The LR Independence test of Christoffersen (1998) aims to test the independence "against an explicit first-order Markov alternative". The transitin probability matrix is defined as:

$$\Pi_t = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \tag{3.5}$$

where $\pi_{ij} = \Pr(Hit_t = j|Hit_{t-1} = i)$. The maximum likelihood estimators of π_{ij} are:

$$\Pi_t = \begin{bmatrix} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{bmatrix} \tag{3.6}$$

with n_{ij} the number of $Hit_t = j|Hit_{t-1} = i$. The LR test of Independence is then defined as:

$$\begin{aligned} LR^{in} = &-2\cdot\left((n_{00}+n_{01})\log(1-\hat{\pi}_2) + (n_{01}+n_{11})\log(\hat{\pi}_2)\right) \\ &+2\cdot\left(n_{00}\log(1-\hat{\pi}_{01}) + n_{01}log(\hat{\pi}_{01}) + n_{10}log(1-\hat{\pi}_{11}) + n_{11}log(\hat{\pi}_{11})\right) \end{aligned} \tag{3.7}$$

where $\hat{\pi}_2 = (n_{01} + n_{11})/(n_{00} + n_{01} + n_{10} + n_{11})$. $LR^{in} \sim \chi^2(1)$. Given that the test is not dependent on the true desired coverage probability, this is only a test of independence.

Statistically speaking, the hit process should be an i.i.d. Bernoulli process with parameter α. Then, it is necessary to test if the VaR model generates a failure rate equal to the desired coverage level, but conditioning on the available information at time $t-1$:

$$E(Hit_t|\Omega_{t-1}) = \alpha, \forall t. \tag{3.8}$$

This can be done aggregating the two tests of before, leading to the Conditional Coverage test, defined as a linear combination of the two tests:

$$LR^{cc} = LR^{uc} + LR^{in} \tag{3.9}$$

Using this test, a VaR forecast model can be rejected if it generates too much, too few or too clustered $Hits$.

3.4 Loss Functions

3.4.1 Regulatory Loss Function

Sarma et al. (2003) on their "Selection of VaR models", used a simple loss function, called it "Regulatory Loss Function" defined as:

$$l_t = \begin{cases} (r_t - VaR_t^{\alpha})^2, & \text{if } r_t < VaR_t^{\alpha} \\ 0, & \text{if } r_t \geq VaR_t^{\alpha} \end{cases} \tag{3.10}$$

Differently from the binomial loss function (i.e. the Hit process), the quadratic term ensure that large failures are penalised more than the smaller ones.

3.4.2 Firm's Loss Function

Sarma et al. (2003) used this loss function, which, similarly to the loss function described above, penalises more the failures greater in magnitude. The difference is that here is taken into account the opportunity cost the institution faces when reserving too much capital due to the VaR estimation. The Firm's loss function is defined as:

$$l_t = \begin{cases} (r_t - VaR_t^\alpha)^2, & \text{if } r_t < VaR_t^\alpha \\ -cVaR_t^\alpha, & \text{if } r_t \geq VaR_t^\alpha \end{cases} \tag{3.11}$$

where c represents the opportunity cost.

Chapter 4

Empirical Application

4.1 The Data

Our data set consists of a long tick-by-tick series for Standard and Poor's 500 Futures index, from 02-Jan-1996 to 14-Dec-2011 (3,956 observations). We used tick-by-tick returns and employed the Zhang et al.(2005) two-scales estimator to obtain daily realized volatility. It has been subsequently aggregated as in equation (1.8) to obtain the weekly and monthly components of the HAR models. Here below is a plot of the return series with the realized volatility.

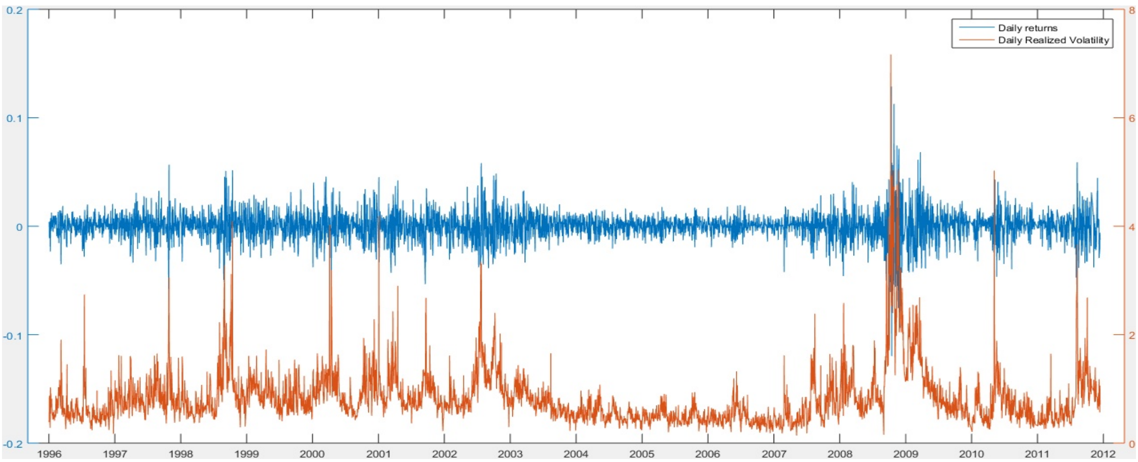

Figure 4.1: Plot of the daily returns together with the measured realized volatility. Blue: daily returns. Red: Realized Volatility

It is interesting to see a graphical comparison between the realized volatility as it is measured and the re-scaled version:

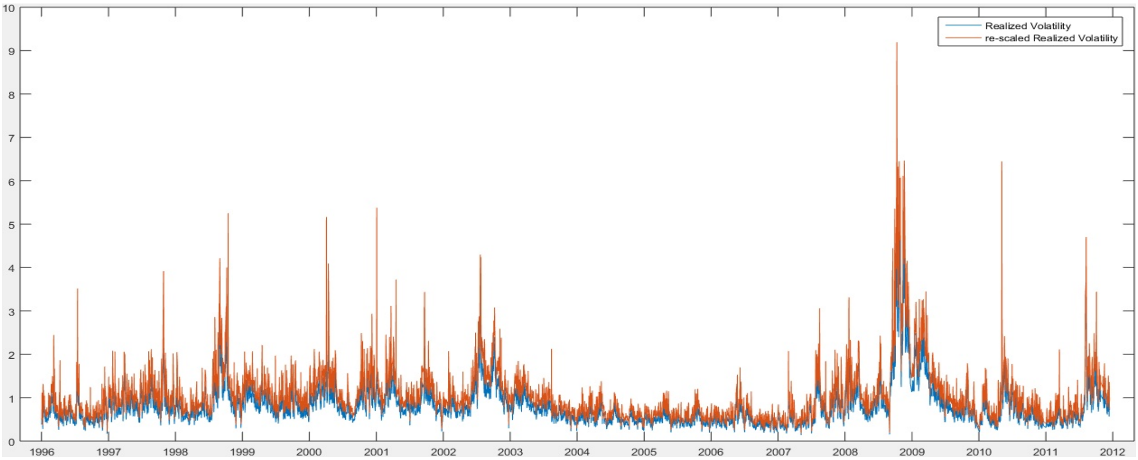

Figure 4.2: Graphical comparison between the realized volatility as it is measured and the re-scaled version. Blue: Realized Volatility. Red: re-scaled RV.

As it was expected, in order to take into account the overnight volatility, the re-scaling process leads to a greater (in magnitude) realized volatility. In fact, the constant of equation (2.5) has resulted to be $\Phi = 1.6460$.

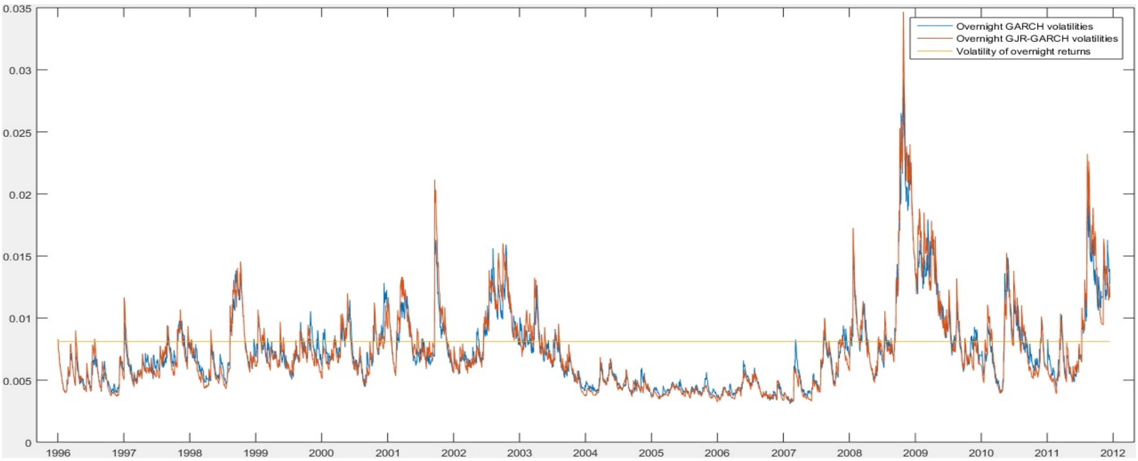

Figure 4.3: Overnight volatilities vs. volatility of overnight returns.

On figure (4.3) are plotted the estimated overnight conditional volatilities from both the GARCH and GJR-GARCH specifications vs. the volatility of overnight returns: The conditional volatilities are clearly not constant, and can be highly inefficient to summarize them with the simple volatility of the overnight returns, used in the re-scaling constant of the realized volatility. It is also possible to see the different response of the volatility estimates corresponding to days (actually, nights) with a high degree of variability: the GJR-GARCH model produces more extreme peaks.

4.2 Models and useful plots

The HAR-RV models are applied using a moving window of 1,000 observations, moving of one observation at a time, and producing 2,596 out-of-sample, one-day-ahead forecasts (from now on, the period considered for the out-of-sample forecasts is 22-Dec-1999 to 14-Dec-2011).

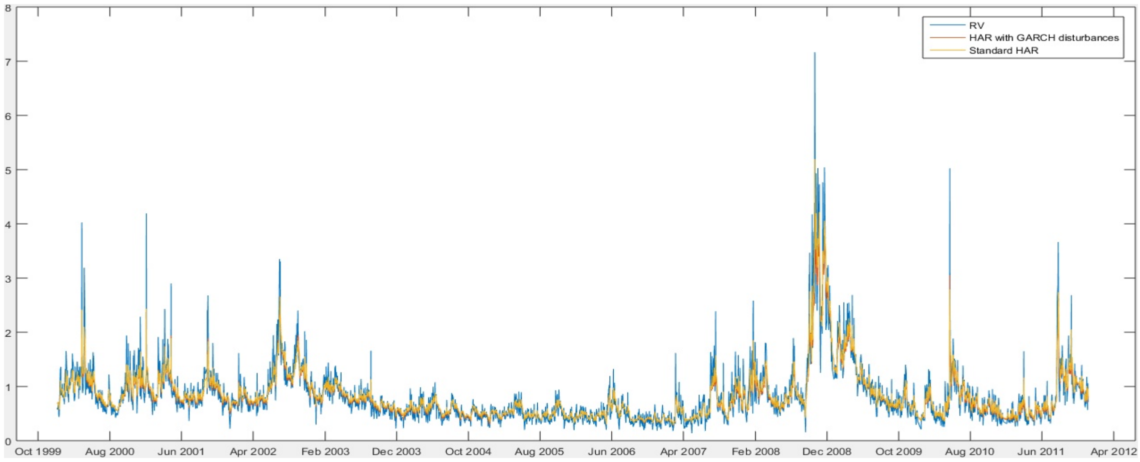

Figure 4.4: Comparison between the standard HAR-RV forecasts and the same model with a GARCH specification for the residuals.

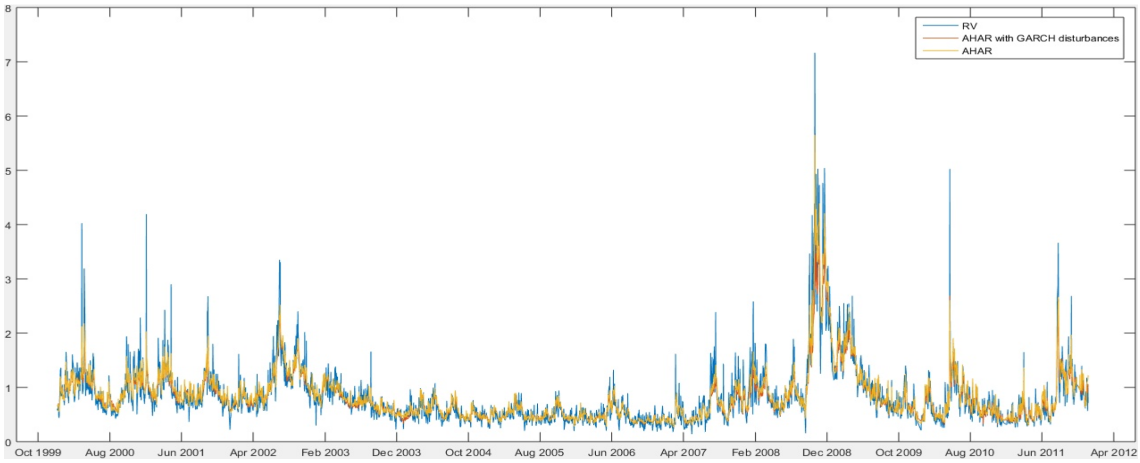

Figure 4.5: Comparison between the Asymmetric HAR-RV forecasts and the same model with a GARCH specification for the residuals.

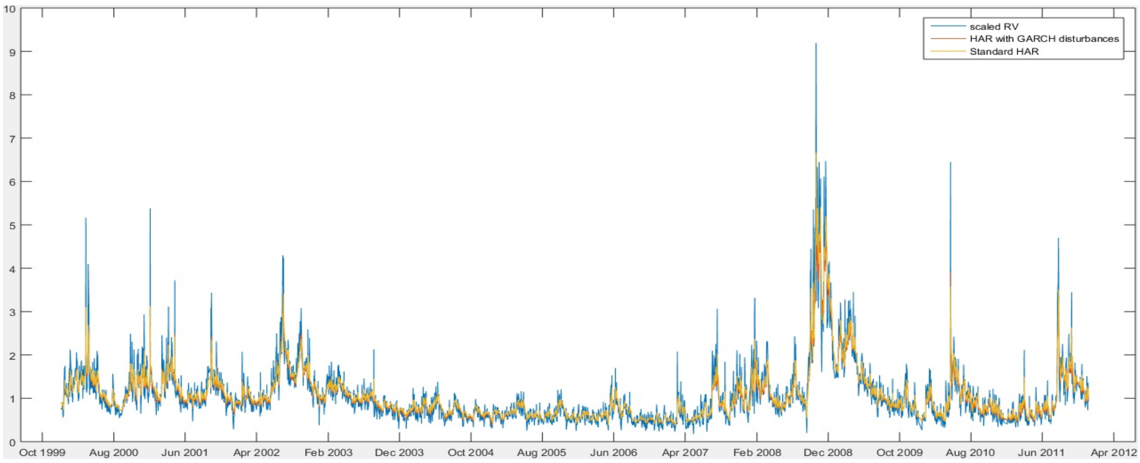

Figure 4.6: Comparison between the standard HAR-RV forecasts and the same model with a GARCH specification for the residuals applied to the re-scaled RV.

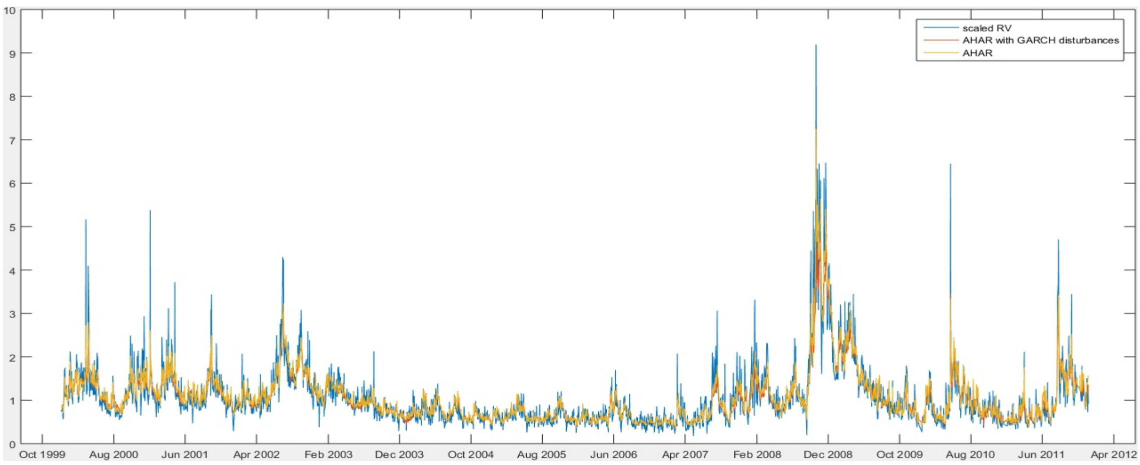

Figure 4.7: Comparison between the Asymmetric HAR-RV forecasts and the same model with a GARCH specification for the residuals applied to the re-scaled RV.

Figures from (4.4) to (4.7) provide a graphical comparison between forecasts produced by the different HAR models employed. Notice how the asymmetric models are more capable to reproduce the peaks than the non-asymmetric ones.

The ARMA-GJR-GARCH model applied to the whole sample and used to produce forecasts for the same for the FHS benchmark of Barone-Adesi et al. has produced the following parameter estimations:

	Estimate	Std. Error	t-test	p-value
constant	2.071990e-04	1.287900e-04	1.608813	0.1076572415
ar	5.135341e-01	2.661516e-01	1.929480	0.0536713429
ma	-5.570538e-01	2.580120e-01	-2.159022	0.0308484250
omega	2.273779e-06	8.693936e-07	2.615363	0.0089132796
alpha	4.210006e-02	1.487446e-02	2.830358	0.0046495913
beta	9.096912e-01	3.998219e-03	227.524082	0.0000000000
gamma	8.617402e-01	2.411280e-01	3.573787	0.0003518549

Only the constant of the mean model is not significant, and some doubt arises for the autoregressive coefficient.

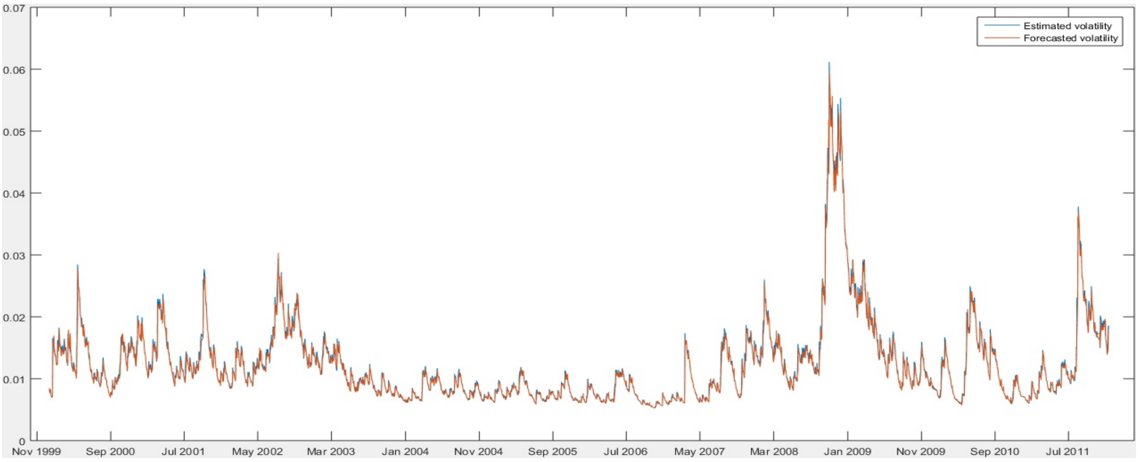

Figure 4.8: Plot of estimated volatilities and forecasted ones. ARMA-GJR-GARCH model.

Figure (4.8) represents the plot of the in-sample forecasted volatilities versus the conditional volatilities obtained by the model. They are in-sample since we have used the whole sample to estimate the parameters of the model.

The application of the HAR models consists of a regression on a moving window of 1,000 observations, to produce 2956 out-of-sample, one-day-ahead forecasts; consequently, given the high number of regression run, the parameters are not reported here. Yet, the parameters are always significant.

4.3 Application and Evaluations

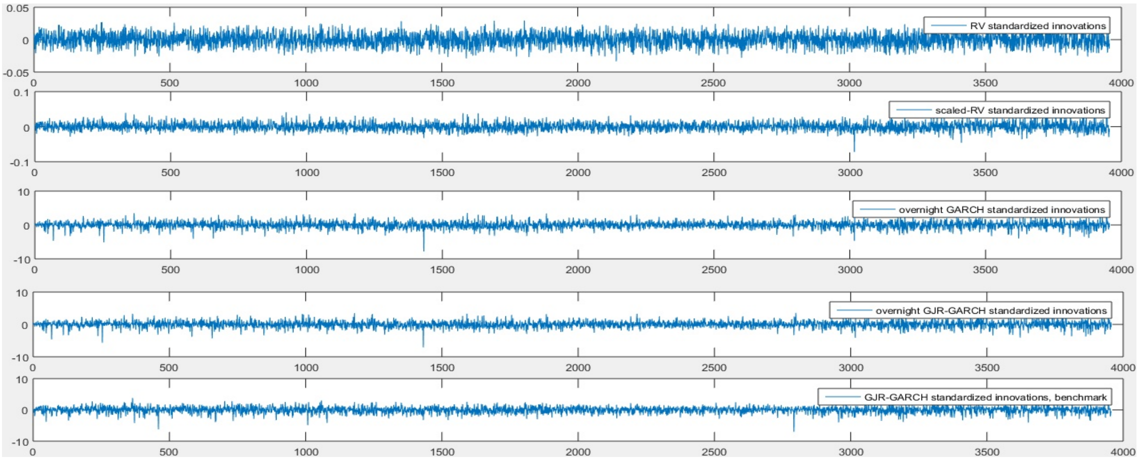

Figure 4.9: Plot of standardized innovations. From the top: realized volatility standardized innovations; re-scaled realized volatility standardized innovations; GARCH (applied to the overnight returns) standardized innovations; GJR-GARCH (applied to the overnight returns) standardized innovations; GJR-GARCH (applied to daily close-to-close returns) standardized innovations, for the benchmark FHS.

Figure (4.9) represents the plots of the standardized innovations used to produce the simulations. Notice how they are almost i.i.d., that is the aim of the filtering procedure. The RV-related innovations have a different scale compared with the GARCH-related innovations, because of the different scales of the two measures themselves and of the open-to-close and close-to-open returns. The issue is solved when the RV-innovations and GARCH-innovations are multiplied by RV forecasts and volatility forecasts, respectively, during the simulations.

As stated before, 10,000 simulations have been run for every time step, in order to ensure a sufficiently high number of observations to produce empirical distributions.

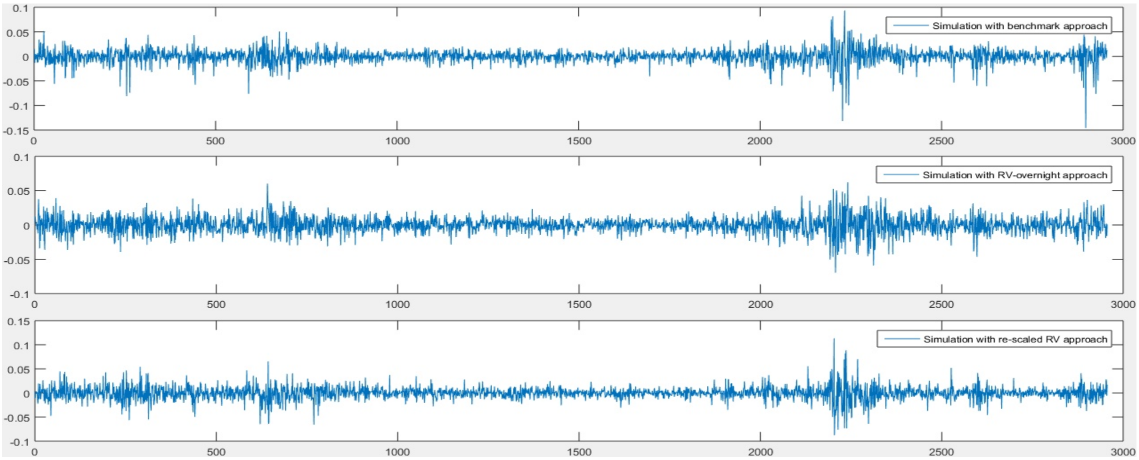

Figure 4.10: Plot of simulations. From the top: Benchmark approach; Daily open-to-close RV + overnight GARCH approach; re-scaled RV approach.

Figure (4.10) provides some examples of these simulations: it is possible to notice how the considered approaches are able to reproduce the market behavior.

After the simulation, empirical distributions are available. Value at Risk is estimated as in formula (3.1), for confidence levels of $0, 1$, 0.05 and 0.01.

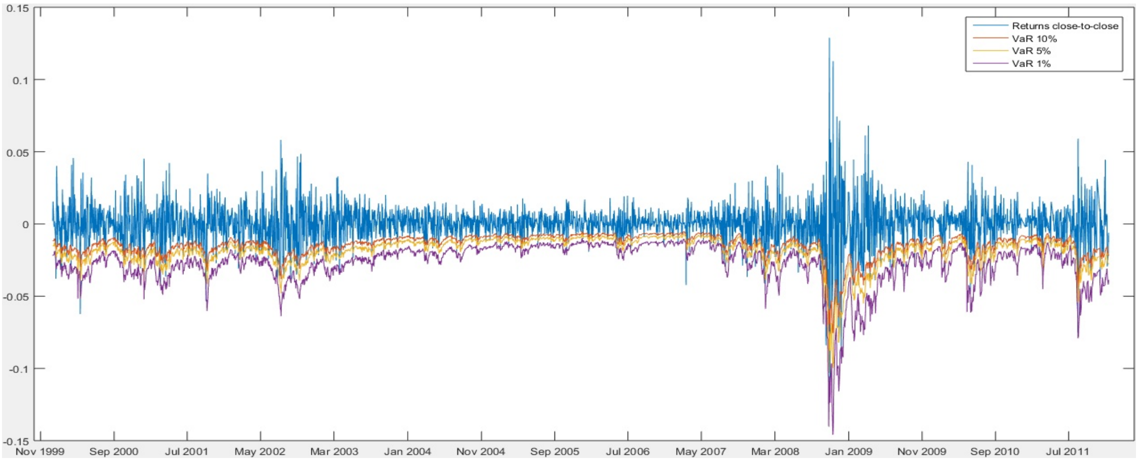

Figure 4.11: Example of plot of VaR estimates. The VaR estimates come from the simulations with AHAR-RV model for RV + GJR-GARCH for overnight volatilities

The plot in figure (4.11) is an example of the VaR produced by the FHS. In particular, are plotted the VaRs obtained by FHS on AHAR-RV for daily RV and GJR-GARCH for overnight volatilities.

On the next subsections will be commented the results of the tests on the models, together with tables representing the results.

The suffix "rRV" denotes the models which are based on the re-scaled RV, which are the simple HAR model (HAR-rRV); HAR with GARCH disturbances (HAR-GARCH-rRV); Asymmetric HAR (AHAR-rRV); Asymmetric HAR with GARCH disturbances (AHAR-GARCH-rRV). The suffix "G" denotes the models with non-re-scaled RV and a GARCHX specification for the overnight returns: namely, simple HAR model plus a GARCHX applied to the overnight returns (HAR-RV-G); HAR with GARCH disturbances plus overnight-GARCHX (HAR-GARCH-RV-G); Asymmetric HAR plus overnight-GARCHX (AHAR-RV-G); Asymmetric HAR with GARCH disturbances plus overnight-GARCHX (AHAR-GARCH-RV-G);The suffix "A" denotes the models with non-re-scaled RV and a GJR-GARCHX specification for the overnight returns: i.e., simple HAR model plus a GJR-GARCHX applied to the overnight returns (HAR-RV-A); HAR with GARCH disturbances plus overnight-GJR-GARCHX (HAR-GARCH-RV-A); Asymmetric HAR plus overnight-GJR-GARCHX (AHAR-RV-A); Asymmetric HAR with GARCH disturbances plus overnight-GJR-GARCHX (AHAR-GARCH-RV-A);. "GBA" is the FHS application of Barone-Adesi and colleagues.

4.3.1 Failure rates

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	**0.1028**	**0.0544**	0.0148
HAR-GARCH-rRV	0.1069	0.05784	0.01623
AHAR-rRV	0.0707	0.0284	0.0040
AHAR-GARCH-rRV	0.0859	0.0412	**0.0098**
HAR-RV-G	0.1140	0.0629	0.0145
HAR-GARCH-RV-G	0.1160	0.0639	0.0162
AHAR-RV-G	**0.0967**	0.0446	**0.0101**
AHAR-GARCH-RV-G	0.1045	**0.0537**	0.0121
HAR-RV-A	0.1106	0.0619	0.0152
HAR-GARCH-RV-A	0.1123	0.0652	0.0162
AHAR-RV-A	0.0947	0.0402	**0.0098**
AHAR-GARCH-RV-A	**0.1004**	**0.0520**	0.0128
GBA	0.1018	0.0504	0.0081

Table 4.1: Failure rates of the models. The bold numbers are the best performers of every group (the re-scaled RV group, the overnight-GARCHX group, the overnight GJR-GARCHX group).

Table (4.1) represents the failure rates of the considered models. The models that present the closest failure rate to the required coverage level, are often Asymmetric HAR models, with the exceptions of the simple HAR applied to the re-scaled RV, that produces the closest failure rate to che coverage levels of 10% and 5%, with respect to the asymmetric models fitted to the re-scaled RV. Notice that the AHAR applied to the re-scaled RV presents a very low number of failures: this is unusual and it needs further investigations. The better models, in absolute terms, are: AHAR-GARCH-RV-A for the 10% VaR; The GBA benchmark for the 5% VaR; AHAR-RV-G for the 1% VaR.

4.3.2 POF test

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	**0.6080**	**0.2718**	0.0128
HAR-GARCH-rRV	**0.2156**	**0.0558**	0.0017
AHAR-rRV	2.476e-08	5.204e-09	0.0002
AHAR-GARCH-rRV	0.0091	0.0249	**0.9172**
HAR-RV-G	0.0128	0.0019	0.0199
HAR-GARCH-RV-G	0.0044	0.0008	0.0017
AHAR-RV-G	**0.5542**	**0.1747**	**0.9353**
AHAR-GARCH-RV-G	**0.4144**	**0.3501**	**0.2496**
HAR-RV-A	**0.0579**	0.0041	0.0080
HAR-GARCH-RV-A	0.0283	0.0002	0.0017
AHAR-RV-A	**0.3350**	0.0119	**0.9172**
AHAR-GARCH-RV-A	**0.9316**	**0.6031**	**0.1351**
GBA	**0.7412**	**0.9194**	**0.2880**

Table 4.2: P-values for the POF (proportion of failures) test. A p-value lower than 0.05 denotes the rejection of the null hypothesis of correct coverage level. The bold numbers represent the accepted models.

Table (4.2) represents the p-values of the POF test. More than half of the models applied to the re-scaled RV presents an incorrect coverage. As suspected, the AHAR applied to the re-scaled RV fails this test, denoting a Failure Rate too far from the desired coverage level. It is interesting to see that almost all of the models that pass this test are the Asymmetric HAR (with or without GARCH disturbances), with a GARCH or GJR-GARCH-overnight specification. This evidences the importance the asymmetries have in fitting RV models.

4.3.3 Conditional Coverage test

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	**0.0521**	**0.0914**	0.0420
HAR-GARCH-rRV	0.0262	**0.0592**	0.0073
AHAR-rRV	4.135e-08	2.330e-08	0.0001
AHAR-GARCH-rRV	0.0082	**0.0721**	**0.5972**
HAR-RV-G	0.0016	0.0023	0.0613
HAR-GARCH-RV-G	0.0009	0.0009	0.0073
AHAR-RV-G	0.0027	**0.2757**	**0.6267**
AHAR-GARCH-RV-G	0.0265	**0.2476**	**0.4042**
HAR-RV-A	0.0093	0.0029	0.0281
HAR-GARCH-RV-A	0.0031	0.0001	0.0073
AHAR-RV-A	0.0042	0.0284	**0.5972**
AHAR-GARCH-RV-A	0.0098	**0.4227**	**0.2713**
GBA	**0.8930**	**0.5897**	**0.2583**

Table 4.3: P-values for the Conditional Coverage test. A p-value lower than 0.05 denotes the rejection of the null hypothesis of correct coverage level. The bold numbers represent the eccepted models.

Table (4.3) represents the p-values for the Conditional Coverage test. The HAR applied to the re-scaled RV fails this test, too, denoting a number of hits that may be too high or too law or denoting too clustered hits: in accordance with the previous POF test, the reason is probably the too low number of exceptions. The AHAR-GARCH-RV-A for the 10% VaR (the best model in terms of failure rates) results with an incorrect conditional coverage. Instead, the GBA benchmark for the 5% VaR and AHAR-RV-G for the 1% VaR are accepted by the test. The accepted models are almost all of the Asymmetric-type, showing, again, the asymmetric behaviour of RV.

4.3.4 ”Regulatory” Loss Function

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	0.02661	0.01218	0.00200
HAR-GARCH-rRV	0.02950	0.01419	0.00247
AHAR-rRV	**0.01558**	**0.00547**	**0.00034**
AHAR-GARCH-rRV	0.02462	0.01113	0.00138
HAR-RV-G	0.03414	0.01563	0.00329
HAR-GARCH-RV-G	0.03674	0.01735	0.00370
AHAR-RV-G	**0.02573**	**0.01032**	**0.00164**
AHAR-GARCH-RV-G	0.03288	0.01515	0.00326
HAR-RV-A	0.03366	0.01559	0.00323
HAR-GARCH-RV-A	0.03581	0.01714	0.00378
AHAR-RV-A	**0.02463**	**0.01012**	**0.00157**
AHAR-GARCH-RV-A	0.03262	0.01516	0.00350
GBA	0.03852	0.01638	0.00241

Table 4.4: Values of the ”Regulatory” Loss Functions. The bold numbers represents the best performes of every group of models.

The loss functions are necessary to understand which is the model that causes the lower losses. Table (4.4) represents the values of the Regulatory Loss Function applied to the VaR models. The AHAR-GARCH model applied to the re-scaled RV produces the lowest losses, in absolute terms. This is, however, an unuseful result: from the previous tests, it generates too few exceptions, and is sistematically rejected from the tests. The model that presents the lowest loss functions of the HAR + GARCHX-overnight group is the AHAR-RV-G, for any coverage level. The best performer of the HAR + GJR-GARCHX-overnight group is the AHAR-RV-A model, for all the coverage levels. The benchmark GBA is outperformed from both the last two models.

4.3.5 Firm's Loss Function

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	0.04418	0.03431	0.03500
HAR-GARCH-rRV	0.04664	0.03575	0.03458
AHAR-rRV	**0.03325**	**0.02894**	**0.03361**
AHAR-GARCH-rRV	0.04181	0.03356	0.03380
HAR-RV-G	0.05080	0.03736	0.03690
HAR-GARCH-RV-G	0.05323	0.03873	0.03688
AHAR-RV-G	**0.04257**	**0.03206**	**0.03549**
AHAR-GARCH-RV-G	0.04932	0.03645	0.03637
HAR-RV-A	0.05036	0.03757	0.03710
HAR-GARCH-RV-A	0.05229	0.03872	0.03687
AHAR-RV-A	**0.04145**	**0.03278**	**0.03540**
AHAR-GARCH-RV-A	0.04922	0.03714	0.03668
GBA	0.05739	0.04421	0.04833

Table 4.5: Values of the Firm's Loss Functions. The bold numbers represents the best performes of every group of models.

The Firm's Loss Function is an improvement of the aforementioned loss function: it incorporates an opportunity cost, due to a wrong capital allocation, which comes from an overestimation of the VaR. The opportunity cost is, usually, a risk-free rate: we have employed the 1-Month London Interbank Offered Rate (LIBOR), based on U.S. Dollar. Table (4.5) represents the values of the Firm's Loss Function applied to the VaR models. Notice how the AHAR-GARCH 1% model applied to the re-scaled RV produce a Firm's Loss Function even worse than the same model for the 10% VaR: this is an additional sign that probably the model is not correct for a VaR estimation application. Something similar, but in a less serious way, happens to the AHAR-RV-G and AHAR-RV-A (again, the best performers of the relative groups): the Firm's Loss Function is slightly better for the 5% VaR for both the models, than the one for the 1% VaR. In particular, the 5% VaR produced by AHAR-RV-G is the best performer in absolute terms. The benchmark GBA is outperformed by all the other models, for all the coverage levels: even if it is correctly specified, it generates a wrong capital allocation.

4.4 A post-crisis Analysis

The analysis has been run on a post crisis basis, too. This has been done in order to show that a simple re-scaling of the realized volatility over the whole sample, is, at least, a limiting procedure to achieve the goal of VaR estimation. Instead, the use of a dynamic model applied to overnight volatilities is a better way.
The same tests have been repeated, considering a sample from 1-Jul-2009 to 14-Dec-2011 (599 observations). The time period considered is more than two years, so the analysis is valid, since a VaR backtest should be run on one year of observations (252), at least.

4.4.1 Failure rates

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	0.1116	0.07	0.03
HAR-GARCH-rRV	0.1133	0.0766	0.03
AHAR-rRV	0.08	**0.045**	0.0016
AHAR-GARCH-rRV	**0.0966**	0.0633	**0.0183**
HAR-RV-G	0.1133	0.065	0.0116
HAR-GARCH-RV-G	0.1116	0.065	0.0183
AHAR-RV-G	0.0883	**0.0483**	0.0066
AHAR-GARCH-RV-G	**0.0966**	0.0583	**0.0133**
HAR-RV-A	**0.1016**	0.0683	0.0216
HAR-GARCH-RV-A	**0.1016**	0.0716	0.0216
AHAR-RV-A	0.0883	**0.0516**	**0.0116**
AHAR-GARCH-RV-A	0.0933	0.065	0.0166
GBA	0.095	0.0666	0.0133

Table 4.6: Failure rates of the models. The bold numbers are the best performer of every group.

Table (4.6) represents the failure rates of the considered models on the post-crisis period. The best performing models are still the Asymmetric HAR, with the exception for the HAR-RV-A models for 10% VaR that are the best, regarding the GJR-GARCH-Overnight-type of models. In absolute terms, HAR-RV-A and HAR-GARCH-RV-A rank first for the 10% VaR; AHAR-RV-A ranks first for both 5% and 1% VaR. The FHS benchmark of Barone-Adesi is always outperformed by AHAR models, for every coverage level.

4.4.2 POF test

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	**0.3488**	0.0335	7.062e-05
HAR-GARCH-rRV	**0.2853**	0.0052	7.062e-05
AHAR-rRV	**0.0917**	**0.5678**	0.0110
AHAR-GARCH-rRV	**0.7844**	**0.1494**	**0.0661**
HAR-RV-G	**0.2853**	**0.1063**	**0.6893**
HAR-GARCH-RV-G	**0.3488**	**0.1063**	**0.0661**
AHAR-RV-G	**0.3320**	**0.8506**	**0.3823**
AHAR-GARCH-RV-G	**0.7844**	**0.3609**	**0.4349**
HAR-RV-A	**0.8920**	**0.0503**	0.0128
HAR-GARCH-RV-A	**0.8920**	0.0218	0.0128
AHAR-RV-A	**0.3320**	**0.8521**	**0.6893**
AHAR-GARCH-RV-A	**0.5824**	**0.1063**	**0.1341**
GBA	**0.6808**	**0.0740**	**0.4349**

Table 4.7: P-values for the POF (proportion of failures) test. A p-value lower than 0.05 denotes the rejection of the null hypothesis of correct coverage level. The bold numbers represent the accepted models.

Table (4.7) represents the p-values of the POF test on a post-crisis time period. The difference between re-scaled RV models and non-re-scaled RV models is evident: no one of the non-re-scaled RV models with a GARCHX specification for the overnight returns are rejected by the test, against 3 rejections on the GJR-GARCHX-overnight group and 5 rejections in the re-scaled RV group. The benchmark presents a correct coverage level.

4.4.3 Conditional Coverage test

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	**0.0779**	0.0390	0.0003
HAR-GARCH-rRV	**0.0576**	0.0126	0.0003
AHAR-rRV	**0.1329**	**0.8181**	0.0003
AHAR-GARCH-rRV	**0.4048**	**0.1937**	**0.0893**
HAR-RV-G	**0.1473**	**0.1247**	**0.2160**
HAR-GARCH-RV-G	**0.1922**	**0.1247**	**0.0893**
AHAR-RV-G	**0.0555**	**0.8873**	**0.0609**
AHAR-GARCH-RV-G	**0.7085**	**0.4245**	**0.2157**
HAR-RV-A	**0.2908**	**0.0553**	0.0277
HAR-GARCH-RV-A	**0.2908**	0.0216	0.0277
AHAR-RV-A	**0.0555**	**0.8121**	**0.2160**
AHAR-GARCH-RV-A	**0.4444**	**0.1247**	**0.1363**
GBA	**0.9049**	**0.0927**	**0.2157**

Table 4.8: P-values for the Conditional Coverage test. A p-value lower than 0.05 denotes the rejection of the null hypothesis of correct coverage level. The bold numbers represent the accepted models.

Table (4.8) represents the p-values for the Conditional Coverage test. The situation is identical to the one of the previous test: in general, the re-scaled RV models are more rejected by the test, in particular, regarding the 1% VaR estimates.

4.4.4 ”Regulatory” Loss Function

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	0.00723	0.00369	0.00065
HAR-GARCH-rRV	0.00765	0.00395	0.00076
AHAR-rRV	**0.00343**	**0.00101**	**4.890e-06**
AHAR-GARCH-rRV	0.00571	0.00243	0.00015
HAR-RV-G	0.00768	0.00373	0.00076
HAR-GARCH-RV-G	0.00807	0.00393	0.00078
AHAR-RV-G	**0.00533**	**0.00203**	**0.00026**
AHAR-GARCH-RV-G	0.00693	0.00321	0.00061
HAR-RV-A	0.00777	0.00365	0.00062
HAR-GARCH-RV-A	0.00801	0.00373	0.00064
AHAR-RV-A	**0.00510**	**0.00184**	**0.00015**
AHAR-GARCH-RV-A	0.00685	0.00291	0.00043
GBA	0.00949	0.00433	0.00036

Table 4.9: Values of the ”Regulatory” Loss Functions. The bold numbers represents the best performes of every group of models.

Table (4.9) represents the values of the Regulatory Loss Function applied to the VaR models. The AHAR-GARCH model applied to the re-scaled RV produces the lowest losses, in absolute terms. It can be accepted for what concerns the 10% and 5% VaR, but has to be rejected the 1% VaR estimation, since this failed the statistical tests. The best performer of the HAR + GARCHX-overnight group is the AHAR-RV-G. The best performer of the HAR + GJR-GARCHX-overnight group is the AHAR-RV-A model. The benchmark GBA is outperformed by the best RV models.

4.4.5 Firm's Loss Function

	VaR 10%	VaR 5%	VaR 1%
HAR-rRV	0.00736	0.00385	0.00094
HAR-GARCH-rRV	0.00777	0.00412	0.00104
AHAR-rRV	**0.00361**	**0.00124**	**0.00031**
AHAR-GARCH-rRV	0.00584	0.00264	0.00045
HAR-RV-G	0.00783	0.00398	0.00112
HAR-GARCH-RV-G	0.00822	0.00412	0.00114
AHAR-RV-G	**0.00548**	**0.00228**	**0.00063**
AHAR-GARCH-RV-G	0.00707	0.00345	0.00097
HAR-RV-A	0.00792	0.00389	0.00098
HAR-GARCH-RV-A	0.00801	0.00373	0.00064
AHAR-RV-A	**0.00524**	**0.00209**	**0.00052**
AHAR-GARCH-RV-A	0.00699	0.00314	0.00079
GBA	0.00964	0.00461	0.00081

Table 4.10: Values of the Firm's Loss Functions. The bold numbers represents the best performes of every group of models.

Table (4.10) represents the values of the Firm's Loss Function applied to the VaR models. The situation is almost identical to the one of the previous loss function. The best model in absolute terms is still the AHAR-rRV; however, given its failures on the statistical tests for what concerns the 1% VaR estimate, the best model on this coverage level is the AHAR-RV-A. The benchmark is still outperformed by all the best RV models.

Conclusions

The main purpose of this research was to check whether the specification of a dynamic model to the overnight returns, in order to obtain an estimate of the close-to-open volatilities, is a better approach than the simple re-scale with a constant over all the sample period, to the aim of VaR forecast and employing the Filtered Historical Simulation as an improvement of the more inefficient Historical Simulation.
The results of the application to the whole sample suggest that the adoption of a dynamic specification to model the overnight volatilities lead to more appropriate VaR estimations, in terms of statistical tests, with respect to the re-scaled RV. We can discard the AHAR-rRV model, given its incorrectness about VaR estimation emerged from the statistical tests, subsequent to a too low number of exceptions generated. Accordingly, the performances in terms of loss functions of the Asymmetric HAR models with GARCHX or GJR-GARCHX overnight models are, at least, comparable to the AHAR-GARCH-rRV model. Specifically, they outperform on the 5% VaR loss functions.
The advantages deriving from the use of such a dynamic specification are no more evident when the models are applied to the post-crisis subsample. Our procedure performes worse in terms of loss functions, but, in general, is more suitable to a VaR estimation purpose, in terms of statistical accuracy tests. In particular, for what concernes the 10% and 5% VaR, the best model has resulted the AHAR-rRV and, given its failures on the tests about the 1% VaR, the best model in absolute term on that coverage level is the AHAR-RV-A.
In addition, what emerges from our analysis is that Filtered Historical Simulation

applied to Realized Volatility models are able to sistematically outperform the Asymmetric GARCH benchmark of Barone-Adesi et al.. Even if the tests tells us that it is well suited to a VaR estimation purpose, it is sistematically outperformed by the other models in terms of loss functions. In particular, Asymmetric-HAR-type of models are able to reproduce better than the others the market conditions, that changes over time.

On practical applications, we would suggest the employment of the Asymmetric-HAR-RV model to fit the open-to-close realized volatility, together with the use of an asymmetric model to fit the overnight volatilities. The application of asymmetries ensures a better response to the modificaions of the market conditions.

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