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Final Thesis

Adaptive Market Hypothesis: a new point in Finance evolution

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Abstract:

The man, from dawn of his birth, has faced an evolution path under all the aspects, genetic, biological, social ones.

Specifically, during the last centuries, the growing process of scientific and socials environments has grown so fast, that from one "generation" to another, a large gap of knowledge and know how was formed.

The purpose of this thesis is to look deeply into the present state of art of economy and finance and try to link the major new theories that are emerging in recent years. This task is really hard because we have understood that each discipline has influence on the other, the interconnectedness between economy and math, biology, psychology, sociology etc.. is so dense and deep that to understand also the apparently simple events beyond economy, we have to take into account the complete environment in where the event occurs, studying all the participants and their relation.

The starting point is to analyse an accepted and taught economics theory and then, to study new theories that can unify all the discordances of existing theories under a single model.

I choose to start with the EMH, efficient market hypothesis, this is not a random choice, although it seems this topic is specific of financial world, I recognize in it a lot of arguments and assertations that engage with numerous other topics. Then it will be presented a possible reconciliation between the traditional theory of the markets and its critics, a new theory made by Andrew Lo, the Adaptive Market Hypothesis, the scope of this thesis is to extend our view over the possible next steps and implication of finance world.

Introduction

This thesis born with the idea of analysing the new theory of professor Andrew Lo: Adaptive Market Hypothesis. This theory was first presented in 2004[1] and after several developments and contributions he give to his theory a more complete form publishing his book: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press). This book inspired me and give me the possibility to have a possible general view of a Finance world in which classic finance theories (Efficient Market Hypothesis, Capital Asset Pricing Model and others), behavioural finance and quantitative trading are linked together.

I have presented the Lo's new theory not on its own but inserted in a development process regarding finance sector from its dawn. Clearly, the world is continuously changing and with him society, laws and so, the man. Finance is a "product" of man and his interactions, and so it has evolved too, together with the different contexts that society has crated. From a first historical analysis what emerges is that, for a long period of twentieth century, after the 1929 crisis, financial markets have remained stable, i.e. with low volatility and moderate positive returns on average. The stability of financial market was the right input for the development of the classical finance theories, I recognized as central in this process, the Efficient Market Hypothesis of Eugene Fama, from which numerous financial models have been developed, both theoretical both with practical application as Capital Asset Pricing Model, Sharpe ratio, Black Scholes option pricing and so on.

These models try to describe financial markets with a rational and efficient structure, in general the empirical proofs seem to follow this one. Nevertheless, towards the end of the 80's, the first signals of inefficiencies in financial markets started to appear: returns volatility increased, a crash of the market occurred in 1987 and so, the classical models started to be called into question. At the same time, after the coming of internet, a technological revolution started and changed radically every aspect of the life of a person. In the last thirty years, the speed by which man has advanced technologically has been so high that, for the other sciences, has been tough to adapt, also because several traditional theories have become unsuitable and obsolete.

The opposite aspect is that, thanks to this progress, numerous new discoveries has been made. The new sophisticated and advanced scientific instruments have led to a better and more complete understanding of human dynamics, both external, namely the relationship of man with the environment and with the other man, both internal, the inner process that regulate the human behaviour. This general context, as said, has influenced ever field of research, so the purpose

[1] The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective (Andrew W. Lo August 15, 2004).

of this thesis is to see the Finance world and theories under this idea of a development process. In the first chapter I start explaining the Efficient Market Hypothesis, its forms, its core concepts and its roots. To define an evolution process it is important also explain how the studies and researches of the past have influenced the traditional theories, not only in theoretical terms but also the quantitative models from which EMH and Random Walk Theory has born.

After some examples of how EMH explains some empirical events, I'll present the principal challenge to Fama's theory: the behavioural finance. I want to highlight how my purpose is not to express a definitive judgement between EMH and its critics, but rather to present these new alternative theories as the result of a development process in which man and its inner behaviours become the new subject of studies. This concept has led to reconsider several traditional theories in a new perspective, and so I examine how traditional finance concept has changed, especially the meaning of rationality. In the second chapter, I present a new theory: Adaptive Markets Hypothesis, developed by professor Andrew Lo, which tries to reconcile EMH and the classical models with the alternative theories.

Always following the idea of an evolution process, I focus on how AMH can help to improve EMH adopting the concepts taken form biology. In doing so a brief description of how Neurosciences affect man's behaviour is explained, adding some relevant aspects of how human brain works, focusing on their application in finance and economy.

The precious contribution of biology and neuroscience to economics and finance is then summarized into the principles of the new theory elaborated by Andrew L. In particular I focus on the mainly insights of it presenting a model developed by Brennan-Lo that simulates the reproductive and evolutive process on a population sample. This model will be further developed in the third chapter with proposal applications in a financial environment.

The second part chapter of part 2 is dedicated to the re-formulation of the traditional investment rules code, under the new concepts expressed by the AMH. I end the chapter 2, disputing about how AMH could explain some financial anomalies, presenting an empirical case: the Quants Meltdown of 2007, a case of anomaly that is not completely understood still nowadays. As said, the third chapter is entirely dedicated to the empirical tests of the binary choice model re-elaborated in a new, financial framework.

1) The Stand: Efficient Market Hypothesis

1.1 Introduction

In an ideologic path toward the deepest knowledge, man reaches a point which seems to be the highest. To find a new stable and higher point gets harder and harder, also because, to leave a safe road (a theory generally accepted by scientific community) for a new, unexplored one is always complicated.

Considering the history of Economic and Financial Markets, now could be the right time to move some little steps ahead. In the next paragraph I'll present the detailed reasons for this statement.

Basically, the economic and financial world is changing very quickly in the last decades, numerous events lead to new challenges, see the subprime mortgages, the south-European union crisis (Italy, Greece Spain and Portugal). Finance is becoming more complex, new financial instruments appears and new technologies are transforming radically the markets, so, in response to this mutating sector, economic academicians and researchers are trying to find new theories that better fit the real world, since the theories developed in the past, could not still be the "most appropriated".

I choose as base point for the finance evolution process, the Efficient Market Hypothesis, which probably reached its height of dominance in academic environment around the 1970s.

In 1970 indeed, Eugene Fama, one of the most influent American economist, he won the Nobel Prize in 2013, established the Efficient Market Hypothesis, which has been widely accepted and many researchers has tried to test it by using different empirical examples.

He coined the terms "market efficiency" and "efficient markets,". They first appear in "Random Walks in Stock Market Prices," paper number 16 in the series of Selected Papers of the Graduate School of Business, University of Chicago, reprinted in the Financial Analysts Journal.

Fama, in his Ph.D thesis of 1965, give a first definition of the term efficient market:

"An efficient market is defined as a market where there are large numbers of rational, profit maximisers, actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants…

In an efficient market, on the average, competition will cause the full effects of new information on intrinsic value to be reflected instantaneously in actual prices."

The efficient market hypothesis (EMH) has been subject to debate for decades.

The theory developed by Fama was primarily focus on the informational efficiency concept: it is not possible to gain extra-profits through financial trades (buying/selling financial assets) based on public available information. This idea represents a concrete situation in which unexploited profit opportunities are cancelled out. The debate about informational efficiency of stock markets started more than 40 years ago, but if we look in a larger historical perspective, Adam Smith in his “The theory of moral sentiments”. Indianapolis: Liberty Fund (1759), and “The wealth of nations”. New York: P.F. Collier (1766), was concerned by the efficiency and the nature of financial and economic markets, or essentially speaking, whether or not stock prices are in line with the intrinsic value of the underlying financial asset.

The efficiency concept has various meaning in Financial Markets:

1) information-arbitrage efficiency; financial market is efficient if, buying and selling financial assets, it is not possible to obtain excess returns based on public available information. Only investor that know private information can use these to gain excess returns, this is called insider trading and it is not legal.
2) fundamental-valuation efficiency[2]; The price of a financial asset must reflect the expected value of the dividends or other cash flows, which represents the fundamentals of that particular asset. The market is so, fundamental-valuation efficient if its quotations reflect the rational expectations of future cash flows.
3) full-insurance efficiency; When the agents of a market can assure the delivery of goods and services in every future circumstance, selling immediately resources or negotiating the future delivery; the market is full-insurance efficient[3].
4) functional efficiency. This concept is related in a concrete way to the financial industries and its economic functions. The services and resources provided in the market should not be directly useful to producers or consumers, but instead should manage more safely risk, help the transaction and the financial network, the management of savings for investments in physical and human capital and so on.

What it is important to disentangle for our discussion is the core idea of efficiency developed by Fama, Efficient markets is “a market in which prices always fully reflect available information”.

[2] In the market, the price fluctuations of the stocks are wider than rational expectations based on fundamentals.

[3] In real financial market these conditions are hard to satisfy, indeed to buy or sell stocks it is need to employ resources and bear the linked cost.

The model developed by Fama based not only on theoretical works but also the empirical ones. The research reviewed some historical studies and also tested the hypothesis by using several models.
The model is based on some hypothesis:

1. There is no transaction cost in trading securities;

2. All available information is costless available to all market participant;

3. Everybody agree on the implications of current information for the current price and distributions of future prices of each security.

According to the EMH, information is unbiased, indicating we cannot use the historical price to predict the future return, and the price should be random. The investors can not speculate by buying the undervalued stocks or selling the inflated stocks. They should trade in the stock market with the fair price. The new information or the signal will appear in the future randomly and in a unpredictable way, and the investors cannot just outperform within the market by using the already released information, unless they have really good luck.

1.2 Market efficiency forms

Fama developed the idea of efficiency into three different types of efficient markets based on three applications of the concept “available information”, each form of efficiency corresponds to successively greater amounts of information i.e. weak form efficient markets (based on historical price information); semi-strong form efficient markets (based on all publicly available information); and strong form efficient market (based on all information, both public and private).

In **weak form** efficient markets, it is impossible to persistently generate portfolio returns higher than the market return by trading on past price information because prices fully reflect available information i.e. technical analysis, such as “head-and-shoulders” patterns and candlestick chart, of stocks is useless.

If prices are **Semi-strong** form efficient then prices reflect all public information. This form of efficient market implies that it is impossible for an investor to use public data as company's earnings, sales, and book-to-market ratios to identify mispriced securities.

In **strong-form** efficient markets all private information is reflected in prices. As consequence the insider trading is no more profitable, whatever actions made, based on preferred information won't lead to excess return; the market has already discounted that information. This theory has been studied through the examination of mutual funds. The objective of that studies was to discover whether fund manager have some preferred information that could allow them to realize returns above the market. The empirical evidence surveyed in Fama (1991) and Fama (1998) generally supports the idea that prices seem to be weak and semi-strong efficient but that that markets are not strong form efficient (there are theoretical reasons why strong-form efficiency is unlikely - Grossman-Stiglitz(1980). Evidence is that insider trading is slightly profitable [Finnerty (1976, JF), Muelbrouk (1992, JF)], but performance of mutual funds [Jensen (1968), Blake, Lehman and Timmerman (1997)] found that they do not generate abnormal returns, which is consistent with strong form efficiency.

So, it seems that all the work of Wall Street's technical analysts, fundamental analyst, proprietary traders, and hedge fund managers is a waste of time!

The efficient market hypothesis applied in real life market consists of many rational investors who are constantly reading the news and react quickly to any new significant information about a security. There are also many funds whose managers are constantly reading new reports and news, and with the aid of high-speed computers, are constantly shifting through financial data looking for mispriced securities.

Mentioning Lucas (1978), all investors have "rational expectations", prices do fully reflect all available information and marginal-utility-weighted prices follow martingales.[4] The EMH has been

extended in many other directions, including the incorporation of non-traded assets such as human capital, state-dependent preferences, heterogeneous investors, asymmetric information, and transactions costs. But the general thrust is the same: individual investors form expectations rationally, markets aggregate information efficiently, and equilibrium prices incorporate all available information. As previously written, in real world, the EMH represents a situation in

[4] The martingale is a stochastic process, a sequence of random variables, xt (where t is a increasing parameter), where, for $r \leq s \leq t$, the expected value of xt conditioned respect to values: xr is equal to xs

which the unexploited profit opportunities are deleted, so the returns of a financial asset greater than the return expected based on his information.
To summarize, EMH rests on the following predicates:

- that information is widely available to all investors;
- that investors use this information to analyse the economy, the markets, and individual securities to make trading decisions;
- that most events that have a major impact on stock prices, such as labour strikes, major lawsuits, and accidents, are random, generally unpredictable events and when they do happen, they are quickly broadcast to investors;
- and that investors will react quickly to any new information.

1.3 Back in the History

"Prices must fully reflect all available information" has been a point of arrival for Fama,
But Efficient Market Hypothesis is not the only "road" that drive to this point,
Cardano's martingale, Bachelier's random walk, Samuelson's "Foundations of Economic Analysis" all lead to same place.
These roads come from the past, from the primitive intuition about a mathematical model of financial markets prices.
We have to go back to 1565 when the prominent Italian mathematician, Girolamo Cardano, in Liber de Ludo Aleae (The Book of Games of Chance) wrote: 'The most fundamental principle of all in gambling is simply equal conditions: of opponents, of bystanders, of money, of situation, of the dice box, and of the die itself. To the extent to which you depart from that equality, if it is in your opponent's favour, you are a fool, and if in your own, you are unjust'.
This idea comes from the world of gambling and this, should not be surprising, since financial investing and gambling both involve calculating-trade-offs between risk and reward.
Cardano had realized that, in a fair game, your winnings or losses can't be forecast by looking at your past performance. If someone could, the game would be no fairer, because one could develop a slight edge over the opponents and increase constantly his gain. Sometimes it has happened, some very clever people in Black Jack have figured out how to make inferences about the cards which remain to be dealt, or in Roulette, with strategies based on past game performance.

Cardano gives us an advice on speculation that is wise to follow even today; this notion of a "fair game" came to be known as a martingale.

Did people follow Cardano's suggestion over the years?

Not at all, over the years many thousands of people have studied, tested, tried to find a way to beat the market, and most of them have failed (the history tells us one of the most important aspect of human investing behaviour: overconfidence[5], I'll discuss it better later).

A possible explanation to this "debacle" by the men versus the market has started to appear in 1900;

a French Ph.D student, Louis Jean Baptist Bachelier published his PhD thesis: "Theorie de la Speculation", and chose to analyse the Parisian stock market, in particular the prices of warrants trading on the Paris Bourse.

We are getting closer to the Fama's theory, indeed the studies and experiments made by Bachelier lead to a discovery as important, as unusual about stock prices.

The price of any stock trade must be a fair price on which the buyer and the seller agree. No-one wants to buy for more or sell for less, so trade has to be fair.

The result for Bachelier was to assert that stock prices must necessarily moves completely random, as a "drunkard's walk". From his studies born what we call "Random Walk Model" of stock prices.

In the simplest terms, a "random walk" is essentially a Brownian motion where the previous change deduced that 'The mathematical expectation of the speculator is zero' 65 years before Samuelson (1965) explained efficient markets in terms of a martingale. Bachelier's work was way ahead of his time and was ignored until it was rediscovered by Savage in 1955. Five years later Karl Pearson, a professor and Fellow of the Royal Society, introduced the term random walk in the letters pages of Nature (Pearson, 1905). Unaware of Bachelier's work in 1900, Albert Einstein developed the equations for Brownian motion (Einstein, 1905).

Brownian motion is a sophisticated stochastic process, based on a process in plants discovered by Robert Brown in 1827. He found that small particles suspended in a fluid were in continuous movement and thus, described it as Brownian motion

The explanation of Brownian motion, given by Einstein in 1905 and based on the kinetic-molecular conception of matter, is considered one of the fundamental pillars supporting atomism.

[5] Overconfidence: the investor is extremely trustful in his skills, he believe that he can beat the market thanks to his superior ability resulting in a too confident behaviour.

We are reaching a point in history in which economics is developing into a science from a more philosophical/abstract subject. Bachelier looking at warrant's prices in stock market discovered something that 5 years later would have become a central theory in physics, it was just question of time before someone started to give to all the ideas in economics a mathematical form.

It was the 1941, a Ph.D thesis, "Foundations of Economic Analysis" (not so modest as title) was published , and it became truly the foundations for the modern economics.

His author, Paul A. Samuelson was deeply inspired by the American mathematical physicist Josiah Willard Gibbs. Samuelson applies ideas from physics across the full spectrum of economics, and he is one of the biggest contributor to the reason why modern economics is so mathematical [1].

He became probably the most influential economist of the second half of the 20th century.

Samuelson won the Nobel Prize in 1970 (he was the first American economist to win it) and he has been considered as the last of the great general economists.

Bachelier's work remained unfortunately and inexplicably in shadow for about five decades.

A statistic professor of University of Chicago, Leonard Jimmie Savage come upon a copy of Bachelier's thesis. Savage alerted several colleagues about this undiscovered important work. Samuelson was one of the colleagues and immediately recognize the significance of that paper, starting to refer to Bachelier always more often, not only, he makes another important step forward, if Bachelier explained the how of the Random Walk Model, he explains why market prices moved as random [2].

Using the mathematical technique of induction, Samuelson showed that all the information of an asset's past price changes is incorporated in the asset's present price.

The price already contains all the known information about the asset until the "present" moment, everything has already been taken into account. As a result, past prices changes carry no information in predicting the asset's next price. [3]

The term "random walk" became popular after Samuelson, when Burton Malkiel, a Princeton professor of Economics, in 1973 wrote his famous book: "A Random Walk Down Wall Street". However, the first to examine stock price series considering the theory that stock prices move randomly was Maurice Kendall in 1953, with his paper: "The Analysis of Economic Time Series, Part 1: Prices". From his study of 22 stock and commodities price series, he got the conclusion that "in series of prices which are observed at fairly close intervals the random changes from one term to the next are so large as to swamp any systematic effect which may be present. The data behave almost like wandering series". From this work, Samuelson's one and all the previous studies, Fama worked out his efficient market hypothesis.

Now we have re-connected with our starting point: Fama's efficient market hypothesis. Before looking at next steps, let give a brief look to how all these processes narrated developed in **quantitative terms**:

Our historic digression started from Cardano, who introduced the notion of fair game, coming close to the actual definition. In his book, previously cited, Cardano wrote: "there is one general rule for calculation: One needs to take into account the total number of outcomes and the number of outcomes presenting interest then to find the ratio of the second and first numbers. The sizes of stacks should be related in the same manner for a fair game."

The concept, later known as martingale, in money terms states that the expected profit at a given time given the total past capital is null with probability one.

If we apply martingale hypothesis to the prices of financial securities it follows some implications that could be surprising, i.e. if stock prices are a martingale, it can be mathematically proven that no linear forecasting rule based solely on historical prices can forecast future price changes[6].

Defining a probability measure P for a random experiment is a real-valued function, consider a probability space specified by the triple (S,A,P) where (S,A) is a measurable space, with S the domain and A is its measurable subsets, and P is a measure on A with P(S) = 1. Then the measure P is said to be a probability measure[7].

The concept of martingale means that under certain probability measures, and assuming that: the asset's price of tomorrow is the best forecast of actual price, asset prices turn out to have the martingale property, assuming that: tomorrow's price is today's best forecast and price changes are uncorrelated at all leads and lags.

1.4 The Quantitative models forefather of Efficient Market Hypothesis

1.4.1"Fair game" model

If $\boldsymbol{\Phi_t = \{ p_0, p_1, \ldots, p_t\}}$ are an asset price history at time t = 0, 1, 2, . . . and p_t is the price of an asset at time t,

[6] CMT Level I 2016: An Introduction to Technical Analysis

[7] Weisstein, Eric W. "Probability Measure." From MathWorld

expressing the relevant information, we have currently regarding the asset price time series. Then the expected next period price at time t+1 is equal to the current price

$$\mathbf{E\ (p_{t+1}|\ p_0, p_1, p_2, \ldots, p_t) = p_t} \quad (1)$$

or $\mathbf{E\ (p_{t+1}|\ \Phi_t) = p_t}$ for any time t

where E is the expected value,

introducing r_{t+1}, a causal variable who state the return of the single asset at time t+1, we can write

$$\mathbf{E\ (p_{t+1}|\ \Phi_t) = [1 + E(r_{t+1}|\Phi_t\,)]\ p_t} \quad (2)$$

If instead asset prices decrease (or increase) in expectation over time, we have a super-martingale (sub-martingale):

$$\mathbf{E\ (p_{t+1}|\ \Phi_t) \leq (\geq)\ p_t}$$

Equation (2) describes a conditional expectation of the relevant information set which is fully incorporated into the price formation. The direct consequence of the fair game hypothesis is that: using some strategies (buy or sell) based on the information set, it is impossible to have profits bigger than equilibrium ones.

Defining

$$x_{t+1} = p_{t+1} - E(\tilde{p}_{t+1}|\Phi_t) \quad (3)$$

x_{t+1} represents the excess market value of a security, equal to the difference between the observed price and the expected value of the prices estimated.

So

$$E(\tilde{x}_{t+1}|\Phi_t) = 0 \quad (4)$$

in other words, the historical series of returns x_t, are a fair game respect to information set Φ_t.

1.4.2 “Random Walk” model

Albert Einstein provided a mathematical foundation to explain Brownian motion in 1905 as the result of the random molecular bombardment of the pollen grains—at any given time, the molecules bombarding the pollen grains on all sides are unbalanced, causing the grains to move one way, then another. Because the bombardment of the molecules was random, so was the resultant motion.

Using Bachelier's work we can determine a random walk model, in addition we have to make some hypothesis:

Every variation of the price (his returns) are independent and identically distributed (as the motion of Brownian particles). So, probability distribution is the same and the information known of one variable (the price variation in this case) does not make any influence on the other variable value.

The mathematical model "Random Walk" consider a density function f, (independent from t), related through this equality:

$$f(r_{t+1} | \Phi_t) = f(r_{t+1}) \qquad (5)$$

This means that for a random independent variable, the marginal conditioned distribution of probabilities is identical with respect the unconditioned one.

The hypothesis of random walk, has to be considered as a restriction respect to the fair game model,which, assuming constant returns on time, can be re-written in this following way:

$$E(r_{t+1} | \Phi_t) = E(r_{t+1}) \qquad (6)$$

This formula tells that the expected value of r_{t+1} is independent from the information set Φ_t, while in (5) the entire distribution (not only his expected value) is independent from Φ_t

Samuelson made an additional development to Bachelier's Brownian motion model, which has not considered that normally distributed asset price could be negative at any one time; (Bachelier realizes this, but assumes it happens with an effectively negligible probability).

To solve this, Samuelsons introduced the **geometric Brownian motion** model in which the asset price P(t) is given by

$$P(t) = P(0)\exp(at + \sigma W(t)) \qquad (7)$$

where W(t) is Brownian motion and a, σ are constants.

Since $W(t) \sim N(0,t)$ we have $E[e^{\sigma W(t)}] = \exp(\frac{1}{2}\sigma^2 t)$, so if we take

$a = \alpha - \frac{1}{2}\sigma^2$ then the expected asset price at time t is equal to:

$$E[P(t)] = P(0)e^{\alpha t} \tag{8}$$

So, the asset price at initial time multiply by an exponential growth factor: α is the expected growth rate, σ is a parameter, that measures the standard deviation of log-returns, so it represents the volatility of asset prices. The log returns are defined as : log(P(t+h)/P(t)) , the log of the ratio between the price of an asset at a time t+h over the price of the same asset at a time t, where for a small h the log returns can be simply defined as ~ P(t+h)/P(t) ; so at a time t we buy the stock at a price P(t) and after a time " h" we sell the stock at a price P(t+h). So, the standard deviation will be $\sigma \sqrt{h}$. This achievement reached by Samuelson not only was an arithmetic solution to avoid the problem of Bachelier's Brownian motion, it had also deep inferences on the mathematical development of asset price's financial models. First, highlight that the exponential function: $e^{\alpha t}$ is not linear, so, in order to analyse a nonlinear transformation of Brownian motion it is needed Itô calculus[8]. If we apply the Itô's lemma[9] to P(t)(7), we obtain a differential function dP(t) which satisfies the stochastic differential equation (SDE):

$$dP(t) = \alpha P(t)dt + \sigma P(t)dW(t) \tag{9}$$

From this formula it is easily to see α as the average growth rate, and if α=0 P(t) is a martingale. The formulas become very important in the History of mathematical finance. The famous Black-Scholes option pricing formula[10] has been developed resting on these models and also actually, in order to understand deeply the trading strategies of long/short an asset, the stochastic differential equation is fundamental. All these past models, have been an important core for the future development of the financial models and that's why it is necessary to describe them if we want to understand where efficient market hypothesis, and the linked theories, have born.

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[8] Itô calculus refer to the Itô stochastic integral, where both integrands and the integrators are stochastic processes, it has powerful applications in Finance and stochastic differential equations and it extrapolate the method of calculus of a stochastic process, as Brownian motion.

[9] Itô's lemma is an identity used in Itô calculus to find the differential of a time-dependent function of a stochastic process. (Kiyosi Itô (1944). Stochastic Integral. *Proc. Imperial Acad. Tokyo* **20**, 519-524)

[10] Black-Scholes model is a formula used to calculate the theoretical price of options. This model takes into account several factors: current stock price, expected dividends, the option's strike price, time to expiration, expected interest rate and expected volatility

1.4.3 A brief formalization of EMH

Finally, the Efficient Markets hypothesis born and it took sum of all the precedent insights, reaching the previous discussed core idea: stock prices instantaneously reflect all available public information.

According to this theory, the stock price movement can be described as a stochastic process where the price is conditioned by the coming of new information. Its formalization is:

$$P_{t+1} = \mathbb{E}(\tilde{P}_{t+1}|\Omega_t) + \varepsilon_{t+1} \tag{10}$$

where t and t +1 indicate two consecutive time instants, $P(t+1)$ is the financial asset price at time t +1, $\mathbb{E}(\cdot)$ is the expectation operator, $\tilde{P}(t+1)$ is the random variable "financial asset price" at time t +1, Ωt is the set of all available public information at time t, and t+1 is the random variable "financial asset prediction error" at time t + 1, with $\mathbb{E}(t+1) = 0$. This last condition implies that it is not possible to gain systematically exploiting prices movements. In other words, in an efficient market that fully reflects all available information, price changes are completely random and unpredictable. Following the EMH, this is due to the financial agents that, by working in a fully rational way, instantaneously incorporate such information into market prices.

1.5 Efficient Market Hypothesis at work

A concrete example of the meaning of Efficient Market Hypothesis could be an event occurred in 1986.

At 11.39 a.m. on Tuesday, January 28, the Space Shuttle Challenger took off from the Kennedy Space Centre at Cape Canaveral, tragically, after seventy-three seconds the shuttle exploded.

A lot of people were watching the event on television, so instantly everybody knows the fact, but no-one knew what had happened, the reason for the failure of the mission.

At the press conference, Nasa's Administrator explained that before to make any hypothesis on the cause of disaster, a full investigation should had done, analysing and reviewing all the data.

For the successive period, no other relevant public information was released, but media began to speculate basing their possible conclusion on few seconds of impact's video.

A commission, called. “Rogers”, of the most expert scientist were established after few days of Challenger explosion. There were, for example, a Nobel prize physicist, the first American woman in space, Neil A. Armstrong, the first human on the moon and several other important and brilliant experts. After almost six months, they reached the definitive sentence: the explosion was caused by the failure of the Shuttle’s O-rings on the right solid fuel booster rocket.

The Challenger disaster had also serious financial repercussion: four major NASA contractors were involved in the Space Shuttle program: Lockheed, Martin Marietta, Morton Thiokol and Rockwell International.

The O-rings of booster rocket, responsible of the incident, were built and operated by the contractor Morton Thiokol, so the Rogers Commission’s release was a truly bad news for the company, whereas for the other companies the news should have been a sigh of relief.

To see how stock market, react, recall that for investors good news means “buy”, bad news means “sell” and the market will incorporate the news into the prices of publicly traded corporations, not only also “rumours” or speculations usually have a great impact on stock prices, similar to the announced news.

Why this story is connected to the Efficient Market Hypothesis?

Because its major statement “in an efficient market, the price of an asset fully reflects all available information about that asset” has vast implications, one of them is evident in Challenger case.

Indeed, the market evaluated the Challenger explosion and incorporated it into the stock price of the Morton Thiokol, not the day after the release of the report, nor during the period of speculation while the commission was analysing the data; but on January 28, 1986 itself, within few minutes after the explosion.

Almost immediately after the accident the price of the four NASA vendors drop, after few hours of trading, only one company was continuously suffering a rising loss (about six percent in few hours, almost 12 percent by the end of the day), with huge volume traded (seventeen time the average). Other three companies, after the first drop started to recover the loss, by the end of the day their losses and overall volume traded were much smaller and within statistical norms. Guess which was the company with the biggest loss? Exactly, Morton Thiokol.

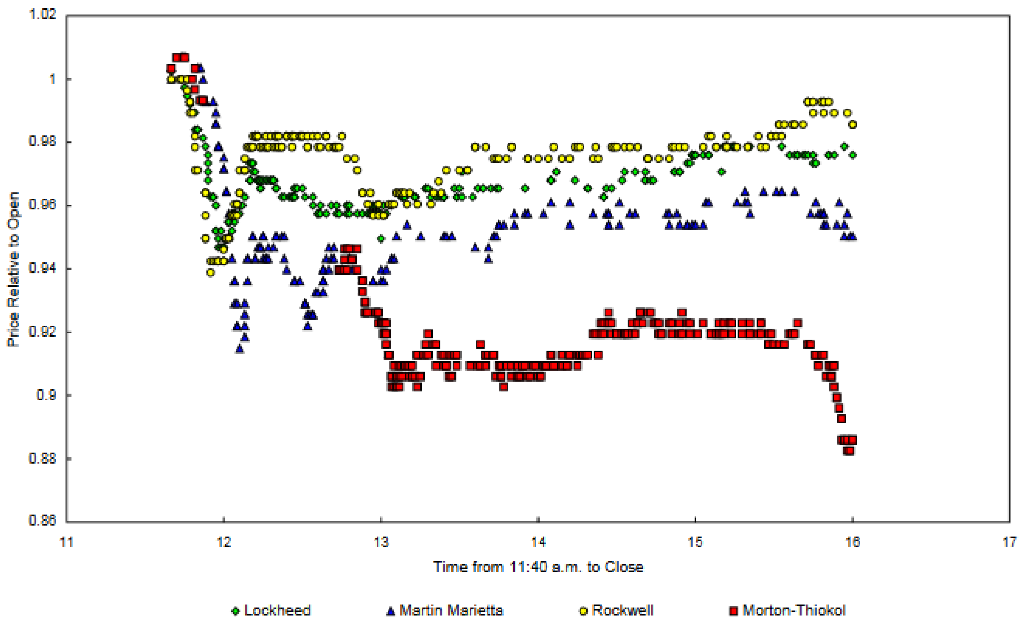
This event represents a certain implication of what economist assert with Efficient Market Hypothesis. A journalist of The New Yorker, James Surowiecki, define a word for this example: wisdom of crowds. Investor were not all aerospace engineer, or the best shuttle experts to analyse the fact, but each one was moved by self-interest to understand what happened and which could have been the cause for disaster.

Each person operating in the financial market that day, tried to use his personal knowledge and experience, and the intuition that, behind the catastrophic explosion of the Challenger, the most probable reason was some failure of fuel tanks. So, the major traders made the same judgment driven by their intuition, which, five months later revealed to be exact, demonstrating that the "crowd" was wisdom and the price of the stocks have reflected their wisdom.

To have an evidence of what happened in the market during the day of the Challenger's crash, it is presented below a figure with stock prices trend of that day for the four companies and a table showing the data collected for four variables: volume per hour, trades per hour, average trade size and stock returns, in three different moments: before the crash, immediately after the crash (when Morton Thiokol trades was halted), and after the re-opening of negotiation for Morton Thiokol.

Figure 1.1 January 28, 1986 stock prices trend for the four aerospace companies.

On the x axis we have the trading time for that day, on the y-axis the stock prices. It is evident how, after the halt of the market, only Morton Thiokol (red symbols) continues to fall.



Source: Maloney and Mulherin (2003)

Table 1.1 Intraday trading data collected before and after the Challenger crash

Intraday Trading Characteristics before and after the Challenger Crash

This table compares trading volume and stock returns of the four major space shuttle firms before and after the 11:39 a.m. crash on January 28, 1986. Data come from Francis Emory Fitch, Inc.

Variable	Morton Thiokol	Lockheed Corp.	Martin Marietta	Rockwell Int'l
Panel A. Pre-Crash Trading Volume: Open to 11:30 a.m.				
Volume per Hour	29,500	77,250	68,900	14,250
Trades per Hour	15	27	31	28
Avg. Trade Size	1,967	2,859	2,259	518
Stock Return	0.67%	1.06%	2.14%	-0.72%
Panel B. Trading Volume during Morton Thiokol Trading Halt: 11:36 a.m. to 12:35 p.m.				
Volume per Hour	Halt	210,300	153,100	301,600
Trades per Hour	Halt	126	94	147
Avg. Trade Size	Halt	1,669	1,629	2,052
Stock Return	Halt	-4.60%	-6.98%	-1.82%
Panel C. Trading Volume after Reopening of Morton Thiokol: 12:36 p.m. to Close				
Volume per Hour	475,571	82,371	43,114	65,714
Trades per Hour	105	53	37	60
Avg. Trade Size	4,523	1,542	1,179	1,090
Stock Return	-12.97%	1.65%	3.03%	0.73%

- We see as at the end of the same day of disaster only one firm has suffered an enormous loss, (-12.97%) Morton Thiokol, the one who built the fuel tanks responsible of failure of the mission.

- Panel B shows that for Martin Thiokol the negotiations were suspended for one hour, caused by the exceptional sell order.
- In Panel C it is shown how the volume of trade for Morton Thiokol after the disaster reached a very bigger quantity than others.

Source: Maloney and Mulherin (2003, table 2)

1.6 The Three P's of Total Investment Management

Let suppose that these models presented are an island where experiences from gambling, physics converge into an economic theory, a few miles far from here there is another island, the finance island, where its citizens (stocks prices fluctuations) act as described in previous models: basically, following a "Random Walk". To be possible, it is needed a "bridge" over the two islands, where the theories can be connected with the citizens. This bridge provides the answer to the question: how prices reflect all the available information? Thanks to the following consideration: in the stock market, security and stocks prices are the result of the equilibrium of supply and demand.

But it is not enough, one more detail is necessary to complete the connection:

it is the *instantaneous* supply and demand that determines actual prices, and at any given time, the supply and demand will differ simply due to chance.

For instance, suppose, on a particular day, that there are 100 investors who want to buy a particular stock and 100 investors who want to sell the same stock, and suppose further that they believe that the opening market price to be a fair price and they place market orders to affect their trades—and these traders are not aware of any news about the company during the course of the day. There is very little chance that these traders will all come to market at the same time, even on the same day, and if some of them do happen to trade at the same time, the number of buyers and sellers probably will not be equal, and that whether there are more buyers than sellers or vice versa will differ throughout the day. Hence, at most times of the day, there will be an instantaneous imbalance of supply and demand for the stock, which will cause the stock price to move randomly throughout the day, this, because even though the stock price is determined by the instantaneous supply and demand of the stock, it is impossible for anyone to determine the equilibrium price ahead of time. More generally, the current EMH paradigm can be summarized in the "three P's of Total Investment Management" (see Lo, 1999): prices, probabilities, and preferences.

The idea that price is determined by supply and demand is both simple and deep. Two opposing forces interact with each other to achieve the equilibrium price. This principle, that it has been described few rows above, is taken from micro-economy: the demand curve represents the aggregation of many individual investors preferences, each one is the result of an optimization subject to a budget constraint that depends on prices and other factors (e.g., income, savings requirements, and borrowing costs). The supply curve is the aggregation of many individual producers' outputs, each derived from optimizing an individual preference subject to a resource constraint that also depends on prices and other factors (e.g., costs of materials, wages, and trade credit). The probabilities influence both consumers and producers, when they start planning their future consumption and production there is always a certain degree of uncertainty that affect their expected income, costs and business conditions. It is the interactions among prices, preferences, and probabilities that give modern financial economics its richness and depth. Formal models of financial asset prices such as Leroy(1973), Merton (1973), Rubinstein (1976), Lucas (1978), and Breeden (1979) show precisely how the three P's simultaneously determine a "general equilibrium" in which demand equals supply across all markets in an uncertain world where individuals and corporations act rationally to optimize their own welfare. The three P's enter into any economic decision under uncertainty, and it may be argued that they are fundamental to all forms of decision-making. So, applying this theory in financial world, the outcome has to be that the interactions between prices, preferences and probabilities, set the robustness and reliability of the basis of modern financial theories. To better understand the practical consequences of Fama theory, it has been developed various type of empirical tests, direct to demonstrate EMH in its three version; numerous authors have made empirical analysis on different data type. The general result that emerge is not exclusive, there is no a complete approval or rejection of efficient hypothesis, this result should be diversified and opportunely contextualize for each analysis. Another important aspect to consider is distinguish between test that have a long-time horizon for investment return, and test with a short time horizon. In addition, the methodology of "event study" has tried to examine the effect of the information spread into the market (information as company's news, announcements and so on...) on stock prices, to see how much the market is efficiency, basing especially on the time in which the new information is incorporated in the stock price.

This thesis has not the purpose to investigate specifically these tests, but they and their results, have been necessary for the researchers to continuously arise new questions and different possible solutions to the trivial problem: is the Efficient Market Hypothesis true in the real market?

1.7 Conflict with behavioural finance

What's next?

"Behavior Finance is a fascinating area, a course of self analysis. The more we learn about it, more we realize that each of us fail in traditional tests of rationality in an unsuspected way. Von Newmann-Morgenstern, despite of their brilliant analysis, omitted relevant pieces of the history."

Peter Bernstein (1997) Against the Gods: The Remarkable Story of Risk.

All the chain seems perfectly oiled, everything seems to work, but unfortunately, real world is different from theoretical one, the assumptions of the efficient market hypothesis have been subject to debate for decades. All these debates, discussions between researchers, professionals and finance and economic experts give raise to the idea that market efficiency and rationality may not be the "baseline" from which start to move through the human behaviour in economy and finance, but it could be necessary to establish a more accurate starting point.

The field of behavioural finance was developed in response to the body of anomalous evidence regarding the EMH.

Behavioural theory of finance which drops the conventional assumptions of expected utility maximization with rational investors in efficient markets. The two building blocks of behavioural finance are cognitive psychology (the cognitive processes that influence expectations about the future) and the limits to arbitrage (when markets will be inefficient) asserts investor sentiment as an irrational factor e.g. DSSW (1990) and Shiller (2003), affects the asset returns and volatility. Additionally, The EMH assumes the market performs on the rational conditions. It assumes in the rational conditions investors drives the prices close to fundamentals. In contrast, the behavioural finance assumes that markets are informationally inefficient, since these rational "economic individuals" often exhibit distort behaviour and adopt decision making process based on heuristics.

Heuristics are nearly innate approaches and brain processes that have been evolved together with human characteristic and ambient conditions. This evolution so, has been determined by the survival aim, (all living beings that tend to this aim). The heuristics are the brain scheme of the physical actions done in an instinctive way, that "suggest" the easiest and quickest way to survive. But if used in different contexts, as for example a financial market, they can induct to wrong (or, as behavioural finance would call them, irrational) actions/behaviour.

Moreover, the behavioural finance retards the EMH. Conceptually, most of the standard asset pricing theories are based on the Rational Expectations Equilibrium framework (REE) which assumes individual investors to be rational. Behavioural finance departs from REE by relaxing the assumption of individual rationality. By behavioural finance empiricists, there is a growing consensus that indeed noise traders can generate price movements and excess volatility at least in the short-run.

The driving force of all these discussions is the desire to understand and explain the impact of investor's decision on financial markets. In parallel this mechanism could explain other decision-making issue, where we have some discrepancy between theoretical and real world.

In particular, Lo [2004] explains that, psychologists and experimental economists have documented a number of departures from market rationality in the form of specific behavioural biases. These, are apparently ubiquitous to human decision-making under uncertainty, several of which lead to undesirable outcomes for an individual's economic welfare.

There are a lot of studies, researches to talk about this argument. Here I present which are the most important steps made, to find a reconciliation between these apparently so far theories.

A fundamental contribution comes from the American economist Richard H. Thaler, recently he has been awarded with Nobel Prize in Economic Sciences exactly for his studies in the field of Behavioural Economics. He, together with other well-known economist such as D. Dreman, R.Shiller, W.De Bondt, integrate in his theories behavioural arguments, especially they studied the work of the famous psychologists P. Andreassen and A. Tversky.

Actual and most believed financial economic theory is based on the assumption that the "representative agent" in the economy is rational in two ways: The representative agent (1) makes decisions according to the axioms of expected utility theory and (2) makes unbiased forecasts about the future. An extreme version of this theory assumes that every agent behaves in accordance with these assumptions. Most economists recognize this extreme version as unrealistic; it's truly hard that common people, when they operate in markets, follow the previous assumptions, for obvious reasons.

Even so, there are defenders of the traditional model, which argue that if some agents in the economy make sub-optimal decisions, these ones do not alter the “rational equilibrium” since they are only “marginal” investor, and their irrational actions are compensated by rational agents.

The argument that asset prices are set by rational investors is part of the grand tradition in economics and is often attributed to Milton Friedman, one of the greatest economists of the past century and one of the greatest debaters of all time. Richard Thaler, the author of the famous book: *The End of Behavioral Finance*, give a precise explanation of this argument claiming that it has two fundamental problems.
First, even if asset prices were set only by rational investors in the aggregate, knowing what individual investors are doing might still be of interest. Second, although the argument is intuitively appealing and reassuring, its adherents have rarely spelled it out carefully. Thaler, in his book, present an example that helps to clarify the critical points of the tradition economic theories regarding this topic. Suppose a market has two kinds of investors: rational investors (rational), who behave like agents in economics textbooks, and quasi-rational investors (quasi's), people who are trying as hard as they can to make good investment decisions but make predictable mistakes. Suppose also that two assets in this market, X and Y, are worth the same amount but cannot be transformed from one into the other. Finally, assume that the quasi's think X is worth more than Y, an opinion that could change (quasi's often change their ideas) while the rational know that X and Y are worth the same. What conditions are necessary to assure that the prices of X and Y will be the same, as they would be in a world with only rational investors? This question is complex, but some of the essential conditions are the following. First, in dollar-weighted terms, such a market cannot have too many quasi's, otherwise the “wrong” decisions made by quasi’s could not be cancel by the rational decision of rational investor, that is, quasi’s must be a marginal percentage of the market’s investors. Second, the short-selling of a security should be allowed without any cost; this to permit to rational investor to correct a over-priced stock, indeed, if prices get too high, rational investors must have the possibility to drive them down. Third, this short-selling can be operated only by rationals, in order to avoid under-priced assets, thinking at previous example: when the prices of X and Y will be the same, the quasi’s will react to this situation short-selling Y, since the quasi’s have the wrong beliefs that Y is worth less than X. Fourth, at some date T, the true relationship between X and Y must become clear to all investors. Fifth, the rationals must have long horizons, long enough to include date T.

1.8 If you’re so smart, why aren’t you rich?

As Lo states, there is no doubt that humans do exhibit certain behavioural idiosyncrasies from time to time. The question that is not yet solved in a unique way is: what the directed consequences on investment management are, decision making et so on.

The counter-intuitive nature of the Efficient Market Hypothesis is that, if the markets would be efficient no one would have any reason to trade and invest money based on their own information,

indeed, their information should be accessible to everyone, and all the technical analysis would be completely a waste of time, since there should not exist price patterns, no mathematical model that can predict future prices.

In the book of Lo (1) “Adaptive Markets”, there is a title of a Chapter, that, despite its simplicity has a deep significance, and personally at the very first sight makes me feel a little be confused.

If you’re so smart, why aren’t you rich?

Simply true and captivating. (We can also write the opposite: if you’re so rich, why aren’t you smart?)

What does it mean, effectively this phrase?

Lo recount an episode occurred when he was a younger assistant professor.

He received an invitation to give a talk at a well-known conference, for this important event he, with his co-author and colleague, Craig MacKinlay, wrote a research paper:” Stock Market Prices Do Not Follow Random Walks: Evidence From a Simple Specification Test”. They developed a new statistical test of Bachelier’s Random Walk Hypothesis

for weekly stock market returns by comparing variance estimators derived from data sampled at difference frequencies.

As the title said, they rejected it; they checked the relation between the variance of two-week stock returns and the variance of weekly returns using real data: a broad U.S. stock market index from September 6, 1962 to December 26, 1985.

The result was surprising, all the academy theories at that time would have proved that the variance of two-week stock was exactly two times the variance of weekly returns, this because if prices follow a random walk, the mathematics of the variance means that investment risk will increase in lockstep with the length of the investment period. But Lo and MacKinley had found that the ratio was about three times; a very surprising result!! that they test it several times to be sure of its validity and of clean of programming error.

When they present it, the discussant (a senior and well-respected financial economist) assert that the two authors made, for sure, some errors.
And at the end he asked them, “If you’re so smart, why aren’t you rich? “.
A logical question arises, if prices of stocks do not follow Random Walk model, they for sure follows some alternative patterns, and if they follow some other patterns why Lo and MacKinlay did not discovered that patterns and invest in stocks to became rich?
This rejection of Random Walk model is an anomaly in the body of market efficiency theory, and it is not the only one: Through the efficiency tests made after the Fama theory numerous anomalies have been spotted, I try to present a summary useful to understand what is the “gap” between Fama’s theory and the reality. Each of these sources of inefficiencies would deserve a longer explanation, but my scope is to present what is in common between all these chinks to reach the intuition made by Lo.

1.9 Anomalies of Market Efficiency

"You don't make money by investing in a good company . . . You make money by investing in a company that is better than the market thinks."
Robert Vishny, Institutional Investor, January 1997.
As mentioned before, during the ‘70s Classical Finance theories become accepted worldwide, over time, there have been several studies that support stock market efficiency and give empirical evidence of it. Nevertheless, various long-term anomalies have been documented in the stock market and these seem to contradict the efficient market hypothesis.
The existence of anomalies in stock prices is now widely accepted, a more threatening debate regard whether investors can exploit anomalies, earning excess returns. The term anomaly was used for the first time by Kuhn (1970) and the discovery of anomalies often represent a starting point for the development of new theories. An anomaly of market efficiency can be identified as an empirical event that seems incompatible with traditional models of asset-pricing. In more specific terms, is the existence of a pattern, in contrast with Random Walk theory, in security returns. This pattern is generally regular, reliable, widely known (many investors can take advantage of it), implying so, a certain degree of predictability. The stock market anomalies can be identified in four groups: Calendar, Fundamental, Technical and Behavioural anomalies. The number of market inefficiencies that have been observed is very high, so it is difficult to list all of them. We have also to consider that a lot of these anomalies are only supposed to

exist, so I'll present only the ones which contain meaningful methods on how to gain excess returns exploiting their patterns.
Remind that these anomalies are appeared from efficiency and, so they constitute empirical situations of not immediate interpretation.

Value

Value effect can be described as the trend of value stocks to outperform the market in the long term. Value stocks are those with high book-to-market ratios[11], and they seem to have higher average returns than growth stocks, those with low book-to-market ratios.
Value effect is one of the most publicized and known anomaly and is often suggested as the best strategy for equity investing. There is a large body of empirical researches indicating the fact that, historically, investors tend to overestimate the prospects of growth companies and underestimate value companies. Professors Josef Lakonishok, Robert W. Vishny, and Andrei Shleifer (of LSV Asset Management) concluded that "value strategies yield higher returns because these strategies exploit the mistakes of the typical investor and not because these strategies are fundamentally riskier."
The most important research on this anomaly has been conducted by Louis K.C. Chan and Josef Lakonishok in their paper: "Value and Growth Investing: Review and Update". After a deep examination of literature regarding the performance of value versus growth strategies[12] they conclude that the reason for this anomaly between value and growth stocks should not be searched in the risk factor. Fama and French indeed affirms that value stocks are riskier than growth ones, so the excess returns generated by low ratio of market value to book value are simply a compensation for risk. They have published a work in which they have examined have examined the performances of value stocks, with low price to book, using data of the period from 1963 to 1990, stocks were picked from NYSE, AMEX and NASDAQ and ranked in ten groups depending on book/market, each year Fama and French have re-classified the stocks. The annualized returns stocks with lowest price to book ratio outperformed the stocks with the highest price to book ratio. Not only, they found that each decile has returns performance worse

[11] Book-to-Market Ratio. A ratio used to find the value of a company by comparing the book value of a firm to its market value. Book value is calculated by looking at the firm's historical cost, or accounting value. Market value is determined in the stock market through its market capitalization
[12] Growth strategies: Investors look for companies that are expected to grow faster in terms of revenues, cash flows, profits and consequently in stock price. Investor don't expect dividends and the risk is higher since there is no certainty about company's growth.
Value strategies: Investor look for companies whose stock prices don't reflect (apparently) their fundamental value.
This strategy is less risky than growth one, but potential profits are limited.

than the previous one. Fama and French also ranked the deciles by beta and found that the value stocks had lower risk and the growth stocks had the highest risk Chan and Lakonishok instead, in their opinion, address behavioural considerations and the agency costs of delegated investment management as the principle causes for this anomaly.

Price over earnings/sales/book ratio and market capitalization

It seems that exist undervalued stocks and we can identify them looking at low P/E values. Research has shown that small-cap stocks tend to generate a higher return on investment than large-cap stocks. The same anomaly seems to be found applying the same concept to stocks with lower Price to Sales, lower price to book. Eugene Fama and Kenenth R. French in their work: "The Cross-Section of Expected Stock Returns", published on 1992. The study had a profound impact in the academic community and made headlines in part because Fama was a long-time champion of the Capital Asset Pricing Model. Some researchers now believe that "value" represents a risk factor that investors are compensated for (just as investors expect higher returns from stocks as opposed to bonds).

Neglected Stocks

Neglected stocks commonly are selected by those that follow a contrarian strategy of buying stocks that are out of favour. Werner F.M. DeBondt and Richard Thaler[13] conducted a study of the 35 best and worst performing stocks on the New York Stock Exchange (NYSE) from 1932 through 1977. They studied the best and worst performers, where the stocks performance were evaluated over periods up to five years. They found that the best performers over the previous period subsequently underperformed, while the poor performers from the prior period produced significantly greater returns than the NYSE index.

Momentum effect (in short run)

The momentum anomaly is one of the most challenging and interesting anomalies. It has been one of the first anomalies to be discovered and even now there is not a certain explanation for it. It is based on the past performance of a stock: when a stock has experienced significantly gains or losses in the past, in the near future (Short-run) it probably continues the tendency. Its price will grow if it is in a positive momentum (positive past performance), or it will continue

[13] "Does the Stock Market Overreact?" Werner F. M. De Bondt; Richard Thaler: The Journal of Finance, Vol. 40, No. 3, Papers and Proceedings of the Forty-Third Annual Meeting American Finance Association, Dallas, Texas, December 28-30, 1984. (July, 1985), pp. 793-805

to fall if its past performance was negative, the significance of this effect has been calculated on a 3- to 12-month horizon. This effect implies a positive autocorrelation.
Several studies have proved that this effect occurs in different stock markets (U.S. European Union, Japan and emerging countries) and it is not influenced by company's capitalization (small or large cap). In academically context, it is one of the mostly analysed- effect which has showed strong persistence. The explanations for anomaly persistence are various, no-one appear to be superior, some proposals are the risky related factor, the persistence of behavioural factors as over-reactions or under-reactions.

Reversal effect (in long run)

It has been argued that there is a tendency for stocks with past long-term poor performance to outperform past long-term good performance stocks over a longer time horizon, that is negative autocorrelation. Such a phenomenon is generally regarded as one of the most serious violations of the Efficient Market Hypothesis (EMH) in the literature (Dimson and Mussavian, 2000).

Calendar Based Stock Market Anomalies

The calendar effect refers to the several theories which assert that during some specific period of time, days or months or times of year, the stock market prices changes are not random but seem to follow some repetitive pattern. This calendar influence on financial market performance have been deeply analysed and I present the most famous and important.

The January Effect

January effect refers to an unusual market trend that exhibits during the month of January. In particular there is historical evidence that on the first month of the year stocks (especially the small ones) generates abnormally high returns. These anomaly is considered as one of the best-known example of anomalous behaviour in financial markets and probably the most exploited by investors[14].
Analysing the historical data professors Robert Haugen and Philippe Jorion Haugen found that after its initial discovery, January effect persisted consistently for a long time, whereas in the recent years it seems that it has diminished. A possible reason is that as more traders try to exploit of an anomaly it tends to disappear. Moreover, many studies affirm that also other market anomalies occur especially during the first month of the year. Certainly, historical data

[14] Robert Haugen and Philippe Jorion: "The January Effect: Still There after All These Years, 1996".

show January as one of the best month for buy stocks, since there is the opportunity to buy them for a price lower than their value, and take profit re-selling them after January at a higher price. In order to analyse this peculiarity Rozeff and Kinney (1976) reported the stock's returns of the New York Stock Exchange (NYSE) from the 1904 until 1974. From their examination, they noticed an increase of about 3% in the average return for the month of January compared to the average performance of the rest of the year (Klock and Bacon 2014).

Turn of the Month Effect

Stocks historically show higher returns around the turn of the month[15]. This phrase, which describes the essence of this anomaly has been coined by Lakonishok and Smidt. They have studied returns of the S&P 500 over a 65-year period finding evidence that U.S. large-cap stocks consistently show higher returns at the turn of the month. Chris Hensel and William Ziemba[16] argue that the principal reason for this calendar anomaly is the end of the month cash flows: salaries, interest payments and so on. They found returns for the turn of the month were significantly above average from 1928 through 1993 and "that the total return from the S&P 500 over this sixty-five-year period was received mostly during the turn of the month."[17] From this study it is possible to affirm that for investors making regular trades on stocks, it could be more profitable schedule them prior to the turn of the month.

In Equity Returns at the Turn of the Month (which earned a Graham and Dodd Scroll Award) John McConnell and Wei Xu studied CRSP daily returns for the 80-year period of 1926-2005. Specifically, "turn-of-the-month is defined as beginning with the last trading day of the month and ending with the third trading day of the following month." They found that the turn-of-the-month effect is pronounced over the recent two decades such that, when they combine their findings with those of Lakonishok and Smidt, the result is that over the 109-year interval of 1897-2005, on average, all of the positive return to equities occurred during the turn-of-the-month interval. They also affirm that this anomaly does not appears to be enclosed in some specific case as small and low-price stocks, calendar year-ends or calendar quarter-ends, or in some specific country as U.S, since trading volume isn't higher and the net flows of funds to

[15] Josef Lakonishok and Seymour Smidt, 1988, Are seasonal anomalies real? A ninety-year perspective *Review of Financial Studies* 1988 1(4), 403-425

[16] Chris R. Hensel and William T. Ziemba, "Investment Results from Exploiting Turn-of-the-Month Effects," *Journal of Portfolio Management*, Spring 1996

[17] "Investment Results from Exploiting Turn-of-the-Month Effects" by Chris R. Hensel and William T. Ziemba: Journal of Portfolio Management, Vol. 22, No. 3 (Spring 1996): 17-23

equity funds is not systematically higher. They concluded that the turn-of-the-month effect in equity returns poses a challenge to “rational” models of security pricing and it continues to be a puzzle in search of a solution.

Other Calendar Anomalies

It seems that also the days of the week have a little influence on markets performance: Monday is the worst day to buy stocks, since in most cases closure stocks prices are lower than the open ones, implying a daily negative return; on the opposite on Friday usually stock prices close with a positive increment of price. Moreover, it seems that some anomalies happen around the end of each month, another inadequacy is registered during the days before holidays, where, the work of Ariel (1984) who has considered the period from 1963 until 1982, has demonstrated that 35% of the increase in share prices happened every year in the eight days before a holiday.

1.10 Behavioural biases

These are the principal anomalies discovered by testing the historical stock prices.

A lot of papers, studies has been written and analysed to understand the main cause of these anomalies, but researchers do not reach a common explanation or a unique theory, but numerous intuitions have the same directions, the one that will be explained in the next part.

At the begin of this chapter I have mentioned “Behavioural Finance”, as the principal theory that reject EMH. A rejection of a theory born both from empirical anomalies (inefficiencies in this specific case) and from theoretically argumentations.

I have presented an insight of the first part, but maybe more important is understand where the EMH has its weaknesses. Dozens of examples of irrational behaviour and repeated errors in judgment have been documented in academic studies. Peter L. Bernstein wrote in “Against The Gods” that the evidence "reveals repeated patterns of irrationality, inconsistency, and incompetence in the ways human beings arrive at decisions and choices when faced with uncertainty”. Many researchers believe that the study of psychology and other social sciences can shed considerable light on the efficiency of financial markets as well as explain many stock market anomalies, market bubbles, and crashes. As an example, some believe that the outperformance of value investing results from investor's irrational overconfidence in exciting growth companies and from the fact that investors generate pleasure and pride from owning growth stocks. Many researchers (not all) believe that these humans flaws are consistent, predictable, and can be exploited for profit.

Daniel Kahneman and Amos Tversky are two experimental psychologists, which have studied and test these systematic biases in an experimental setting and in so doing, they radically changed how scientist view the human decision-making process.

After the 1960s they started to gather information about all the example of human errors in mathematical judgment, errors that diverged from the rational solution.

The most common behavioral biases summarized, are the following:

Overconfidence

Overconfidence is an "unwarranted faith in one's intuitive reasoning, judgments, and…abilities," cognitive and otherwise"[18]. Most people can probably recount times when they may have exhibited overconfidence. A simple example is asking to someone if he believes to be an above-average driver, a lot of people would answer "yes, I am". Indeed about 80% of drivers share the same belief. Investors are not immune to this phenomenon and it is considered as one of the most detrimental bias for investment results; overconfident investors tend to chase returns and underestimate risk, The overconfidence models have a theoretical foundation [Daniel, Hirshleifer, and Subrahmanyan (1998), Odean (1998), Hirshleifer and Luo (2001), and Garcia, Sangiorgi and Urosevic (2007)] and have been widely applied empirically to explain market anomalies [Chui, Titman and Wei (2003), Statman, Thorley and Vorkink (2006), Chuang and Lee (2006) and Glaser and Weber (2007)]. The models provide several testable hypotheses. First, investors overreact to private information and under-react on average to public information [Daniel et al (1998)]. Second, trading volume increases when traders are overconfident [Odean (1998)]. Third, overconfident traders increase volatility [Odean]. Fourth, overconfident traders underestimate risk and hold more risky assets [De Long et al (1991)]. Chuang and Lee (2006) empirically test these hypotheses and overall find support for all of them. The works of several other researchers also lend support to the prescriptions of the overconfidence paradigm.

Overreaction and Underreaction

"A common explanation for departures from the EMH is that investors do not always react in proper proportion to new information" (Lo, 1999)

[18] Pompian, Michael M. *Behavioral finance and wealth management: how to build optimal portfolios that account for investor biases*, John Wiley and Sons, 2006. p. 51

When news come out and market’s investors are aware of them, it seems that the reaction tend to be exaggerates or underrated. Individuals indeed often over or under-react to news. These reactions could still be consistent with the EMH if we consider them divided and random. However there are evidences that suggest that systematic patterns of over-reaction and under-reaction may exist. Psychological factors are the drive of these anomalies. For example, the under-reaction to news cab be caused by individuals tendency to be conservative and to rely too much on their prior beliefs.

On the opposite, news that are outstanding and prominent grab people’s attention and becomes more relevant in the decision-making process. People are so tempted to assign heavier weight to such information in forming new beliefs, resulting in over-reaction. Prices can therefore deviate temporarily from their fair or rational market value.

After several psychological evidences, Barberis, Shleifer and Vishny (1998) developed a model of investor behaviour that shows an anomaly in investor’s reaction to news of the market. More precisely they affirm that: in the long-run, investors tend to react with excessive optimism to a series of good news and with too pessimism to a series of bad news (overreaction). In the short-run investor tend to underreact the stock’s news, for example earnings announcements. Daniel, Hirshleifer, and Subrahmanyam (1998) claim a theory of stock markets by which individual behaviours as overconfidence and biased self-attribution (which causes changes in investors' confidence as a function of their investment outcomes) contribute to market under- and overreactions. For example, in some cases investors may overreact to performance, selling stocks that have experienced recent losses or buying stocks that have enjoyed recent gains. Such overreactions tend to push prices beyond their 'fair' or 'rational' market value. In order to bring prices back to their fair value, rational investors are needed to tale the other side of the trade. Another implication is that *contrarian*-investment strategies, strategies in which 'losers' stocks are purchased and 'winners' ones are sold, will earn superior returns.

Loss aversion and Prospect Theory

Prospect Theory is a model of analysis of decision under risk ideated by Daniel Kahneman and Amos Tversky[19] and it is an alternative model to the expected utility

[19] “Prospect Theory: An Analysis of Decision under Risk” by Daniel Kahneman and Amos Tversky: Econometrica, 47(2), pp. 263-291, March 1979

theory (the classical rational choice theory). It gives a description of how investors effectively behave facing a decision in risk conditions, where the probabilities of outcomes are known.

Prospect Theory, as elaborated by Tversky and Kahneman, divide the decision process into two phases, both these phases can be affected by the formulation of the prospect, and also by the decision makers expectations (Starmer 2000 p352-353). The first one is the phase in which, when the subject analyze all the possible outcomes. This analysis is not subject to a maximization function, as EUT states, but, every outcome is weighted based on a mix of heuristics and rules of thumb. In the next phase, each outcome analyzed is valued according to the following model.

Considering an objective function:

$$\sum_i w(p_i)v(x_i) \tag{11}$$

Where index *i* represents the number of possible outcomes,

$w(p_i)$ is the weighting function, an increasing function of probability p_i: the higher is the probability of an outcome, the weight given to that outcome is higher. The function satisfies the extremes w(0) = 0 and w(1) = 1, which imply that impossible outcomes are discharged and that certain outcomes is treated as certain (Kahneman & Tversky 1979 p280-284).

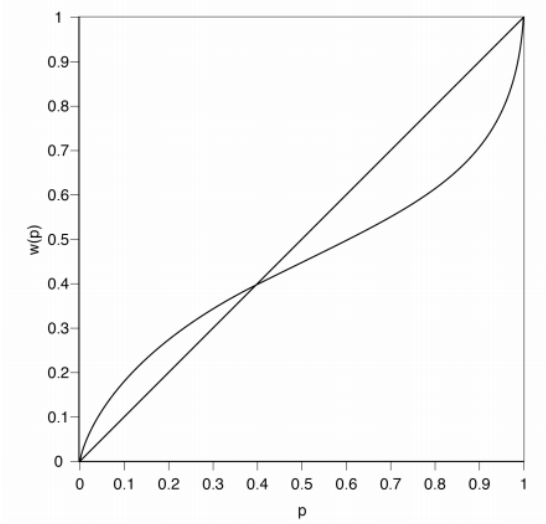
The important property of weighting function is that the transformation of the probability scale is not linear. This means that, when the probability of a positive event[20] increase from, let's say, 20% to 30%, the effect on the weighting is lower than an increase in winning probability from 90% to 100%. This property gives to the weighting function an inverted s-shape, concave for low probabilities and convex for large ones.

The following figure illustrated a graphically example[21] of weighting function:

Figure 1.2 On the x-axis there is the probabilities p_i and on the y-axis the decision weights $w(p_i)$. This figure shows how individuals place different weights to different probabilities.

[20] For a positive event we could think a possible lottery winning, or a positive return from an asset investment

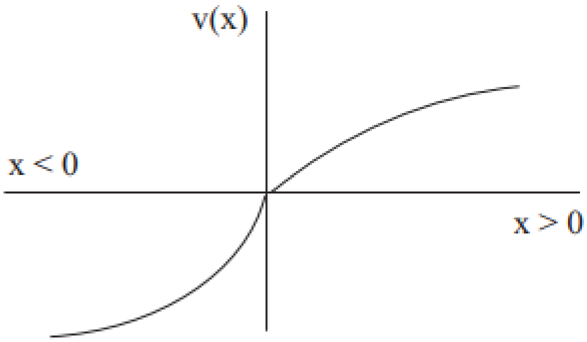
[21] There is not a unique graph of weighting function, numerous studies has proposed different graphs, but with the same structure.



$v(x_i)$ is the value function, where x_i are the shifts in outcomes (generally wealth or income or consumption quantities) defined on deviations from the reference point, not on the final outcomes. The reference point is the point to which profits (x>0) or losses (x<0) are evaluated. The peculiarity of valuation function is that it is thought to be concave for profits and convex for losses, with a kink at the reference point[22]. A concrete example is to consider the difference in value for a gain of $100 compared to a gain $200, that is greater than the difference between a gain $1100 and a gain $1200 (Kahneman & Tversky 1979 p277-278).

The value function so, is s-shaped.

Figure 1.3 The graph of a valuation function: convex for x<0 (loss) and concave for x>0 (profit):



[22] A function f: **I->R** is defined convex if, considered two points x and y belonging to interval **I**, and for every t ∈ [0.1]:
f(tx + (1-t))y) ≥ tf(x) + (1-t)f(y). If, instead f(tx + (1-t))y) is lower or equal than tf(x) + (1-t)f(y) the function is concave.

Tversky and Kahneman after several psychological experiments, found that in a decisional process an individual give contrary to expected utility theory, different weights on gains and losses, not only, the weights change according to the values of probability: small probabilities are over-weighted and high probabilities are under-weighted. They concluded that individuals feel much more sorrow by expected losses than how much they are excited by equivalent gains. Some economists affirm that investors generally conceive the loss of $1 dollar twice as painful as the pleasure from a $1 gain. They also found that individuals will respond differently to equivalent situations depending on whether it is presented in the context of losses or gains. Tversky and Kahneman indeed, were the first to introduce the theory that people are willing to take more risks to avoid losses than to realize gains. Faced with sure gain, most investors are risk-averse, but faced with sure loss, investors become risk-takers. This important psychological concept is known as loss aversion (Venkatesh, 2002).

Fear of regret

Every Investment decision imply make a personal prediction on future, to buy or to sell a stock is a decision based on our expectations. The crucial point is that, since this the decision is taken with incomplete information, investors make choices in a contest of uncertainty. It is in this context that investors are affected by emotionally distress. Let's consider this practical example: An investor is considering two possibilities of investment: stock X and stock Y, they both have a similar set up and expectations. The investor chooses stock X, but its price starts to decline, at a certain point (typically when price hit stop-loss target if settled) he liquidates his position. Stock Y instead, has increased its value considerably. The investor feels regret for having choose stock X over stock Y. He lost money but simply choosing the other opportunity he would have gain money. This is just one example, but there are several ones. Another one very common is when an investor feels regret for having close a positive (it means he is gaining money) position on a stock too early. After his decision to close the trade indeed, stock price continue to move in the same direction, so investor loose the possibility to gain more. In these examples, the past choices made by the investor have caused him to feel regret, so for the future decisions, investor will be affected by the fear of regret. The future decisions will be influenced by the regret felt in the past and this can lead the investor to make not the best or optimal decision.

People tend to feel sorrow and grief after having made an error in judgment. Shefrin and Statman[23] (1985) affirm that the fear of regret cause investors to put off the realization of losses and, on the other side investors tend to realize profits too early. Investors when face the decision whether to sell a stock are typically emotionally affected by potential profit/loss. One theory is that investors avoid selling stocks that have gone down in order to avoid the pain and regret of having made a bad investment. Some researchers theorize that investors follow the crowd and conventional wisdom to avoid the possibility of feeling regret in the event that their decisions prove to be incorrect. Many investors find it easier to buy a popular stock and rationalize it going down since everyone else owned it and thought so highly of it. Buying a stock with a bad image is harder to rationalize if it goes down.

1.11 The man is the subject

How to consider these (and all the small others) behavioural bias into the complex system of economic and finance?

As Lo tells us, they can be compared with optical illusions, they are not a full theory of human vision, but their effects are still important in the real world.

In the same way behavioural biases are not a fully theory of economic behaviour but they still have important effects in the real world, and through them we can define a complete theory of economic behaviour. To well-describe all the anomalies and market inefficiencies that could predict stock prices patterns revealing gain opportunities for investor, it should take a lot of efforts and time and the result would not be sure. These anomalies indeed, predict some prices patterns that are not really recognized with certainty, and ultimately might self-destruct. It is fundamental to be careful in exploiting these supposed predictable situations.

The evolution of our discussion has reached a new subject, necessarily, the man.

The social sciences have started to examine him from the past, and each discovery made about the human brain and its functions has led to important consequences in all the fields with linkage to the social science. Finance too, indeed have switched from the Efficient Market theory and its implications, developed under a view based on economic theories, event study, mathematical

23 Hersh Shefrin and Meir Statman: The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence, The Journal of Finance Vol. 40 (1985).

model and so on, to the subject who really carries out in practice these theories: The human being.
That’s why we need to explore deeply how man think, act, make decisions, how he feels emotions and consequently how these factors could influence the economic and finance world, not only in macro terms, but also in micro ones. For example, in a simple financial trade, the subjects could not act as perfectly rational, but instead their decision will be the output of several behavioral factors that could not lead to the best choice. In the next paragraphs I’ll explain the main aspects of this revolution.
Exactly this conception of man is the point that researchers of behavioral finance are challenging strongly. The EMT explains that the subject of the “efficient economic world” is the so called: “Homo economicus”.

Homo economicus is a fundamental concept of the classic economic theory. His principal features are: rationality (intended especially as decision-making precision) and the exclusive concern about his own purposes. As consequence his preferences are constant, and he always have complete information, (incomplete information can lead to not optimal decision). This model of man, states that empirically Homo economicus acts exclusively in his own interest and every action is subject to a criterion: the maximization of his individual utility, which in economic context coincide with net economic gain. This criterion is true both for consumers and producers, buyers and sellers, every individual that operate in a market strives for maximizing his profits. This is the concept of homo economicus, a prototype of an economic person and also a starting point for model formulation, for several economic applications models still considered valid, and for a large part of experimental games. During years the concept of homo economicus has been subject to some developments and changes , nonetheless all its evolution have maintained a common core (Manstetten, 2000, p. 20). The model does not represent all the characteristics of a real individual since it would be impossible, but it is thought as a simplified representation of the typical actors seen in the economy. The main ability of homo economicus.is his fully rational behaviour, in every context and time. He is also continuously informed about all decision alternatives and their consequences. Thus, homo economicus acts as an objective function seeking to optimize wealth or income (Lofthouse and Vint, 1978, p. 586). The model of homo economicus can thus be regarded as a simplified model of the real man, which serves to explain certain social realities and human behaviours and their consequences (Tietzel, 1981, p. 118).

These assumptions seem to be far from the real subjective human experience, but, surprising most of the time, they explain most economic behaviour reasonably well. They capture human behaviour well enough for economist to use them for build economic methods and models; despite the fact that few economists really believe that individuals actually behave like homo economicus.

So, summarizing, EMH and his implications like Homo Economicus turns out to have too many “weaknesses”. Indeed, humans exhibit reinforcing irrational behaviours in the marketplace.

But behaviourist haven’t produce a theory or a model that outgun the followers of EMH. They haven’t yet come up with a complete alternative, so the debate between EMH and its behavioural critics is still open.

Before to see a possible reconciliation, it is necessary to see briefly the potential roots which cause the controversy. It is clear that there are a lot of factors contributing to this debate, and one possible explanation is carried out by the key differences in the cultural and sociological aspects of economics and psychology. Despite the fact the subject of both fields is the same, human behaviour, the differences are larger than what we would expect.

As Lo states, the principal characteristics of psychology are in contrast with the comparable characteristic of economics:

Psychology	Economics
• Psychology is based primarily on observation and experimentation. • Field experiments are common. • Empirical analysis leads to new theories. • There are multiple theories of behaviour. • Mutual consistency among theories is not critical.	• Economics is based primarily on theory and abstraction. • Field experiments are not common. • Theories lead to empirical analysis. • There are few theories of behaviour. • Mutual consistency is highly prized.

These points describe the basic disagreement between economics and psychology. Note that, these are generalization and exist exceptions, but the divergence remains wide. For example, even though occasionally, new behavioural theories are proposed, the majority of academic psychologists develop their researches through experiments and trials; whereas, economic and finance professionals and academicians generally focus especially on new studies of traditional theories and relative empirical research, setting aside the branch of experimental economics.
Let me write a brief recap of what I have present. Started from EMH, I walk through history to understand its origin, then I have presented the critics (empirical and theoretic) made by behaviourists to the theory, finally I have understood why it is difficult to have a new "complete financial model" from psychology and its derivates. Now, it is time to present the theory elaborated by Andrew Lo, who has tried, and is trying to solve the puzzle and add the missing pieces to understand completely, or at least, better the financial world.

1.12 A New Meaning of Rationality

To reconciles the idea of rationality, proper of EMH, with the behavioural finance a proposal is made using a concept expressed By Damasio, a neurologist, who came to the profound conclusion that the role of emotion in human cognition is central to rationality; in other word to be fully rational, we need emotions. (LO, 2017).
This conclusion is still surprising many people, but reality shows us that fear and greed cause prices to deviate irrationally form market fundamentals, maybe without the emotions our rational brans could come to the correct conclusion, without any behavioural biases.
A possible solution is given by neuroscientist and psychologists, through their researches they are trying to give a more structured model of what emotions are and the role they play in decision making. The solution reached by them, is simply but forceful:
"Emotion is a tool for improving the efficiency with which animals -including animals- learn from their environment and their past. We're more efficient learners with emotions than without".
From a neuroscientific perspective, emotions help to form an internal reward and punishment system that allows the brain to select the more suitable behaviour.
Instinct and sensitivity are significance factors during the activity of risk management, trading etc.

The ability of control inner feelings and emotion is fundamental for a financial agent, since he is continuously under pressure, and market conditions change rapidly.
Emotionality, from an evolutionary point of view, represent a core psychological characteristic for the learning and sequentially, the improvement of our species. Feelings as *fear* or *avidity*, are the principal causes of the temporary absence of rationality during the process of decision making and reasoning, these two emotions can be seen as the representatives of the evolutionary forces that allow the individual survival. They "appears" or better, we feel it because of the survival instinct.

Fear is a very efficient mechanism for learning

By drawing on recent research in psychology, neuroscience, evolutionary biology, and artificial intelligence it is showed that human behaviour is the result of several different components of the brain, some of which produce rational behaviour while others produce more instinctive emotional behaviour. These components are interconnected and work together, but the roles are not so well-defined, or rather, rationality is not the "boss" of emotions and instinct. There are circumstances in which our decision making is driven by instinct and feelings, our best response so, becomes no more the most rational but the more adapt to the environment, these responses, the already mentioned heuristics are based on evolutionary purpose.
The problem is that these hardwired responses to physical threats are also triggered by financial threats and freaking out is generally not the best way to deal with such threats. Therefore, investors and markets have a split personality: sometimes they're quite rational but every so often, they freak out. The conclusion that can be drown from these concepts applied to economics are:
Neuroscience and evolutionary biology confirm that rational expectations and the Efficient Market Hypothesis capture only a portion of the full range of human behaviour.
Although this idea seems to be already reached from various other topics, it has an insight explained by Lo, that start the reconciliation between EMH and its opponents: the portion of the full range of human behaviour captured by EMH is not small or unimportant, but it provides an excellent first approximation of many financial markets and circumstances, and should never be ignored[24] The Theory expressed by Fama is simply incomplete, because it misses that market

[24] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press).

behaviour, is the outcome of human behaviour applied and this one is the result of forces like evolutionary which are factors that can't be ignored.

The work that Lo is trying to do is to find the "theory that can beat a theory", behavioural finance literature has not yet present a complete alternative; my personal idea of his work is that Lo, is really trying to connect all the tiles of the puzzle of Finance, and sequentially describe the picture that will emerge from the union.

My metaphor wants to explain how Emh can be considered a part of the great picture of finance world, as Lo states, and only with the help of the other subjects (the tiles) from psychology to neuroscience, we can have a more complete and accurate picture, obviously the though work of researchers, professors and academics in general is to connect all the pieces in the right way.

1.13 Maximization or Satisfaction?

Previously I talked about one "product" of the EMH, homo economicus, and I presented the weaknesses of this concept of human behaviour.

One of his vulnerability is the idea of optimal decision making, in sense of utility maximization. This is our starting point for the description of the new theory introduced by Lo.

The reason is that, the first alternate theory to this consequence of Fama's theory is traced back to 1952 when Helbert Alexander Simon proposed his theory of bounded rationality: "A behavioural Theory of Rational Choice".

The neoclassical view assert that individuals maximize their expected utility function through rational expectations. Simon, who was an economist, psychologist and informatic, give born to the idea of a "satisficing" (a mix of "satisfy" and "suffice" utility, so, individuals did not optimize they make decision that are not optimal, but good enough):

Prior to Simon's model, traditional finance was based on the model presented in 1947 by John von Neumann[25] and Oskar Morgenstern: Expected Utility Theory (EUT). It was supported strongly by professionals and researchers reaching a popular consensus remaining for thirty years at the basis of the economic theory of behaviour (Fishburn 1989). Expected Utility Theory can be recognized as the foundations of traditional economic models of how people makes choices and in this theory, it is implied that the preferences of individuals are stable and

[25] John von Neumann**,** original name János Neumann, (December 28, 1903—February 8, 1957) was a Hungarian-born American mathematician physicist and computer scientist. is one of the most

coherent. That mean, concretely that when an individual is facing a choice he considers and analyse all the possible alternatives before selecting the one judged to be the best.

According to Expected Utility Theory (EUT) an individual facing a choice (so, acting as decision maker) in a condition of uncertainty, evaluate risks and expected payoffs of each possible scenario, so, his utility will be the result of the weighted average of the utilities in every possible decision.

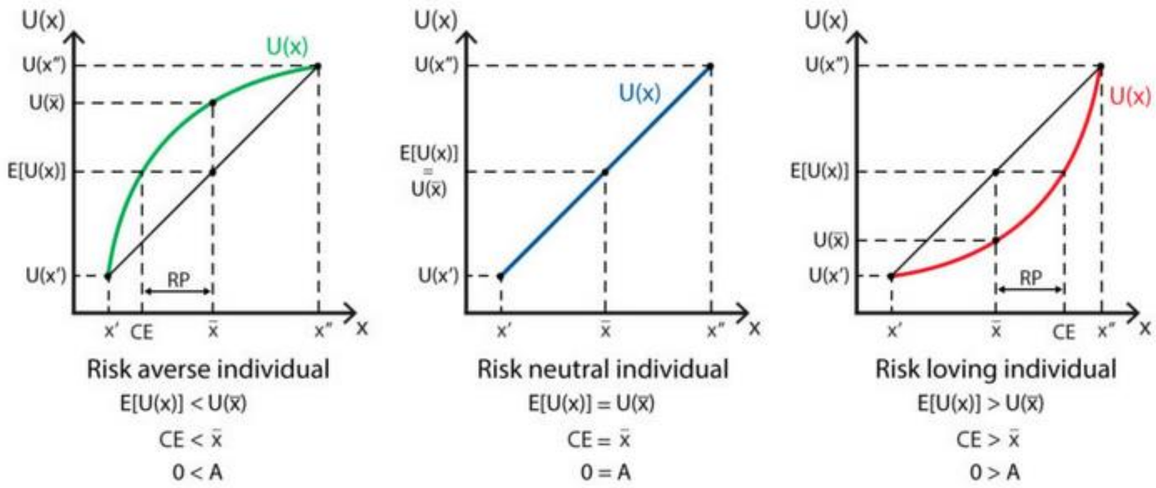
Daniel Bernoulli, the Swiss mathematician was the first, in 1738, to develop a theorization of the EUT. He thought this theory as a key to solve the Paradox of Saint Petersburg, the specific game of chance based on a random variable with infinite expected value that resulted in infinite expected payoff. Even if a rational individual should be willing to pay infinite price to play this game, actually just a very little amount appears to be worth to the players. Bernoulli considered as a solution to this paradox a function where he proved that the utility of a game is finite, in spite of the fact that its expected value is infinite (Graham 2005). Based on this theory, two centuries later, von Neumann and Morgenstern developed a theory that defined the utility as the cardinal point of the subject's preferences in a situation of uncertainty (Fishburn 1989).

The principal aspects of the EUT according to von Neumann and Morgenstern are the following:

Economic agents are rationals, and so they prefer to have higher wealth than lower wealth. As consequence the marginal utility function of wealth is always positive. Economic agents make decisions according to a utility function, which is a measure of individual preferences and according to their personal risk attitude. The utility function is continuous and its form depends on the preferences of the economic agent. The first case is when the agent is risk-averse, i.e. the investor between two investments with comparable expected return will choose the one with lower risk. The second case is when the agent is risk-neutral, and the third case is the opposite of the first one, the agent is risk-lover. He prefers an investment that for higher expected return, require taking additional risk. The following Exhibit shows the utility function drawn in these three cases:

Figure 1.4 Utility function: indifference curves.

The first graph represents a convex utility function, since the agent is supposed to be risk-averse. The second graph represent a straight utility function, the agent is risk-neutral and the third one shows a concave utility function for a risk-lover economic agent.



EUT "looks" very well, it could explain a lot of situations in which an economic agent is involved, nevertheless Simon identify some concepts that could be not so realistic. Let's going back to Simon's model to see what the main idea is and how this can be used and applied in the modern behavioural theories.

During the decision-making process, an individual has some limitations: he does not have the complete information, he has some cognitive boundaries, also subjective, and temporal limits, so, when he makes decision, he calculate he best solution until he reaches a breakeven point, where he is satisfied. He does not move towards the maximum point because this process carries some cost: to get the information needed costs, he need computational skills too complex, and a lot of time for evaluate all the possible alternatives and choose the best one. Here it comes again the "heuristics" , we can view them as the fast and cheap way to get to the point where the individual's utility is satisfied; and the precious insight we can learn from Simon's view of rationality first and then developed by Lo, is that: " heuristic can evolve at the speed of thought"[26]This mechanism I feel like is extraordinary: humans, Homo sapiens scientifically speaking, can adapt almost instantaneously his mechanism of learning, decision-making, being based on the feedback received from the environment. We have the ability to engage in abstract thought, to imagine counterfactual situations, to analyse the response of our action and to model a new, more efficient heuristic, individually and collaboratively, and according to Simon's thought reach a new satisfying utility.

His theory did not became strengthen, on the contrary, economist rejected Simon's theory resting on a simple critique, there no is possibility to know if a decision is good enough without already knowing the optimal one. To understand if my decision is satisfying, I need to reach

[26] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press).

my optimal solution, doing so, I can understand the further benefits I could get switching from satisfying point to the optimal one, implying that satisficing require optimizing. Simon never dismissed his idea, he believed that the point of satisficing should be determined empirically, so he used the concept of bounded rationality in other fields of studies like artificial intelligence research.

In economic field his research was rarely taken into account. its critics won over Simon reasoning, instead, this concept become relevant for theories alternative to orthodoxy Efficient Market Hypothesis. In particular, Lo recognize in Simon’s work the joining link between the critics of “homo economicus” as described by classical economist and his idea of an environment adaptive man. To answer the most challenging critic made towards bounded rationality, Lo explains how it is impossible to know when in the satisficing process, you reach a decision good enough, and it’s impossible because our inner rules are developed by a trial and error method[27], so at the same time it’s hard to know exactly if a decision is optimal, all depends upon our experience and feedback received by our experience, “we learn and adapt to the current environment”.

Only when environment becomes stable we can learn enough through experience to act like “Homo economicus”, any new change in environment, we start to experience new circumstances and again receive feedback form our decisions, which will be elaborated for construct a better mechanism of decision making.

After a sufficient number of trials and errors we have constructed an efficient heuristic.

[27] Learning by trial and error is characterized by repeated and varied attempts to find a solution until when the correct answer is found. A subject confronted with a new task or problem tries out sequentially new strategies and rejects one by one the ones that are not successful. This kind of learning is best suited to situations in which there is uncertainty about the rules or variables that are going to influence the payoffs. In such situation it is wise to use a strategy that does not imply any knowledge of the mechanism behind the payoff and is instead purely based on the payoff itself. For example, in a unstable financial market in which it is not possible to fully predict the prices variations and the reactions of the other investor, a subject is going to rely on the learning by trial and error in order to build up a successful strategy. This strategy is then, going to be generalized to new situations in within the same market, giving birth to heuristic.

2) Adaptive Markets Hypothesis

2.1 One More Step (Towards a financial market theory)

"Graham's observations that investors pay too much for trendy, fashionable stocks and too little for companies that are out-of-favour, was on the money…why does this profitability discrepancy persist? Because emotion favours the premium-priced stocks. They are fashionable. They are hot. They make great cocktail party chatter. There is an impressive and growing body of evidence demonstrating that investors and speculators don't necessarily learn from experience. Emotions over-rides logic time after time."

David Dreman, Forbes 5/6/96.

After our journey from the origin of the EMH to its critics, it's time to present the new theory elaborated by Andrew Lo, an economist and the director of MIT's Laboratory of Financial Engineering, "Adaptive Markets Hypothesis". The first, fundamental definition of this theory is that it is a "reconciliation theory", and I'll explain why in the next paragraphs.

It is based on the principles of evolutionary biology (competition, mutation, reproduction, adaptation, survival of the species and natural selection), and their application on financial markets with the aim to analyse them giving a description the most possible similar to the reality.

This thesis has started with the explanation of the Efficient Market Hypothesis, which has been for years the fundamental theory of the financial markets and I have talked about how Fama had come up to this theory; then I have investigated why academic and professional sectors need to move towards a more complete theory. I have presented numerous bias from real world that EMH can't explain. Finally, the point of arrival of our discussion has come. In the following paragraphs I'm going to present the theory of Andrew Lo, which intend to be a new base point for the study of financial market, without completely discharging the previous one. So, during this process of development, my idea was to try to explain why a new theory is needed, then I'll write how this new theory can solve the criticalities emerged from EMH. Before to start these investigations, it is fundamental to understand and analyse the roots of Adaptive Markets Hypothesis. Lo, indeed, in his work view financial markets from a different perspective: new subjects are considered as influencer of financial markets and as said, "evolutionary psychology" is the new "seed" from which Lo "built" his new theory. This emerging discipline was developed by his father: E.O. Wilson (1975) an American biologist, researcher, theorist, naturalist and author. He applied the principles of competition and natural selection to social interactions yielding numerous insight about several aspects of human behaviour, as altruism, fairness, abstract thought, fear (see, for example, Barkow et al., 1992; Pinker, 1993, 1997; Crawford and Krebs, 1998; Buss, 1999; and Gigerenzer, 2000).

Applying Wilson powerful ideas to economic and financial contexts, Lo find a possible reconciliation between EMH with its behavioural alternatives theories. But the origins of this new theory are various and all interconnected- Starting from the past, we have to recall that Thomas Malthus took into account biological arguments (the fact that populations increase at geometric rates whereas natural resources increase at only arithmetic rates) also the works of Darwin and Wallace do not ignore the consequences of the biology on the other contexts (see Hirshleifer, 1977, for further details).

Another important “input” comes from Joseph Schumpeter. His theories on business cycles, entrepreneurs, and capitalism, contributed to the foundation of Adaptive Markets Hypothesis; through notions of “creative destruction” and “bursts” of entrepreneurial activity he established a comparison to natural selection and Eldredge and Gould’s (1972) notion of “punctuated equilibrium”[28].

More recently, economists and biologists have begun to explore these connections in several veins: direct extensions of socio-biology to economics (Becker, 1976; Hirshleifer, 1977; Tullock, 1979); evolutionary game theory (Maynard Smith, 1982; Weibull, 1995); evolutionary economics (Nelson and Winter, 1982; Andersen, 1994; Englund, 1994; Luo, 1999); and economics as a complex system (Anderson, Arrow, and Pines,1988). Hodgson (1995) contains additional examples of studies at the intersection of economics and biology and publications like the Journal of Evolutionary Economics and the Electronic Journal of Evolutionary Modelling and Economic Dynamics now provide a home for this burgeoning literature. (Andrew Lo, The Adaptive Markets Hypothesis: Market Efficiency From an Evolutionary Perspective Pag. 17).

So, researchers have understood how deep the inter-connections between different fields of study are, and the basic reason of this is that humans constitute a very complex system that interact constantly with all the others. Now, it has become fundamental look under the hood, and emerging interests in “evolutionary psychology” is not casual. It is the result of a desire to

[28] *Punctuated Equilibrium* is a theory about how the evolutionary process works, based on patterns of first appearances and subsequent histories of species in the fossil record. The theory holds that species originate too rapidly to enable their origins to be traced by palaeontologists (punctuation), and then persist unchanged through geological time in stasis (equilibrium). All is due to a mysterious shared homeostasis that is postulated to regulate the collective morphology of individuals.

understand why human acts as they do. It sounds not as a big news, everyone knows that at the begun of 1900 Sigmund Freud tried to give an answer to previous question and all the psychology and psychoanalysis after him; but now science is moving a step forward: thanks to the innovative technologies being developed, sciences are trying to connect the dots in order to have the more complete picture of human being.

For these reasons, it is necessary to give a brief description of what is hide behind the cognitive processes and behavioural scheme that influence finance environment; the Human Brain.

2.2 Behind the Theory: How Neuroscience has affected Economics and Finance

The more appropriate science to understand the human brain and all his functions and processes is the cognitive neuroscience, a branch of neurosciences. It has been developed at the beginning of the 1980, the subject of its research is the analysis of the neuro-anatomical basis of cognition, in order to highlight the anatomical changes of the brain, associated to evolutional alterations.

The last discoveries of cognitive neurosciences have led to a re-formulation of the general neuro-psychologic models for the decision-making process, analysing the link between human behaviour and cerebral functions.

The progress in bio-medical engineering has given to these researches the most technological advanced instruments, like the PET (Positron Emission Tomography) and fMRI (functional Magnetic Resonance Imaging).

Brain imaging is currently the most popular neuroscientific tool. Most brain imaging involves a comparison of people performing different tasks—an "experimental" task and a "control" task[29]. The difference between images taken while subject is performing the two tasks provides a picture of regions of the brain that are differentially activated by the experimental task.

There are three basic imaging methods. The oldest, electro-encephalogram (or EEG) uses electrodes attached to the scalp to measure electrical activity synchronized to stimulus events or behavioural responses (known as Event Related Potentials or ERPs). PET measures blood flow in the brain, which is a reasonable proxy for neural activity, since neural activity in a region leads to increased blood flow to that region. The newest, and currently most popular, imaging

[29] Experimental task refers to the task in which the variable of the experiment (i.e. the attention of the subject) is involved and measured.
Control task, is a task that the subject of experiment performs without involving the measured variable in the experimental task.

method is functional magnetic resonance imaging (fMRI) which tracks blood flow in the brain using changes in magnetic properties due to blood oxygenation (the “BOLD signal”[30]). Simultaneous direct recording of neural processing and fMRI responses confirms that the BOLD signal-reflects input to neurons and their processing (Nikos Logothetis et al. 2001).
The psychologic and psychiatric research was heavily influenced by these instruments, a lot of question that were unresolved found solutions thanks to these technologies, so, psychology research has improved rapidly expanding his research possibilities, like, investigate and analyse the neuro-psychologic and physical aspects of cognitive processes and behavioural schemes. As result, new important discoveries were made, that influenced all the related fields to psychology and sometimes also field which seemed far for it. Reconnecting to the core of this thesis, an example of new discovery is: the financial decision-making process was believed not to be linked at all with emotivity, since it is based on rational behaviour which was supposed to be opposite to emotivity and without neuropsychologic connection. Damasio, in 1994, conduct his studies on a patient who had been subject, due to a tumour, to a surgically remove of the brain frontal lobe. The patient, after the operation, lost his ability to feel emotions, but, not only, unpredictably he lost the capacity to make rational decisions, also in daily domestic activities he behaved taking irrational choices. The studies of Damasio demonstrated that the ability to feel emotions is connected to the rational behaviour, so the belief before Damasio study was not correct: emotions has a fundamental role in the human rationality and represent its complementary. The neuroscientific literature lists several causal factors of the human irrationality.
First of all, the brain is not merely a set of nerve endings, it is a much more complex structure, composed by areas with specific functions and behavioural schemes that interact with each other’s.
A well-known example of this complex approach to the functional anatomy of the brain it the triune brain model, proposed by Paul MacLean in (1990). He indeed proposed that our skull holds not one brain, but three, each representing a distinct evolutionary stratum that has formed upon the older layer before it, like an archaeological site. He calls it the "triune brain."
MacLean claims that three brains operate like "three interconnected biological computers, [each] with its own special intelligence, its own subjectivity, its own sense of time and space and its own memory"(The Triune Brain in Evolution: Role in Paleo-cerebral Functions By P.D. MacLean 1990). The triune brain is divided in three sections: the reptilian complex, the

[30] BOLD (Blood Oxygenation Level Dependent) signal is the vase for the MRI signal variations. It measures inhomogeneities in the magnetic field due to changes in the level of O2 in the blood

paleomammalian complex (limbic system), and the neo-mammalian complex (neocortex), viewed as structures sequentially added to the forebrain in the course of evolution (see picture 2.1). Each of the three brains is connected by nerves to the other two, but each seems to operate as its own brain system with distinct capacities.

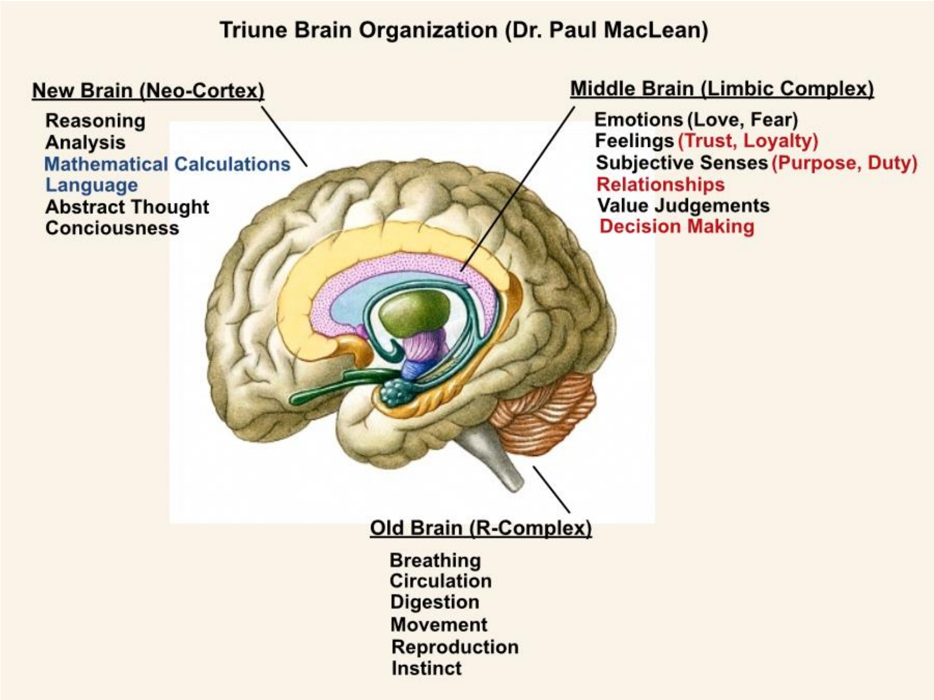
This hypothesis has become a very influential paradigm, which has forced a rethink of how the brain functions. It had previously been assumed that the highest level of the brain, the neocortex, dominates the other, lower levels. MacLean has shown that this is not the case, and that the physically lower limbic system, which rules emotions, can hijack the higher mental functions when it needs to.

The Reptilian Brain: The reptilian brain, was called originally by MacLean the “R-complex”, now it also called archipallium or primitive or "Basal Brain". It includes the oldest brain: the cerebellum and also the brain stem. The reason for which we refer to this part as “reptilian brain” is that in animals such as reptiles it is the dominant one. It consists of the structures of the brain stem - medulla, pons, cerebellum, mesencephalon, the oldest basal nuclei - the globous pallidus and the olfactory bulbs. It controls vital autonomic functions, such as breathing and heartbeat.

The Limbic System or Paleomammalian brain. It is the middle part of the brain, characterized by being common within mammals and therefore called also aleopallium or intermediate (old mammalian) brain. MacLean coined the name "limbic system" in 1952. This part of the brain is deeply involved in elaboration of emotions and instincts, feeding, fighting, fleeing, and sexual behaviour. According to MacLean, in this emotional system categorizes each stimulus either as "agreeable or disagreeable". Survival is encoded as avoidance of pain and search of pleasure.

The Neocortex, cerebrum, evolutionarily the youngest structure of the brain, is also known as the superior or rational (neo-mammalian) brain and includes the most external part of the brain (Cortex and Neocortex) and some subcortical neuronal groups. Cognitive functions such as abstraction, attention, modulation of lower functions and complex thinking are performed by the neocortex. According to the triune model, human brain is the result of an evolutionary process: in the early stage only survival functions were developed. Slowly the brain evolved and became more complex, developing emotions and functions related to social interaction and last the high cognitive functions emerged.

Figure 2.1 Mac Lean's Triune Brain Model and its functions



This theoretical approach can be applied to interpret the behavioural distortions during finance activities, especially when it comes to decision making. Different behavioural choices can reflect the overcome of the output of one of the brain sections described above, as these sections interact, cooperate or even hinder each other's.

Several neuroscientific studies prove how under specific conditions. Human behaviour is not primarily ration, it is indeed driven by emotions. Instinctive reactions aroused by emotional stimuli can be much more rapid compared to a rational choice made by analysing the situation and the all of involved stimuli.

Through evolution a specific path of response has established in dangerous situations: survival instinct takes control and prevail over rationality. A typical response patter caused by this process is the fight-or-flight response. This term was first coined in 1915 by Walter B. Cannon to describe an animal's response to threats in" Bodily Changes in Pain, Hunger, Fear and Rage: An Account of Recent Researches into the Function of Emotional Excitement". Cannon later developed the Cannon-Bard theory with physiologist Philip Bard to try to explain why people feel emotions first and then act upon them. The fight-or-flight response is a physiological reaction that occurs in response to a perceived harmful event, attack, or threat to survival. During the classic "fight-or-flight" stress response, sympathetic nervous system activation leads to catecholamine release, which increases heart rate and contractility, resulting in enhanced cardiac output leading animals to be ready to “fight or flight” as response of a dangerous stimulus. This complex reaction has also specific cognitive traits, such as an attentional drift towards negative stimuli, an under- or overestimation the control on the situation and is linked with an increase of negative emotions. An individual that finds himself in a social relevant and risky situation is therefore prone to experience fear, anxiety and aggressiveness and an acute physiological arousal. In a not physically dangerous situation the combined effect of physiological activation and negative feeling causes often an overrating of the real danger and hostility of the situation and the resulting response of the individual can be negatively affected by this process, for example in a social situation in which high cognitive functions are needed in order to analyse all the variables (such as choosing how to respond to a not foreseen happening in the stock market).

2.3 Explaining the AMH Neuro-Economics basis

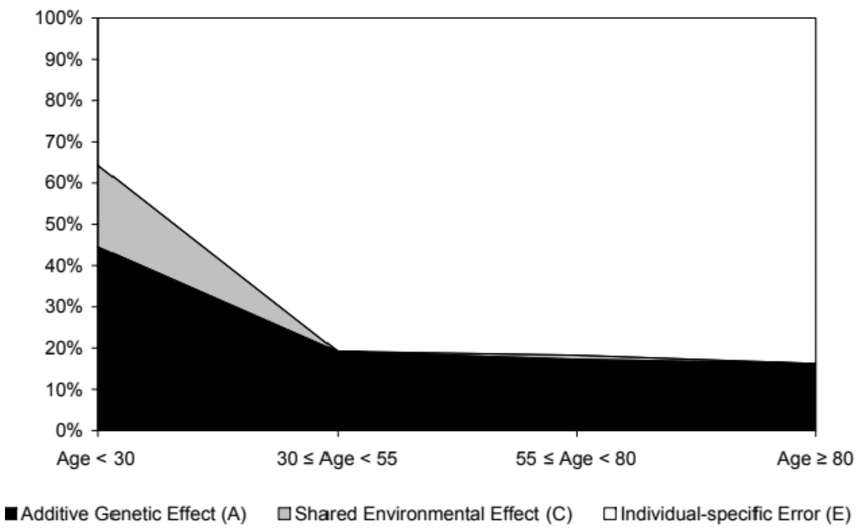
2.3.1 Nature or Nurture

One very important example that helps to understand the Adaptive Market Hypothesis neurological basis is the paper Nature or Nurture[31]. Barnea at co. used date from the Swedish Twin Registry, -the world's largest database of twins- and the Swedish Tax Agency, allowing them to analyse the portfolio of 37504 twins. Their goal was to estimate the extent to which “Nature i.e., genetic variation across individuals, versus Nurture or other environmental treatments explain the observed heterogeneity in investment decisions“. Their line of thoughts was straightforward: if an investment behaviour is caused/influenced by a genetic factor, individuals who are more closely related genetically (e.g., identical twins) should present more similar performances. Their focus was on two important financial decisions: the decision to invest in the stock market and the choice of asset allocation. The authors decompose the variance in each of the measures of investment behaviour into three components: an additive genetic component (A), a common environmental component (C), which is shared by both

[31] Nature or Nurture: What Determines Investor Behavior? By Amir Barnea, Henrik Cronqvist, and Stephan Siegel (2009) Journal of financial)

twins, for example their parental upbringing, and a non-shared environmental component (E). They found that about a third of the cross-sectional variance in the examined investment decisions is explained by genetic differences (A), even after controlling for individual factors such as age, gender, education, and wealth. The effect of nurture was also reported, specifically was the non-shared environmental component (E) the one with a bigger influence in explaining the cross-sectional variance in investment behaviours. Furthermore, they also show that genetic influences are stronger than the common environment influences: analysing data from twins reared apart (that means component C was close zero) they found that those twins still share a remarkable component of their investment choices. The family environment (C), i.e. nurture, showed also a significant but not long-lasting effect on the investment behaviour of young individuals. The effect was indeed affected by age: gaining own experience made the family environment influence no more relevant. The genetic component also decreases with age but give still a major contribution to the variance. Looking at the following figure, it shows the decrease of the effect of family environment throughout life: the experience gained during lifetimes allows individuals to surpass belief and influences originated by nurture. In the first decades of life there is also a smaller decrease of the genetic component.

Figure 2.2 Shares in Equities: Variance Components by Age Group



Altogether these results show how one third of the variance in investment behaviour can be explained by genetic factor, whereas nurture has only a small influence that strongly decreases with age. With age indeed increases experience, and experience modulate both components (nature and nurture, genetic and family). These data support those models that integrates evolution into a classic economical approach in order to explain investors' behaviour highlighting how the coexistence of rational choices, learning experience and behavioural biases can and should be embedded in economic models.

2.3.2 Camerer, Loewenstein, and Prelec: Neuroeconomics

Historically the function of the brain was thought not be possibly known: the brain was considered an impenetrable black box. As William Jevons in 1871 stated: "I hesitate to say that men will ever have the means of measuring directly the feelings of the human heart. It is from the quantitative effects of the feelings that we must estimate their comparative amounts".
Neuroscience has proved Jevons's pessimistic prediction wrong; new technology is allowing the study of the brain and nervous system with direct measurement of thoughts and feelings. These new methods are challenging our understanding of the relation between mind and action, leading to new theoretical constructs and calling old ones into question and this is affecting also the economic field.
Neuroscience points out two generic inadequacies of the common economic theories: their inability to handle the crucial roles of automatic and emotional processing. "Automatic" processes are fast, occur with little or no awareness or feeling of effort (John Bargh et al. 1996; Bargh and Tanya
Chartrand 1999; Walter Schneider and Richard Shiffrin 1977), while emotions strongly influence behaviour and is common to humans and many animals (Joseph LeDoux 1996; Jaak Panksepp 1998; Edmund Rolls 1999).

Table 2.1 The two dimensions of neural functioning

	Cognitive	Affective
Controlled Processes ▪ serial ▪ effortful ▪ evoked deliberately ▪ good introspective access	I	II
Automatic Processes ▪ parallel ▪ effortless ▪ reflexive ▪ no introspective access	III	IV

Camerer and co. (Neuroeconomics: How Neuroscience Can Inform Economics) organized the brain processes involved in decision making see, Table 2.1. Controlled processes, as described by the first row of Table 2.1, are serial (i.e. Step by step computations), usually invoked deliberately by the agent confronted with a challenge or surprise (Reid Hastie 1984) and are often associated with a subjective feeling of effort. People are aware of this kind of processing and can thus provide a good introspective analysis. Standard tools of economics, such as decision trees and dynamic programming, can be viewed as stylized representations of controlled processes.

Automatic processes are the opposite of controlled processes: they are parallel, effortless and not accessible to consciousness. Being parallel allows rapid response and multitasking, as it has been broadly studied and applied using the "Connectionist" neural network models as framework (McClelland & Rumelhart, 1986).

McClelland has been one of the most prominent contributor to psychology: he is the father of parallel distributed processing (PDP). PDP was an artificial neural network approach that stressed the parallel nature of neural processing, and the distributed nature of neural representations. It provided a general mathematical framework for researchers to operate in. A lot of the research that led to the development of PDP was done in the 1970s, but PDP became popular in the 1980s with the release of the books Parallel Distributed Processing: Explorations in the Microstructure of Cognition - Volume 1 (foundations) and Volume 2 (Psychological and Biological Models), by James L. McClelland, David E. Rumelhart and the PDP Research Group. The books are now considered seminal connectionist works, and it is now common to fully equate PDP and connectionism, although the term "connectionism" is not used in the books.

People often are unaware of automatic processes and cannot explain why automatic choices or judgments were made.

Automatic and controlled processes can be roughly distinguished by their location in the brain (Lieberman et al. 2002). Cognitive automatic activities are concentrated in the back (occipital), top (parietal), and side (temporal) parts of the brain. The amygdala, located deep and medially within the temporal lobes and being part of the limbic system, is suggested to serve as a key structure in the emotional brain, implicated in diverse automatic affective processes, especially fear. Controlled processes occur mainly in the front (orbital and prefrontal) parts of the brain. The prefrontal cortex (pFC) is known to be the "executive" region, playing a key role in integrating information from all over the brain and planning strategies according for both short and long-term goals (Timothy Shallice and Paul Burgess1996).

The two columns of table 2.1 make a distinction between affective and cognitive processes. Most affect probably operates below the threshold of conscious awareness (LeDoux 1996; Piotr Winkielman and Kent Berridge 2004).

These distinctions are to be made in order to make clear that although in economics it is generally assumed that cognition is typically controlled, and affect is automatic, most behaviour results from the interaction of all four quadrants of the previous table. For example, a lot of cognitive processing is automatic as well—e.g., visual perception or language.

Camerer and colleagues debated four neuroscientific topics that are relevant to economics: intertemporal choice, decision making under risk and uncertainty, game theory and discrimination.

2.3.3 Intertemporal Choice and Self-Control

Intertemporal choice is seen in economics as a trade-off of utility at different points in time. Discount rate is supposed to explain individual differences in the way that people make this trade-off. This concept being however based only on its convenient similarity to financial net present value calculations (Loewenstein 1992).

Humans have the unique ability to take long-term consequences of our behaviour into account in terms of caring about, making immediate sacrifices for, and flexibly responding to, desired future consequences. As suggested by Hersh Shefrin and Thaler (1988), intertemporal choice can be viewed as a result of two processes: an impulsive, affective, process and a more far-sighted process guided by the prefrontal cortex.

This account has been investigated with brain scanning technique by Samuel McClure et al. (2004). They scanned subjects using fMRI while they made a series of preference judgments between monetary reward options that varied by amount and delay to delivery. One reward can be immediate or both rewards were delayed (though one by more than the other). The results showed that parts of the limbic i.e., affective, system associated with the midbrain dopamine system were preferentially activated by options involving immediately available rewards, whereas the activation of regions of the lateral prefrontal cortex and posterior parietal cortex typically viewed as more cognitive regions was not influenced by of delay. In addition, it was possible to predict the choices made by the subjects according to the relative activity of the two systems: greater relative activity in affective systems was associated with choosing earlier rewards more often. Taking into account the interactions between affect and cognition can help to explain not only impulsivity, but also why many people have self-control problems of the opposite type of those typically examined in the literature, like not being able to stop to spend (or invest) money when they are spending more than they can. The new data obtained by neuroscientific experiments points to some deficiencies in the way that economists currently model intertemporal choice and also suggests directions for future modelling. Even if some intertemporal decisions are well represented by the discounted utility model, specifically those involving detailed deliberation but minimal effect, there is however a wide range of other intertemporal choices that are not. These are for examples the ones influenced by affectively "hot" processes such as drives and emotions. Models which focus on how these discrepant processes interact are promising (e.g., Bernheim and Rangel 2004; Loewenstein and O'Donoghue 2004).

2.3.4 Decision-Making under Risk and Uncertainty

According to the expected utility model decision making under uncertainty is a trade-off of utility under different states of nature—i.e., different possible scenarios. People however react to risks at two different levels. As predicted by traditional economic theories using cognition, people evaluate the objective level of risk that different hazards could pose. On the other hand, there is a reaction at an emotional level that can strongly influence the behavioural answer of individuals (Loewenstein, Weber, Christopher Hsee, and Ned Welch 2001).

The anatomical bases of risk averse behaviour are known to be located in the amygdala, due to the large role of fear played in such behaviour. In fact, the amygdala scans all incoming stimuli for indications of potential threat and can potentially elicit a fear response to inputs both from automatic and controlled processes in the brain. The response is modulated by the prefrontal cortex, as the data obtained with animals and patients suggests. Furthermore, Bechara et al. 1997 presented healthy subjects and patients with prefrontal damage with task in which they have to choose a sequence of cards from four decks. The payoffs of the decks could be learned only from

experience (a "multiarmed bandit" problem). Two decks had more cards with extreme wins and losses (and negative expected value); two decks had less extreme outcomes but positive expected value. Skin conductance was recorded as measure of fear, and even if both groups exhibited similar values after large loss cards were encountered, patients reacted differently afterwards.

Compared to normal, prefrontal subjects rapidly returned to the high-paying risky decks after suffering a loss and, as a result, went "bankrupt" more often. Although the immediate emotional reaction to losses was the same for both groups, the damaged patients apparently did not store the pain of remembered losses as well as normal, so their skin conductance rose much less than normal when they resampled the high-risk decks.

Judgments of probability is also an example of a process in which the brain reaction systems diverge: as shown in several studies, there is a systematic divergence between explicit judgments of probability in different settings (presumably the product of controlled processing) and implicit judgments or judgments derived from choice (which are more closely associated with automatic processing and/or emotion). An example of this behaviour is the following: subjects are required to choose to draw a bean from a bowl containing either ten winning beans and ninety losing ones, or bowl containing one winning bean and nine losing beans. When performing this experiment, the result is that the subjects chose more often the first bowl (the one with 100 beans), even if they admitted knowing that the probabilities of winning are the same. This result can be explained by an automatic cognitive preference for the bowl with more winning beans (10 to 1) (Kirkpatrick and Epstein,1992).

Game theory has the main following assumptions about players: (1) they have accurate beliefs about what others will do (i.e., players are in equilibrium); (2) have no emotions or concern about how much others earn (a useful auxiliary assumption); (3) plan ahead; and (4) learn from experience. Neuroscience data can give new insight about the above mentioned assumption. For example, using the "ultimatum game", a game in which a "proposer" offers a division of a sum of money, generically $10.00, to another "responder" who can accept or reject it, ending the game. If the responder will accept the offer, the money will be split between the two participants according to the percentage offered by proposer, if the responder will reject the offer, no one will receive any money. According to game theory, responder and proposer have no emotional reactions (guilt for the unfair proposer and disgust for the responder). In this case, the proposer should divide the sum unequally, thus earning more money than the respondent, who is supposed to accept the smaller offer. In reality this happens rarely: the proposer offers usually 40–50 percent and about half the responders reject offers less than 20 percent. Following the prediction of game theory results confusion and low pay off. This was the case of an Israeli college student, whose low offer in a $10.00 ultimatum game was rejected (from Shmuel Zamir 2000).

He objected: "I did not earn any money because all the other players are stupid! How can you reject a positive amount of money and prefer to get zero? They just did not understand the game! You should have stopped the experiment and explained it to them".

A behaviour that matches game theory sound “autistic”, as if the subject would not be able to understand the feeling of the other player (disgust for not being treated fairly) and react consequentially.

Brain imaging allowed further understanding about the brain structure involved. McCabe et al. (2001) used fMRI to measure brain activity[32] when subjects played games involving trust, cooperation, and punishment. The results showed a higher brain activity in players who cooperated more often. On the other side, players who cooperated less often showed no systematic activation.

They found that very unfair offers differentially activated three regions: Dorsolateral prefrontal cortex (DLPFC), anterior cingulate (ACC), and insula cortex. While DLPFC is an area involved in planning, the insula cortex is known to be activated during the experience of negative emotions like pain and disgust. ACC is an “executive function” area involved in problem solving and integration of information coming from other areas.

This specific pattern of activation can be interpreted as follow: after an unfair offer, the brain (ACC) struggles to resolve the conflict between wanting to accept the money because of its planned reward value (DLPFC) and disliking the “disgust” of being treated unfairly (insula). Not only the activated areas support the key role of emotion in game theory, the data also allows to predict rather reliably (a correlation of 0.45) if a subject is going to accept or reject an unfair offer by the level of their insula activity. (It is noteworthy to speculate that the insula is a neural locus of the distaste for inequality or unfair treatment posited by models of social utility, which have been successfully used to explain many varying patterns in experiments — robust ultimatum rejections, public goods contributions, and trust and gift-exchange (e.g., Bazerman, Loewenstein, and Leigh Thompson 1989).

An important principle in game theory is “backward induction”: figuring out what to do today by and reasoning how others will behave at all possible future points and working backward. According to behavioural studies people have trouble doing more than a couple of steps of backward induction (e.g., Johnson et al. 2002) but after getting instruction they can learn it pretty quickly and with little effort. This highlight, in the game theory context, the important distinction between controlled and automatic behaviour: thanks to the backward induction becoming automatic the answers get faster and correct at the same time. According to an

[32] In these specific brain regions: Brodmann area 10 (thought to be one part of the mind-reading circuitry) and in the thalamus (part of the emotional “limbic” system)

economical point of view backward induction has high cognitive costs that can be overridden after it becomes automatics.
Knowing how the brain solves problems, and which specialized systems it has at its disposal to do so[33], challenges some of our fundamental assumptions about how people differ from one-another when it comes to economic behaviour. Economists currently classify individuals on such dimensions as “time preference,” “risk preference,” and “altruism.” These are seen as characteristics that are stable within an individual over time and consistent across activities; someone who is risk-seeking in one domain is expected to be risk-seeking in other domains as well.
But empirical evidence shows that risk-taking, time discounting, and altruism are very weakly correlated or uncorrelated across situations. This inconsistency results in part from the fact that preferences are state-contingent, but it also may point to fundamental problems with the constructs that we use to define how people differ from each other.

2.4 Combining evolution, fight or flight response and finance

A key concept in evolution is adaptation. Adaptation is the adjustment or changes in behaviour, physiology, and structure of an organism to become more suited to a dynamic environment. According to Charles Darwin's theory of evolution by natural selection, organisms that possess heritable traits that enable them to better adapt to their environment compared with other members of their species will be more likely to survive, reproduce, and pass more of their genes on to the next generation. Adaptations can take many forms: a behaviour that allows better evasion of predators, a protein that functions better at body temperature, or an anatomical feature that allows the organism to access a valuable new resource — all of these might be adaptations. Humans, at the actual state as Homo sapiens, are the only ones that possess a specific adaptation ability. This is due to cognitive, decisional and behavioural processes that can be adjusted to the environment requests in a quick and efficient manner. Humans are indeed equipped with the extraordinary ability of abstraction, powerful communication skills and the ability to plan future steps in order reach a goal. According to neuroscience these skills are located in the youngest -last evolved- brain structure, the neocortex. Before that the behaviour

[33] How the different parts of the brain are involved in the processes for the problem resolution. So, which specific part is responsible for a determinate reaction and how it is linked to another part

of our ancestor was dictated by instincts and their physiological reactions, such as the flight or fight response. These instinctive responses didn't disappear, they integrated with the new functions that the brain developed during evolution. These concepts can be applied to economical science, since individual preferences are a result of the above described interaction between "old" instinctive responses and "new" rational ones. This interaction explains how individual preferences change over time: at the beginning they are dominated by the instinctive inputs, they are then modified by the environmental conditions, as described in evolutionary process, leading the better adapted ones to overcome the others. In the same way in financial market, a highly competitive environment, a "survival of the fittest" happens, letting only the best adapted to the fast-external changes. This "natural selection" in financial market let only the most -best- skilled operators survive, while the less adapted exit the market.

This evolutionary view of financial market takes into account concepts from several disciplines: psychology, cognitive neuroscience and last but not least evolutionary biology and bring them together, integrating neuroanatomical brain functions and development with adaptation and evolution, the effect of natural selection on individual preferences and the newly adapted cognitive rational skills. This all-together prompted the development of a new school of thoughts different from the efficiency of the market: the Adaptive Market Hypothesis (AMH).

In conclusion, thanks to neuroscience and its discoveries, sciences in general are developing new models and theories, considering new factor and re-viewing the traditional ones, as, for example: the interactions among individuals, which now are perceived in a different way. From this perspective, the society is a complex system characterized by competition, diversity, and natural selection; these essential components, become the fundamental bases of the Adaptive Markets Hypothesis.

2.5 The Principles of the new theory developed by Andrew Lo

We have seen the origin of the Adaptive Market Hypothesis: the historical and cultural background in which it has been developed by Andrew Lo, and its "roots" in the neuro-economics field.

Now I re-connect these basis with the finance development process, writing about what's new in the Lo's theory.

The first words that I want to spend regards its innovative aspect; as I have written in previous paragraphs, there are a lot of researches, theories that are alternative to EMH, they started from

all its critics and propose new models with the purpose to substitute EMH, but it seems none is enough complete and powerful. Why Adaptive Market Model should be different?
Because it is not an alternative theory that reject a previous one but, it is a *reconciliation theory*.
I have recognized in it two very starting point from which Lo has built his reasoning:

1) Efficient Market Hypothesis is not completely wrong, it has to be considered as the maximum level of rationality that man aim to reach when acting in the market. I would like to compare it to an asymptote, where the evolution of man could be moved next to him it is touching since our human condition.
2) Market inefficiencies do exist, Homo Sapiens is not Homo Economicus and we are neither entirely rational nor entirely irrational.

I think they are the natural conclusions of all the analysis made by Lo and its collaborators regarding EMH, behavioural theory, human brain, decision making, investments and the other topics related to. For this reason, I choose to present them initially in this chapter.
Andrew W. Lo, during the 30°th anniversary of "The Journal of Portfolio Management", in 2004 presented his theory: *Adaptive Markets Hypothesis.*
This new theory born with the hard challenge to establish a new point of equilibrium. Lo tries to summarize the new insight of the theory in five key principles:

1. We are neither always rational nor irrational, but we are biological entities whose features and behaviors are shaped by the forces of evolution.

2. We display behavioral biases and make apparently sub-optimal decision, but we can learn from past experience and revise our heuristic in response to negative feedback

3. We have the capacity for abstract thinking, specifically forward-looking what-if analysis; predictions about the future based on past experience; and preparation for changes in our environment. This is evolution at the speed of thought, which is different from but related to biological evolution.

4. Financial market dynamics are driven by our interactions as we behave, learn, and adapt to each other, and to the social, cultural, political, economic, and natural environments in which we live.

5. Survival is the ultimate force driving competition, innovation and adaptation."

These principle lead to a very different conclusion than either the rationalists or the behaviorists have advocated. As I have explain talking about Simon's bounded rationality: Individuals never know for sure if their current heuristic is "good enough".

They come to this conclusion through trial and error. They make choices based on their past experience and their "best guess" as to what might be optimal, and they learn by receiving positive or negative feedback from outcomes. As a result of this process, individuals will develop new heuristics and rules to help solving their various economic challenges. As long as those challenges remain stable over time, their heuristics will eventually adapt to yield approximately optimal solutions to those challenges[34]-

We have seen economic behaviors that are approximately rational, so close to be, but there's still a gap, this can be explained easily by Adaptive Market Hypothesis together with the previous theories that have tried to add more realistic concept than EMH and its implications. One example is the previously explained Simon's theory of bounded rationality. Until now, economic behaviors that look completely irrational or too far from an efficient perspective[35] are still "unsolved". Lo proposes to go further with his theory and explain also that kind of behaviors. Individuals and species adapt to their environment, if the environment changes, the heuristic of the old environment might not be suited to the new one.

The great innovation in this theory is the refuse of the word "irrational "applied to such behaviors, that won't lead to an optimal solution.

Lo calls these behavior, borrowing from evolutionary biology, "mal-adaptive". Some examples proposed by Lo that explain the meaning of this word are: the sea turtle that instinctively eats plastic bags because it evolved to identify transparent objects floating in the ocean as nutritious jellyfish, or the investor who buys near the top of a bubble because he first developed his portfolio management skills during an extended bull market. The point is that, these behaviors may have a valid reason to be adopted, but not in the current environment, where they seem to be totally wrong; maybe in different environment these behaviors could be ideal, but not in the current one.

Evolution in complex and randomly changing environments can yield surprisingly complex and undetectable behaviors; Herbert Simon (1969) said about it: "An ant, viewed as a behaving system, is quite simple. The apparent complexity of its behavior over time is largely a reflection of the complexity of the environment in which it finds itself."

[34] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press) (pag.188).

[35] Here we're referring to event as financial crisis, or in smaller measures, irrational behaviours that lead to unfair value of financial assets. Events that, it is hard to explain under a "efficient and rational" view.

This phrase refers to Simon's example about an ant on a beach looking for food. If we graph the ant's path it would look contorted and complex. If instead, we graph a picture of the entire beach, we would realize that there is nothing special about the ant, she is only trying to avoid obstacle.

From this simple example we can derive that, in order to understand current behavior, we need to understand the past environment and the selection processes that gave rise to that specific behavior over time and across generations of trial and error[36].

This idea is the essence of the Adaptive Market Hypothesis, which tries to be more complete than EMH or its behavioral critiques, filling the gap between the two ways of thinking; as Lo affirm, AMH, "offers conditions that give us rationality as well as irrationality, and both can coexist for a period of time as natural selection works its magic on behavior".

If the real-world imperfections, the laws of natural selection or, as Lo define, "survival of the richest", are considered the agents who determine the evolution of markets and institutions, behavioural biases could be considered as simply heuristics that have been "used" in the wrong context, not necessarily counterexamples to rationality. Through sufficient time
and enough competitive forces, any not appropriated (to the context) heuristic will be reshaped to better adapt to the current environment.

2.6 Maximizing or Surviving?

To understand why some behavioural biases might happen, a lot of experiments has been made, the academic literature has already presented evolutionary models, but they are often very sophisticated and hard to apply or to analyse. For example, the selective process that take an individual to choose which utility function to maximize or the developing of complex trading strategies.

The researchers Thomas J. Brennan, tax and finance professor at Harvard Law School and public finance expert, and Andrew Lo has developed a binary choice evolutionary model, defined as: "Evolutionary Model of Bounded Rationality and Intelligence[37]". They have started

[36] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press)

[37] Thomas J. Brennan and Andrew W. Lo: An Evolutionary Model of Bounded Rationalityand Intelligence (March, 2012)

with the idea of H. Simon described before, the bounded rationality, for which human is not fully rational but he has some limits (cognitive, psychological, temporal); then they have applied some concepts of the evolution theory together with the researches of the socio-biology and evolution-psychology on the effects of the natural selection forces.

Here I present a summary of their model: “binary choice model”. The purpose is to see if AMH can explain behavioural biases, so explain for example why we don’t maximize, but rather we optimize; simulating and analysing the decision-making processes and the behavioural types shown by individuals belonging to a population.

This model describes the evolution of a species in which individuals have the chance to make their decision about where to build their home, between two possibilities: a valley (choice a) and a plateau (choice b). The first generation of individuals, make his choice at a time t = 1, then if they survive (this probability is conditioned by their choice) a new generation will be originated, the second and this will make their own choice about where to live for time t = 2 and so on, each subsidiary population make a new choice at t* = t + 1.

This decision will have a positive or negative effect for the survival of the population, in relation to weather that will occurs. If the weather is sunny, the individuals who decided to live on the plateau won’t survive because of the exposure to the sun’s deadly rays and the lack of water (on contrary, valley offers its inhabitants shade to repair from the sun and water (thanks to a river). The situation is exactly the reverse when it rains: the rain causes floods in the valley that will draw all the people living there, whereas plateau eliminates any possibility of flooding thanks to its elevation. Individuals who are in a situation in which they survive (sunny day and valley / rainy day and plateau) give birth to a new generation: each individual has an expected offspring value equals to 3.

In this model it is supposed that probability of a sunny day is 75%, whereas rain occurs with a probability of 25%.

Individuals choose where to live randomly: they choose valley with a fixed probability *f*, the plateau with a probability 1-*f*. An individual is represented with the variable x, where when he chooses to live in the valley he is indicated as x_a, when instead he chooses to live in the plateau he is indicated as $x_{b.}$

Summarizing:

$$\begin{cases} \text{Sun} \ \ (p = 0.75) & \rightarrow \mathrm{E}[x_a] = 3 \quad \mathrm{E}[x_b] = 0 \\ \text{Rain} \qquad (1 - p = 0.25) & \rightarrow \mathrm{E}[x_a] = 0 \quad \mathrm{E}[x_b] = 3 \end{cases}$$

where, the individual's probability of choice for each alternative is:

$$\begin{cases} \text{Choice of a (valley)} & = f \\ \text{Choice of b (plateau)} & = 1 - f \end{cases}$$

Now, the fundamental question is finding the optimal decision for where the individuals should live.

Subject to? Let's give to the individuals a biological instinct: the reproductive instinct, which is supposed to make people act for species survival. In more mathematical words, individuals want to maximize the average number of its surviving offspring.

In these conditions, the behavior that maximize the survival of the species is to choose to live always in the valley (choice a), so f = 1, because the probability of a sunny day is much higher than a rain one. Nevertheless, if all the individuals would choice a, in case of a rainy day, that is not impossible, since 25% is not enough closer to 0, all the population would be "extinguished", without a new generation of offspring.

This situation leads to a conclusion: the optimal decision is not equal to f* = 1. This scenario is very similar to another famous example of a behavioral bias intrinsic in man, the probability matching.

Before explaining the reason of this result, I'll present the data set outcome of the simulations made by Lo and Brennan, in order to have a concrete interpretation of the model:

Table 2.2 Data set outcome of simulated population size for evolutionistic binary-choice model

Table 1. Simulated population sizes for binary-choice model with five subpopulations in which individuals choose a with probability f and b with probability $1-f$, where $f=0.20,0.5,0.75,0.9,1$, and the initial population is 10 for each f.

Generation	f = .20	f = .50	f* = .75	f = .90	f = 1
1	21	6	12	24	30
2	12	6	6	57	90
3	6	12	12	144	270
4	18	9	24	387	810
5	45	18	48	1,020	2,430
6	96	21	108	2,766	7,290
7	60	42	240	834	21,870
8	45	54	528	2,292	65,610
9	18	87	1,233	690	196,830
10	9	138	2,712	204	590,490
11	12	204	6,123	555	1,771,470
12	36	294	13,824	159	5,314,410
13	87	462	31,149	435	15,943,230
14	42	768	69,954	1,155	0
15	27	1,161	157,122	3,114	0
16	15	1,668	353,712	8,448	0
17	3	2,451	795,171	22,860	0
18	3	3,648	1,787,613	61,734	0
19	9	5,469	4,020,045	166,878	0
20	21	8,022	9,047,583	450,672	0
21	6	12,213	6,786,657	1,215,723	0
22	0	18,306	15,272,328	366,051	0
23	0	27,429	34,366,023	987,813	0
24	0	41,019	77,323,623	2,667,984	0
25	0	61,131	173,996,290	7,203,495	0

Reproductive uncertainty is systematic and also binary, with $Prob(\mu_a=3,\mu_b=0)=0.75$ and $Prob(\mu_a=0,\mu_b=3)=0.25$. In this setting, probability matching $f^*=0.75$ is the growth-optimal behavior.
doi:10.1371/journal.pone.0050310.t001

Source: Brennan-Lo (2012)

The authors start their calculation hypothesizing an initial population of 10 persons, and the total offspring generated are 25. They have calculated the population considering different values of individual's choice, *f*: 0.20, 0.50, 0.75, 0.90 and 1.

The first visible aspect is that only in three cases the population lived until the twenty-fifth offspring, for value of *f* equal to 0.20 population died at the twenty-second run (they chose too often to live in the plateau). For *f* equal to 1, the population died at the fourteenth run, when the first rainy day occurred. Based on previous conditions, numerical and probabilistic elements, applied to an ecologic context, Lo and Brennan obtained this formula to calculate the population for each period.

$$n_t(f) = x_{at} \sum_{i=1}^{n_{t-1}(f)} I_{it}^{f} + x_{bt} \sum_{i=1}^{n_{t-1}(f)} \left(1 - I_{it}^{f}\right) \tag{1}$$

I choose not to show all the steps necessary to reach this formula because the core of the discussion is the result obtained by Lo and Brennan in their model; but a bit explanation is needed:

- x_{at} and x_{bt} are casual variables supposed to be identically and independently distributed (IID)[38] from a generation to another and between the individuals of the same generation *t*; they are also independent from other casual variables for every choice probability, *f* , and for every individual. Variable x_{at} represent the proportion of individuals who have chosen to live in the valley *a* at time t, x_{bt} represent the remaining individuals, the ones that have decided to live in the plateau *b* at time t.
- I_{it}^{f} is a Bernoulli distribution random variable[39] which describes the decision-making process of an individual, it assumes value 1 when the individual chooses *a* (valley) with a probability *f*; and value 0 when instead the alternative *b (plateau)* is chosen with a complementary probability *1 – f*. The subscript *i* is a counter linked to the individuals of precedent generation *t – 1*.
- $n_t(f)$ represents the number of individuals belonging to offspring *t* and type equal to *f*
- The identical distribution of the function Φ (x_{at}, x_{bt}) between people of the same generation, imply that they belong and live in the same ecosystem and they generate the same quantity of offspring : x_{at} and x_{bt} depending on the choice made (a or b) , additionally x_{at} and x_{bt} will be equal for every generation t

[38] Casual variables are said to be IID (identically and independently distributed) if their probability distributions are the same and they statistically independent each other, so their correlations and covariance is equal to 0.

[39]To define a Bernoulli distribution, consider x as a discrete random variable and Rx the set of values that the random variable can take, where Rx = {0,1}. Consider a parameter p ∈ (0,1), random variable x has a Bernoulli distribution if its probability mass function, a function that associated a number to the probability that a discrete random variable is exactly equal to some value (Stewart, 2011), is:

$$p_X(x) = \begin{cases} p & if\, x = 1 \\ 1-p & if\, x = 0 \\ 0 & if\, x \notin R_X \end{cases}$$

The aim of the model is to identify the optimal value of *f*, intended abstractly as behavioural phenotype (v. Brennan-Lo, 2012*),* for which the highest geometric growth rate for the population is obtained. To highlight this concept, Brennan and Lo refer to *f* as the "growth-optimal" behaviour.

Analysing the data set outcome obtained, we can see as the optimal value, *f**, belonging to "growth-optimal" behaviour is equal to: = 0.75. For this parameter indeed, after twenty-five generations (so in the long-run environment) we have the highest number of individuals, so the highest population growth rate and best species reproductive success. Exactly the value of the probability that a sunny day will occurs, $p = 0.75$.

This is not a coincidence; Probability matchers is the winner![40]. Probability matching, a concept derived by "Matching Law" of R.J. Hernstein, is a typical human behaviour, rather, psychologists has documented this behaviour also in primates, pigeons, fish, bees and ants. It explains what happen when humans have to make a repeated decision between two mutually exclusive alternatives: following to decision theorists, humans should select the option with maximum probability of occurrence, so a called all-or-nothing allocation strategy. Applying this concept to the previous example, it means that individuals should choose the valley 100% of times. Since probability of a sunny day is greater than a rainy day, individuals want to maximize the chance of winning according to this behaviour model.

Here it comes the "bug". Indeed, numerous experiments have shown that humans (Lee. 1971) decision makers tend to choose between the two alternatives with a probability of selection almost equal to the probability of the "most probable outcome", so in other words we tend to match our responses probabilities to the payoff probabilities. This behaviour has been recognized as a "behavioural bias", due to an inner heuristic.

Coming back to the Brennan and Lo particular experiments it is possible to conclude for the authors that the result of the experiments is perfectly coherent with the fundamental concepts of Adaptive Market Hypothesis, in which evolutionary forces and the adaptation are the principle roles in evolutionary process of financial markets and operators and it can explain this behaviour, apparently irrational. From the experiment we learn that, when the ecosystem is characterized by randomness and this can lead to extreme consequences after some decisions or particular choices, a deterministic or all-or-nothing behaviour as $f^* = 1$, would lead to extinction of the whole species.

[40] "it turns out that in an environment that's sunny 75% of time, the group with the heuristic f=75%, in other words the probability matchers is the winner!", Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press) pag 193.

It is only through adaptation and past experience that people can learn and understand that to maximize their chance of survival, the best possible choice is to randomize, namely to adapt the behaviour to the ambient causality, in this experiment $f^* = p$, rather than optimizing the individual's expected numbers of offspring, probability matching optimize the growth rate of the entire group, leading this "heuristic" and the people following it, become dominant after several generations.

In this scheme, despite its simplicity and basic environment, we can observe the result of natural selection forces; these forces indeed highline specific behavioural biases and decision-making processes disadvantageous for the individuals of the species.

Additionally, the natural selection gives raise an intelligent way of thinking and acting into the groups of individuals (intelligent behavior, v. Brennan-Lo, 2012). Lo and Brennan explain this "intelligence" not as an absolute rationality but as a cognitive process influenced continuously and directly by the environment and by the physical characteristics of the species. If the environment plays a fundamental role for the individual behavior and his survival (it exhibits through the reproductive success), we need also to take into account the differences between idiosyncratic risk and systematic one, and the consequences of these for evolutionary dynamics[41].

These concepts can be transposed to the portfolio managing in financial sector: the idiosyncratic risk, which represent the specific risk component of an asset (since it depends of the specific characteristics of the asset), could be lowered or eliminated by diversification of a portfolio. Systematic risk, derived from the risks factors that involve the entire financial market and the entire environment correlated; this risk could not be eliminated, and it should be always engage in the whole analysis. A possible application of this it is described in chapter 3, where, as just written the features of Brennan-Lo are transposed to a financial environment.

With reference to the experiment exposed, it is the short version of the one conducted by Brennan and Lo. Here I don't present the complete, since it could just one thesis itself, it is important where this experiment conduct the idea and the consequences of an approach different from a one based on full rationality and efficiency.

Through the binary choice model elaborated by Brennan and Lo, it is possible to obtain a framework able to explain the deviations from the neoclassic utility-based economic theory, so,

[41] The basic idea is that, referring to the previous binary choice model, there is a specific risk for the evolution of the species, which can be reduced differentiating the choice of individuals about where to live. A systematic risk is correlated to the entire system, or environment, so it could be considered the risk of an epidemy that could reduce the number of population independently form where they live.

the major conflicts between individual rationality and human behavior can be understood and reconciled thanks to the Evolutionary models.

The core aspect in binary choice model is the Mutation. Lo et al. find in it the re-connection between rational and irrational behaviors in an evolutionary context.

Summarizing: considering a specific environment, and assume it is stable over a period of time, the evolutionary optimal behavior will emerge leading to the rational behavior.

When the environment changes, also evolutionary optimal behavior will change, so, irrational behavior become necessary to give solidity to the process of population growth (i.e. in the binary choice model, choosing rationally 100% to live in valley will lead to extinction on the first rainy day). The authors also demonstrated as for the entire population it is possible to determine an evolutionary optimal degree of irrationality.

More unstable environments imply more irrational behaviors in the population and more new entry over time.

The evolutionary origins of strategic behavior have also been considered (Robson, 1996b; Skyrms, 2000; Skyrms, 2014), and natural selection can also produce more sophisticated behaviors such as overconfidence (Johnson and Fowler, 2011), altruism and self-deception (Trivers, 1971; Becker,1976), and state-dependent strategies like the Hawk-Dove game (Maynard Smith, 1984), which emerge as a result of more complex environmental conditions.

The Hawk-Dove game is a concept proper of Game Theory literature (Besanko-Braeutigam, 2009); it describes a fight strategy between a hawk and a dove, (predator versus prey) or between the animals belonging to the same species (hawk vs hawk and dove vs dove).

In the following table it is depicted the payoff for the four types of fights. V stands for value of the winning (the value of the prey defeated), C is the cost for the escalated fight and it is intended that C is greater than V (otherwise this case would be equal to the prisoner’s dilemma).

When the fight id between two-different species: hawk vs dove, hawk wins and take the all value V, if the battle is between the two predators, no-one wins, and the payoff is the value V minus its cost C divided by 2, and finally, if two doves fight each other, value V is divided equally by them. This strategy describes the meaning and the influence of species identity.

Table 2.3 Payoffs for hawk and dove

	H	D
H	$(\frac{V-C}{2};\frac{V-C}{2})$	(V;0)
D	(0;V)	$(\frac{V}{2};\frac{V}{2})$

In the framework presented, assuming that one individual's action is correlated with the reproductive success of another individual, individuals engaging in strategic behavior will reproduce more quickly than those with simpler behaviors such as probability matching. If the actions of individuals in the current generation can affect the reproductive success of individuals in future generations, even more complex dynamics are likely to emerge as in the well-known overlapping generations model (Samuelson, 1958). In a resource-constrained environment in which one individual's choice can affect another individual's reproductive success, strategic interactions such as reciprocity and cooperation will likely emerge within and across generations (Trivers, 1971; Nowak and Highfield, 2011). In contrast, the model considered does not require any strategic interactions, and individual decision-making is deliberately mindless, allowing us to determine the most primitive and fundamental links between stochastic environments and adaptive behavior. Even in such a simple setting, we find a range of behaviors—behaviors that do not always conform to common economic intuition about rationality—can arise and persist via natural selection. Simon (Simon, 1981) illustrated this principle vividly with the example of a single ant traversing a mixed terrain of sand, rocks, and grass. The ant's path seems highly complex, but the complexity is due more to the environment than the ant's navigational algorithm. Much of the rationality debate among economists and psychologists focuses on whether the rational models can help people make better inferences and decisions in the real world (McKenzie, 2003). Instead, our framework provides an evolutionary explanation of irrational behaviors and different degrees of irrationality in the population. The results suggest that irrational behaviors are necessary even if they are seemingly inefficient in the current environment, and the nature of stochastic environment determines the degree of irrationality, and the number of new entrants into the population. From a policy perspective, our results underscore the importance of addressing different human behaviors in different environments. For example, the financial market is considered to be efficient most of the time (Samuelson, 1965; Fama, 1970), and participants with irrational beliefs constitutes a minimum part in the market. However, in periods of economic turbulence and financial crisis, irrational behaviors are much more prevalent than usual. Our results also highlight the importance of entry of new actors into the market even if they appear suboptimal in the current context and suggest that the optimal number of new entrants depends on the degree of environmental stability. On the other hand, if not properly managed, volatile environments can lead to increases in the degree of irrationality, implying higher social costs and lower economic growth. However, our results also highlight the potential dangers of sustained government intervention, which can become a source of systematic risk and cause volatile environments in its own right (Acharya, Richardso, Van Nieuwerburgh, and White, 2011; Lucas, 2011).

Adaptive Market Hypothesis in front line

The new theory elaborated by Lo, seems to be a perfect conjunction between the traditional theory of efficient markets and the new, behavioural theories. But a theory, to be accepted in real world needs to be proved in practice. This new conceptual “framework” it is not only a descriptive and qualitative analysis of financial markets and economic systems, but it has been developed empirical implications, like investment management.

2.7 The Traditional Investment Paradigm

Efficient Market Hypothesis carries out core beliefs and principles, used by different type of financial workers, finance professors, investment managers, brokers and so on.

These principles are the results of all the various aspects of the theory elaborated by Fama applied to a practical and real context, as the investment sector is. Rather, when I have discussed about the theories on which efficient markets hypothesis is founded, its consequences applied to the real financial market shown some principles that, during time has become fundamental for people who want to confront with the financial investments.

Note that, I’ll present only the core aspects of each one principle, the reason is that my purpose is to focus on the adaptive difference from EMH and AMH and their practical implications.

These are:

The Risk/Reward Trade-Off.

In financial investments risk and return are positive correlated: it takes higher risk to invest in assets with higher expected returns and so, if an investment carries higher risk of a potential loss, the investor should expect a higher return in order to be compensated for the higher risk. If the risk is higher but there is not the expectation of additional reward, no one (since humans are rational) will invest.

Alpha, Beta, and the CAPM

The Capital Asset Pricing Model[42], and the factors alpha and beta. Where alpha represent risk-free interest rate, so the coefficient of the intercept of an ordinary least square regression, the *beta* is the covariance between the security return and the market return divided by the variance of the market return.

Portfolio Optimization and Passive Investing

A portfolio manager can build diversified long portfolio of financial asset, which are optimal for investor in terms of risk and return, the optimization uses statistical estimates derived from CAPM.

Passive investing is a based on the belief that alphas of financial investments (excess returns) are very hard to obtain, also statistic support this idea since most of portfolio's alpha are very small. As a result, passive investing is created with the purpose to reply the returns of a particular benchmark or index fund lowering so the cost of an active managed portfolio. Passive investing represents the best form of investing for who strongly belief in efficiency of the markets, so that it is impossible to beat the market on average. The direct consequence is that the alpha of a passive portfolio is equal to 0, only beta is considered.

[42] The Capital Asset Pricing Model is a mathematical model applied to financial markets, it has been introduced by William Sharpe in the 1964. It describes the relationship between the expected return of an asset with the expected return of the market including the return of a risk-free asset through the coefficient β which express a measure of the risk of the asset. The model is represented by this formula:

$$E[r_i] = \beta_{im}(E[r_m] - r_f) + r_f$$

Where $E[r_i]$ represent the expected return of the asset i, $E[r_m]$ is the expected return of the market and r_f is the return of an asset without risk, for example a treasury bills.

Asset Allocation

To manage the risk of an investor, it is necessary to define how much invest in each security, instead of picking individual stocks. Investors should focus on managing their portfolio through a strategic asset allocation[43] based on factors as risk attitude, time horizon, investor's expectations.

Stocks for the Long Run

Investor should hold stocks with a long-time horizon. We could say that this one, is a relative modern principle. In 1994, economist Jeremy Siegel wrote a book: "Stocks for the Long Run", it has become one of the most influential book of the investment management industry. Siegel affirms that, considering data from the born of financial markets, there is empirical evidence that historical performance of U.S. stocks has been highly profitable over enough long holding periods.

These five principles have become the foundation of the investment management industry, every product or service offered by financial professionals is based and influenced by them, and a lot of investors have earned rewards over the years. Clearly, they should be considered approximations of the much more complex reality, and these principles are the direct conclusions of all the process that leads to the birth of Efficient Market Hypothesis. As consequence, these principles are valid as guidance for the investors, they clearly rely on some core assumptions derived from EMH which can be summarized in six points:

I. The risk-return relationship is linear;
II. The relationship is stable in time and in every situation;
III. The parameters can be estimated;
IV. All the investor has rational expectations;
V. The returns are stationary (their joint distribution does not change in time);
VI. The markets are efficient.

We have to consider the fact that these principles aren't the same things as physical laws, they don't necessarily have the same permanence as, for example the law of gravity.[44] The new concept expressed by Lo, here apply in this way:

[43] For strategic asset allocation it is intended a portfolio constructed of various asset classes: stocks, bonds, treasury bills and so on.

[44] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press)

From an ecological perspective, these assumptions are about the stationarity of the environment and the rationality of the investors, while the environment might fluctuate and the assumptions of Homo economicus are not realistic, so these principles should be considered as heuristics, approximations to a more complex system.

The finance literature asks if the approximation errors associated with these assumptions are small enough to be ignored. The answer proposed by Lo is: "the emerging narrative from the perspective of the Adaptive Market Hypothesis is that these errors used to be small but have grown considerably in recent years".

This concept does not come from an abstract thinking, indeed Lo has observed in macro terms the stabilization of U.S. financial market in the last century, finding that the period from the Great Depression (caused by the financial disaster of 1929) to the mid-2000s (before the dot.com bubbles and a series of changes in regulations) has been relatively stable, so, long term investments in a well-diversified portfolio could generate profitable returns. Indeed, looking at the biggest collapse of stock indexes, happened on the famous black Monday 19 October 1987, Wall Street dropped by 22.6% of its value in just one day, we notice as the recovery of that big loss was quickly, at the end of 1987 the Dow Jones index quoted slightly higher than the previous end-year.

This long period. called by Andrew Lo "The Great Modulation", was characterized by financial markets stability, a stability that nowadays we could call unusual if we think how much markets have been subject to changes in the last two decades. To give an idea of this we can look at the following graph: Figure 2.3 shows the annualized volatility of daily U.S. stock returns (the value weighted CRSP index) over 125-day rolling- window[45] in order to have a measure of the short-term volatility. During the financial crisis of 1929, the volatility reached extremely high levels, then after the recovery, volatility decline, and remains into a narrow range, with very few exceptions. This low level of volatility is partly due to margin requirements imposed by the FED on stocks purchase. After the 1934 indeed, to purchase stocks you should had a minimum amount of capital deposited in your account. This amount has been subjected to fluctuations, it has reached the no-leverage level: 100%, then since 1974 it has been fixed at 50%.

[45] A rolling window analysis conduct the analysis of data constructing the new observations over samples of consecutive observations. They are statistical analysis methods, which consist in analysis of temporal period of historical series. In this case, a 125 days rolling window means that each day of my window I will move the starting point and the end point of the analysis, so a new performance, obtaining all the possible records.

This is only a possible reason for explaining the stability during the Great Modulation period, indeed the most volatile period of U.S. stock period coincide with the global financial crisis of the 4th quarter of 2008. Lo, conclude his analysis pointing out a fact that surely has affected the economic stability: population growth: in the last century, human population has seen its number grow faster than ever and along with them, the world has changed rapidly, new economic powers have born, like China, India, Brasil. In the figure 2.4 we can have an idea of this change.

As consequence the interactions among the actors of the economic world stage become much more complex, and as Lo says:" The Great Modulation seems to be giving way to a new world order".

The extraordinary progresses in the field of technology, the new developments in all the scientific sector, the innovations in the productive process: from the industrial till the agricultural, all synonymous of a strong evolutionary force, that drives the evolution of man to a new environment.

Figure 2.3 Annualized volatility of daily U.S. stock returns (the value-weighted CRSP index)

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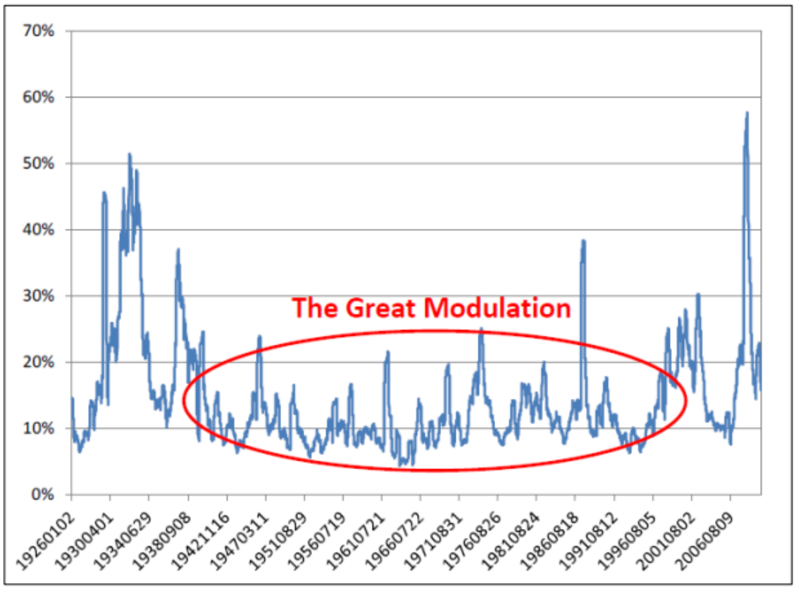
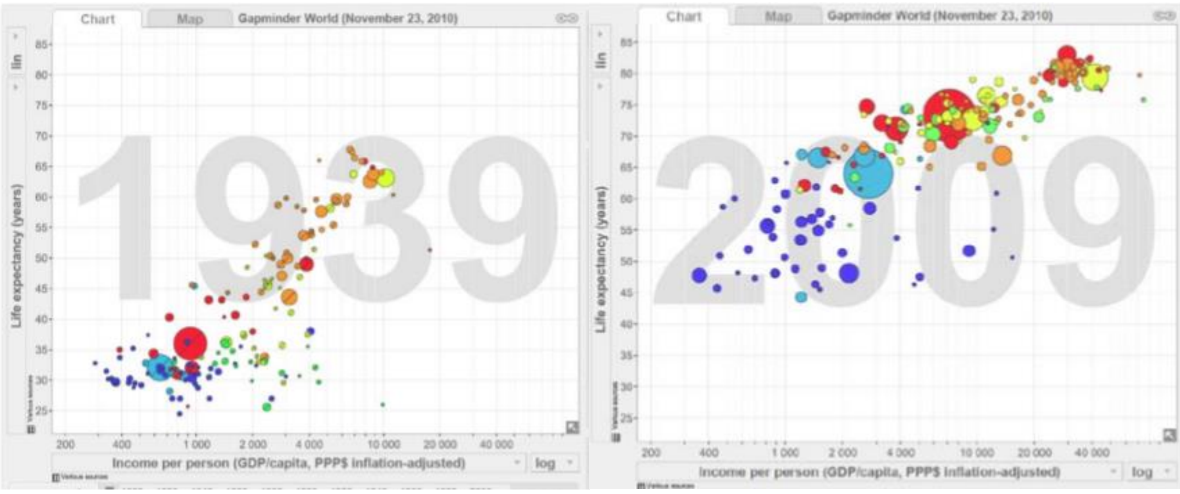


125-day rolling-window annualized volatility of CRSP daily value-weighted return index, from January 2, 1926 to December 31, 2010. Source: CRSP and Lo's calculations

Here in the figure 2.4 is presented how world has changed in the last century in terms of population size, richness and healthy, these factors are illustrated using as parameters: Population, through the size of the circles, per-capita GDP (x axis), and life expectancy (y axis).

Figure 2.4 Population, per-capita GDP, and life expectancy for countries of the world in: (a) 1939; and (b) 2009

Each geographic region is represented with a coloured circle: yellow colour represents America continent, Orange is for Europe and Asian Russia, Red is for East Asian and Oceania, Africa is divided in middle-south part (Blue) and North



On the x-axis is plotted GDP per capita (inflation adjusted), on the y-axis the life expectancy in years. The left graph illustrates the data for year 1939, the right one for year 2009. It is evident how both parameters have increased significantly, Africa is still the poorest continent and has the lowest Life expectancy, but on average individual's welfare has made great progresses (and there are still a lot of improvements to do).
Source: http://gapminder.org.

2.7.1 Risk reward and punishment

If, as Lo asserts, world is changing at high speed, the dynamics of economic environment follow this "evolution", it may happen that new economic species appear into the environment and some other extinguish. My comparison with evolutionary biology is exactly the point that professor Lo wants to tell: evolutions means change, change means adaptation, species who can't/don't adapt could be extinguished, and this create an opportunity for other species to make their appearance into the new environment. To be honest, and not because of some personal "reward", for me these intuitions are easily applicable to the financial world.

Again, if we consider it as an ecosystem, we may think all the investors/managers who make wrong decisions repeatedly for long time, as species that has not the ability to learn from ambient feedback, to change their decision-making process, to be rational in a new sense ("to be fully rational we need emotions") and so on.

These people will be dropped out from the market, and this will create new opportunity for new investors. The same thought could be applied also to financial products, which have literally evolved in years, think at instruments as credit default swap, new hedge funds and all the new financial products, always more sophisticated that have substituted some more "static" products.

Let's see in practice an implication of all these concepts.

In first chapter I have written about the wisdom of crowds, showing the example of the Challenger disaster. Following Lo reasoning, we know that wisdom of crowds does exist in real life, but only in some circumstances that we have seen deeply, I summarize them with one word: stability.

If we translate this in investment terms and looking for example on the principle of risk/return, which, in the traditional form is: the riskier is an asset, the more its expected return (the profits that investor expect to earn) is higher, this because I want to be compensated for the risk I am assuming in investing in that one. A rational investor indeed when faces the decision of which stock to pick, look at the expected return of each stock, so if one stock has the same expected return of another but it is riskier (it has more volatility so more possibilities that its price drop) the rational investor will certainly choose the less riskier stock. To be bought the riskier stock should have higher expected return.

This notion has strong empirical evidence and if we measure risk by volatility of returns we obtain an additional confirm. Table 2.4 is a summary that shows the data collected for six financial products: large and small stocks, long-term corporate bonds, long-term and intermediate-term government bonds and Treasury Bills for the U.S. stock market. small-cap stocks are riskier (they have a high volatility) than large cap stocks and on average small-cap stocks in one year earn almost two base point more than large one.

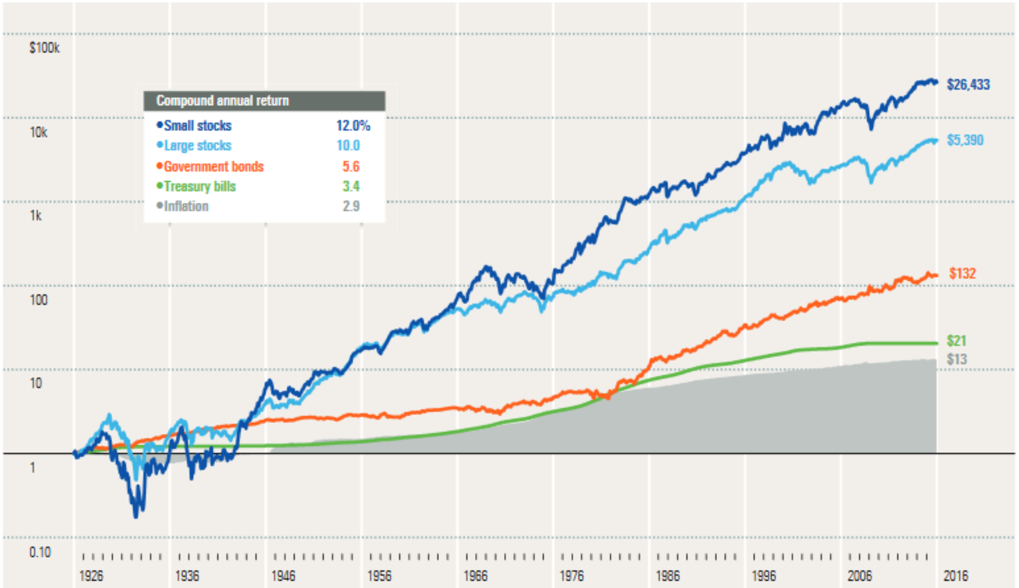
Bonds are financial products that are less risky thank stocks, and as consequence the average returns are lower. The same for the U.S. Treasury bills.

Table 2.4 Performance summary of stocks and bonds from January 1926 to December 2015

	Large Stocks	Small stocks	Long-term corporate bonds	Long-term government bonds	Intermediate-term government bonds	T-bills
Average Returns	10.0%	12.0%	6.0%	5.6%	5.2%	3.4%
Volatility	20.0%	32.0%	8.4%	10.0%	5.7%	3.1%
Cumulative return	5390 $	26433 $	188 $	132 $	94 $	21 $

The average returns are geometrically compounded and annualized; volatility is based on monthly returns and annualized by multiplying monthly estimates by $\sqrt{12}$, Lo (2017)
Source: Ibbotson (2016)

Figure 2.5 Performance summary of stocks and bonds from January 1926 to December 2015



This graph is the result of a data analysis conducted by Ibbotson associates, they analysed the cumulative return for one dollar invested in different financial products in the year 1926.
The blue line represents the return for small stocks, it tells that if we had invested one dollar in small stocks, on average we would have earned an annual return of 12% and in 2016 we would

have 26433$. The light blue line represents the return for one dollar invested in large stocks and so on, whereas the green line stands for the Treasury bills return.

Source: Ibbotson(2016)

The results of the table and figure leave no-doubt about the linear risk-return relationship. Where is the “bug”? If we consider long run investments, with horizon of almost ninety years as Table 2.4, these assumptions are clearly true, but since only few of us has this long enough period of time so it might be more useful look at data with a shorter horizon.

Figure 2.6 shows an unexpected result: risk/reward trade-off seems not to be consistent.

Indeed, there are two graphs in the figure, first is the average return for the stocks, second one is the corresponding volatility; both calculations are made over the same window of time: 1250-day-rolling (about five years).

We can see as, often, there has been period where the average return and volatility move in opposite direction. More precisely LO has estimated a correlation between the two curves of: -58%! This means that an investor sometimes instead of receiving some premium for taking more risk, obtain a lower return thank if he would have taken less risk.

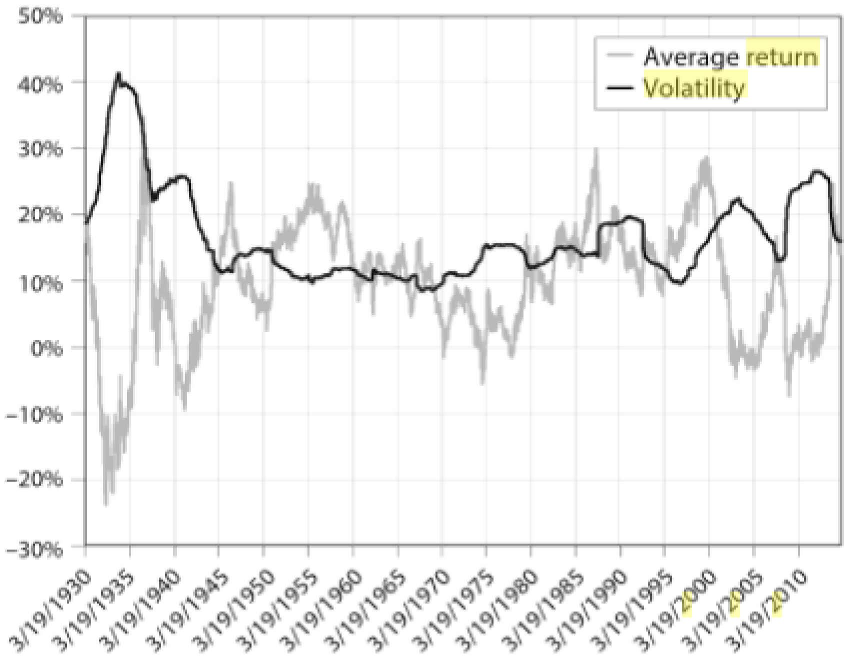


Figure 2.6 1250-day rolling-window annualized compound return and volatility of daily CRSP value-weighted stock market return index from January 2, 1926 (first 1250-day window ends Marchs 19, 1930) to December 31, 2014

This surprising relationship was already discovered by the father of Black-Scholes/Merton option-pricing formula: Fischer Black. He provided a possible explanation of this negative correlation between risk and reward, the "leverage effect". Thinking to a business example, when a corporation is financed with debt, its equity capitalization could decrease (due to a fall in equity value[46], Lo uses mortgage example where the house value falls) leading to higher volatility.

For an investor that use leverage, this means that his rate of return is much more volatile given the same level of fluctuations of stock prices.

This is a really good explanation but can't be the only one since the leverage effect is even stronger among companies that carry no debt[47]. To solve this problem, Lo proposes an alternate explanation: also in this concrete example, the new insights of Adaptive Markets Hypothesis can apply, indeed, we must never forget that we're not robot, but human that act in response to our brain processes, which are clearly influenced by emotions; I am repeating the same concept expressed before in this thesis, but it is necessary in order to understand how really AMH works in practice.

When an investor's equity decreases, and its volatility instead increases; generally, the first response, is a natural heuristic called in nature: "fight or flight response". In financial terms, is better known as "freaking out" where a large part of investors starts quickly sell their holdings putting a downward pressure on equity prices, and on the other side the demand for safer assets increase putting an upward pressure on their prices. This phenomenon is also known as the "panic selling", and it is temporary, once this emotional response calm down, wisdom of crowds get ahead of madness of mobs and the equilibrium is recovered, and with equilibrium, the correlation risk/rewards turn to be positive.

Looking again at the table with a 90-year horizon we see how this behavioural bias does not stronger affect the relationship between return and volatility but, considering a more realistic time-horizon for an investor, this anomaly can't be ignored; analysing too long time-horizon could lead to miss some important factor of financial environments.

During normal business environments, the principles of Efficient Markets reveals to be an excellent approximation to reality: the U.S. stock market had relatively consistent average returns and volatility, Lo considers that a long-only passive investment strategy of 60% stocks

[46] An example from house market: I buy a house for 300000$. The down payment I have to give initially is the 20%: 60000$. For the remaining 270000$ I establish a mortgage. The day after my purchase the price of the house decline by 10%: 30000$, my mortgage remain 270000$ but my equity is no more 60000$ but it has fall to 30000$.

[47] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press) (page 265).

and 40% bonds produced pretty decent returns, particularly for those who were investing over a 10- or 20-year horizon. Thinking that this approach could always work, reflecting the real world, could not be correct, since we must consider that market conditions are not fixed, they change, and we can experience large macro shocks like the financial crisis of 2008. When these changes happen, the simple heuristics like 60/40 no longer work as well because the dynamics of financial markets are different form previous period. Today's markets are now much more responsive to intervention by governments and their central banks and punctuated by the irregular cycle of fear and greed. So since 2007 and 2008, we've seen a very different market dynamic than over the previous six decades. The point of *Adaptive Markets* is not simply to be wedded to any static theory, but rather to understand how the nature of markets can change. And once it does change, we need to change with it. Mentioning John Maynard Keynes, whom in responding to criticism about his overturn on gold investment said: "When the facts change, sir, I change my mind. What do you do?"

2.8 Implications of adaptation and selection for the financial system

Practical implications of this new theory are what investors and economic agents want to know. Clearly the theoretical basis of every financial theory, are fundamental but market operators and professionals are focused of how to take advantage in practice of the new concepts, insights and consequences of a new theory. Andrew Lo tries to feed their needs, showing the results of the considerations and studies made on the financial markets under the AMH point of view.

"Finally, he discovered a simply formula to make all the investor rich standing easily sited on own sofa. No, I have confused it with one of the many advertisers we found every day in website."[48]

Lo first of all write about the differences in practical implications between the traditional finance paradigm and Adaptive Markets Hypothesis.

In order to understand this, he has re-considered the five principles of the Efficient Market Hypothesis, developing them including the new insights of Adaptive Market Hypothesis:

[48] This a personal reference to the so common advertising that we found every day surfing on the web. Unfortunately, a lot of people become interested in Finance with the wrong (personally) idea that exist some magic theory or formula that can make them rich. The concept I want to express here is basically that, the empirical concepts that Lo gives to AMH are not a sort of assumptions which, if followed, can make us everybody rich. I will explain it deeply further.

Principle 1A: The risk/Reward Trade-Off.

During normal market conditions, the principle of the traditional paradigm is true, positive and linear relationship between risk and reward for all financial investments; but this is not stable over time and circumstances, it depends on factors as population of market participants and the business environment. There exist periods where the population of investors face extreme financial threats, leading the individual to act in an irrational way, for example when fear dominates the market, reducing the average return on risky assets and increasing the average return on safer ones. These periods can last. In extreme cases for years. It is exactly during such periods in financial history bubbles and crashes emerge and EMH shows its weakness.

Principle 2A: Alpha, Beta and the CAPM

Personally, also in this re-formulation of the CAPM principle Lo shows its great intuition, he does not reject CAPM, rather, he underlines its usefulness but set a change of view: maybe it is more important focus on knowing the environment and population dynamics of market participants than any single factor model. CAPM, Alpha, Beta and other factors based on economic assumptions that are not a good estimate in certain market environments.

The Adaptive Market Hypothesis so, intend to focus on market dynamics than any static final state or equilibrium[49].

The consequences of this change of view are directly observable on investor's strategies. Recalling the traditional theory EMH and the Capital Asset Pricing Model, we know that: it is impossible for an investor to have a portfolio which generate consistent return above the CAPM (the market portfolio) without taking more risk.

Now, with the Lo's theory, we have said that we should focus on market dynamics, so, it is possible to gain a sustainable risk premium for investors for a period of time, because as we have seen financial environment is not always stable and perfectly efficient, and Adaptive Markets Hypothesis implies that market efficiency is not an all-or-nothing condition, but a continuum, for every trading period, we may consider a day or a month or whatever interval of time we can measure a "degree of market efficiency" based on some conditions. Market efficiency depends on the relative proportion of investors who make investment decision based on their "feelings". It mays seems wired because in the previous paragraph I have presented a precious insight of AMH: to be fully rational we need emotions, but as counterpart, there are investors that act on the base of unappropriated heuristic, that derives from instinctive faculties.

[49] Andrew Lo: Adaptive Markets Financial Evolution at the Speed of Thought (2017, Princeton Press, page 269)

If this amount of "irrational" investors is big enough, the degree of market efficiency will be lower, and in that case, there will be possibilities for an extra-return, beating the market. If this behavior become frequently and massive, it can lead to market bubbles or crash; and so, the possibility to determine a degree of market-efficiency would help us to choose the optimal investment strategy.

Principle 3A: Portfolio Optimization and Passive Investing

Portfolio optimization tools are only useful if the assumptions of stationarity and rationality are good approximations to reality. The notion of passive investing is changing due to technological advances and risk management should be a higher priority, even for passive index funds.

One important implication of *Adaptive Markets* for investors and portfolio managers is that passive investing is changing, and we have to adapt. John Bogle—the founder of the Vanguard Group and the father of passive investing and index funds—had an incredibly important insight in the 1970s which he calls the "Cost Matters Hypothesis": he claimed that the reduction of trading costs can lead to remarkable positive effect on wealth accumulation. With his work he has contributed in large measure for individual investor environment democratizing the investment process.

Here we can see the work of the Adaptive Market Hypothesis: at first, indexes and their corresponding mutual funds used equal weighting[50] (giving the same importance to each stock in portfolio or index fund). This strategy requires a large number of trades[51], raising the fees that investors should pay; often it happen that fees completely overcome the return of the portfolio (if present). The evolutionary process driven by innovation and natural selection, leads to solve this emergence for the index-fund industry switching to the market-cap weighted indexes: the single components of the portfolio are weighted proportionally according to security's market capitalization; i.e. the biggest components of portfolio in capitalization's terms will have higher percentage weightings. Numerous advantages derive form this technique: no rebalancing is required because the weights of the stocks[52] adjust automatically as price changes (if prices increase, market-cap increase too and so the portfolio weight). The

[50] Equal weighting means that for each stock of the portfolio (or the fund) the proportion of the money invested is equal to the other portfolio's stocks. The amount of money invested is equally distributed between the managed stocks

[51] It requires a large number of trades because when prices fluctuate, there is no longer an equal weighting in the portfolio: the proportion of money invested change for the stocks. If the price of a stock increases, it will have higher weight in the portfolio than a stock with a declining price. As consequence, portfolio must be rebalanced often in order to maintain an equal-weighting, causing the high number of trades executed.

[52] The amount of money invested in each stock

amount of trading needed is consequently reduced, the strategy of market-cap weighted are usual “buy-and-hold”.
For these reasons, after numerous processes of trial and error, market-cap weighting becomes the standard for the passive investing and its instruments (mutual funds, exchange traded funds exc..). Technological innovations like automated trading, electronic market-making, and big data analytics, could represent the next evolutionary step of Bogle insight (Cost Matter Hypothesis, creating personalized index for individual investor aimed to achieve specific goals, for example, the trend in healthcare towards personalized medicine. The goal that Lo suggest achieving, is to manage the individual portfolio’s risk more actively. This means to try to adapt the passive investment management to a more suited on individual characteristics, as risk tolerance, aims, job specifics and so on, in order to reduce the “freaking out” effect.

Principle 4A: Asset Allocation:
If we look at the current financial environment, asset classes are strictly correlated and their boundaries are no more well-defined. New financial institutions and macro factors have created new links across precedent unrelated assets. A direct consequence is that diversification is less effective, offering lower benefits than in the past (as Great Modulation period). Diversification it is harder to achieve in today’s macro-factor-driven markets. Its implementation must be adapted to the current environment, so, following Lo, investors should not be focused only on diversifying across asset classes, but they should give more importance to diversification across wide variety of investments in multiple countries, and according to factor exposures.
The asset allocation implication of the traditional investment paradigm is valid only under a stable and stationary environment where the assumptions I-VI becomes reasonable. When the environment changes, and there is a significant variance of volatility (volatility of volatility) and the same for risk premium, the construction of an asset-based portfolio could be not appropriate from a decision-making point of view.
Using Lo’s example, a portfolio with 60% weight in equities may yield a volatility of $0.6 \times 0.20 = 12\%$ during normal times, but during the fourth quarter of 2008, when, due to crisis, volatility reached peak of 80%, such an allocation would have yielded a volatility of approximately 48%.
The Adaptive Market Hypothesis takes care about investors preferences, in the sense that investors are more concerned with risk and reward than the numerical values of their portfolio weights.
So, quoting Lo:” the AMH suggests that denominating asset allocations in risk units may be more useful and more stable, example, if an investor is comfortable with an annualized return

volatility of 10% for his entire portfolio, this can be the starting point of an asset-allocation strategy in which a risk budget of 10% is allocated across several asset classes, say 5% risk to equities, 3% risk to bonds, and 1% risk to Commodities".

(Lo, Andrew W. "Adaptive Markets and the New World Order (corrected May 2012)." Financial Analysts Journal 68.2 (2012): page15–16).

This new concept expressed by Lo could be an opportunity to manage the risk in more effectively way. Prices of underlying assets fluctuate over time, not only, also the correlations between them do not remain constant and so, Lo suggests allocating asset weights according to their risk. Maintaining constant risk weights, it is possible to avoid volatility excesses reducing so any possible behavioral biases.

Principle 5A: Stocks for the Long Run

This principle is a direct consequence of the precedents. It points out that, as traditional investment paradigm says, holding equities over the very long run increase the chance to achieve bigger returns. The addition made by Lo is that in real life, few investors can afford to wait that big returns, the real investment horizon for bigger part of investor is shorter, so risks are greater. Investor so, should be more proactive about managing their risk/reward trade-off.

These new principles clearly show how Adaptive Market Hypothesis lead to different practical implications, even if the theory presented by Lo started from an extended number of researches, theories alternative to EMH also going back in the past. Its development is still in its primary steps.

For each of this principle here discussed, there is need for further investigation and demonstration of the implications of adaptation and selection for financial system.

2.9 Can you beat the Market? part two

In this paragraph my intent is to present a story about a fact happened in the August 2007: "The Quant Meltdown"; then to understand the possible causes and consequences.

Quants is a word used to describe the field of Quantitative trading; where the trading strategies rely on quantitative analysis; through complex statistical and mathematical computation models the quantitative analyst try to study mathematically the reality identifying trading opportunities. Its techniques include high-frequency trading, algorithmic trading and statistical arbitrage

Quantitative trading is in general adopted by financial institutions and hedge funds (but it can also be used by individual traders) which normally use as data inputs, a lot of data where the most common are Stock prices and volume.

The Quant Meltdown of August 2007 refers to a remarkable event that happened during the second week of August: several prominent Hedge Funds collapsed, suffering unprecedented losses.

The Hedge Funds that were damaged more by this recession has some peculiar aspects: they were all employing quantitative strategies; going more in details, the specific strategies were the so-called Long-Short Equity strategies: the most used were the Pairs Trading, Value Strategy, Momentum Strategies, Short-term Reversal and Contrarian Strategies.

They basically apply the concept of Long-Short, an investor buys or sell a stock when its price has a standard deviation *k* from the mean, so for each strategy the investor can consider different time intervals, and when the prices deviate from the trend in significantly terms, the investor should act on the market. A congruent example is the basic mean-reversion strategy.

The idea behind this strategy is a perfect example of how investor tries to take advantage of the short inefficiencies of the market: daily fluctuation of a share's price is the result of the ever-changing investor's sentiment and this sentiment, we have seen in this thesis is not always perfectly rational leading so, to temporary mispricing of the security.

2.9.1 Contrarian Trading Strategy

In order to give a quantitative formalization to Contrarian trading strategy, I present the following model described by Andrew Lo and MacKinley. The notation used are: (1) all vectors are column vectors unless otherwise indicated; (2) vectors and matrices are always referred with boldface characters, so: R is a scalar, whereas **R** is a vector or matrix.

To start, define $\mathbf{R}_t$ as a vector of N rows and 1 column, where the N rows are the N random variables defined on a probability space. The random variables represent the N returns of a collection of N stocks, evaluated at a period t. $\mathbf{R}_t$ is equal to: $[R_{1t} \cdots R_{Nt}]'$.

Now let's consider the following assumption:

"$\mathbf{R}$t is an array generated by a jointly covariance-stationary stochastic process with expectation $E[\mathbf{Rt}] = \boldsymbol{\mu} \equiv [\mu_1\ \mu_2\ \cdots\ \mu_N]'$ and autocovariance matrices $E[(\mathbf{R}_{t-k}-\boldsymbol{\mu})(\mathbf{R}_t-\boldsymbol{\mu})'] = \mathbf{\Gamma}_k$ where, with no loss of generality, we take $k \geq 0$ since $\mathbf{\Gamma}_k = \mathbf{\Gamma}'_{-k}$."

This assumption has been constructed with a simple notation, indeed, the joint covariance stochastic process that construct **R**t, is stationary and so, population moments $\boldsymbol{\Gamma}_k$ and $\boldsymbol{\mu}$ are independent from time.

According to the logic of all contrarian strategies, consider buying at time t stocks that were "losers" at time t−k, and selling at time t stocks that were "winners" at time t−k, where winning and losing is determined with respect to the equal-weighted return on the market.
The formalization of this concepts starts form the notion of $\omega_{it}(k)$, the fraction of the portfolio devoted to security i at time t, the "weight" of a single a stock *i* at time *t*:

$$\omega_{it}(k) \quad = \quad -\frac{1}{N}(R_{it-k} - R_{mt-k}) \qquad i = 1, \ldots, N \ , \tag{2}$$

Where $R_{mt-k} \equiv \sum_{i=1}^{N} R_{it-k}/N$ is the equally-weighted return for the index chosen.

$\omega t(k) \equiv [\omega_{1t}(k)\omega_{2t}(k)\cdots\omega_{Nt}(k)]'$, the vector of the weights of the portfolio has been constructed as a "dollar-neutral" or "arbitrage" portfolio since the weights sum to zero. In order to have the most clear and suitable explanation, the weights are defined as the actual dollar positions in each stock; since any multiple of the weights will sum to zero. The total dollar investment long (or short) in this case is defined by $I_t(k)$, at time *t*.

$$\mathrm{I}_t(k) \quad \equiv \quad \frac{1}{2}\sum_{i=1}^{N} |\omega_{it}(k)| \tag{3}$$

Contrarian investments strategies have become "popular" between trading strategies because of their relative simplicity and for their apparent profitability[53]. Their positive extra-returns are justified by the fact that these strategies take advantage of stock market overreaction. For example, when investors "overreact" a good news relative to a stock, they strongly buy it, pushing its price over its fair value. The Contrarian trading strategy bet on the temporariness of this event, in other words it bet on the following decrease in stock's price. In order to exploiting efficiently this opportunity, the weights of the portfolio are determined through formula (10): proportional to the differences between the market index and the returns. Lo and MacKinlay[54] assert that overreaction could not be the only reason for the profitability of contrarian strategies. They have demonstrated that the presence of stock market overreaction is not necessary to yield positive expected returns, this because when returns are positively cross-autocorrelated, they

[53] There is not a certain proof that contrarian strategies are always profitable, (otherwise everyone would adopt it).
[54] "When are Contrarian Profits Due to Stock Market Overreaction?" Andrew W. Lo; A.Craig MacKinlay, (the review of financial studies, 1990)

show on average positive profits, clearly for a contrarian strategy. Not only, this is true even if the returns of single securities are serially independent. The statistical properties of the contrarian strategy are not complex to derive, since its linear form. Lo and MacKinlay (1990) show that the strategy's profit-and-loss
at date t is given by:

$$\pi_t(k) \quad = \quad \boldsymbol{\omega}_t'(k)\mathbf{R}_t \tag{4}$$

and re-arranging it and taking expectations yields the following[55]:

$$\mathrm{E}[\pi_t(k)] \quad = \quad \frac{\boldsymbol{\iota}'\boldsymbol{\Gamma}_k\boldsymbol{\iota}}{N^2} \;-\; \frac{1}{N}\mathrm{trace}(\boldsymbol{\Gamma}_k) \;-\; \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 \tag{5}$$

where μm= E[Rmt] = μ'í/N and tr(.) denotes the trace operator[56]. The first term of the precedent equation is the k-th order autocovariance of the equally-weighted market index. The second term is the cross-sectional average of the k-th order autocovariances of the individual securities, and the third term is the cross-sectional variance of the mean returns.

From this formula elaborated by Lo and MacKinlay, the main conclusion for the aim of our investigation is that contrarian strategy's expected profits are an explicit function of the means, variances, and autocovariances of returns. To investigate deeply the statistical details and properties of this strategy see Lo and MacKinlay (1990, 1999) which also give an empirical analysis of its historical returns. Algorithmic trading tries to take advantage of this price far from average, because there is the belief that market efficiency force will drive the price back to its mean. The proportion of stocks to buy or sell is given by the amount of the standard deviation from the mean (considered into a specific time-interval).

In order to better understand the impact of the recession occurred in the days from 6 August to 10, 2007 for these category of hedge funds, I present a specific strategy elaborated first by Lehmann (1990) and Lo and MacKinlay (1990), then used as research tool by Khandani and Lo (2007) with the purpose to explain the Quant Meltdown.

They have applied the long/short market-neutral equity strategy; a mean-reversion strategy where beta, the systemic risk of portfolio is always kept near to 0, to U.S. stocks market,

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[55] The re-arrangement of the equation is derived from the population counterpart of Lehmann's (1988) sample moment equation divided by N. See Appendix 1 of paper "When are Contrarian Profits Due to Stock Market Overreaction?" Andrew W. Lo; A.Craig MacKinlay.

[56] The trace operators of a square matrix, ($\boldsymbol{\Gamma}_k$ in this case) is defined as the sum of the elements on its principal diagonal.

analyzing the stocks returns. Their strategy is to consider N stocks, and “go long” for stocks that have lower than average returns, “go short”, with the same amount of money used to buy stocks, for stocks with higher than average returns, where the average is considered on the base of benchmark as S&P 500 index. Each position is weighted in proportion to the amount of std. deviation of the specific stock from the index. This means that: the more a stock’s returns deviate from the index ones, the more weight (money) is placed on it. The reason is straightforward: when the returns of a stock are largely positive or negative than the index ones, a larger weight on that stock can takes more advantage of a possible trend’s inversion. Specifically, the portfolio weight of a stock: ω_{it} is calculated through the equation:

(2)
$$\omega_{it} \;=\; -\frac{1}{N}(R_{it-k} - R_{mt-k})$$

where the weight of a security *i* is evaluated at a date *t* and it is equal to the negative value of the difference between the return of security *i* at date t: R_{it-k}, and the mean return at same date t:

R_{mt-k} both are evaluated considering k periods ago, this means that for each value chosen by the investor, the result will be a different strategy. Andrew Lo, has also tested the mean-reversion strategy for different values k-minute periods, in this example we considered the value k equal to one day. Note that ω_{it} expresses how much the stock i, outperform the market for the k period.

Mean return is so defined:

$$R_{mt-k} \;\equiv\; \frac{1}{N}\sum_{i=1}^{N} R_{it-k} \qquad (6)$$

This means that R_{mt} represent the average of all the N securities returns considered, so if we considered all the stocks of an index R_{mt} represent the return of the market.

The property of the strategy defined by Lo and Khandani is that the portfolio is “market-neutral” (or “arbitrage strategy), meaning that the long positions established must be equal to the short one, in order to have the sum of the weights ω_{it} as defined by (1) equal to 0. To be precise, to be “market-neutral” the beta of the portfolio should be equal to 0, otherwise the strategy assumes the correct term of “dollar-neutral”; however, the two terms are usually used as synonymous.

An example of how a portfolio could be constructed with this strategy is showed by Lo and Khandani, in their paper. The fundamental concept to understand is that to define the returns of a portfolio so constructed, there is the need to define the leverage ratio.

- With 1000$ of initial capital, a leverage ratio of 2:1, or a 50% margin requirement, we can buy stocks for 1000$ and short sell stocks for other 1000$, so portfolio value would be 2000$. If in one day, the profit earned is 20$, the return will be 20 over the initial capital, 1000$ so equal to 5%.

Leverage can increase returns in huge way, but in parallel also the exposure to losses and risks.

Some leverage can require only a 10% of initial margin, that mean a ratio of 10:1, so for 1000$ it is possible to establish a position of 10000$ value.

Indicating the degree of leverage with teta θ, the portfolio returns with a leverage ratio of θ:1; $L_{pt}(\theta)$, are equal to:

$$L_{pt}(\theta) \equiv \frac{(\theta/2)\sum_{i=1}^{N}\omega_{it}R_{it}}{I_t} . \tag{7}$$

where, I_t is the gross dollar investment.

For each interval period, Lo and Khandani buy the losers and short sell the winners of the previous one, this mean-reverse strategy takes advantage from the re-balancing that occurs within the interval period. As Lo and Khandani state: weighting formula “has been called a “contrarian” trading strategy that benefits from market overreaction, i.e., when underperformance is followed by positive returns and vice-versa for outperformance”.

The contrarian trading strategies used by hedge fund carry out an important role in financial market, indeed they provide liquidity to the marketplace (3). Recall that, by definition, “losers are stocks that have under-performed relative to some market average, implying a supply/demand imbalance, i.e., an excess supply that caused the prices of those securities to drop, and vice-versa for winners” (Khandani and Lo, 2007). These strategies as said, expect to buy losers stocks, adding the demand for them, and, on the contrary, sell the winners stocks increasing their supply; through this process, hedge fund stabilizes the imbalance between supply and demand in a very similar way to the market-makers. They buy low and sell high in each trade, giving liquidity to market, for doing so their reward is represented by the bid/offer spread.

Table 2.5 An example of this Quantitative Market-Neutral Strategy applied by Lo and Khandani:

Ticker	R_{t-1} (%)	$R_{t-1} - R_{mt-1}$ (%)	I_t ($MM)
CEC	1.55	1.62	-45.53
IBM	-0.89	-0.82	23.15
INTC	-0.97	-0.90	25.32
MCD	-0.18	-0.11	3.03
MRK	-1.79	-1.73	48.50
MSFT	1.87	1.94	-54.47
Average:	-0.07	Sum:	100.00
		Sum:	-100.00

.

Source: Quants in August 2007 by Khandani and Lo

The first ticker, CEC has obtained a positive return of 1.55% in the previous period (t-1), and overperformed the average return of the benchmark (-0.07) by 1.62%. So the investment strategy will go short on CEC, selling the stocks for a value of -45.53 on the total capital invested (200). For the second ticket, IBM, the strategy is the reverse, since in the previous period it has obtain a negative return, now it is time to buy its stocks for a value always based on the weight calculated with formula (1). This strategy is applied for every stock considered, at the end the total value of capital invested long will be equal to the total capital invested short.

2.9.2 What Happened in August 2007?

Lo and Khandani, decided to apply their strategy for the month of August in 2007, among all the possible choices for a single quantitative strategy, recalling that they used to buy and sell the largest stocks (S&P1500) based on the returns of the day before, rebalancing the portfolio once a day. The results were more than remarkable, if for the first week of August, the reached returns were within the normal range, the three days of the second week, precisely: August 7th, 8th, and 9th were extraordinary. The first of the three days, on Tuesday, the return of the strategy was – 4.64%; on Wednesday -11.33%; on Thursday 9 August -11.43%; these returns should have been impossible according to strategy construction. During these days no one knew the reasons of that tremendous losses, so the fund managers act in what seems to be the most rational way, they cut their risk exposure, selling stocks they held long and bought back the stocks they had shorted, so they have tried to minimize their losses converting their stocks into cash. Nevertheless, the abnormal events for this week weren't finished, on 10 August indeed, the strategy simulated by LO and Khandani rebounded, but most unpredictable was the fact that rebounded with a one-day return of +23.67%, it seems that managers who exit from their strategies on 9 August made a big mistake, missing the rebound on Friday. These events, happened in just four days, weren't due to a failure of strategy or an FDA announcement, Lo highlights how the probable cause was a "liquidity spiral". Liquidity is a measure of the degree of how easy is to buy or sell an asset or security in the market without affecting asset's price. Cash is the most liquid asset, while real estate, fine art and collectibles are all relatively illiquid. Traditionally, market makers, like NYSE/AMEX specialists and NASDAQ dealers provide liquidity to market place, practically, when the demand for a stock is increasing they provide supply and vice versa, so they sell when investor want to buy, and they buy when investors want to sell. clearly, they are rewarded for doing this through the bid-offer spread. Lo explain how the contrarian trading strategies made the same "work", namely, provide liquidity to marketplace: losers stocks have underperformed the market, so the supply were in excess, the contrarian strategy, buying that stocks, do exactly the same role of a market maker, that is, buy when others sell adding the demand for the under-performing stocks, the same reasoning apply to the winners stocks, where the contrarian strategy sell these stocks, adding supply for them, with the result that it stabilizes the imbalance between supply and demand. To explain the phenomena of "Quant Meltdown", Lo and Khandani, has developed the theory of "Unwind Hypothesis". They argue that the large losses occurred from Tuesday, August 7 to Thursday August9, are the result of at least one large statistical arbitrage portfolio who has liquidated all its position in very short time, Lo and Khandani also hypothesize that liquidation of this portfolio could have been made by a large commercial bank that needed to raise cash; they recall that in that summer subprime mortgages and mortgages related securities started their collapse. The price impact of the unwind on August 7–8, and other behavioural phenomena as panic selling, caused a number of other types of equity funds—long/short, 130/30, and long-only, to cut their risk exposures or "de-leverage", liquidating their position, intensifying the losses of many of these funds on August 8th and 9th.

Another evidence that support the theory of "Unwind Hypothesis" is the massive reversal happened on Friday 10 August for the leveraged contrarian strategy. If the precedent losses would be caused by some structural change in the equilibrium of long/short equity strategies, the effect on price would have last longer, probably with a permanent change on price levels.
David Viniar, Chief Financial Officer of Goldman Sachs, affirmed that "We were seeing things that were 25-standard deviation moves, several days in a row... There have been issues in some of the other quantitative spaces. But nothing like what we saw last week" (Thal Larsen, 2007).
Lo and Khandani proposed an explanation for the rebound of Friday, probably, the quantitative hedge funds involved in this "thriller" week, after three days of "unwinding", reached the desired level of risk exposure and liquidity on Friday; moreover, considering the price volatility of the precedent three days, and summing up the losses, (-27.41%), these cumulative returns has generated stronger trading signals for the long/short equity strategies that lost the most.
If we refer as example, to a contrarian strategy like (1), For example, in the case of the contrarian strategy (1), consider the weighted percentage of security i to the profits at date t,

$$\omega_{it}R_{it} = -R_{it}(R_{it-1}-R_{mt-1})/N.$$

If this position has obtained uncommon large losing return for a given portfolio weight ω_{it}, R_{it-1} would be greater than R_{mt-1} and R_{it} would be large and positive, or, R_{it-1} would be less than R_{mt-1} and R_{it} would be large and negative. In either case, the loss is due to persistence or momentum in security i's price—the bigger the loss, the more significant the momentum. This, in turn, implies a much bigger position of the same sign for security i at date t+1 on average since $\omega_{it+1} =-(R_{it}-R_{mt})/N$ and R_{mt} has much lower volatility than R_{it}. Therefore, large losses will, on average, yield bigger bets for the contrarian strategy[57] Together with "Unwind Hypothesis" other crucial factors should be considered to depicting the complete picture of this extraordinary event.
(a) The very fast growth in the amount of funds and assets under management in the Long/Short Equity Hedge and Equity Market Neutral categories. Hedge Funds Sector in general has grown quickly, in the years previous to 2007, but the enormous growth has been in assets devoted to long/short equity strategies in the last decade and, more recently, to various 130/30 and active-extension strategies; total assets in the Long/Short Equity Hedge and Equity Market Neutral categories grow up to over $160 billion.

[57] Andrew Lo Amir E. Khandani What Happened To The Quants In August 2007?

(b) If the number of hedge funds and assets under management has increased rapidly, on the contrary the profitability of quantitative equity market-neutral strategies suffered a constant decline, the average daily return of the contrarian strategy has reached a low value of 0.13% in 2006. It may seem counter-intuitive that asset under management increase into hedge-fund contrarian strategies with declining expected returns, the reasons of this decline in profits are found in the increasing competition, technological advances, institutional and environmental changes such as decimalization, the decline in retail order flow, and the decline in equity-market volatility (Lo 2007). In the Figure 2.6 it is illustrated this trend in quantity and profitability for the year from 1994 to 2007.

(c) In order to maintain the expected return stable and profitable, year after year, hedge funds manager had to increase the leverage ratio. The hedge funds investors require a certain level of expected return, the managers so faced the decrease in profitability using more leverage.

TABLE 2.6 Required leverage ratios for Contrarian Strategy to yield 1998 level of average daily return

Year	Average Daily Return	Return Multiplier	Required Leverage Ratio
1998	0.57%	1.00	2.00
1999	0.44%	1.28	2.57
2000	0.44%	1.28	2.56
2001	0.31%	1.81	3.63
2002	0.45%	1.26	2.52
2003	0.21%	2.77	5.53
2004	0.37%	1.52	3.04
2005	0.26%	2.20	4.40
2006	0.15%	3.88	7.76
2007	0.13%	4.48	8.96

From this picture, which illustrates the average daily return calculated on annual basis from Lo and Khandani, becomes clear how hedge fund using contrarian strategy, must increase the degree of leverage in order to maintain about the same average level of returns obtained in 1998, when the leverage ratio was only 2.00. For the year 2007, contrarian strategies obtained the average daily return of 0.13% but using a return multiplier of 4.48, so a leverage ratio of almost 9!

Leverage is a powerful financial tool, it can transform small expected profit opportunities into greater ones, expanding both side of the financial trade: expected returns and risk, indeed leverage can expand small losses into larger losses. A consistent portion of hedge funds adopt large leverage in their strategies, so, the capital posted to support the position of these strategies is copiously smaller than the size of the positions. Andrew Lo and Khandani, in order to (8) give an insight about the consequences of the increasing leverage ratio used by the long/short equity strategies for the hedge funds sector, computed the necessary amount of leverage required in each year after 1998 to yield an expected return for the contrarian strategy equal to level of 1998. Mathematically speaking they used the data collected to find a value of leverage ratio: θ* such that:

$$\begin{aligned} \mathrm{E}[L_{pt}] &\equiv \frac{\theta^*}{2}\,\mathrm{E}[R_{pt}] = \mathrm{E}[R_{p,1998}] \\ \theta^* &= \frac{2\,\mathrm{E}[R_{p,1998}]}{\mathrm{E}[R_{pt}]} \quad , \quad t = 1999, \ldots, 2007 \end{aligned} \tag{8}$$

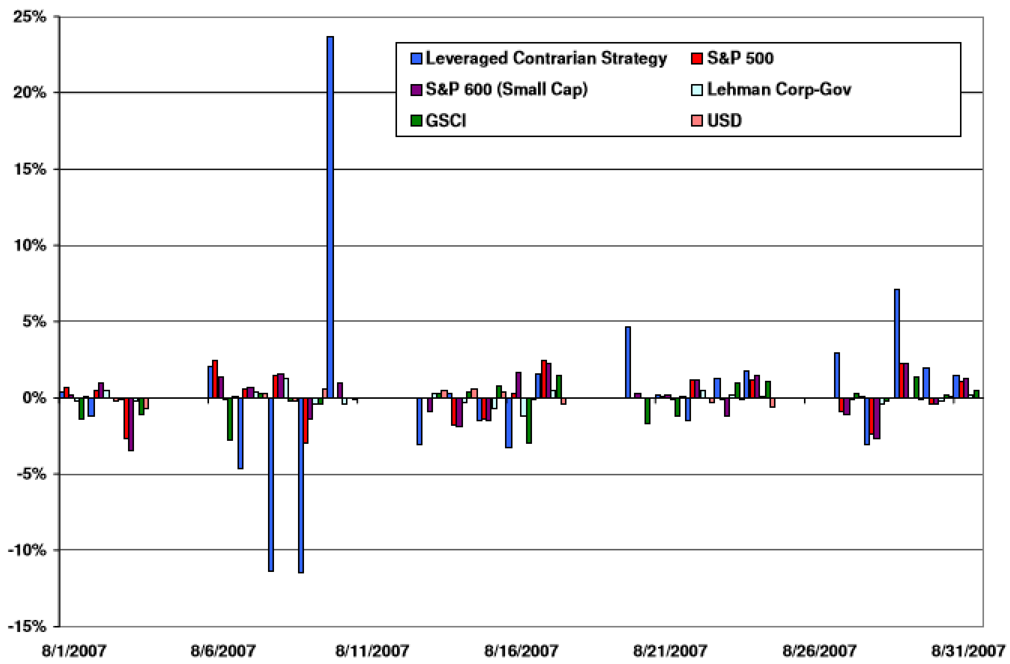
where (8) defines the expected return of the leveraged portfolio ($E[L_{pt}]$) equal to the expected return of the portfolio in year 1998,constructed without leverage ($E[R_{p,1998}]$), t represent the year considered for which find the leverage ratio. The leverage ratio required to make the expected return of year t equal to the expected return of year 1998, θ* is divided by the factor of 2, since it follows from the definition of leverage as the sum of the gross long and short positions (which are equal in the case of market-neutral portfolios) divided by the investment capital.

(d) After a deep study of the Quants Meltdown, especially by academics, it has been asserted that most quantitative hedge funds managers followed strategies with the same structure, so very similar each other. The position of these strategies has become interrelated and highly correlated since the trades has been made under similar quantitative portfolio construction techniques. The consequences of this similarity are that, the decisions made by the quants managers, to buy or to sell the same stocks are based on the same historical data and technical signals, so the same empirical anomalies are exploited, for example, the value premium or the size premium. Another important fact, also pointed out by Cliff Asness at AQR in his letter to his investors, is that: the competition among hedge funds managers, in particular the ones that apply contrarian strategies, has increased a lot during the years precedent to 2007, due to the widespread use of standardized factor risk models. An intensified competition, plus the large amount in the use of these strategies lead to a limited anomalies opportunity available for managers, "leaving the exit door overcrowded" (Lo, 2017). This explains the sudden and quickly liquidation of large position that has led to financial panic in the Quants World.

(e) The lack of knowledge (at least prior to Quants Meltdown 2007) about the historical liquidity of U.S. equity markets (due also to lack of transparency) and how the amount of long/short equity category had increased, becoming “crowded”.

(f) The unknown size and timing of new sub-prime mortgage-related problems in credit markets, which created a climate of fear and panic, heightening the risk sensitivities of managers and investors across all markets and style categories.

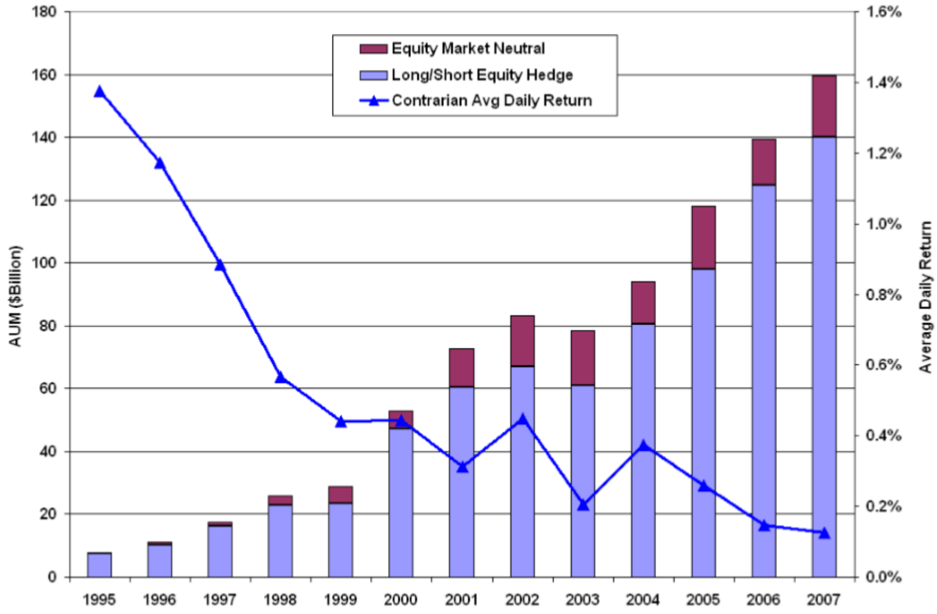
Figure 2.7 The Daily returns of The Leveraged Contrarian Strategy and most important indexes



Source: Quants in August 2007 by Khandani and Lo

From this picture it is evident how the volatility occurred for quants strategies during the second week of August was extremely high and so extraordinary. The blue columns represent the returns for Contrarian strategy (with 8:1 leverage ratio). The returns of this quantitative strategy were so abnormal also respect to indexes that are by definition more volatile, like the S&P 600 for Small Cap.

Figure 2.8 Assets Under Management in Lipper Hedge Fund Database (TASS) and the profitability of the Contrarian Trading Strategy from 1995 to 2007



This picture shows two important factors for the Quants Meltdown: on the y axis we have the Asset Under Management (in billions of $) for the hedge funds sector from 1995 to 2007, represented with the vertical bars: it is clear how it constantly increases. On the other side, the profitability of contrarian strategies, represented by the blue line has constantly decreased.

Source: Quants in August 2007 by Khandani and Lo

2.9.3 Illiquidity Exposure

We have seen how, for Long/Short Equity Hedge and Equity Market Neutral strategies, the degree of liquidity has significantly decreased over the last years; due to the previous described factors:

The huge increase in hedge funds ad assets per fund amount, and so their massive use of this specific quantitative strategies, and the consequent rapid growth of the leveraged needed to maintain positive return. For These reasons, liquidity becomes a fundamental factor in the financial market analysis, a factor that needs to be measured. The central question is how to measure illiquidity, the solution proposed by Lo and Khandani has the basis of its development on the works made by Lo (1999) and Getmansky, Lo, and Makarov (2004), Nicholas Chany, Mila Getmansky, Shane M. Haas, and Andrew W. Lo (2005).

In the 2005 paper: Systemic Risk and Hedge Funds, the authors has defined the first-order autocorrelation coefficient for fund i: $\rho_{1t,i}$, where t is the interval of time considered, for their calculations they use t equal to a month; then calculations are made using a rolling window of past returns. They reach an aggregate measure of illiquidity ρ_t^* using cross-sectional weighted average of these rolling autocorrelations:

$$\rho_t^* \equiv \sum_{i=1}^{N_t} \omega_{it}\rho_{1t,i} \tag{9}$$

ω_{it} are the weights of the single fund, calculated as the proportion of asset under management for fund i.

$$\omega_{it} \equiv \frac{\mathrm{AUM}_{it}}{\sum_{j=1}^{N_t} \mathrm{AUM}_{jt}} \tag{10}$$

where Nt is the number of funds in the sample in month t, and AUM jt is the assets under management for fund j in month t.

From these equations, Lo and Khandani computed the following equations:

$$\hat{\rho}_{1i} \equiv \frac{(T-2)^{-1}\sum_{t=2}^{T}(R_{it}-\hat{\mu}_i)(R_{it-1}-\hat{\mu}_i)}{(T-1)^{-1}\sum_{t=1}^{T}(R_{it}-\hat{\mu}_i)^2} \tag{11}$$

Where R_{it} is the return of the fund i's evaluated at time t, μ_i is defined as the average of the fund i's returns

$$\hat{\mu}_i \equiv T^{-1}\sum_{t=1}^{T} R_{it} \tag{12}$$

ρ_{1i} is the correlation factor between the return of fund i and the return obtained from the previous month. Getmansky, Lo, and Makarov (2004) followed a basic intuition: a common knowledge is that, the historical returns of residential real-estate investments are more highly autocorrelated than the S&P 500 index returns, which in turn are more autocorrelated than the returns of S&P 500 futures contracts.

The real estate market is the less liquid, (to buy an house is extremely harder and longer than buy a stock) and, the future contracts are the most liquid in previous example.

What Getmansky, Lo, and Makarov has demonstrated is intuitive; they have shown that there is a negative correlation between funds liquidity and funds autocorrelation.

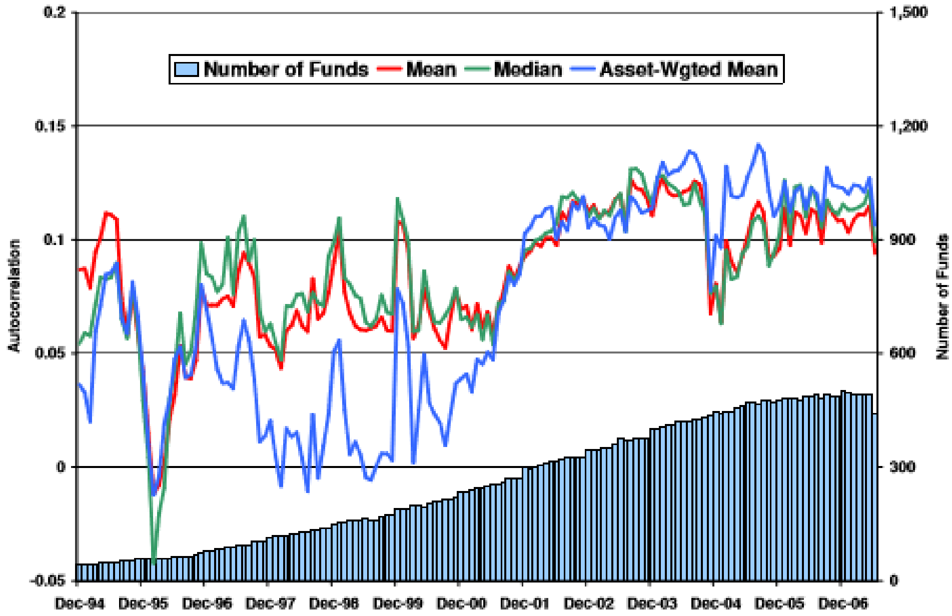
So, for great values of ρ_{1i} funds will be less liquid. In order to make evident the changes in estimated illiquidity risk of the assets, the authors suggest using a rolling window to estimate the asset's autocorrelation coefficients.

Using ρ1i as a measure of the illiquidity of each fund 'i', we can construct three aggregates measures of the illiquidity exposure of long/short equity funds along the lines of Chan et al. (2006, 2007), i.e., by computing the mean and median of rolling-window ρ1i's over all funds 'i' in the TASS Long/Short Equity Hedge and Equity Market Neutral categories month by month:

$$\begin{aligned} \hat{\rho}_{at} &\equiv \frac{1}{n}\sum_{i=1}^{n}\hat{\rho}_{1it} \quad \text{(equal-weighted mean)} \\ \hat{\rho}_{bt} &\equiv \sum_{i=1}^{n}\frac{\text{AUM}_{it}}{\sum_j \text{AUM}_{jt}}\,\hat{\rho}_{1it} \quad \text{(asset-weighted mean)} \\ \hat{\rho}_{ct} &\equiv \text{Median}(\hat{\rho}_{11t},\ldots,\hat{\rho}_{1nt}) \end{aligned} \tag{13}$$

For individual hedge-find returns Lo and Khandani have calculated the equal-weighted and asset-weighted means and the median of 60-month rolling-window autocorrelations; the results are shown in the next figure 2.9, where the data are collected from December 1994 to June 2007 using the TASS database. All funds in the two equity categories, that report asset under management in US dollars has been considered.

Figure 2.9 Mean, Median and Asset-Weighted 60-month Rolling Autocorrelations for TASS Long/Short Equity Hedge and Equity Market Neutral Funds, December 1994 to June 2007



Source: Quants in August 2007 by Khandani and Lo

The graphical result suggests how for all the three-series computed, the global performance is similar, since the beginning of 2000, they raise continuously, with an exception of a short decline on the late 2004. An increase in aggregate autocorrelation of Long/Short Equity Hedge and Equity Market

Neutral, imply a significant decline in the liquidity of this sector. Specifically, Lo has calculated, under the assumption of cross-sectional independently and identically distributed autocorrelations the approximate standard error for the equal-weighted mean of 400 60-month rolling autocorrelations, which is 0.65%, implying an high statistical significance for the levels of autocorrelations. This represent another indicator of the raise of systemic risk in the hedge fund industry. However, it is necessary recall that the absolute level of illiquidity exposure in Long/Short Equity Hedge and Equity Market Neutral Funds, is generally lower than many other categories as for example, Convertible Arbitrage[58] or Emerging Markets[59]. (Lo and Khandani, 2007).

[58] Convertible Arbitrage is a strategy in which the investor tries to generate profits from the convertible securities of a company. The investor establishes a position in which typically buy a fixed income security (a convertible bond for example) and short-sell the stock of the same company in order to hedge from market volatility

[59] This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to

2.9.4 Conclusions

The financial markets are a very complex system, that is always evolving, becoming larger, faster, more connected, and technologically advanced, for these reason new variables must be considered into the financial models which turn to adapt to the changes of the system.
For example, considering only a single measure of systemic risk wold lead to exclude from risk management, some dangerous threats of the markets, (as just seen in “Quants Meltdown”). Lo proposes to consider as alternative, a collection of measures, each one would be specifically designed to catch a specific risk exposure of the whole financial system, these characteristics could be: ▪ Leverage ▪ Liquidity ▪ Correlation ▪ Concentration ▪ Sensitivities ▪ Connectedness.
This is clearly an idea, that, if the “Unwind Hypothesis” will be confirmed need to be explored deeply and structured in a mathematical model. The purpose by which I choose to write about Quants Meltdown is not specifically a new model of hedge fund’s risk exposure or to criticize the quantitative strategies. The central point is the evolution of the financial system and its instruments, the Quants Meltdown, despite its short duration, has been an important event for several reasons and above all it is a very good example of how Adaptive Market Hypothesis work. Briefly I re-write the history of this event under a different point of view. Before Quants Meltdown, the hedge fund industry was growing so fast that it was hard to understand all its instruments and implications. The number of hedge fund, the volume of stocks traded by them, the leverage used, the quantitative strategies correlation are all factors that were growing, but, in parallel there was not enough information about hedge fund. Here it is necessary tell few information about hedge fund culture, the hedge fund culture is a culture of secrecy and it is almost impossible to know what happen in their management. Hedge funds “live” in a lack of transparency and they are largely unregulated. Nevertheless, they cover a fundamental role in the financial environment, both for their volume of stocks traded, both for the people employed for them. So, going back to our Quants Meltdown summary, when it happens in that specific days of August 2007, only few people understand what was going on, the most people did not recognize any signal of a possible crisis. After this event things do not change a lot, but, as Lo states it takes a lot of time to make deep change in a large structure as financial market. The very important aspect of this fast crisis is that from it the financial system has started to interrogate on what it should be changed in order to avoid this phenomenon. This is the Adaptive Market Hypothesis doing practically his work, I have summarized the most important

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hedge, emerging market investing often employs a long-only strategy. (This description is taken directly from the CS/Tremont website: www.hedgeindex.com).

changes and improvements that have been discussed during these years[60] (from 2007 to 2017). This summary talks about factors that are in common with the most famous sub-prime mortgage crisis happened just one year later than Quants Meltdown. My personal intent is to show what effectively AMH means and so the primary aspect is to see this process of adaptation: from where it has its origin[61] and how it develops. Since the world of finance is high correlated it is too hard to distinguish this process for a single topic, as Quant Meltdown. However, in the next paragraph I will present a short description of the crisis of 2008.

- Risk[62] are one of the most interesting challenge in every field since its importance is recognized to be crucial. Economic and Financial world is not different, rather, risk, its management and measurement represent a daily challenge. Events as Quants Meltdown, but not only, have highlighted the possibility that the actual definition of risk and its measure are not appropriate. AMH seems to well work here: the evolution of hedge fund industry has led to a state in which the risk measures are no more able to capture all the important factor of risk. It seems that risk needs to evolve in order to effectively avoid or at least detect a financial crisis. Related to the risk, we know, there are a lot of behavioural factor that man develop and so also our behavioural sensibility to the risk become no more appropriate. In other word, also our decision-making process, when facing risk decisions could lead us to the wrong decision. Also if, as Quants Meltdown has showed, we relate on quantitative methods we are not out of risk: Hedge Fund Managers did not recognize some form of risk in contrarian strategy, even though, each year, their leverage exposure needed to increase. A possible evolution on risk factor could be to operate a priori, establishing new risk factors and new risk measures in a formal way, but also sufficiently practical to be used by policymakers and by the public. Another important aspect is that, as Lo states[63]: "Such measures may require hedge funds and other parts of the shadow banking system to provide more transparency on a confidential basis to regulators, e.g., information regarding their assets under management, leverage, liquidity, counterparties, and holdings".

[60] Basically, they are some measures of innovation proposed by Andrew Lo and other researchers in order to improve the financial system and to avoid inefficiencies that could lead to a crisis.
[61] As example, for the Quants Meltdown it could recognized as the origin of the process of the adaptation all the factors described precedingly that led to the "Unwind Hypothesis" and the consequent illiquidity of hedge funds.
[62] For risk, in this case, I intend all the related aspect of this big topic.
[63] Hedge Funds, Systemic Risk, and the Financial Crisis of 2007–2008 Written Testimony of Andrew W. Lo∗ Prepared for the U.S. House of Representatives Committee on Oversight and Government Reform November 13, 2008 Hearing on Hedge Funds

- As already written, for the hedge fund industry public information available are very few, and this represent a concrete obstacle. To detect a possible crisis, it is needed to have a complete scenario with all the critical information. In the Quants Meltdown for example the public information was so few that before to provide a possible explanation, a lot of time has passed. Unfortunately, this lack is not restricted only to hedge fund sector, but it is spread (in moderate measure) all over the financial markets. A recent and well-known example is the subprime mortgages crisis were a lot of information about the value of mortgages were hide or false. When information remains private, only few people can take advantage of them, but the history has shown that for all the other people this limit represents a unrewarded risk. It seems that, to become more efficient and rational, financial system has to be more transparent, providing the public with more information[64] about the real state of art. A possible accomplishment of this purpose is through the establishment of an independent investigatory agency or department. patterned after the National Transportation Safety Board, e.g., a "Capital Markets Safety Board", in which a dedicated and experienced team of forensic accountants, lawyers, and financial engineers sift through the wreckage of every failed financial institution and produces a publicly available report documenting the details of each failure and providing recommendations for avoiding such fates in the future
- A critical part of any crisis management protocol is to establish clear and regular lines of communication with the public, and a dedicated inter-agency team of public relations professionals should be formed for this express purpose, possibly within the Capital Markets Safety Board
- Current accounting methods, for this case I refer to U.S. GAAP[65], but we can extend the concept to the global accounting measures, are backward-looking by definition. This imply that they are not ideally suitable for yield risk transparency. accounting measures are the primary inputs to corporate decisions and regulatory requirements. In order to measure and manage systemic risk efficiently, there is the need for the development and the implementation of a new sector of accounting: "risk accounting". Accounting measures and methods are the driven for regulatory requirements and corporate decisions. Without an effective risk accounting method, it won't be possible

[64] For example, other than previously cited, financial system should provide information on institutions that have failed or risk to fail.
[65] General accepted accounting principles The common set of *accounting* principles, standards and procedures that companies use to compile their financial statements. *GAAP* are a combination of authoritative standards (set by policy boards) and simply the commonly accepted ways of recording and reporting *accounting* information

to detect and prevent events as Quants Meltdown, or generally financial crisis anomalies or fraud.

- The industries of the technologic sector, especially the ones focused on it, face a risk strictly correlated with the degree of technological innovation. New technologies need to be employed wisely, but sometimes our ability is not enough to understand the correct use of them, causing bad consequences on the specific sector. Thinking at the financial technology sector the precedent statement is very important, if we don't understand instruments as subprime mortgages or CDO's for example, the finance will be lead to a non-optimal environment. For this reason, Lo suggests to governments to invest more in education and information of the financial technology sector.
- The capacity of regulators to make progress at the same rate of innovations is fundamental for an efficient management of financial markets complexity. When the actual regulatory bodies were first created, several financial products and technological innovations were not considered. New regulations should be adaptive and focused on financial functions rather than institutions, making them more flexible and dynamic. An example of an adaptive regulation is a requirement to standardize an OTC contract and create an organized exchange for it whenever its size—as measured by open interest, trading volume, or notional exposure—exceeds a certain threshold

2.9.5 A Network View of the Financial System

The financial framework has become very complex, I have already discussed about how the technologic revolution has literally changed the financial markets, its instruments and its operators, also the Quants Meltdown could be considered as an empirical consequence of this complexity.

Complexity, itself does not mean "bad" or, it should not lead to a financial crisis, in the sense that, the increase in the degree of complexity has certainly created new opportunities but also new threats which after enough time has become real damages for investors, the LTCM failure or the Quants Meltdown are clear examples. Under Adaptive Market Hypothesis complexity assumes a little different meaning: for Andrew Lo, complexity means that the narrative is not good enough for the system. Concretely, as financial system becomes more complex, for investors becomes harder to understand and manage it, making the best decisions: In the Quants Meltdown, the narrative adopted by quantitative hedge fund manager was not appropriate, since they have not enough information to understand the liquidity spiral, and their decisions to continuously raise the leverage ratio was not the best response to the environment. Complexity can be reduced through a deep knowledge of the system's structure, otherwise only the few people who are in possession of the complete information can manage the situation properly, but this "elites" often will act abusing of this knowledge, (think at those people who knew how to price mortgage-backed securities or the credit default swaps before the sub-prime mortgages crisis in 2008).

One example of information that can help us to understand the structure of a financial system is the study of the connectedness in the financial products industries.

I have written how this is a crucial factor for the systemic risk in hedge fund industries. Generally speaking, to better identify and measure the systemic risk in a financial system, the concept of network model is developed and applied to the specific systems.

Networks can give a whole picture of a system, focusing no more on the specific parts, but on the linkages between each other. In financial system is the same, and they can be very helpful to extract some precious information about risk, since, focusing on a specific part, or better, a specific dot of the financial network can provide information about its systematic risk, but for idiosyncratic risk it is necessary to identify the linkages between financial institutions.

Professor Andrew Lo, Monica Billio and Loriana Pelizzon (Ca'Foscari University), Mila Getmansky from University of Massachusetts, in their work: "Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors "have suggested several econometric measures of connectedness between financial products and institutions. Lo and colleagues in their research affirm that complexity is an inevitable product of competition and economic growth, and one direct consequence is more interdependence between financial institutions.

What this research or related studies can be useful for? at the state of art there is no certainty about it, but the possibility to measure properly the degree of linkage of financial system and to establish a better measure of idiosyncratic risk could be extremely important as an alarm detection for a financial crisis, a sort of "early warning system" (Lo 2017).

The concept of systemic risk has always been related with bank or currency crises, but, especially after the financial crisis of 2007-2009 it has started to be applied to other "dots" of the financial network, since, by definition systemic risk spreads around the entire financial system. Institutions as commercial paper, money market funds, repurchase agreements, consumer finance and OTC derivatives markets are interconnected, and shocks can propagate through them in particular periods of financial crisis, so, the business relationship would become a way by which, factors as illiquidity, insolvency and losses will expands in the system.

Lo, Pelizzon Billio and Getmansky have divided the financial system into four major categories: banks, broker-dealers, insurers and hedge funds, then for each category they have analysed the monthly returns of the twenty-five largest financial institutions. Every single monthly return calculated has been compared to the other ninety-nine and for all the almost ten thousand possible interconnections, only the returns that have shown strong statistically significant correlations has been considered. To measure these correlations and so the degree of connectedness, the authors have used two econometric methods: the principal components analysis, for estimate the common factors (their number and their importance) that are the drivers of the financial institutions returns; the Granger-causality tests to define the Granger-causal relations between these institutions. The conclusions of their empirical research are important and consistent with the idea of network model and systemic risk here presented. They indeed have shown how the interdependence between the four largest categories has raised during the last decades. The econometrical methods employed for the quantifications of these correlations have detected the intricate net of statistical relations among individual companies in finance and insurance industries. As consequence, the results obtained could be used to capture specific features of finance and insurance sectors. These sectors were the protagonist of the recent sub-prime financial crises, and the empirical results of this research show how these two sectors are the more important sources of connectedness than the others. These factors lead to higher systemic risk, together with the illiquidity of banks and insurance assets which cause a difficult management of rapid and large losses of values. A graphical example of how the interconnection between the four financial groups change in the last decade is presented in Figure 2.10 and 2.11[66]

The nodal point of the discussion is that the complexity of the markets has two, strong aspects. One is negative, and it refers to the inability of investors to fully understand the environment in which they are, leading so to possible wrong choices. A clear example is what happen in last subprime mortgages crisis, where almost nobody fully understands the subprime mortgages instruments and their consequences on the environment. Often the knowledge of the innovative financial instruments, and their consequences is a precious treasure that only few of us own. The financial networks represent a clear example of this: it is a recent feature of markets, it is hard to understand and only few people can take advantage of them. The opposite aspect of

[66] The two figures are the result of complex statistical methods, which details are not useful for the aim of this thesis. So, in order to give the complete information, the network diagram represents the linear Granger-causality relationship that are statistically significant at the 5% level among the monthly returns of the 25 largest banks, brokers/dealer, insurers and hedge funds. The data are computed for two different periods: 1996-1998 and 2006-2008.

complexity is that it can positive, since it can represent a vector for evolution. Financial networks if completely understood and managed can be a valid instrument for the detection of a financial crisis. Every innovation in the financial market has potentially positive insights, the key trade-off is that we need to adapt and evolve according to environment and this is not immediate and effortless. We have to invest time and resources to understand the market evolution, we need to reach a good heuristic through trials and errors. Under this view events as financial crisis can be seen as a way to improve our economic/financial behaviour understanding the mistakes made and make a step forward, evolving to a more efficient condition. This is the process of evolution inner to financial markets, quantitative hedge funds, financial network are just few examples of the countless innovative aspects that are changing the finance environment. The complexity of finance so is one aspect that is perfectly explained by the AMH, with suggest a possible key of understanding and overall of preventing of the reasons why some anomalies happen in the market. Next paragraph will focus on how man can exploit the future possibilities of this theory and how, at the end of all the discussion, under AMH it is the relation between us versus the environment the most powerful engine for the future.

Figure 2.10 Network Diagram for the monthly returns of the twenty-five largest (in terms of average assets under management) banks, brokers/dealers, insurers and hedge funds over January 1994 to December 1996. Colours indicates the type of institution that cause the connectedness. Green is for broker/dealers, red for hedge funds, black for insurers and blue for banks.

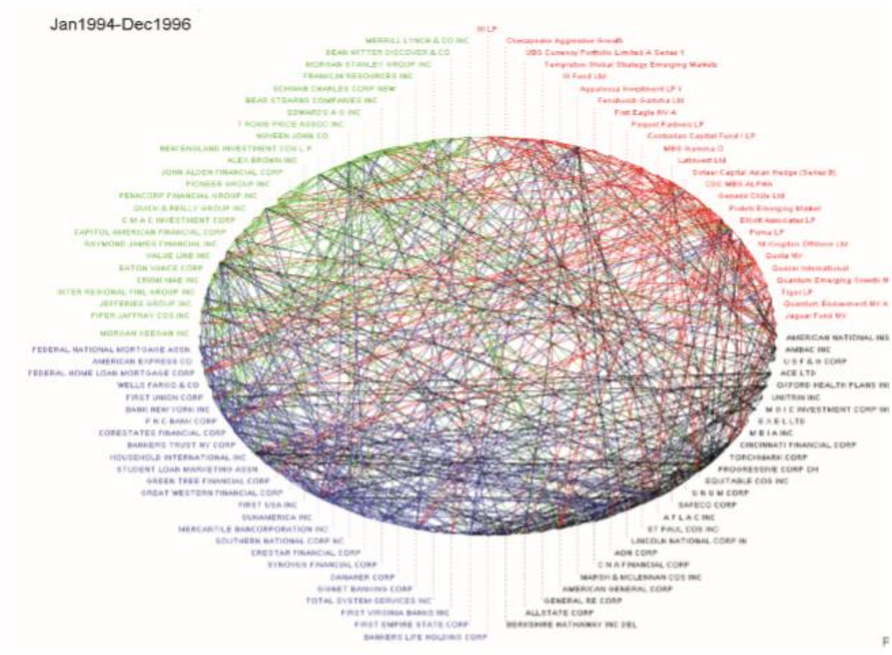
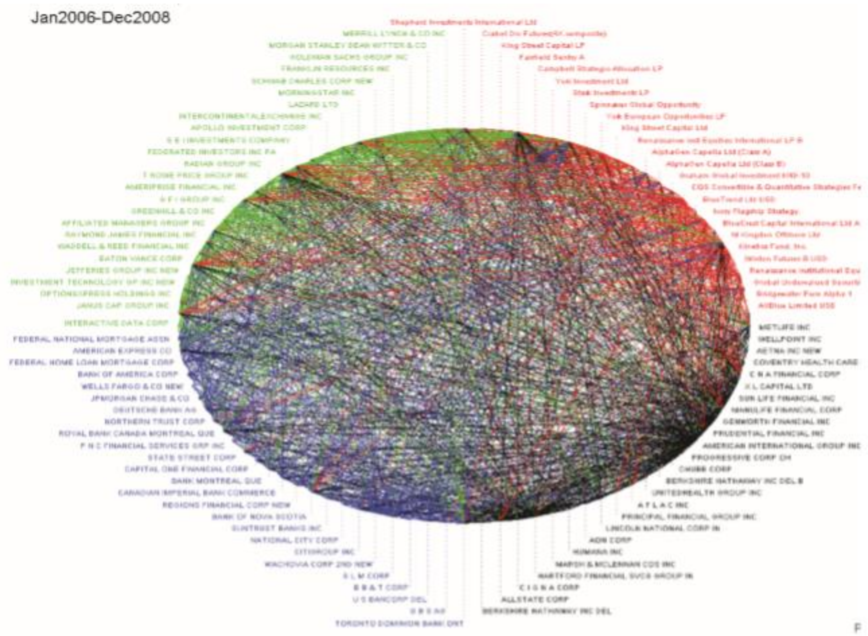


Figure 2.11 Network Diagram for the monthly returns of the twenty-five largest (in terms of average assets under management) banks, brokers/dealers, insurers and hedge funds over January 2006 to December 2008. Colours indicates the type of institution that cause the connectedness. Green is for broker/dealers, red for hedge funds, black for insurers and blue for banks.



The source of the figures is the paper "Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors "by Lo, Getmansky, Billio, Pellizzon.

2.9.6 Conclusion:The problem is us

Everybody knows about financial crisis, there are people who are expert about them, people who simply hear about it, but a financial crisis has always a big impact in our mind.
Financial crisis of 2008 has been subject to so many discussions, analysis, studies; there has been a moment during while media, newspapers talks about it every day and inside people were born feelings of fear, greed. When a crisis became known, a spontaneous question arises, why there is crisis? Where does it comes from? People of different ages, educations try to find a solution listen to politicians and economist, choosing a tendency or another. If, after years, the factors involved in the cause of crisis are well defined, it remains open the debate about how to predict a financial crisis and how to avoid it. For these reasons, every financial theory should "test" itself against a financial crisis and see if it has something useful to say about it, otherwise, it could be not taken seriously, so I present Lo explanation about what Adaptive Market

Hypothesis can tell us about the origin and nature of financial crises. Lo defines a starting point for this examination:

"the financial system isn't a physical or mechanical system, but an ecosystem, a collection of interdependent species all struggling for survival and reproductive success in an ever-changing environment."

The direct consequence of Lo words is that we should change our way to approach the study of financial crisis, that means we should not focus on the immediate and most evident causes of the crisis, i.e. subprime mortgages, under-capitalized banks, securitization and liquidity spirals. What we should analyzed is based on concepts of AMH, so it is the environment, it is the behavior and how the two interact over time, our subjects of study.

The conduct of study that Lo suggest, is to obtain all the information about the environment that led to the crisis; to document the behaviors of the many "species" in the ecosystem, so banks, hedge funds, insurance companies, regulators, legislators and investors, so again: environment and behavior of its inhabitant.

Another important aspect of the AMH is intrinsic in its definition. This theory it is not only a classical definition of theorem plus a demonstration with the relative empirical studies. It is something more, it represents a bundle of concepts and events that if put together with the right mix can lead to a better future for finance, and so for the entire society. These words seem too idealistic, but, after all ideas are the most powerful engine of the world. Re-connecting with the idea that financial markets are a product of human interactions and that we can't perfectly rational it is logical to think that there is always something that we can do better. The key aspect is that the process of evolution needs to be driven toward a positive adaptation for all the mans. Using a less philosophical language I explain why AMH can be more than a simple theory. I have already explained how finance is following a technological revolution that has totally changed our life, and how this revolution can have some negative aspects. Man is trying to adapt itself to this "new" environment but it is clear that this is process is long and that there are some unexpected events. Financial crisis is one example, but it is not the only one. Basically, people invest their money with desire to become richer, but our environment is saying to us to be careful about the way we try to do that. Finance is not fair, its culture is often based on "greed", one of the most famous movie on finance, perhaps the number one, "Wall Street" is based on the figure of Gordon Gekko, an unscrupulousness financier which strong believe that "greed" is good. The movie has been directed in 1987, after thirty years how much is finance culture changed? Maybe not so much but AMH tell us that also culture is subject to a process of evolution, mutation, selection as a mental narrative. Culture is very important also in the

Finance sector, because it has a large influence of the context in which man act. One of the most famous fraud in Finance history was the decades long, multi-billion-dollar Bernie Madoff one. And it is a classic example of how a bad culture can influence the environment (or a part of it). Madoff cheated a lot of people with a giant Ponzi scheme. He has the ability to make people, especially potential investor to trust him, with no distinctions between friends, mentors and charities. This system is called Ponzi as his inventor, an Italian immigrate that, at the beginning of 1900s developed and applied it for first, promising fraudulently higher profits to investors. He paid the interests accrued using the money of the new investors. The same concept has been applied by Madoff, which guaranteed constant annual profits despite the market's trend. Madoff did not invest actually the money from investors, only a very small portion of them, and he justified the returns of his investments with false documents. Madoff was arrested in December of 2008, when his investors start to demand more refunds than the Madoff's liquidity from new investors; it has been calculated that the value of the fraud was about 65 billion dollars. This story tells us two important aspects: the first one is that culture is not only an innate result of the tendencies by its members, rather it's the outcome of the iteration of its members with the environment id highly peculiar contexts. Finance so, is not fair or better, Finance can be fair but since the culture, the values of man are different each other's, it is impossible to have only fair investors. For this reason, the natural selection, subject to man intervention through authorities control and regulators, must be erase these bad behaviors from the market. We must create a control system which prevent the "reproductive success" of Gordon Gekko's culture or Madoff fraud. The other aspect of the story indeed is the inadequacy of authorities responsible for the legality of the investments. This aspect is a warning for the future, for the need of more efficient controls in order to fix finance.

3) An experimental application of the" binary choice" evolutionistic model

3.1 Introduction

In this chapter I present a personal application of an evolutionistic model, elaborated through a calculus software: Matlab[67]. As reference model, I have been inspired by Lo and Brennan binary choice model[68]. I have already presented the framework and how the authors developed this model, it is also necessary to highlights that the aim of their model is not to give forecast or prediction about future data time series, but instead it is a descriptive model. So, the model has the purpose to give a description of the dynamics that occur in a specific environment. Where, for specific I mean that the environment is subject to well-established characteristics: the parameters, the analytical constraints and the mathematical formulas used by the authors. In the evolutionistic model the individuals of a population have to make choice that determines, depending on weather trend, reproductive success. Supposing to view this context in a financial market perspective, it is possible to develop some interesting interpretation. The choice between two alternatives can be thought as a choice between deafferents financial instruments or portfolio on which invest or the choice between investing strategies. In the context of a financial market, the single investor and large investment group, all the market operators in general, have to make investment choices between a very large number of financial products. For each of these assets it is possible to study the correlation with other general market variables, so the market and sector indexes, interest rates, commodities price, foreign exchange. The following considerations are referred to the most interesting features of the model and to its experimental application in order to understand the financial markets framework of the evolutionistic model "under" the AMH point of view. The distinctive features are the: randomness of the environment and the choice that has to be made by the population. Remind that in Brennan-Lo model these two features are represented respectively by the weather trend and by the choice between two possibilities about where to live (choice made by the individuals). To start the interpretative description of the evolutionary model in a financial markets perspective I describe

[67] Matlab is an acronym for Matrix Laboratory, it is a numerical computing environment written in its own programming language.
[68] Described in "An Evolutionary Model of Bounded Rationality and Intelligence" by Thomas J. Brennan and Andrew W. Lo (2012).

the features of the environmental context in which the evolutionistic simulation is conducted. One of the main theory of the adaptive market hypothesis, as explained in previous chapter, is that the decision strategies and the behaviour adopted by the market operators are heavily influenced by the context in which they apply. Since market operators are not perfectly rationals, and they haven't the full features of homo economicus, they can make "not efficient"[69] decisions when facing a variable context as financial market is. Variability is a fundamental parameter for the model to be developed. When performing a simulation, the idea is to understand the consequences of this instability on the financial market. Remember how Brennan-lo express variability through a real parametrical variable: the weather trend: it is a random element of the model and it determinates the output of each generation, according to the decision of individuals about where to live (valley or plateau). I have considered the volatility of the market as the parameter on which define the different possibilities of outcomes. Market's volatility for this model has to be considered as the sum of all the factors that lead to the instability of the financial markets, making its operations and dynamics generally not efficient. Volatility is associated to the fluctuations of the financial instruments prices, it defines the variations of the price in a specified temporal interval, the more the prices variate, the higher is the volatility. I have applied this concept into the model considering the trend of financial market in the same way that Brennan-Lo consider weather trend: market trend can have two possible outcomes: positive or negative. The determination of the output for each generation[70] is random with a probability of a positive market-trend that recall the probability of a sunny day in the original one, but with some differences that I'm going to explain further. As introduced before, the other fundamental feature of the original model is the choice made by each generation's individuals. It is implemented through the choice about how to divide the capital. Investor's capital, which represents the entire "population" have two choices: be used for buying stocks or be used for buying fixed assets. The capital is considered exactly as population, so we consider theoretically as if each unit of capital, i.e. 1 dollar, must choose between "being invested" in stocks or fixed assets. This process simulates the Brennan-Lo model, where each individual of the population makes a choice about where to live. Considering the real framework, it is obvious that it is an investor that decides how much of its capital to invest in

[69] I use the term "not efficient" relating to the concepts expressed in the previous chapter, so for not efficient I means a decision that result not to be the best in the context in which it is applies. It would be not correct use the term "wrong" since a decision could still be correct despite the fact it is not the best one. See the concept of satisfying expressed in previous chapter.
[70] Generation is the correct word to be used in Brennan-Lo model, for this model it would be more correct to say: "for each time -interval". This because we are considering an investment that each time-interval, as defined, accrues some return. This part will be discussed deeply in next paragraph.

stocks or fixed assets. The result is so a portfolio in which, at each new “generation” the entire amount of money is invested between stocks and fixed in a defined proportion. The choice made by the investor lead to different payoffs, depending upon the output of the variable in the model (the market trend). choice has some payoffs, exactly as in Brennan-Lo model choose to live in valley or in the plateau could lead to different payoffs according to the weather trend.

3.2 Model analytical description

In this paragraph I explain how I develop the model and the choices made in order to build an original application of Brennan-Lo’s model.

First of all, I have considered a basic model for the simulation of an evolution process applied to financial market, then I have developed different models with less assumptions and more realistic.

The reason for this choice is that my purpose is to investigate how the Brennan-Lo model can be applied to financial market and which conclusions are possible to define. Starting from a basic model it is necessary to extrapolate the core concepts and compare them to the basic idea of Adaptive Market Hypothesis. The next step is to introduce more conditions and variables to the model, in order to make it more complex but also more specific, with the aim to obtain other precious insights.

Before to introduce the analytical description of the model I have implemented I present an analytical summary of Brennan-Lo one. Despite its apparently simplicity Brennan-Lo model has a complex structure of quantitative formulas and data simulations. The authors have considered four cases for the model. Each case refers to the same binary choice model but with different hypothesis:

- The first case describes the general evolution of irrational individuals which actions are taken dependently from the others belonging to the same generation.
- The second case eliminate the specific reproductive risk since it hypothesizes that individuals act independently
- The third one introduces the concept of evolutionary intelligence
- The fourth add the existence of costs for evolutionary intelligence.

Considering the first case, we know[71] that under the following hypothesis: random variables x_{at} x_{bt} are identically and independently distributed from a generation to another, between the individuals of the same generation t and also independent from other random variables for every choice probability f and individuals i. Remind that the decisional process of the individual I is described by a Bernoulli variable with distribution I_{it}^{f} that assumes value 1 when the individual choices a with probability f and 0 when he choices b with probability 1 – f.

The formula for the number of individuals at generation t (for population type f) is the (10):

$$n_t(f) = x_{at} \sum_{i=1}^{n_{t-1}(f)} I_{it}^{f} + x_{bt} \sum_{i=1}^{n_{t-1}(f)} \left(1 - I_{it}^{f}\right) \qquad (1)$$

Increasing the generations t, we can apply the law of large numbers: the variable α(f), which represents the geometric growth rate[72] converge for every value of 'f' to the following limit:

$$\underset{t\to\infty}{plim}\ log\left[\frac{n_t(f)}{t}\right] = \alpha(f) = E[\log(f^*x_a + (1\text{-}f)^*x_b)] \qquad (2)$$

Following the law of large numbers, if a sample size tends to infinite ($t \to \infty$), random variables are IID and have the same probability distribution, the distribution average ($\frac{n_t(f)}{t}$) is next to the real expected value. The aim of the model is to determine analytically the optimal value for the behavioural phenotype of the population, the one for which individuals survive to natural selection without extinguish and increasing the population size. Every population indeed, aims to maximize the reproductive success, so we can describe it through a growth parameter, subject to maximization.

In order to obtain the optimal value f* we need to maximize population growth rate α(f), described in (3) respect to f. We have to differentiate three conditions on the base of the expected ratios between the offspring sizes: $E[x_a/x_b]$ and $E[x_b/x_a]$, which determine the reproductive success (on unsuccess) of choice a over choice b. Maximizing α(f) we obtain:

$$\alpha'(f) = d\alpha(f)/df = E[(x_a\text{-}x_b)/(f^*x_a + (1\text{-}f^*)x_b)] = 0 \qquad (3)$$

and solving this equation we have:

1) $f^*=1$ for $E[x_a/x_b] > 1$ and $E[x_b/x_a] < 1$
2) $0<f^*<1$ for $E[x_a/x_b] \geq 1$ and $E[x_b/x_a] \leq 1$ (solution to (3))
3) $f^*=0$ for $E[x_a/x_b] < 1$ and $E[x_b/x_a] > 1$

where the expectations in the precedent formulas are respect to the joint distribution $\Phi(x_a,x_b)$.

[71] See paragraph 2.6

[72] Geometric growth rate for a population [0;t] is defined as the square root of the ratio between population at generation t and population at generation t_0, minus 1: $\sqrt[t]{\frac{n(t)}{n(1)}} - 1$

The first part of solution, 1) f*= 1 if $E[x_a/x_b] > 1$ and $E[x_b/x_a] < 1$ implies that for population, choice ‘a’ determine a reproductive success higher than choice ‘b’ since x_a, the offspring size for choice a is greater than x_b, the offspring size for choice b. Third part of solution, 3) f*=0 for $E[x_a/x_b] < 1$ and $E[x_b/x_a] > 1$ implies that the best reproductive success is reached for choice ‘b’, it guarantee the greatest number of offspring. For the second part, 2) $0 < f^* < 1$ if $E[x_a/x_b] > 1$ and $E[x_b/x_a] < 1$ we determine the optimal value of f*, which is between 0 and 1, through the resolution of equation (3). In this case neither choice between ‘a’ and ‘b’ is dominant, so the optimal behaviour f* is determined by random decision-making processes. This solution seems to be inconsistent with classical economic theory since it claims the maximization of individual wealth, through the expected utility function, which maximum is obtained for deterministic behaviour, i.e. f*= 0 or f*=1).

Under the theory of AMH, we can interpret this optimal f* as an analytical representation of the behavioural distortion of altruism, since a non-optimal action for the individual result to be optimal for the society (the population) and the entire species, allowing its survival. From these considerations I developed analytically the following exposed model.

Formalization of the model:

Considering an investor that is deciding how to build a portfolio, he has the possibility to invest in two different products: stocks and fixed asset securities[73]. The key question is how much invest in one or another. At time 0 the investor built his portfolio according to the proportion chosen, he also decides the temporal horizon of his investment. Considering time interval of one month, the investor after one interval will see the percentage of return gained (or lost) on its initial capital. I have considered as a binomial random variable the market’s trend: market can be in an up-trend or in a down-trend. The returns in this model are fixed: if the market is in up-trend the return gained on the capital invested in stocks is clearly higher thank the return gained on the capital invested in fixed assets. On the opposite, when market is in down-trend, return on stocks are negative but the returns on the fixed asset are still positive. The basic assumption here is that risk-return relationship is linear, the reward for having invested in riskier assets (stocks) is higher than reward of less risky assets (fixed assets). For the stocks returns I use as reference the idea of a stocks index: a stocks index includes the value of a basket of financial securities. The variations of an index represent a good approximation of the change in prices of stocks included in the index. This is the general context, next section goes further into details of the model, explaining the variable chosen and the assumptions made.

[73] For simplicity here we are assuming that the investor goes “long”, i.e. he buys and hold the securities. This assumption remains true for all the models proposed.

Variables adopted in the basic model:

- 'f' represents the 'theoretical' proportion of the investor's initial capital invested in stocks. It is theoretical because the exactly amount is defined through a binomial distribution where f is the parameter that express the probability of success (further explanation will be provided when explaining the random variable t). So, it is the probability of the proportion of initial capital invested in stocks, clearly 1-f represents the probability of the proportion of initial capital invested in fixed assets. Recall that for Brennan-Lo f represents the most important parameter of the model, since the model's aim is to find the optimal f* (the optimal behavioural phenotype) that maximize the number of individuals, so the best reproductive success. In this model, it is interesting to find the same optimal f* for which the investor will maximize its capital. For this simulation I have considered the following values for f ∈ [0, 0.25, 0.5, 0.75, 1] and for each of these values I have run the simulation 100 times[74]. 'f' is a random variable with the following distribution:

  $P(X=1) = f$ and $P(X=0) = 1-f$ for some $0<f<1$ is called a Bernoulli random variable and X can be written as X~Ber(f). Where X is the random variable of the experiment and P(X=1) is the probability of X to be equal to 1.
- 'T' represents the total number of time's interval considered, T=1 is assumed to be one month after the initial investment. For this model I have chosen a period of 120-month, T=120, so 10 years of investments horizon. This choice is not random, in order to evaluate a simulated evolutionistic process for financial instrument, the duration of the investment should be long enough to exhibit significant outcomes[75]. Another important reason is that, as seen in chapter 2, under AMH when investments time horizon is short-medium (five to ten years) risk-return relationship may not be linear. This prevent to choose a longer period than 10 years. For example, for T=360, thirty years, becomes improbable to test eventual anomalies in the traditional finance paradigm. But, as Lo states, the average of investors has shorter time-horizon for their investments. In addition, 10 years is a reasonable period for fixed asset products, as bonds, Treasury Bills and so on.
- 'k' is the parameter linked to the T=120 iterations.

[74] I execute the Matlab script one hundred times for each value of f, in order to calculate some statistics of the outputs.

[75]Significant outcomes

- ‘n‘ is the amount of capital invested. Initially at time T=1 it is fixed to an arbitrary value of 1000.
- ‘xs1’ is the return gained on the capital invested in stocks, after one month of positive market trend. In this basic model I choose it as a fixed value, equal for all the runs: +3,5% (0,035).
- ‘xs2’ is the return lost on the capital invested in stocks, after one month of negative market trend. Its steady value is: -4% (-0,04), it is expressed as a percentage of lost.
- ‘xf1’ is the monthly return gained on capital invested in fixed asset when market trend is positive, its value is steady for all the simulations: +0.5% (0,005).
- ‘xf2’ is the monthly return gained on capital invested fixed asset when market trend is negative, its steady value is: +0.4% (0.004).
- ‘p’ is the probability that the monthly market trend is positive. Clearly 1-p represents the probability of negative monthly market-trend.
- ‘r’ is a random variable distributed as a binomial distribution subject to the probability of success ‘p’ it determines the trend of the market: if the output is 1, market is in up-trend, if 0 market is in down-trend.
- ‘t’ is the matrix that represents the effective percentage of capital invested in stocks according to probability ‘f’. t(k) is equal to a random number generated from the binomial distribution with the following parameters: ‘n(k)’ represent the number of trials and it is equal to the capital invested for each iteration k; ‘f’ is the probability to invest in stocks. This variable ‘t’ is important in the model since it adds a certain degree of “randomness”. The proportion in which invested capital is allocated, indeed, is not perfectly constant. For example, if f is equal to 0,5 and n(1)=1000; the capital invested in stock is not surely 500, the exact amount depends upon the output of variable t(1)=binornd(1000,0.5) which is random (but subject to parameters n(k) and f). At the next iteration, for k=2 the proportion of capital invested will be different, since the output of t will be different. Clearly, t for each iteration is equal to a different amount of capital invested in stocks also because the total capital n(k) change according to the sum of returns. The reason of this choice, that does not reflect exactly the reality,[76] is based on the fact that, for this specific model my intent is to keep my assumptions similar to Brennan-Lo model in order to consider financial market very close to the idea of environment in which population evolve and change.

[76] In real markets when an investor faces the decision of how to allocate its capital, usually he don’t decide base on a random variable generated by a binomial distribution.

- 'n(k+1)' is the outcome matrix, for each iteration k (from 1 to T) it is equal to the sum of the capital invested plus the return gained or lost on it. In Matlab code the formula is so expressed:

$$n_{k+1}=t_k*(1+x_{s1})*r_k + (n_k-t_k)*(1+x_{f1})*r_k + t_k*(1+x_{s2})*(1-r_k) + (n_k-t_k)*(1+x_{f2})*(1-r_k) \tag{4}$$

when r(k)=1, market is in a positive trend, so:

n(k+1)= t(k)*(1+xs1)*r(k) + (n(k)-t(k))*(1+xf1)*r(k). This means that after one month the value of the capital invested is equal to the amount invested in stocks (t(k)) the precedent month, plus the return gained on it (xs1*t(k)), then add to this value, the amount of capital invested in fixed asset (n(k)-t(k)) plus the return gained on it (xf1*(n(k)-t(k))).

When r(k)=0, market is in a negative trend, so:

n(k+1)= t(k)*(1+xs2)*(1-(r(k))) + (n(k)-t(k))*(1+xf2)*(1-(r(k))), this means that after one month the value of capital invested is equal to the amount invested in stocks (1-t(k)) the precedent month, plus the return gained on it (xs2*t(k)), note that in this case the return is negative, stocks portfolio has lost value, so the addition will become a subtraction. Then, add to this value the amount of capital invested in fixed asset (n(k)-t(k)) plus the return gained on it (xf2*(n(k)-t(k))). This formula has been implemented starting from Brennan-Lo generalized model equation (see formula (2), cap.3):

$$n_t(f)=\sum_{i=1}^{n_{t-1}(f)}[I_{it}^f x_{at}+(1-I_{it}^f)x_{bt}] \tag{5}$$

Further annotation about the choice of variables are the following.

The values chosen for the returns xs1,xs2,xf1,xf2 are not random. To establish the most significative (for this model) values, I have calculated the average return of two indexes for the last ten years. The first index I have analysed is the FTSE MIB[77] and second is the Standard & Poor 500[78]. For the FTSE MIB I have found the data[79] relative to monthly returns from 31/01/98 until 31/01/18, so for twenty years. I have calculated the average returns for window of 10 years, obtaining so 120 outcomes, then I have calculated the average of these 120 average

[77] The FTSE MIB (acronym of Financial Times Stock Exchange Milano Indice di Borsa) is the most important stocks index of the Italian Stock Exchange. It is the basket of the forty Italian companies, listed on MTA (telematic stocks market), with highest market capitalization, floating and liquidity.

[78] Standard & Poor 500, note as S&P 500 is a stock index composed of a basket of the five-hundred U.S. companies with the highest floating capitalization. It is the most significative North-American stock index.

[79] As source of data i have used Factset, a financial data and software company.

monthly returns on 10 years window- The result is that on average, investing in stocks, replicating the FTSE MIB index, with 10 years as time-horizon, the monthly return is equal to -0.3%. I also calculated the average of positive returns during periods of ten years, starting from 1998 and it is equal at 5,11%, whereas for the negative returns, the average is equal to -5,41%. At the end I have calculated the average number of months with positive returns for ten years: it is about 50%.

At the first sight, these results seem surprising, but during this period of twenty years, several negative events has led to intervals of time in which stocks have lost significant percentage of their value. The principal negative events occurred in these 20 years is basically the global financial crisis, from which Italy has took long time to recover, then an internal political situation of instability together with Italian banks crisis have been the other principal negative influence on Italian stock market. This data collected seem consistent with the Adaptive Market Hypothesis: Italian stock market, in the last twenty years, has appeared to be far from efficiency. This is due by an evolving environment, both the global financial market, both the specific Italian one which have led to higher volatility[80], poor stocks returns, numerous anomalies.

So, for my model, in order to make a choice less specific than using average FTSE MIB ones, I repeat the same calculus also for the S&P 500 with the same interval of time and the same conditions: the average monthly returns for ten years window of time, is 0,56%. It is evident how S&P500 index has constantly grown during the last twenty years (and also before), with however, same exceptions as the fall during the 2008 financial crisis.

I also calculated the average of positive returns during periods of ten years, starting from 1998 and it is equal at 3,10%, whereas for the negative returns, the average is equal to -3,62%. At the end I have calculated the average number of months with positive returns: it is about 62%.

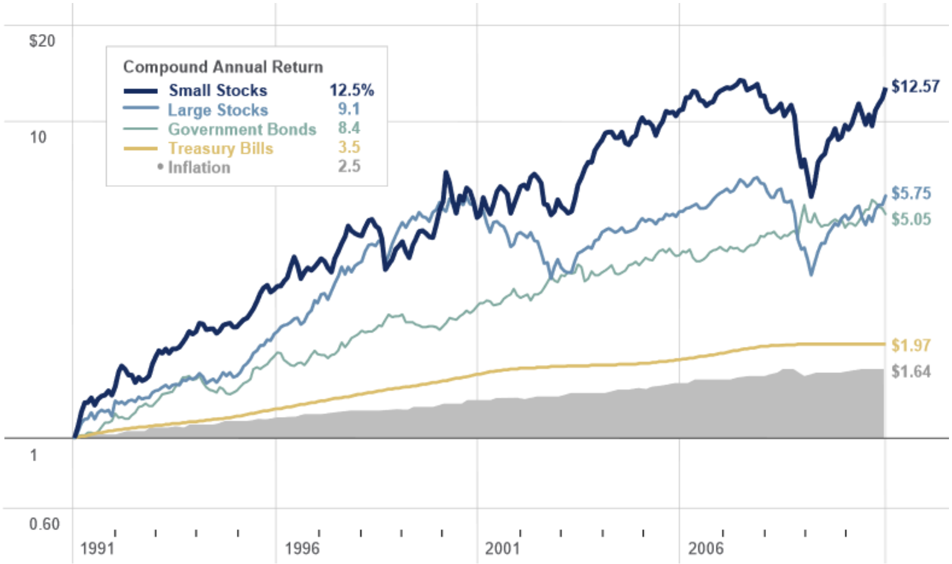
Putting together this data I have chosen to assign the following values: when market has a positive trend, the monthly return earned on the stocks is equal to: $xs1 = +3{,}5\%$, when market is negative it is equal to $xs2 = -4\%$. The percentage of positive months id equal to $p = 0.62\%$

For the returns of the fixed assets, I have used the data collected by Ibbotson and presented in Table 2.4, so when market trend is positive, the monthly return $xf1 = +0.5\%$, when it is negative: +0.4%. It is slightly lower in the negative case, since I discounted the risk of a fixed-asset default. I choose not to consider transaction or other costs and taxes on capital gain since they would be irrelevant to the purpose of this model.

[80] I have calculated a standard deviation for the 20 years period of 8980,208, a volatility on FTSE MIB prices of about 33%.

In the appendix I show the tables of the historical returns for S&P 500 and FTSE MIB with the respective calculus I have made.

Figure 3.1 Returns of a 1$ investment made in 1991 for stocks, bonds, T-bills, and inflation value in the U.S. market



Source: Created by Raymond James using Ibbotson Presentation Materials • © 2011 Morningstar

3.3 Elaborating the model

In this paragraph I will show the result obtained running the simulation for the various values of 'f'.

First of all, I present here the result for a single run of the script, in order and understand how the model works in Matlab. For this single example, I choose T, the investment time horizon, equal to 36 months, for simplicity and practicality. Whereas, for the real simulation I establish a time horizon T of 10 years, 120 months. Then, I choose 'f' equal to 0.75, recall that, this is only a demonstration, this result is not included in my data set, all the other variables are set equal to the real simulation ones. The Table 2.5 collect the data for one run of the following described process.

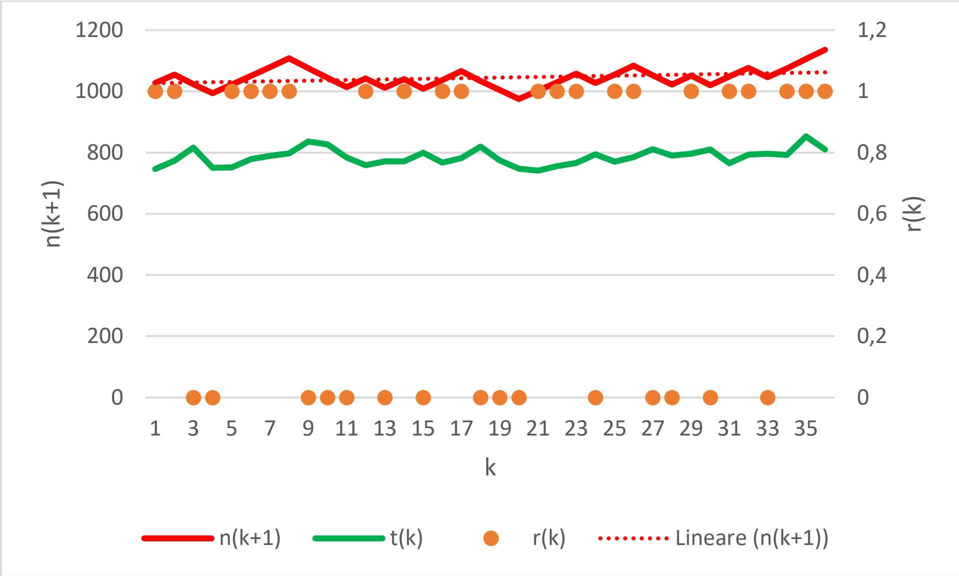
At time k=1 an investor decides to create a portfolio divided in stocks for the 75% of its initial capital and 25% in fixed assets securities. He decides to invest 1000$ and to keep the investment for 3 years (in this example). Since, for this run, 'r' has an output equal to 1, market trend is positive. The exactly amount of money invested in stocks, following the binomial distribution of random variable, is t(1)=746$, so after one month its capital n(2) is equal to:

746(1+0.035)+254(1+0.005) = 1027,38$. The amount is rounded to nearest integer, so 1027$. The capital invested in month k=2 is so, 1027$, again it is divided for the 75% in stocks and 25% in fixed assets, r(2) is equal to 1 also this time so market is positive, returns are positive and n(3)=1055. Looking at the data, it is evident how, when market is negative, for r(k)=0, the value of capital invested decrease, since the return on stocks' capital is negative. However, this effect is compensated by the greater probability of a positive market trend for one month and by the return on fixed assets, which is always positive. So, at the end of the 36 months, the investor will have 1136$ in his account. Clearly, for every run of the code the result will be different, since there are random components as the arrays 'r(k) and 't(k) which lead to slightly different results. This will be investigated in the next paragraph.

Table3.1 Variables output for one simulation of the Matlab Code, with parameters T=36 and f=0.75

k	n(k)	n(k+1)	r	t(k)	xs1	xs2	xf1	xf2
1	1000	1027	1	746	0,035	-0,04	0,005	0,004
2	1027	1055	1	773	0,035	-0,04	0,005	0,004
3	1055	1023	0	816	0,035	-0,04	0,005	0,004
4	1023	994	0	750	0,035	-0,04	0,005	0,004
5	994	1022	1	751	0,035	-0,04	0,005	0,004
6	1022	1050	1	779	0,035	-0,04	0,005	0,004
7	1050	1079	1	789	0,035	-0,04	0,005	0,004
8	1079	1108	1	797	0,035	-0,04	0,005	0,004
9	1108	1076	0	836	0,035	-0,04	0,005	0,004
10	1076	1044	0	827	0,035	-0,04	0,005	0,004
11	1044	1014	0	784	0,035	-0,04	0,005	0,004
12	1014	1042	1	759	0,035	-0,04	0,005	0,004
13	1042	1012	0	771	0,035	-0,04	0,005	0,004
14	1012	1040	1	771	0,035	-0,04	0,005	0,004
15	1040	1009	0	800	0,035	-0,04	0,005	0,004
16	1009	1037	1	767	0,035	-0,04	0,005	0,004
17	1037	1066	1	782	0,035	-0,04	0,005	0,004
18	1066	1034	0	819	0,035	-0,04	0,005	0,004
19	1034	1004	0	774	0,035	-0,04	0,005	0,004
20	1004	975	0	747	0,035	-0,04	0,005	0,004
21	975	1002	1	741	0,035	-0,04	0,005	0,004
22	1002	1030	1	756	0,035	-0,04	0,005	0,004
23	1030	1058	1	766	0,035	-0,04	0,005	0,004
24	1058	1027	0	794	0,035	-0,04	0,005	0,004
25	1027	1055	1	770	0,035	-0,04	0,005	0,004
26	1055	1084	1	785	0,035	-0,04	0,005	0,004
27	1084	1053	0	811	0,035	-0,04	0,005	0,004
28	1053	1022	0	790	0,035	-0,04	0,005	0,004
29	1022	1051	1	796	0,035	-0,04	0,005	0,004
30	1051	1020	0	810	0,035	-0,04	0,005	0,004
31	1020	1048	1	765	0,035	-0,04	0,005	0,004
32	1048	1077	1	793	0,035	-0,04	0,005	0,004
33	1077	1046	0	796	0,035	-0,04	0,005	0,004
34	1046	1075	1	792	0,035	-0,04	0,005	0,004
35	1075	1106	1	853	0,035	-0,04	0,005	0,004
36	1106	1135,83	1	810	0,035	-0,04	0,005	0,004

Figure 3.2 Values of n(k+1), r(k) and t(k), the graph is built on excel using Matlab outcomes.



3.4 Data analysis and numerical outcomes descriptions

The aim of the model proposed by Brennan-Lo is to find the optimal value of f, the one that lead to the maximum number of individuals at the last generation (the authors fixed the number of generations equal to 25). In other words, the value of f that leads to the best reproductive success of the entire population in the environment. In this model, applied to financial market, the aim is the same, since I want to investigate the influence of possible "evolutionary processes" and how these eventually could support the AMH. I search a value of f which lead to the maximum value of capital after 10 years, maximizing the profit. However, since the environment of the experiment has same peculiar characteristics the results must be interpreted taking into account these factors. In Table 3.2 I report the data relative to the simulations conducted. Given the parameters and variables defined in the previous paragraph, I have run 100 times the Matlab code for the following values of f: '0', '0,25', '0,5'. '0,75', '1'. I obtained so, 100 different values for the capital at final generation k=120, after ten years of the initial investment; then I have calculated the average of these values, showed in the second column. Third column represent the standard deviations of the 100 values of n(k+1) for each parameter f; fourth represent the maximum value obtained and the fifth, the minimum one.

Table 3.2 For every value of f it has been simulated 100 runs, from these I take: the average of n(k+1), the standard deviation, the maximum and the minimum value obtained for n(k+1) in the 100 runs; where k is equal to 120, since I consider the last month of investment.

f	Average n(k+1)	Dev.std	Max value of n(k+1)	Min value of n(k+1)
0	1734,126	10,1494	1759,755	1707,804
0,25	1839,896	217,7052	2511,352	1380,900
0,5	1929,958	432,5201	3719,520	1039,920
0,75	2090,337	712,5163	5306,232	1027,868
1	2151,396	934,5286	5188,800	682,065

From this result it seems that, contrary to Brennan-Lo model[81], the optimal value for which the investor will obtain the best profit, maximizing his capital, is f*=1, indeed I obtain that on average n(121) after 10 years is equal to 2151$.A possible reason for this result can be found the properties of the causal variables xs1, xs2, xf1, xf2, which represent the percentage of return gained or lost on stocks and fixed assets. These casual variables are identically independently distributed (IID) from a month to the other and are IID between the components of portfolio belonging to the same month k. As consequence of this property, the idiosyncratic risk, the one relative to the specific stock or fixed asset, is cancelled out through the entire portfolio. In this case the amount of capital at the end of the ten years is maximized for f*=1 if xs1>xf1 and probability p is >0,5. Nevertheless, analysing the data deeply several considerations must be explained. It is true that on average the choice of investing all the capital in stocks would generate the maximum amount of capital after ten years, but if we consider also the standard deviation, we can see how for f=1 it is the highest: 934$. This means that, over the 100 simulations the values of n(k+1), for k=120, are most spread when f=1, in other words, investing completely in stocks would lead, on average, to best profits but with the highest volatility. How is it possible since the returns considered have fixed value? The randomness introduced into the model with the arrays r(k) and t(k) could lead the amount of capital to deviate a lot. The output of r, as seen in the simple example in previous paragraph, could be 1 (or 0) several times in a row, increasing at each run the amount of money gained (or lost). Clearly, investing in fixed assets prevent from this volatility, since they have always a positive return independently

[81] They found that the optimal value of f, f* is equal to 0,75, the same value of the probability of a sunny day (see previous paragraph 2.6) since for f=1 population was being extinguished.

form market trend r. Referring again on Brennan-Lo model, f=1 (all population choose to live in the valley) was not optimal for their model since at the first rainy day, all the population was erased by the flood, stopping the reproduction process. This concept could be applied also in the financial market model. Looking at the fourth and fifth column of table 2.6 we see the in the 100 simulations run for f=1 the minimum value of the capital at T=120, n(k+1) is 682$! Whereas for the other values analysed of 'f' the minimum value of n(k+1) obtained is higher than the initial capital. For f=1, on average an investor will earn more money after ten years, but there is also the possibility to lose money on the initial capital. In this basic model, it is not possible to replicate exactly the idea of Brennan-Lo model: the extinction of the population for f=1, but it is possible to show that there is the possibility to lose money if operating am exclusive choice, as invest only in stocks. From the simulation this situation happened 6 times on 120 simulations, 5% of times. These results support the AMH: the relation risk-return is linear when environment is stable. Investing only in fixed asset is the safest choice since standard deviation is the lowest, but as investor raise the amount of money invested in stocks both average returns and standard deviation increases. Nevertheless, when extraordinary events occur, as financial crisis of 2008, taking more risk could lead to negative, unexpected consequences. The investor who has to face the choice between f=0,75 and f=1 in this model, knowing that choosing to invest only in stocks will lead, on average to an extra-profit of 61$ (2151-2090) respect to f=0,75 but with higher risk and with the probability to lose money will probably decide that the optimal choice is f=0.75. The optimal choice so, during stable period and so stable environment would be f=1 but we know that these conditions happen rarely. To understand the challenging consequences of the results think I make a comparison with Brennan-lo model. If population of that environment choose to live all in the valley for example (f=1) we know that when the first rainy day occurs, population are completely erased. Luckily when a negative market trend occurs in a month, the stocks index does not loose all its value, but typically investors are not all so patient to see if the stocks market "adapt" to new environment and evolve, recovering their precedent losses. This concept goes a little bit out of my model, since I haven't allowed "dollars" (the population) exit from the market, but to explain the model in real terms we have also to consider these aspects (which I try to deeply analyse in the next presented model). Concluding, under the AMH concept of not satisfy individual expectations instead of maximizing, optimal solution would be f=0.75. The values chosen for the variables clearly heavily influenced the model, if we choose a higher value of p, the probability of a monthly market trend, investing in stock would be always more remunerative and preferable, but we would deviate too much from reality. If instead, I would have chosen a higher value (in absolute terms) for the monthly return of stocks when market trend is negative, xs2, for example -4%, we would have observed that also on average terms it is optimal the choice of f=0.75 instead of f=1. Moreover, the choice of xs2=-4% would not be "extreme" since in the last ten years, as discussed before, for the Italian stock market the average monthly returns during negative market trend is about the -5%.

Figure 3.3 The simulations outputs for capital n(121) after 10 years of investment for the five f values tested.

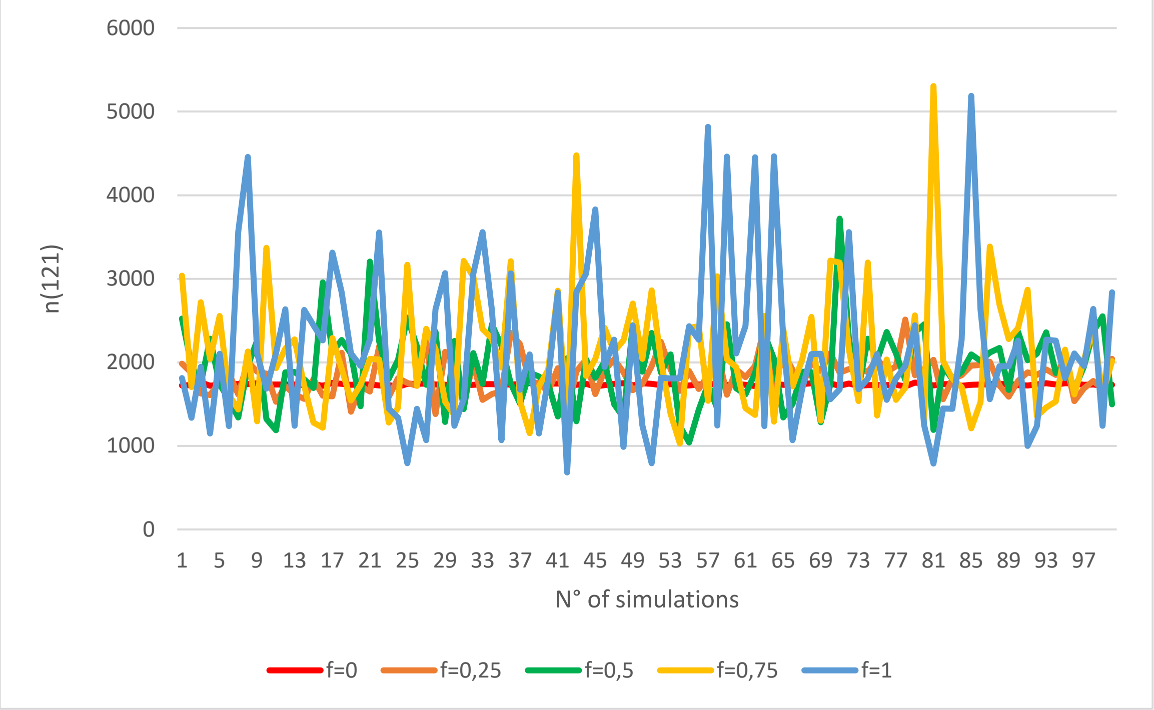
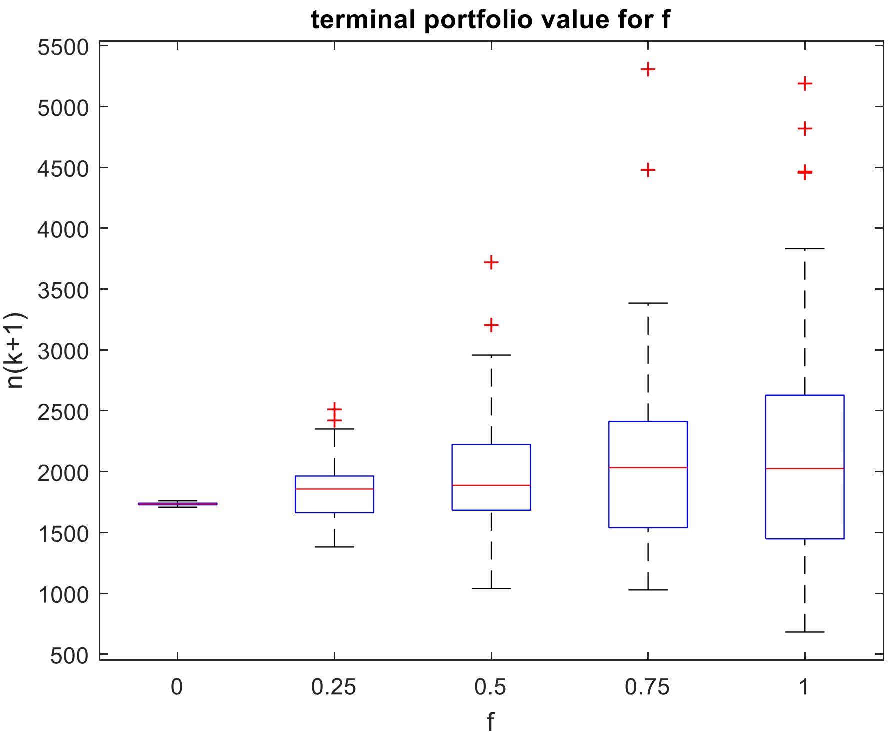


Figure 3.3 shows how for each value of f: 0; 0,25; 0,5; 0,75; 1 the 100 outputs of n(k+1) obtained running one-hundred times consequentially the Matlab Code. Figure 3.3 is very useful to better understand graphically the results. On the x-axis is represented the number of simulation, while on the y-axis it is represented the corresponding output. It is possible to see the extreme variance of the simulation's results obtained for f=1 (light blue line), which are for six times over 4000$ but also five times under 1000$ (the initial capital).

A very useful graph to better understand the outcomes is Figure 3.4 a box-plot for the terminal value of capital invested. Through Matlab I have constructed a boxplot graph in which, the variance, the median, the 25th and 75th percentiles are represented separately for each value of f. On the x-axis we have the 'f' values, on the y-axis the terminal value (at T=120) of capital invested.

Considering a single box, the central red line indicates the median, the bottom and top edges of the box indicate the 25th and 75th percentiles. The whiskers extend to the most extreme data points not considered outliers, and the outliers are plotted individually using the '+' symbol[82].The width of the box is the "interquartile range", it is equal to the third quartile minus the first one (75^{th} percentile-25^{th} percentile). Since statistics hypothesize that data points are clustered around a central value, the interval defined by "interquartile range" is used to define a measure of the dispersion of the values. Data points that are out of the box but are not considered outliers are indicated in this graph by the whiskers and they are the values that are inside the interval defined by the end of the box (upper or lower) and one and a half times[83] the length of the box (IQR). The values that are below the line of the 25^{th} percentile (lower end of the box) minus one and a half times IQR or above the line of the 75^{th} percentile (upper end of the box) plus one and a half times IQR are considered as "outliers", too far from the box interval, so, with lacking significance.

Figure 3.4 Box plot of n(k+1) for five values of f



It is evident how raising the proportion of capital invested in stocks, the width of results obtained for n(k+1) raise too, and additionally for f=1 we have the high number of values

[82] https://it.mathworks.com/help/stats/boxplot.html

[83] This factor: 1.5 has been established historically, and it is based on empirical tests and it is still considered a valid parameter.

“outliers”. This can be interpreted as: the higher volatility of stocks returns in a random environment can lead to “unrealistic” outcomes. Where the meaning of “unrealistic” is different from impossible, these outcomes indeed on average will have a minimal probability to realize. In real world we have example of stocks that constantly outperform the market, the most famous are Apple, Tesla, Amazon. Here in the simulated model we are assuming that the investor tries to reply a market index however, in the past we have had evidence of prolonged boom period for a specific financial market and its relative index.

3.5 The Kolmogorov-Smirnov test

In order to give a statistical significance to the model’s results I have conducted the Kolmogorov-Smirnov test. It is a non-parametric test based on the empirical distribution function (ECDF) which verifies the form of the data (the samples) distributions.

I have applied the specific one-sample Kolmogorov-Smirnov test, verifying if the data of n(k+1) (the matrix containing the one-hundred outcomes for the terminal value of portfolio) come from a standard normal distribution.

The test is defined as follow:

H0: the data samples come from a standard normal distribution

H1: the data samples do not come from a standard normal distribution.

The test statistic is the maximum absolute difference between the empirical cumulative distribution function from the sample data, matrix n(k+1) in our case and the hypothesized cumulative distribution function (in our case we test the normal distribution): $D^* = \max_{x} (|F^*(x) - G(x)|)$, where $F^*(x)$ is the empirical cumulative distribution function and $G(x)$ is the cdf od the hypothesized distribution. In our model we can write: $D^* = \max_{x} (|F^*(n(k+1)|)) - G(n(k+1)|)$, for k=120 where G is the cdf of the standard normal distribution.

The significance level is: $\alpha = 5\%$

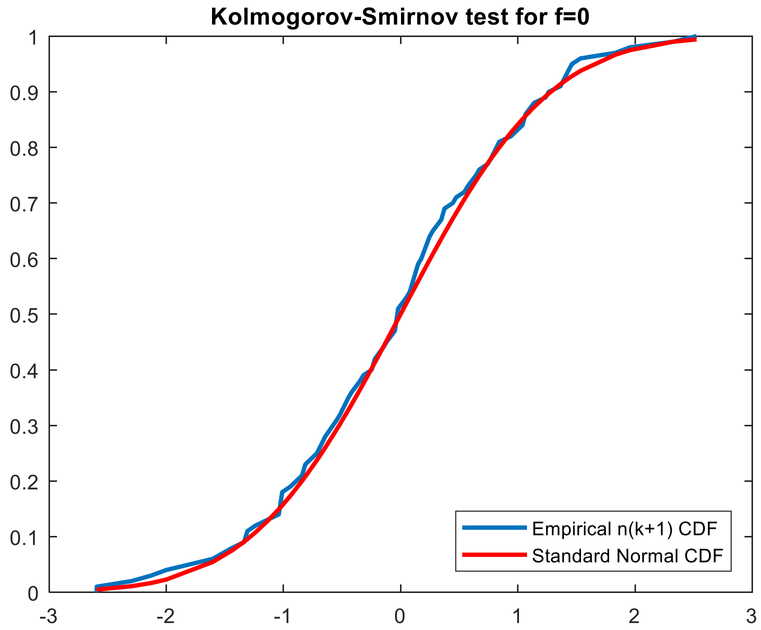
The critical value: the hypothesis about the distributional form of data samples is rejected if the test statistic, D^*, is greater than the critical value, which is obtained by interpolation from a table of values used as reference.

I have implemented the test for each of the five values of ‘f’ through the Matlab software.

Case f=0

Result: hzero = 0

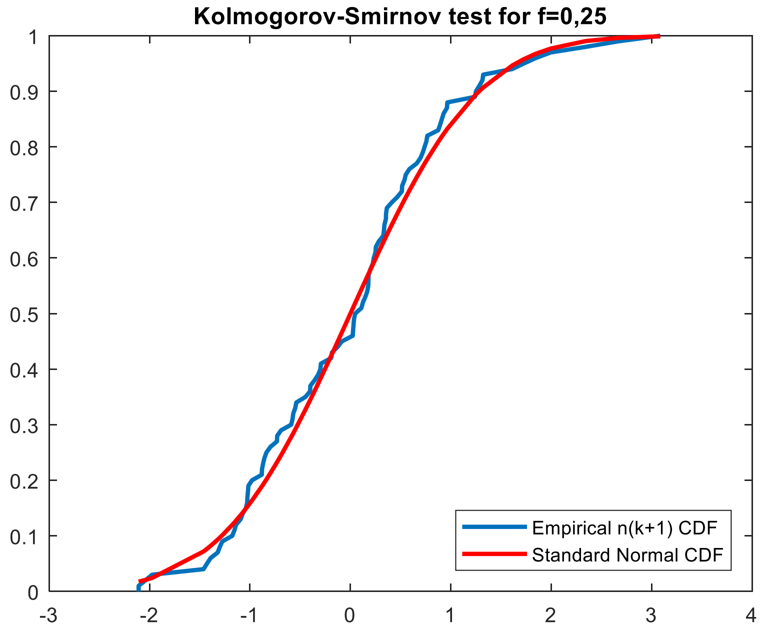
It failed to reject the null hypothesis at the 5% significance level. This means that the distribution of our data follows a standard normal distribution



Case f=0.25

Result: hzero = 0

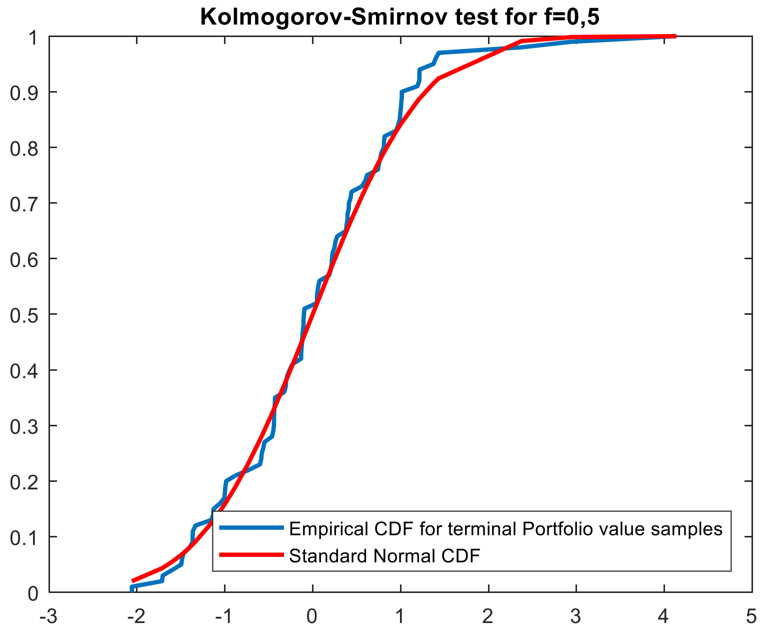
It failed to reject the null hypothesis at the 5% significance level. This means that the distribution of our data follows a standard normal distribution



Case f=0,5

Result: hzero = 0

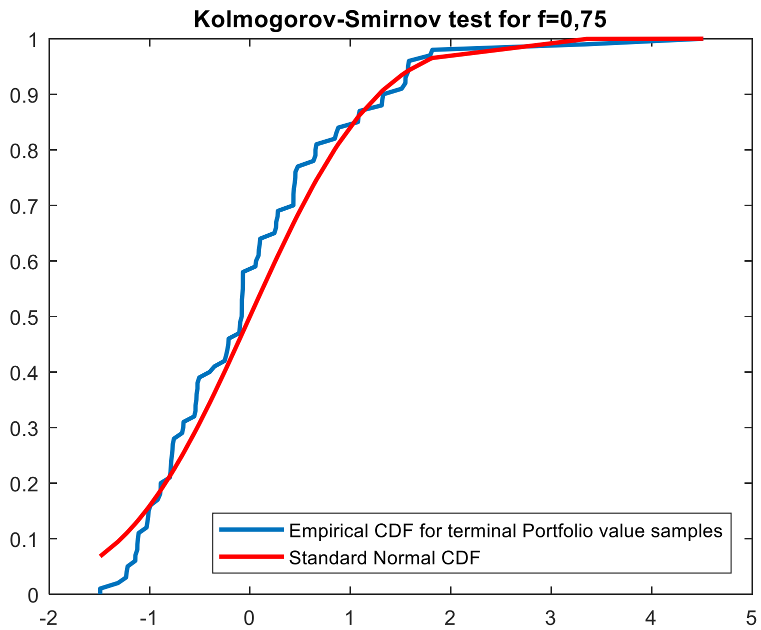
It failed to reject the null hypothesis at the 5% significance level. This means that the distribution of our data follows a standard normal distribution



Case f=0,75

Result: hzero = 0

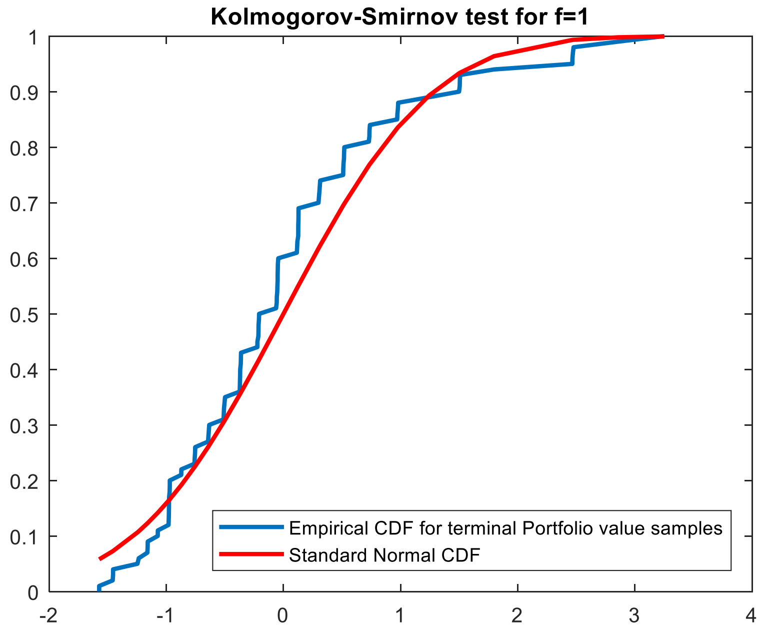
It failed to reject the null hypothesis at the 5% significance level. This means that the distribution of our data follows a standard normal distribution



Case f=1

Result: hzero = 1

It rejects the null hypothesis at the 5% significance level. This means that the distribution of our data doesn't follows a standard normal distribution



Analysis on the results of the Kolmogorov-Smirnov test:

We have seen that for four values of f: [0; 0,25; 0,5; 0,75] we can conclude that our data samples come from a standard normal distribution. Only investing the total amount of capital entirely in stocks leads to model's outcomes that aren't distributed as a normal one. To understand this result, I propose two explanations: one is related to the statistics and the structure of the model, the other one to the consequences about an evolutionary model applied to financial market, under the AMH.

The first one is the central limit theorem: it tells us that if the size of a random sample (for independent variables and with same distribution) is large enough[84], then the sample mean follows an approximate normal distribution. This theorem can be applied regardless of the sample distribution type: discrete or continuous. For this model our samples are the terminal value of portfolio after ten years: n(k+1), the sample size is equal to the number of test run, so n=100. The randomness of n(k+1) is generated through binomial distributed random variables (market trend and the exactly amount of money invested in stocks). So, it is possible to conclude

[84] The term "large enough" is relative; the more the population distribution is different from a normal one, the bigger the sample size has to be in order to apply the central limit theorem. The experimental rule is usually to choose a sample size greater than 30.

that the theorem applies to the four values of ‘f’ for which Kolmogorov-Smirnov test return the rejection of the null hypothesis. The reason for which, for f=1, the central limit theory is not valid will be explained further.

The second possible explanation for the Kolmogorov-Smirnov test can be recognized in the characteristics of the simulated market of the model developed. We know that one implication of the EMH is that the returns on stocks are normally distributed since stock prices moves randomly and the relation between returns and risk is linear. In our model we are examining the mean of returns and standard deviations for our population n(k+1), so the results obtained are consistent with the EMH for the four values of ‘f’. Under an evolutionary model of the financial market we can’t completely discharge the EMH. Considering instead the case of f=1 we have seen that the sample data do not come from a normal distribution. The more we invest in stocks, the more we expose the ‘population’ (portfolio value, literally, the dollars$) to the randomness of the model. This one should not be confused with the randomness of the stock prices fluctuations. The reason is that over the one-hundred simulation conducted, investing totally in stock prices has led to results largely far from the average, increasing standard deviation a lot. This confirm that the environment is not stable and efficient, and it has a deep influence on its inhabitants (stocks and fixed assets in these case). For the other four values of ‘f’ the proportion of money invested in fixed assets attenuates this influence since they can adapt both to the positive both the negative market trend. We can think of them as people in Brenna-Lo model that can survive both in a sunny and rainy day but with low reproductive success. For this basic model it would be interesting to add a ‘mutation’ variable which can add another evolutive aspect and repeat the test to see the results. This idea could be further developed with the implementation of genetic algorithm for example.

3.6 The development of more sophisticated models

From the previous basic model several point of development can be analysed. I present here some more sophisticated models with the aim to understand more specifically some aspects and consequences of the evolutionary model. Starting for the basic model explained before, I decide to release some specific assumptions focusing on a particular aspect of the evolutionary model. From the previous model data analysis, we have seen how the standard deviation grow together with the value of ‘f’, this is not surprising, since stocks returns depend on market trend. This volatility can lead to wide fluctuations of the capital invested, leading to high profit or high lost.

Since in the real finance world, investors are subject to their emotions and as discussed in the second chapter, some anomalies in financial market cause inadequate emotive responses, these wide fluctuations of price can lead to behavioural anomalies.

My aim is to build a model in which try to replicate behavioural anomalies as overreaction and underreaction.

In order to have an evidence of these concepts applied to the financial market model I apply some developments to the basic model in Matlab, calling this new model "Adaptive re-balance model".

- I have introduced a new parameter: 'rk'. It represents the degree of investor's risk tolerance. We know that investors are usually divided in risk-adverse, risk-neutral and risk-lovers.

  For this experiment we consider an investor which is risk-adverse (as generally is) and set rk equal to 15%. This means that, when the value of capital invested has grown more than 15% or has lost more than 15% the proportion of capital invested between stocks and fixed asset changes.

  Going into details, I want to replicate the behaviour of an investor which sees his capital growing or losing. When the return gained on his initial capital exceed +15%, the sentiment of the investor could be too positive, overestimating the recent positive trend, this would make him desire to invest more on stocks, because of their profitability.

  The opposite reasoning is applied when the lost suffered of initial capital exceed -10% the investor could feel sentiment as fear, leading to the famous "panic-selling". He would sell a certain amount of stocks, investing more in the safer fixed asset securities.
- I have introduced new actions in Matlab code. Based on the conditions:

  1)Value of capital after k times < value of initial capital*(1-risk tolerance)→ n(k+1)<n(1)*(1-rk)

  2) Value of capital after k times > value of initial capital*(1+risk tolerance)→ n(k+1)>n(1)*(1+rk)

  When one of 1) or 2) is true a new f is set:

  For case 1) the new proportion of capital invested in stocks is equal to the initial one minus the risk tolerance: f_new=f_initial – rk

  For case 2)) the new proportion of capital invested in stocks is equal to the initial one plus the risk tolerance: f_new=f_initial + rk. Clearly in this case if the value of f should exceed 1, it is set equal to 1.

After the first time that 1) or 2) are true, a new "referring initial capital" is set. For example, if I invest 1000$ and after 10 months the value of my portfolio decreases to 890$, condition 1) is true since 890<900 my portfolio is rebalanced, and the next time n(1), the initial capital to be compared will be 890, not 1000 anymore. In addition, a new variable "nreb" is established; it initial value is 0, each time that one of the two conditions 1) or 2) are verified it increased by one.

This variable is useful to understand how many time in a simulation the precedent conditions are verified and so the parameter f change.

- I have change the returns on stock when market trend is negative: xs2 from -0.035 (-3.5%) to -0.04 (-4%), in order to simulate more the effect of a possible period of a financial crisis.
- As consequences of precedents point, parameter f is no more static, its value changes. So we could think to an initial value of f, and a final one calculated according to the rules expressed in previous conditions 1) and 2).

I present in the appendix the detailed development made in the Matlab code for this model.
I have repeated the experiment of the basic model with this new model: all the other parameters remain unchanged.
Table 3.3 reports the Matlab outputs obtained for a single run of the model. This is an example where I have fixed T=36, three years, and f=0.5 for convenience.
The purpose is to show the data elaboration process of model by which it reaches the final output: n(k+1) for k=T. In the following table it is showed:

- the number of iterations k (which represents also the months after the initial investment)
- the capital at the beginning of month k, n(k) and the capital after one-month n(k+1)
- the outcomes of the binomial random array r(k)
- the amount of capital invested in stock for each month: t(k)
- the number of times that conditions 1) or 2) are verified to be true. So, when profits or losses exceed the risk factor rk=15%
- the variable cap: is the comparing parameter of conditions 1) and 2), it represents the amount of money on which calculate the risk tolerance. Clearly at the beginning it is equal to the initial capital n(1). Then, every time that conditions 1) or 2) are true it is set, after the re-arranging of capital proportion f, equal to the corresponding n(k+1).

- Z is a parameter I have used for displaying the number of iteration at which conditions 1) or 2) become true.
- The last column shows parameter f and how it changes during the iterations.

Analysing this simple example, we can notice that for two times condition 2) has been verified. First time occurred after 19 months, k=19, the capital increase reaching the amount: 1167$ which is greater than 1150. So, having supposed that investor feel excite because of his recent gains, he decides to invest more on stocks. Last column shows indeed that at k=19 f changes from 0,5 to 0,65 which means that the amount of capital invested in stocks pass from the 50% to the 65%.

By now, investor will evaluate his risk tolerance (rk=15%) no more on the initial capital, 1000$ but on the capital's value at time k=19, so 1167$. For the next iterations, we see the array r(k), responsible for market trend, has an output equal to 1 (positive trend) for several times in a row. This is due to the randomness nature of r(k), not to some specific assumptions and so, it is a good simulation of what happen in reality. The effect of this sequence of positive trend is that capital value continues to raise and after 26 months condition 2 is true again. For k=26 indeed, the amount of capital is 1352$, so greater than 1167*(1,15)=1342$, again, the investor seeing the performance of stocks prefer to invest more in them respect to the fixed asset, which have a constant, but lower return. This time, the percentage of capital invested in stocks is equal to 0,8. From this moment until the end of the simulated period, positive market trend months become rare, we can say that after an excess of optimism (or overreaction) the market reacts with several negative trend months, establishing a new equilibrium.

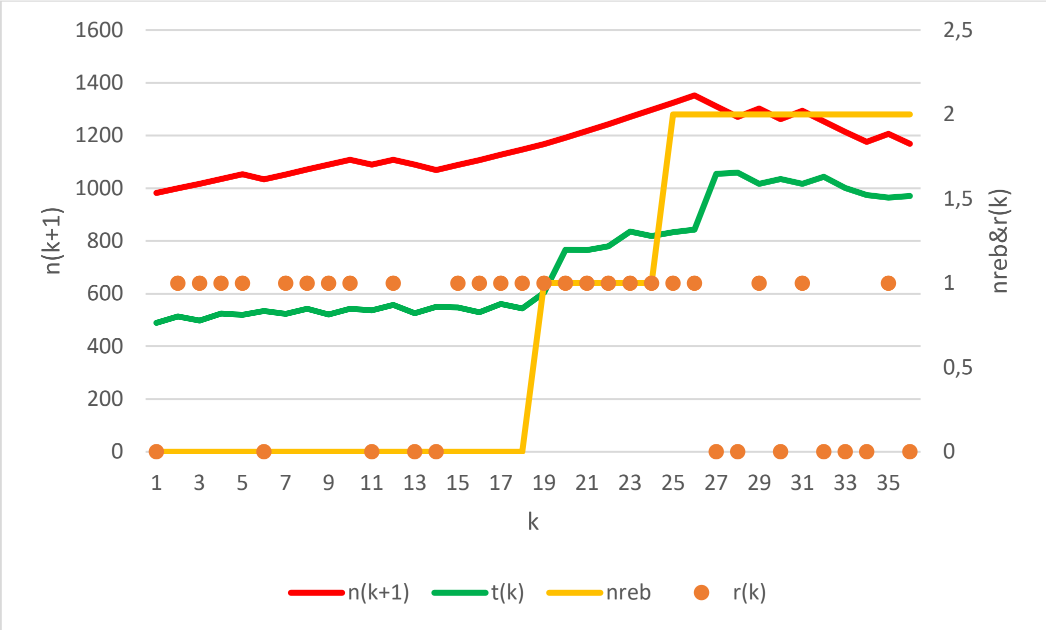
At the end of the 36 months the investor has 1168$. He still gains a positive return on its initial capital, but the over-trust in market has led to rapid losses.

For this simulated model the effects of these behaviours are smoothed, but in the real world, with more capital invested and different environment conditions, they can lead to heavier losses and to the sell-off of the stocks in portfolio.

Table 3.3 Data outputs for Adaptive re-balance model. The rows in reds highlights when a re-balance has occurred (i.e. condition 1) or 2) becomes true).

k	n(k)	n(k+1)	r(k)	t(k)	nreb	cap	Z	f
1	1000	982	0	489	0	1000		0.5
2	982	1000	1	513	0	1000		0.5
3	1000	1017	1	497	0	1000		0.5
4	1017	1035	1	524	0	1000		0.5
5	1035	1053	1	520	0	1000		0.5
6	1053	1034	0	534	0	1000		0.5
7	1034	1052	1	523	0	1000		0.5
8	1052	1071	1	543	0	1000		0.5
9	1071	1089	1	521	0	1000		0.5
10	1089	1108	1	543	0	1000		0.5
11	1108	1089	0	536	0	1000		0.5
12	1089	1108	1	557	0	1000		0.5
13	1108	1089	0	526	0	1000		0.5
14	1089	1069	0	550	0	1000		0.5
15	1069	1088	1	548	0	1000		0.5
16	1088	1107	1	529	0	1000		0.5
17	1107	1127	1	561	0	1000		0.5
18	1127	1146	1	544	0	1000		0.5
19	1146	1167	1	603	1	1167	n°of iterat	0.65
20	1167	1192	1	766	1	1167		0.65
21	1192	1217	1	765	1	1167		0.65
22	1217	1243	1	780	1	1167		0.65
23	1243	1270	1	835	1	1167		0.65
24	1270	1297	1	818	1	1167		0.65
25	1297	1324	1	833	2	1167		0.65
26	1324	1352	1	843	2	1351	'n°of itera	0.80
27	1352	1311	0	1054	2	1351		0.80
28	1311	1270	0	1059	2	1351		0.80
29	1270	1302	1	1017	2	1351		0.80
30	1302	1262	0	1035	2	1351		0.80
31	1262	1294	1	1016	2	1351		0.80
32	1294	1253	0	1043	2	1351		0.80
33	1253	1214	0	1001	2	1351		0.80
34	1214	1176	0	974	2	1351		0.80
35	1176	1206	1	964	2	1351		0.80
36	1206	1168,144	0	970	2	1351		0.80

Figure 3.4 The outcomes of n(k+1), t(k), nreb, r(k) for one simulation with T=36 and f=0.5 The first y-axis is referred the capital invested in each iteration: n(k+1), the second one to the number of rebalance occurred and the outcome of r(k).



This graph explains properly how the re-balance model works. It is showed the evolution of the amount of capital invested: n(k+1), red line. From k=15 it is possible to see how it constantly increase in value, since for twelve times in a row the outcome of r(k), the orange point, is equal to 1, i.e. the market's trend is positive. This particular succession of positive months has led to two re-balance of portfolio allocation. For k=19 condition 2) is verified (n(k+1)=1167$>1150=(n(1)*(1+rk)), so the new f is equal to 0.65 and one re-balance has been operated by the investor, yellow line shows nreb going from 0 to 1. Green line represents t(k), the amount of money invested in stocks, and we can see as for k=19 it has a large increase in value, reflecting the new proportion f=0.65. We can notice how this occurred a second time, for k=26, consequently f becomes equal to 0.80, t(k) increase and so the capital invested. After this two re-balance due to the persistent positive market's trend, several negative months occurred, leading to rapid losses of capital due to the high amount of it invested in stocks (the returns on stocks for negative months is -4%).

Now I present the simulation conducted with same modality of the first basic model, so for each value of f I have run one hundred time consequentially the Matlab script and I have reported the results in the next table. For every value of f, displayed in the first column, I show:

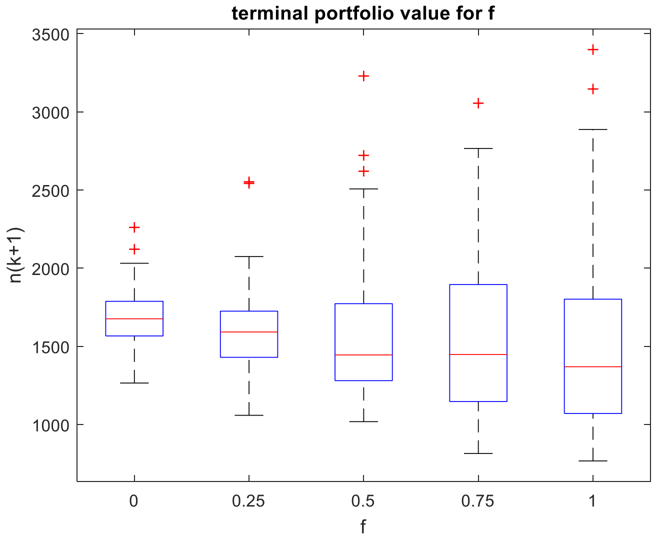
- the average (between the one-hundred simulations) of the final amount of capital n(k+1) in the second column

- the average number of re-balance occurred: each time that conditions (1) or (2) are verified, a variable 'nreb' increase by one. So, for each simulation I know how many times the conditions are verified, then I take the average of the one-hundred simulations. The result is reported in the third column.
- The standard deviation of the results for the one-hundred simulations.
- The maximum and the minimum value respectively for the final amount of capital
- On the last column it is reported the average final value of f. From the model we know that f is not static, since it changes when conditions 1) or 2) are verified. For every initial value of f, each simulation conducted led to possible different final values of it, so I took the average of "final" f for the one-hundred simulations.

Table 3.4 Data Results for the re-balance model

f	Average 'n(k+1)'	Average re-balance 'nreb'	Standard deviation	Maximum Value of n(k+1)	Minimum Value of n(k+1)	Average final f	Average 't(k)
0	1685,93	3,29	159,5799	2260,85	1265,625	0,4935	835,26
0,25	1598,87	3,13	245,147	2552,34	1059,252	0,6925	1127,18
0,5	1561,866	3,42	395,8899	3229,05	1018,74	0,847	1349,46
0,75	1538,346	4,5	478,1053	3054,98	815,288	0,9145	1444,18
1	1479,181	5,31	552,9139	3397,97	767,696	0,931	1479,181

Figure 3.5 Box plot of n(k+1) for five values of 'f'. Graph is drawn from the one-hundred simulation of re-balance model, run for each value of 'f'.



Analysis of the results

First evident result is that the optimal parameter f* that maximize, on average, the amount of capital after ten years of investment is equal to 0. Optimal f*=0 means that, for this model the best choice is to invest all capital in fixed assets. If we compare this result with the one obtained in the precedent basic model it can be very surprising, since it is exactly the opposite and we do not change a lot of the model's structure. The reasons or the determinants of this result can be summarized as follow.

I have changed the value of the return for stocks when market is down-trend, from -3.5% to -4%.

This parameter has an obvious influence on the model: summing up the returns on positive and negative market's trend, stocks are less profitable than the basic model and so it was foreseeable that invest only in stocks (f=1) did not lead to the maximum amount of capital after ten years.

Considering that other parameters (time, returns, probability of a positive market's trend) has remained the same of the basic model, at the first look the logical idea is that the new optimal parameter f * is a value lower than 1 and surely higher than 0[85].

To understand the core determinant of this result we must think about what happen when conditions 1) or 2) are verified and the consequences on this model. Recall that the market's trend outcome is generated by a binomial distributed random variable, so it may happen the outcomes is the same several times in a row, for example 1111111 or 0000000.

This lead the capital to increase (in the case of repeated positive market's trend) or to diminish (for repeated negative market's trend) fast. As consequences, at a certain time condition 2) (positive case) and 1) (negative case) are respectively verified. We already know that f changes: it becomes larger[86] for the positive case and smaller for the negative one. In real terms, the investor wants to take advantage of the positive higher returns on the stocks investing more on them; this for the positive case.

For the negative market's trend case he wants to sell-off a portion of stocks in order to stop the losses and invest in more safe products as the fixed assets. These actions which seems reasonable, have instead negative effects.

For the positive case we may think to this as an overreaction effect or a fear of regret one: it is highly improbable that after several positive months in row the market still continues to be

[85] This personal consideration is motivated by the fact the respect to the first basic model implemented, where on average the optimal choice is f*=1, a slightly change in return for stocks when market is down-trend do not lead to think that the new optimal f is the opposite: f*=0.

[86] Until it reaches the value 1 which means 100% invested in stocks, it is not possible to invest more than the capital owned (no leverage admitted).

positive. It may happen and this lead to higher profits, but on average this lead to diminishing profits since it is more likely that market's trend changes stabling more fair stock prices.

For the negative case, we may think to this behaviour as on other case of over-reaction, but this time related to the recent negative market's trend and to the losses suffered. When selling-off the stocks in this case the investor may loses the opportunity to take advantage of the possible "rebound" of the markets.

He certainly earns more returns on fixed assets, but in comparison to the possible returns of the stocks when market's trend becomes positive, he, on average, chooses the less profitable action.

Reconnecting with the results of the model, we can conclude that generally the more times the investor "re-balance" his portfolio, the lower will be the value of his portfolio at the end of the ten years since, generally, the act of "re-balance" so structured lead to less-profitable action.

Looking at the third column, we can confirm that for f=1 the number of "re-balance" executed is on average the highest for each value off. This because investing all the capital in stocks means a higher volatility and so higher fluctuations of prices, exactly these fluctuations are the responsible for the investor's behaviours.

A direct consequence is that 'nreb' is decreasing until f=0.25. For f=0 the 'nreb', number of times that conditions 1) or 2) are verified, is higher than the one for f=0.25.

The reason is simple: for f=0 the capital is entirely invested on fixed assets, so the investor, independently from the outcome, earn a positive return on its capital every month. When the first re-balance occurs, when condition 2) is true[87], f becomes equal to 0.15, so it is still a little fraction of capital that is invested in stocks.

The principal positive returns for investor comes again from fixed asset, so this lead to a new 're-balance'.

In conclusion the number of "re-balance" is higher than the one for f=0.25 since investing principally in fixed assets generate certain positive returns, and the losses suffered for capital invested in stocks are minimized.

These aspects are consistent with the result obtained for this model: f*=0 is the optimal choice because in an evolutionistic model, as this one:

- it avoids the behavioural anomalies induced by the random market's trend
- The lowest value of standard deviation. It reduces the volatility investing at the beginning all in fixed assets and then only small proportion of capital in stocks. In the last column of table 3.4 we see how the average final f is equal to 0,4935, the lower

[87] The investor's capital can only increase since, its returns are always positive.

one. This is also the largest increment in f: (final f – initial f) during the ten years of investments. The result is due to the certain positive returns of fixed assets which makes the initial capital grows faster avoiding big losses.

- Despite the fact that the maximum value of n(k+1), the amount of capital after ten years, registered during the one-hundred simulations is the lowest: 2260,85$, the minimum value registered is the highest: 1265,625$

To complete our analysis, I have to recall that investing in stocks in proportion above the 50% (f>0.5) can lead to great profit opportunities, the maximum values obtained for f=0.75 and f=1 are above 3000$. Clearly, an investor that choose this strategy take a higher level of risk which can sometimes cause to negative returns on average: the minimum value obtained for f=1 is largely under the initial capital of 1000$.

The results of this model are consistent with the idea of Adaptive Market Hypothesis. Behavioural anomalies that are induced by our inner feeling, are the product of heuristic that may be inadequate to an evolving environment leading to not optimal decision. An investor when facing a dynamic environmental ad financial market should take into consideration that several variables are random, and this randomness is truly hard to be managed. I am not referring only to positive or negative returns, the financial market is the output of a really complex variables, as for example human emotion that can cause stock prices to move fare from their value. Referring to table 3.3, in our example (that is only a simulation) we have the proof that several positive months in a row have lead the capital to increase constantly, probably raising the stock price over their fair value[88].

3.7 A multi-period model

The next model I'm going to present is also a further development of the first basic model which I have presented at the beginning of this chapter. The structure and the reference framework are the same, for this model I decide to release one assumption and to see the results. My aim, indeed, is to investigate the effects of not fixed returns. Until now, Brennan-Lo model and the ones elaborated by me have always considered the returns as fixed during the simulations, I try to release these assumptions introducing different values for returns according to a random

[88] Clearly this consideration assumes that the market tends to move similarly to a binomial distribution with probability p.

variable. I call this model multi-period because my idea is to define five possible periods in which the market can be, so for every month market's trend is determined by one of these five periods. Each period determines different returns based on its characteristics. Going into details I explain the idea behind this model.

Financial market can be in five different periods:

1) average, for average I mean that the market's trend is quite linear, without any significant event which can determine a strong trend's direction (neither up or down). The volatility during this period is low, so the returns are considered to be smooth.

2) above average, during this period market trend is positive, stocks prices are increasing on average, the investors have generally a positive sentiment[89] and they buy larger amount of stocks than average period.

3) under average, when market's trend is under average it means that it is slightly negative, stocks prices are diminishing, the investor have a generally negative sentiment and they tend to sell stocks more frequently.

4) boom, this is an extraordinary period for the financial markets, it happens rarely and due to particular positive events with a larger impact on the economy and on the financial markets. The stocks indexes raise their prices rapidly and consistently since investors are almost all oriented to buy them.

5) recession. this is another extraordinary period for the financial markets, we may think to it as a financial crisis in which stocks prices fall down and the investors sell-off larger amount of stocks, "escaping" from the market.

In order to implement this environment in the model I developed the basic model introducing new variables and parameters, here I present the new multiperiod model explaining its process. Recalling that, this model as the others takes origin from the Brennan-Lo one and they are adapted to a financial environment.

The preconditions remain the same of the basic model: an investor want to invest an amount of money in a portfolio constituted by stocks and fixed assets, for a defined time-horizon.

- 'f, T, n(1), n,t' are the same of the precedent models. 'f' remain fixed for all over the period of the investment, as the basic model, and I have run the simulation for five different values of f (0; 0,25; 0,5; 0,75; 1).

[89] When the general forecast about the market are quite good, investors have a positive sentiment.

- For this model market's trend is no more determined by a binomial distribution function, I don't use both r(k), the array which established market trend, and p, the probability of a positive market trend.
- For each iteration, so for each month the market trend can be of five types with different returns each one. In order to determine randomly the outcome of market trend I consider a new variable; 'trend'. In Matlab code I establish 'trend=rand(T,1)', 'rand' command returns a single uniformly distributed random number in the interval (0,1). So, rand(T,1) returns a T rows and 1 column matrix of random numbers, i.e. an array of 120 random numbers with a uniform distribution. This means that if we graph a histogram of these random numbers, it is roughly flat, the frequency of samples is uniform. This array represents exactly the output of market's trend, more precisely, each position of the array determines the month specific trends. If, for example trend(5)=0,6 this means that for month = 5 (T=5) the outcome is determined by the value 0,6. Now, I'm going to explain the way by which this value (0,6) determines the possible market trend I have established five conditions based on the value of trend(i), where i represents the specific month of the iteration from 1 to T=120.
  1) When the value of trend(i) is between 0,3 and 0,7 (0,3<trend(i)<0,7), market is in an average period. The returns are so established: xs1, the return for capital invested in stocks is equal to the +1% (0,01), whereas xf2, the return for capital invested in fixed assets, is equal to +0,3% (0,003). Considering the uniform distribution of 'rand' function, this mean than on average the market trend is in this period (average period) about the 40% of times[90] during the simulation.
  2) When the value of trend(i) is between 0,7 and 0,9 (0,7<trend(i)<0,9), market is in above average period. The monthly returns for this situation are: xs1=+2% (0,02) and xf2=+0,4%(0,004).

     This situation happens, on average, the 20% of times during the simulation.
  3) When the value of trend(i) is between 0,9 and 1 (0,9<trend(i)<1), market is in "boom" period. The monthly returns for this situation are: xs1=+4% (0,04) and xf2=+0,8%(0,008).

     This situation happens, on average, the 10% of times during the simulation.

[90] Considering an uniform distribution of random numbers between 0 and 1, it means that each number has the same probability of the other to be the outcome. So, considering the interval 0,3-0,7 they represent the 40% percent of the entire sample population.

4) When the value of trend(i) is between 0,1 and 0,3 (0,1<trend(i)<0,3), market is in under average period. The monthly returns for this situation are: xs1=-2% (-0,02) and xf2=+0,2%(0,002).

   This situation happens, on average, the 20% of times during the simulation.

5) When the value of trend(i) is between 0 and 0,1 (0<trend(i)<0,1), market is in recession period. The monthly returns for this situation are: xs1=-5% (0,02) and xf2=+0,1%(0,001). The returns for fixed assets is now very low, but still positive since I consider a case in which investor buy various type of fixed assets, from the riskier bonds to the safest Treasury bills.

   This situation happens, on average, the 10% of times during the simulation.

- A direct consequence of this model is that, the formula for calculating n(k+1), the value of capital after k month of investment, is changed. In this model it is simply: n(k+1)=t(k)*(1+xs1) + (n(k)-t(k))*(1+xf2); since the variable for returns are only two: xs1 and xf2. They change their value according to market trend period, without considering different variables as in precedent models, where I have considered different variables for different returns when a negative market trend occurred.

The model so it is still quite simple, but the principal aim is to investigate what happen when we introduce several possibilities of market trend and consequently of returns.

This new framework replies more accurately the real market where often it is possible to define peculiar market's periods. I maintain a random structure for the returns, trying to give the probabilities for each period outcome the most similar to reality.

In an evolutionary context the optimal response is the one the best adapts to the environment, for this reason the introduction into the model of periods as boom or crisis is, in my opinion, so interesting. Especially during periods of crisis, which in the last decades becomes more frequently, the "survival" of the individuals (investors in this framework) is determined by whom can adapt fast, avoiding threats of big losses. I present a single run of the code for this multi-period model in order to understand the process of evolution of the capital invested at the beginning.

For this example, I choose as usually T=36, three years of investment horizon and a value for f=0,5.

Table 3.5 reports the value, for every iteration from k=1 to k=36, of the following parameters. The capital invested every month: n(k). The amount of money invested exactly in stocks each month 't(k)'. The value of the uniformly distributed random variable 'trend(i)' (for i=1:T) and the consequently market period, represented by the five arrays av, abav, boom, unav, rec. I have indeed created five arrays to report for every iteration the outcome of market period.

When value of 'trend' between 0,3 and 0,7 the market period is "average" and the array 'av' assumes the value 1, while the other arrays remains 0. For example, for i=5 and trend(i) =0,2; unav(i)=1. (av(i), abav(i), boom(i), rec(i) are all equal to 0).

The array 'av' is valued when market period is average; the array 'abav' is valued when market period is above average ('trend(i)' between 0,7 and 0,9).

The array 'boom' is valued when market period is boom ('trend(i)' above 0,9). The array 'unav' is valued when market period is under average ('trend(i)' between 0,1 and 0,3). The array 'rec' is valued when market period is recession ('trend(i)' below 0,1).

In the respective column of Table 3.5, next to the name of these arrays I also show the sum of the number of times that each array has assumed valued 1 i.e. it has determined the market period for that specific month 'i'.

Analysing the result for one simulation of 36 months, we see that the capital at the end of this period has increased from the initial value of 1000. It seems that on average market trend is positive, and this is not surprising, since the largest probability of 'trend' outcome is the period 'average', which means low, but positive returns on invested capital (recall that xf1=+1% and xf2=+0,3%). Nevertheless, the randomness of market period's outcome can lead to losses on initial capital.

It is possible to notice that after one year the value of capital was 973$ and after a brief increase at the 19th month, the value of capital was again under the initial value, 998$.

However, on the third year the trend has been positive and the capital value raise until the final value of 1089%.

The outcomes of market being randomly and independent from the past lead to a higher volatility of the portfolio value, when periods of boom or recession occurs the returns are larger and so the variation of capital's value.

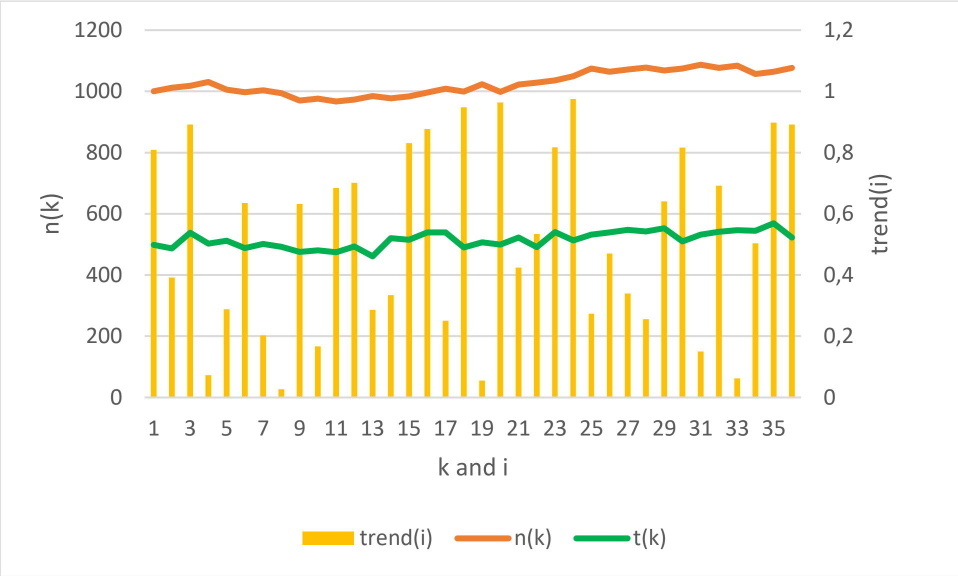
For this simulation the number of times that market period has been "average" have been: 12 (the sum of the 'av' elements). 9 times has been above average ('abav'), 3 times has been in a boom period ('boom'). Market period has been 8 times under average ('unav') and 4 times it has been in a recession period ('rec').

Table 3.5

k	n(k)	n(k+1)	t(k)	trend(i)	av (12)	abav (9)	boom (3)	unav (8)	rec (4)
1	1000	1012	498	0,809129	0	1	0	0	0
2	1012	1018	487	0,391734	1	0	0	0	0
3	1018	1031	538	0,89203	0	1	0	0	0
4	1031	1006	503	0,073007	0	0	0	0	1
5	1006	997	512	0,287921	0	0	0	1	0
6	997	1003	488	0,635329	1	0	0	0	0
7	1003	994	502	0,202843	0	0	0	1	0
8	994	970	492	0,026963	0	0	0	0	1
9	970	976	475	0,632659	1	0	0	0	0
10	976	967	481	0,166723	0	0	0	1	0
11	967	973	474	0,684179	1	0	0	0	0
12	973	985	493	0,701264	0	1	0	0	0
13	985	977	461	0,285831	0	0	0	1	0
14	977	984	520	0,334071	1	0	0	0	0
15	984	996	515	0,830411	0	1	0	0	0
16	996	1009	539	0,876713	0	1	0	0	0
17	1009	999	539	0,250638	0	0	0	1	0
18	999	1023	490	0,948109	0	0	1	0	0
19	1023	998	507	0,054889	0	0	0	0	1
20	998	1022	499	0,963648	0	0	1	0	0
21	1022	1029	522	0,424367	1	0	0	0	0
22	1029	1036	491	0,534044	1	0	0	0	0
23	1036	1049	540	0,816816	0	1	0	0	0
24	1049	1074	513	0,975112	0	0	1	0	0
25	1074	1064	532	0,273491	0	0	0	1	0
26	1064	1071	539	0,470401	1	0	0	0	0
27	1071	1078	548	0,339193	1	0	0	0	0
28	1078	1068	542	0,255566	0	0	0	1	0
29	1068	1075	553	0,641004	1	0	0	0	0
30	1075	1087	510	0,815961	0	1	0	0	0
31	1087	1077	532	0,150564	0	0	0	1	0
32	1077	1084	541	0,69197	1	0	0	0	0
33	1084	1057	546	0,061948	0	0	0	0	1
34	1057	1064	544	0,503575	1	0	0	0	0
35	1064	1077	569	0,897749	0	1	0	0	0
36	1077	1089,66	522	0,891158	0	1	0	0	0

The evolution of the portfolio's value and the capital invested in stocks is depicted in the next graph, where I have also inserted the values of 'trend(i)' for the secondary y-axis. It is possible to see that when yellow vertical lines are very low, indicating a period of recession, the red line of n(k) (capital value) decrease consistently. On the opposite when yellow vertical lines are high, n(k) increase rapidly.

Figure 3.6 Output trend for n(k), trend(i) and t(k).



The results of the simulations are presented in the next table. I conduct the simulations in the same way I have done for the precedent experiments. For each value of f: 0; 0,25; 0,5; 0,75; 1 I have run one-hundreds of times the Matlab Script, then I took the average of the results. In addition to the standard average values of simulations, n(k+1),t(k) standard deviation, maximum and minimum value, I have also reported the average number of times that a specific period has occurred.

Table 3.6 Data outcomes for multi-period model

f	Average n(k+1)	Average t(k)	Max Value	Min	Standard Deviation	av	abav	boom	unav	rec
0	1478,108	0	1565,683	1399,576	31,953	48,39	24,02	12,12	23,63	11,84
0,25	1476,214	369,14	1877,904	1184,500	121,6076	48,34	24,1	11,72	24,35	11,49
0,5	1445,523	719,86	2244,481	927,116	231,0621	47,52	24,35	11,74	24,20	12,19
0,75	1390,324	1035,71	2127,744	750,136	276,2536	47,71	23,23	12	24,77	12,29
1	1382,534	1386,63	2329,680	735,98	358,1431	48,27	23,56	11,82	23,95	12,4

Data analysis:

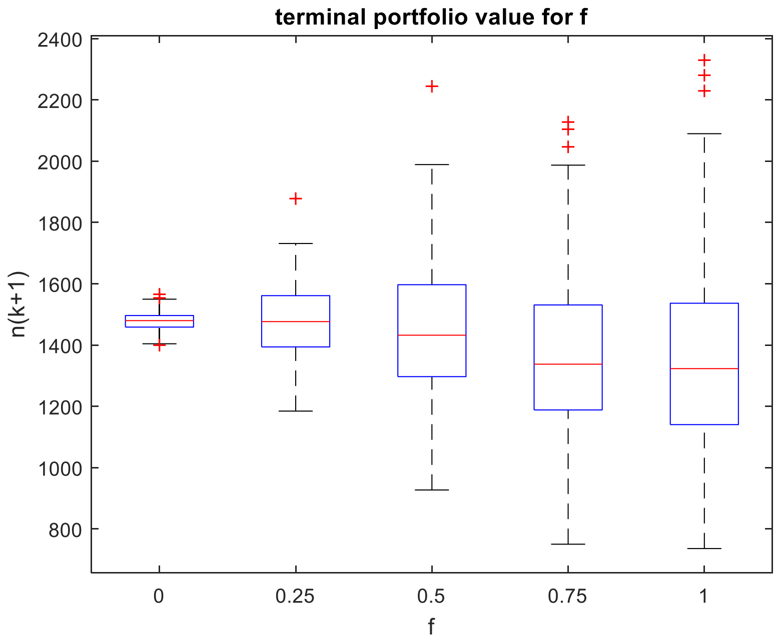
The first fundamental aspect to analyse is that the results of the terminal value of portfolio for the five values of ‘f’ are very similar each other indicating a substantial reproductive equilibrium. The reasons for this particular outcome is inside the model structure. Positive and Negative market periods tend to be equally proportioned: above average and under average periods have the same probability to occurs (20% each one), and the same for boom and recession periods (10% each one).

So, the terminal value of our portfolio is principally due to the average period, which have positive returns. Stock returns neutralize in the case of above/under average market periods: they are +2% and -2% respectively, and they are almost equal on case of boom/recession (+4% and -5%). In this context, the best reproductive success is determined by f=0, investing in fixed assets is the optimal solution. They guarantee the highest profit in ten years of investment, with the lowest standard deviation, so with the lower risk. This result is clearly determined by the fact that, I have assumed that for every period the returns on fixed assets are positive. I have not considered the possibility of a company or nation default since this probability is too low compared to the probability of a financial crisis (in term of stock prices). From this model however, a precious insight is possible to be extract. Recall that under the AMH, the EMH and the random Walk theory are not completely wrong and this concept is fundamental for this model. EMH and Random Walk Theory indeed, affirm that when financial market's framework is characterized by randomness it is impossible to gain consistent profits over time. On average, our model give consistency to this idea, showing the highest profitability on long-run time horizon of fixed assets over the stocks. The crucial point is that, under an evolutionistic view, we can affirm that the best reproductive success is given by fixed assets, but this only assuming that the environment is completely random. When, over time the environment changes new opportunities arise for who can adapt faster to these one, whereas for individuals (stocks or other financial products) that can't adapt, the risk to be "erased" by natural selection forces increases. In our model, investing in stocks on average is less profitable, but it also allows to exploit bigger returns, gaining at the end of the ten years consistent profits. For example, for f=1 (investing all capital in stocks) the maximum value reached is 2329,68$, about 800$ more than the maximum value obtained for f=0. Nevertheless, looking at the minimum value obtained for each 'f' of this simulation, it is clear how investing more than 50% of capital can lead to dangerous conditions for the survival of species. The minimum value obtained for f=1 is indeed 735,98$, a loss more than the 25% of initial capital. Clearly stocks are penalized by their negative returns when market is in a "bad" trend. However, considering that for the 70%[91] of time market trend is positive and stocks return are higher than fixed assets, these results demonstrates how fixed assets can adapt better the random situation of the markets, generating less returns but still positive. We can also conclude that in this model, risk of investing in stocks, on average do not lead to an optimal outcome, but the possibility to gain higher profits than fixed assets is not completely excluded. In this context it could be interesting to insert a

[91] It is the sum of probabilities for an average, above average and boom period, in which the stocks returns are positive and greater than fixed assets ones.

mutation variable, in order to see, also on average how the outcomes change and the adaptive response of individuals (stocks and fixed assets returns) to these mutation in the markets.

Figure 3.7 Box plot of n(k+1) for five values of ‘f’. Graph is drawn from the one-hundred simulation of multiperiod model, run for each value of ‘f’.



3.8 A hedging model

I have implemented a new model, this time keeping all the assumptions made for the first basic model about the environment. The further development I have made in this model regards the possibility of choice by the population: we have always considered a binary choice between two alternatives: stocks and fixed assets.

Now I’m going to introduce a new alternative: the possibility to invest into derivates.

Derivates instruments are contracts or assets which prices is based on the market value of another underlying financial instrument, frequently called only “underlying”, for example stocks, indexes, currencies, commodities and others.

Derivates can be based on three macro-categories of contracts: futures, option and swap.

The common applications of derivates are: the hedging of a financial risk; the arbitrage (the purchase of a product in a market and its selling on another one); and the speculation. In this model, I have considered the choice in which the derivates can be traded for hedging the risk.
So, if an investor decides to buy a stock, he could hedge its risk by short-selling a future contract (or buy a put options). Without going too much into the details of the derivates contracts, I'll present how I build this model and its principal aim, explaining how it works and the reasons to compare stocks and fixed assets with derivates.
Analytical description of the model:
The structure of this model is the same of the previous, so we assume that an investor has to make a choice regarding its initial investment in a portfolio which can be constituted by three products: stocks, fixed assets, derivates. They represent the population of our experimental environment, at every month they can be increase (in case of positive returns) or decrease (in case of negative returns).
We assume that investor wants to buy stocks in order to exploit the possibility of positive returns when market trend is increasing. Since he does not like to risk too much, he prefers to invest also in derivates hedging his position, so, he decide to own a product, let's say a put option on the stocks he bought. So, when market trend is decreasing derivates on the purchased stocks earn positive returns. In this case It is not important which specific type of product the investor own, the aim of the model is to consider a larger population in which a part of it, the derivates takes advantage of the negative market trend. In the same way as when I talked about fixed asset I don't specify if it is a corporate or a Treasury Bills, because I consider a mix of these products, in order to define average returns on a well-diversified portfolio. We have two possibilities for market trend: positive or negative, which are determined as the first two models by a binomial distributed random variable 'r'. When market is positive stocks and fixed assets will earn positive returns whereas derivates value suffer a loss; when market is negative, the stock's value decrease, the derivates earn positive returns and fixed asset earn smaller but still positive returns.
The key aspect of this model is related to a property of derivates. In this model indeed, we allow the investor to buy (or short-sell) derivates "on margin", this means that to establish a position on derivates he doesn't need to have the total value of position, which is usually the price of the underlying asset times its amount. Investor is only required to deposit initial margin, called for the option case, the option premium, which is a percentage of the total value.

The only limitation to investor is that these contracts require a maintenance margin[92].

The aim of this model is so, to see the evolution of three different species in a dynamic context, characterized by randomness, analysing which species can adapt better to their environment. The main difference from the previous model is that here there is part of population, the derivates which not "fit" the positive market trend since its reproductive success is determined by a negative one. I want to investigate how this new feature affect the entire population.

Formalization:

- As in the previous models, the following parameters has remained equals: T = 120, n(1)=1000 (initial amount of capital), p=0,62 (probability of a positive market trend, and the binomial distributed random variable 'r'.
- Parameter 'f' is equal 1 and it represents the sum of the three new parameters for the allocation of capital between the three products: pst, pfa and pder. Which are respectively the proportion of capital invested in stocks, fixed assets and derivates.
- The returns are changed, this time I consider larger returns in order to have strongly evidence on the results and in particular on the hedge 'property' of the derivates.

  For this reason, I set, for positive market trend:

  xs1 = 6%; xf1 = 0,5%; xd1 =- 4% respectively the returns for stocks, fixed assets and derivates

  xs2 = -7% xf2 = 0,4%; xd2 = 5% respectively the returns for stocks, fixed assets and derivates

  We can see how the derivates can't hedge completely the risk of a negative market trend on stocks, since usually we have also to discount the premium that investor pays in order to establish a derivates position.
- In this model I consider three matrixes of T rows and 1 column: 'st', 'der', 'fixass'.

  They represent the value for each product: stocks, derivates and fixed assets at each iteration. The total population for each generation t is given by their sum. At the beginning st+der+fixass=1000$, so they represent the specific product value of portfolio.

[92] The minimum account value an investor is required to maintain to continue holding one or more futures contracts. The dollar value for maintenance margin varies by the specific commodity or financial instrument. An account that falls below the combined maintenance margin for all positions in the account will receive a margin call and be subject to full or partial liquidation
Source: http://www.businessdictionary.com/definition/maintenance-margin.html

- For the derivates I introduce the margin, so a leverage effect. It is a parameter called 'lev' in the code and it is equal to the ratio between the money invested in stock and money invested in derivates. For example, if he investor buy 600$ on stock and 200$ on derivates the 'lev' is equal to three. This parameter represents the leverage ratio, so from the initial amount of money invested in derivates, initial margin, the investor will own derivates for a value three times bigger than its initial margin. Making a numerical example. If I buy 500$ of stocks and invest 250$ in derivates, lev = 2, and I will own 500$ of derivates. I will earn returns on 500$ of derivates, but at the end of the month I assume that I have to give back the loan (250$ in this case). Then for the next month I will establish a new loan of always 250$. I have assumed that during the investment period of ten years the loan is always the same, whereas the ratio changes according to derivates values. This assumption will not change my results, indeed the reason for which at the end of the month investor has to give back the loan is just to calculate the evolution of population 'n(k)' with the right values. The real population indeed has to be evaluated for the initial value of 1000$, the leverage effect will raise it, but it will be only temporary, since it is a loan that should give back. Our reference point it is the initial value, where in order to protect the long position on stocks, I invest the same amount of money in derivates.
- 'loan' is the value of the money borrowed for establish a derivates position equal to the stocks one.
- Maintenance margin is set at 25% This means that if the ratio between: derivates equity value over total value of derivates (equity value plus the loan) falls under the value 0.25, a margin call is request to the investor to continue to invest. In reality an investor could decide to exit from the market and register the losses, here the investor sell some stocks in order to re-establish the maintenance margin. So, the new value of stocks will be:

$$st^*_{k+1} = st_{k+1} - (0.25*(der_{k+1}+loan)-der_{k+1})$$

and the new value of derivates will be:

$$der^*_{k+1} = der_{k+1} + (0.25*(der_{k+1}+loan)-der_{k+1})$$

-

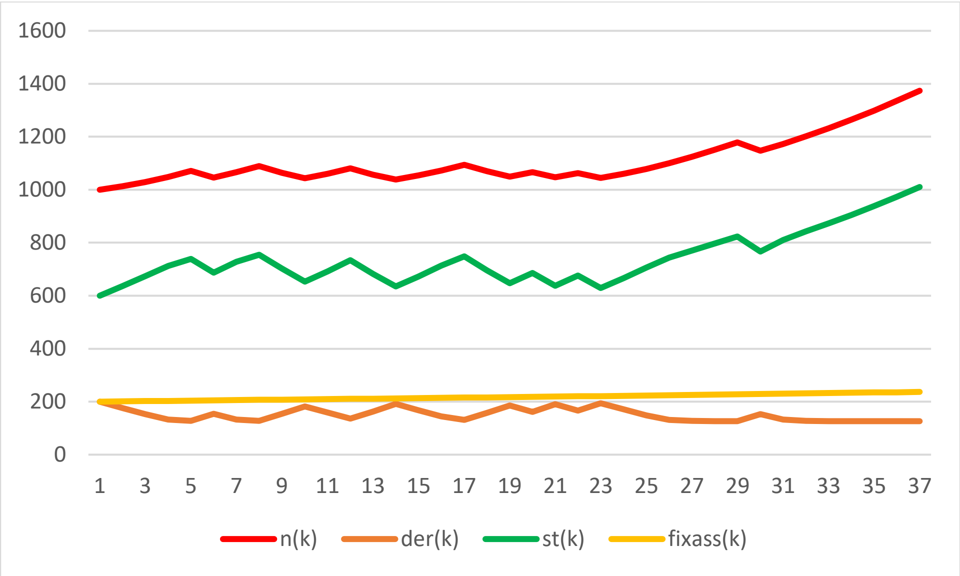
The following Table showing an example of one simulation of the code. I have considered only for this case a time of three years, T=36 months and the 60% of capital invested in stocks, 20% in derivates and the remaining 20% in fixed assets, so 'loan' is equal to 400$

Table 3.7 Data output for one simulation of the hedging derivates model, with parameter T=36.

k	f	T	n(k)	der(k)	st(k)	fixass(k)	pst	pder	pfa	maintenance	r(k)
1	1	36	1000	200	600	200	0,6	0,2	0,2	0	1
2	1	36	1013	176	636	201	0,6	0,2	0,2	0	1
3	1	36	1029,125	152,96	674,16	202,005	0,6	0,2	0,2	0	1
4	1	36	1048,466	132,7104	712,7408	203,015	0,6	0,2	0,2	1	1
5	1	36	1070,937	127,8505	739,0567	204,0301	0,6	0,2	0,2	1	1
6	1	36	1046,412	154,243	687,3228	204,8462	0,6	0,2	0,2	0	0
7	1	36	1066,506	133,0183	727,6171	205,8705	0,6	0,2	0,2	1	1
8	1	36	1089,872	127,9244	755,0473	206,8998	0,6	0,2	0,2	1	1
9	1	36	1064,242	154,3206	702,194	207,7274	0,6	0,2	0,2	0	0
10	1	36	1043,635	182,0366	653,0404	208,5583	0,6	0,2	0,2	0	0
11	1	36	1060,579	158,7552	692,2229	209,6011	0,6	0,2	0,2	0	1
12	1	36	1080,81	136,405	733,7562	210,6491	0,6	0,2	0,2	0	1
13	1	36	1057,11	163,2252	682,3933	211,4917	0,6	0,2	0,2	0	0
14	1	36	1038,35	191,3865	634,6258	212,3377	0,6	0,2	0,2	0	0
15	1	36	1053,834	167,731	672,7033	213,3994	0,6	0,2	0,2	0	1
16	1	36	1072,554	145,0218	713,0655	214,4664	0,6	0,2	0,2	0	1
17	1	36	1094,609	130,8052	748,2651	215,5387	0,6	0,2	0,2	1	1
18	1	36	1069,633	157,3455	695,8866	216,4008	0,6	0,2	0,2	0	0
19	1	36	1049,654	185,2128	647,1745	217,2664	0,6	0,2	0,2	0	0
20	1	36	1066,162	161,8043	686,005	218,3528	0,6	0,2	0,2	0	1
21	1	36	1047,105	189,8945	637,9846	219,2262	0,6	0,2	0,2	0	0
22	1	36	1062,885	166,2987	676,2637	220,3223	0,6	0,2	0,2	0	1
23	1	36	1044,742	194,6136	628,9252	221,2036	0,6	0,2	0,2	0	0
24	1	36	1059,799	170,8291	666,6608	222,3096	0,6	0,2	0,2	0	1
25	1	36	1078,077	147,9959	706,6604	223,4212	0,6	0,2	0,2	0	1
26	1	36	1099,674	131,519	743,6171	224,5383	0,6	0,2	0,2	1	1
27	1	36	1124,153	127,5646	770,9278	225,661	0,6	0,2	0,2	1	1
28	1	36	1150,435	126,6155	797,03	226,7893	0,6	0,2	0,2	1	1
29	1	36	1178,326	126,3877	824,0149	227,9232	0,6	0,2	0,2	1	1
30	1	36	1147,876	152,7071	766,3339	228,8349	0,6	0,2	0,2	0	0
31	1	36	1172,892	132,6497	810,263	229,9791	0,6	0,2	0,2	1	1
32	1	36	1201,352	127,8359	842,3866	231,129	0,6	0,2	0,2	1	1
33	1	36	1231,937	126,6806	872,9717	232,2846	0,6	0,2	0,2	1	1
34	1	36	1264,409	126,4033	904,56	233,4461	0,6	0,2	0,2	1	1
35	1	36	1298,794	126,3368	937,844	234,6133	0,6	0,2	0,2	1	1
36	1	36	1335,184	126,3208	973,0772	235,7864	0,6	0,2	0,2	1	1

From this table we can understand the process of evolution in monetary terms of the three species here considered. Fixed assets increase their value at each iteration, since their return are always positive, they can be compared to those animals which reproductive success is certain but it is not dominant, and for this reason they will not "colonized" the environment. The stocks result to be the dominant species, they exploit the higher positive market trend percentage increasing consistently, this is due to the higher positive return that every positive month is earned. On the other side, derivates value decrease in three years of about the 40%. It is certain a lot and in part is due to the very large number of positive month. We can notice how the margin call, in order to re-set the maintenance margin occurs very frequently. After just one month indeed, the derivate value drops from 200$ to 176$, a loss of 24$, exactly the negative return of 4% on the 600$ dollars invested in leverage. From this simulation it seems that stocks are dominant, and the hedging is not necessary, but this is only a short example.

Figure 3.8 Trend analysis of the three products and the portfolio value for T=36



From this graph we can see how the total value of portfolio is influenced mostly by the stocks, in addition we see that, when market trend is negative, the portfolio losses are smoothed since the value of derivates increases.

I present the data collected for one-hundred simulation run, according to different values of pst, pder, pfa.

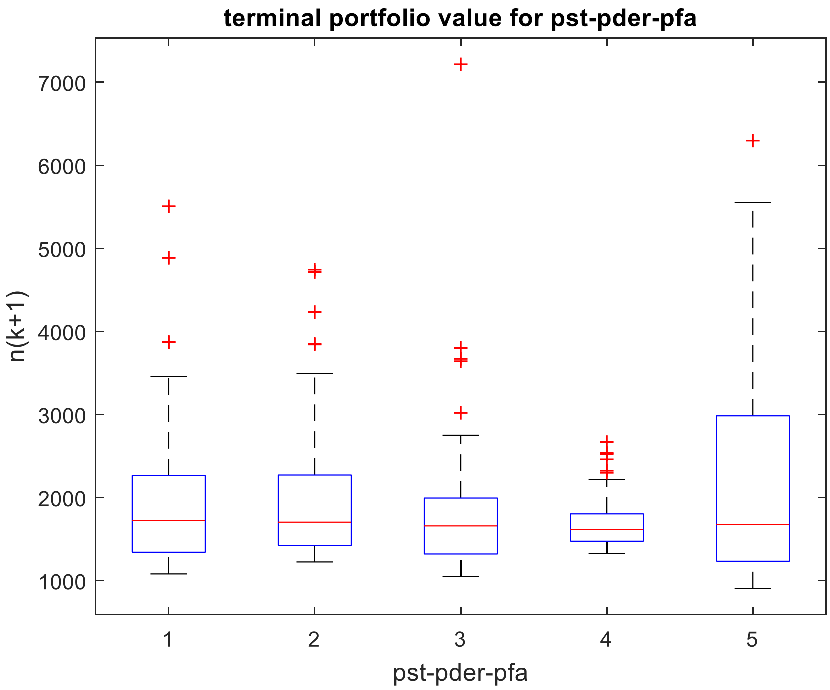
Table 3.8 Data outputs for hedging derivates model,

pst	pder	pfa	average n(k+1)	std. deviation n(k+1)	maximum value of n(k+1)	minimum value of n(k+1)	average st(k+1)	average der(k+1)	average fixass(k+1)	loan
0,4	0,4	0,2	1994,52	967,0765	5505,333	1081,193	1450,324	196,3845	347,8112	0
0,4	0,2	0,4	1992,904	788,8894	4743,978	1225,341	1194,697	102,7514	695,4562	200
0,6	0,2	0,2	1811,615	786,6084	7215,525	1049,715	1251,634	212,3533	347,6275	400
0,4	0,1	0,5	1692,258	288,6163	2668,179	1327,006	675,8196	146,6352	869,8035	300
0,6	0,4	0	2161,839	1252,017	6297,092	905,5664	2011,757	150,0823	0	200

Data Analysis:

The first analysis regards the general result: it seems that the hedging of derivates allow the investor to increase his average returns over ten years of investment since the highest value for the terminal value of portfolio are obtained when initially the proportion of capital invested in derivates is 400$. Despite the increase of risk associated to leverage effect on derivates, in four cases out of five there are no minimum value of terminal portfolio under the initial investment. This is due also to our assumption not to consider all the cost related to the margin calls and the number of trades made to re-stablish the maintenance margin. In addition, I have deliberately chosen higher return for this model both when market is positive, both when is negative. Define an optimal solution for this model become harder since we have several variables to consider. The immediate result is that when in the environment there are no fixed assets, the population on average grow at the best rate. Looking at the fifth row indeed, we see that the terminal value of portfolio when investor started at t=0 with 600$ on stocks and 400$ on derivates, is the highest: 2161,839$. It is important to highlight that despite the fact that the value of derivates from the initial 400$ has gone to 150$ (on average), the terminal value of stocks, on average is more than three times the initial one: 2011,57$. For sure more realistic data could be obtained if we consider cost of transactions and taxes, especially on the derivates. For this model however, it is more important to consider the entire population, which has shown to adapt to a dynamic environment. When we look at the data we can see how the hedging position works for the population as a sort of assurance. We could imagine the population of Brennan-Lo model that built a protection against the possible floods. This new defence against the natural selection forces has a cost that reduce the reproductive success: if we confront these results with the first basic model we see that they are slightly lower. In addition, introducing a new species in the environment add new possibilities of evolution: standard deviation indeed is much higher than other models (this is due in part due to the higher returns of this model), and so the outcomes of the model can be extremely different. For example, the third row which has this asset allocation: 60% stocks, 20% fixed assets and 20% derivates and the maximum value obtained for terminal portfolio value is above 7000$! We can conclude that if species can adapt together to the environment, without searching the personal maximization there could be the possibility to reach higher return for the entire population. Surely if I had decided that once the value of derivates falls under maintenance margin, derivates would have exited from market, the terminal value of population would be lower. Clearly this not apply in real market where sometimes exit from the market and stop the losses could be a good idea. But under an evolutionistic view the idea is to preserve the species, understanding the influence of the environment on them and then to begin a path of adaptation.

Figure 3.9 Box-plot for the 5 cases. The x-axis represents each case of asset allocation: stocks/fixed assets/derivates. case 1: 0,4/0,2/0,4 case 2: 0,4/0,4/0,2 case 3: 0,6/0,2/0,2 case 4: 0,4/0,5/0,1 case 5: 0,6/0/0,4



Conclusions

It is still too early to affirm that finance have reached a new equilibrium through a theory which can explains the several anomalies and unsolved questions that actually challenge the EMH and the finance itself. An enormous amount of studies and empirical tests have been made but the results are not homogenous: in particular some "event study" see to confirm the idea of an immediate reflection of the information into stocks prices just when information become public. On the contrary, several calendar anomalies have been reported into stock prices historical series, showing how the markets would be far from efficiency. Adaptive Market Hypothesis is a new theory elaborated by Andrew Lo with the hard drive to be the reconciliation theory between the two opposites: EMH and behavioural economics. We have seen how this theory exploits concepts belonging to different disciplines: behavioural finance, psychology, cognitive neurosciences and socio-biology applying them to economic and financial world. The AMH it is based on biological principles which, all together, represent the core of the evolutionistic theory: competition, mutation, reproductive success, adaptation, natural selection, survival and extinction. The answer to the fundamental question: "how can AMH represent a reconciliation theory and why?" is encloses exactly in the precedent concepts. When Lo applies them to financial markets there is the possibility to describe more properly the dynamics and the behaviour of the investors, obtaining new and precious insights about the most challenging question around EMH and its critics. Clearly, there are some obvious differences between the evolution of a biological environment and the evolution of financial markets, but the intent of Lo is not to represent markets as an exact copy of it. He tries to analyse the possible similitudes between the systems and once applied to financial markets look at them under a different "light", so re-considering themes as investment strategies and portfolio management.

The core idea of the adaptive markets hypothesis is that laws of biology ruled financial markets more than the laws of physics. We can summarize five basic concepts of adaptive markets:

1. People act in their own self-interest.
2. People make mistakes.
3. From those mistakes, they learn, adapt, and innovate.
4. As they experiment and fail or succeed, the process of natural selection operates on individuals, institutions, and markets just as it operates on bacteria, sea slugs, and chimpanzees.
5. This evolutionary process is what determines financial market dynamics.

The AMH applies the framework of evolutionary biology to specific financial contexts. Following this new approach, for Any anomaly or issue in the world of finance we obtain insightful conclusions that are different than what we would obtain either from the behavioral finance or from the Efficient Market Hypothesis. I have present numerous empirical examples for which AMH can reach new insights that can used for a better management of finance activities, reaching a more efficient condition. The traditional investment paradigm, a bundle with the main rules of the investment world has been re-considered under the AMH, theories as the CAPM, asset allocation have been analyzed and shaped in a more dynamic structure. For example when we have to determine our asset allocation between stocks and bonds, under EMH, which states that prices fully reflect all available information, we should not try to pick winners stocks and bonds or timing the market. We should just consider our own risk preferences, our age, our income, and the kind of retirement we would like to have in order to determine our asset allocation and maximize our possibilities of achieving these goals. The AMH observes that the returns on equities or bonds are not guaranteed since their performance depends on specific market conditions, which evolve over time. Operating in a dynamic environment requires a dynamic management of asset allocation for retire with a certain level of wealth. This means to monitor the whole financial ecosystem, considering financial markets as an ecosystem indeed, allows us to detect and exploit the relation between investment performance and the interactions of the different types of investors. Following this reasoning, we may not be able to take advantage of daily trading, but we can exploit trends over longer holding periods. We can certainly state that this new theory shows both theoretical and practical consequences, the next question I have investigated is how to apply them correctly. When Lo suggests to monitor the entire financial ecosystem, we must ask our-self if we have the instruments to do that, the answer that Lo give to us is that we have some tools, but to make them adequate to our purpose we need to collect all the necessary data to use them. For example, we have seen how over the past 20 years, huge amounts of assets have flowed into passive index funds, especially in this thesis I have studied the quantitative hedge funds and its Meltdowns of 2007. These passive funds have earned positive expected returns on average, indicating a positive market trend that should last until a negative event with large impact occurs. In general, we refer to financial crisis, but sometime events of few days as the Quants Meltdown can have a deep negative impact for the environment and its inhabitants. AMH tell us to measure these possible disrupting events as they begin to show, in order to prevent and possibly avoid the damage of a financial crisis. For this reason, the development of new tools, instruments are fundamental in finance ecosystem. It is hard to analyze correctly but since technological revolution has evolved it, we need also to evolve. In this thesis I have talked about some new features presented by Lo and other academicians: the measure for illiquidity, the financial networks and its measure and there are several other tools that still need to be improved. The problem is often related to how we look at the markets, according to Lo indeed we focus too much on prices and other economic fundamentals, but they may not be the most important drivers of the markets. To understand on which factors we should give more attention, Lo suggest to study the different "species" living in the financial systems in the same way as the biologist does with a new ecosystem. The idea is to create a catalogue of pension funds, hedge funds, mutual funds, banks, broker/dealers, insurance companies, and so on, analyzing size, growth rate and the specific features for each "species". After this first reporting work, the next step should be analyzed the behavior of these species in their environment, by studying this we can understand the reaction of them in different market circumstances. A practical example is to consider a product as pension fund and determine their features: how often they make investment decisions, their risk tolerance, their financial aims and so on. If we put together this information across all the species (products) we can obtain a meaningful picture of the trend of financial markets and their possible reaction to eventually negative events as crisis or shocks. This process however, requires an additional improvement for the data collection and repository, since often it happens that some precious information, especially about financial institution, are lost. The whole process of evolution and improvement is the outcome of a deep understanding of the relation that exist between man and financial environment. In this thesis at the begin of the second chapter I have presented how AMH has its roots also on a new field that is called Neuro-economics. Studying the decision-making processes of the man, we hate to take into consideration that we are rational with emotions. The financial decision making is one feature of human decision making, from this powerful concept Lo applies the theories, the insights of different disciplines: anthropology, sociology, economics, psychology and biology. The reason is that all these subjects has a common denominator which is human behavior, so the models of how people make decisions should not be restricted to a specific sector but have to be exploited also in Finance. starting from biology we have to create analytical methods and models for these processes understanding finally the reasons of several "anomalies" that occurs in financial markets. Human behavior become so, the center of the AMH, his relationship with environment, his ability to feel emotions and to be rational, his inner heuristic and brain processes are the main factors that influence the economic and financial sector. From neuroscience and artificial intelligence (AI) literature it comes the idea that human decision processes are very similar to the elaborations of internet search engines. This new insight has large implications in several sectors. The modern expert systems, opposite to the past ones, use relatively simple algorithm processing an extraordinary amount of data (the famous “big data”) and giving do an algorithmically decision-making process very similar to the human brain. Humans brains indeed have a enormous “database”, represented by the experiences we have faced in our lives, from which, through relative easy algorithms we make predictions, take decisions and actions, often without any detailed information, but just based on big data extrapolation. This is how man adapt to different circumstances, these processes allow us to take fast decisions that allows to have satisfying outcomes, from the less important decision to the survival one. Lo states that If we act as Homo economicus analyzing every possible situation and optimizing the possible outcomes, probably the reproductive success of our species would not have been so dominant. This is the significance of the subtitle of his book: “Financial evolution at the speed of thought”. Lo recognizes in this mechanism the fundamental problem of theories as EMH and relative, indeed it works properly fir allowing us to be alive and maintain the survival of our species, but at the same time it is not optimally suited for decision-making mechanism as, for example determining our asset allocation. The challenge that AMH highlight is understanding the limitations of human cognition and developing methods to improve that mechanism. The first improvement we should do according to Lo and his new theory is that when we make decision most of the time they are the result of our emotions more than rationality. For this reason, for any financial instrument or investment strategy we decide to exploit we should consider our emotional reactions, and the respectively financial products should take into account also the human factor.

AMH is so a work in progress, which objective is not to be a substitute of EMH or behavioural finance, but the possible reconciliation between the opposite sides of the same coin. They indeed reflect the dualistic nature of human behaviour since we are not fully rational neither fully emotional, but we are a complex system of iteration between both aspects. AMH tries to enlarge the critical point of view about Economy and Finance without saying that EMH or behavioural finance are wrong, but it states their incompleteness and the impossibility to apply them in each situation.

AMH seems to work very well empirically, from its first publicization several tests have been conducted, usually we can find academic literature that test the performance of stocks prices returns and their independency, with results that support AMH. However, a lot of empirically studies must be done in order to give robustness to this theory. Especially from an academic point of view, AMH needs quantitative forms, comparing it with theories as CAPM, Black

Scholes-Merton option pricing model, EMH which have all strongly mathematical expressions, AMH result hard to adopt by academic world.

Personally, I think that AMH has so much to offer and the best part is that the benefits of this new theory can be enjoyed by a lot of people, not only academicians or finance professionals.

Probably I am optimistic but having the consciousness that environment is changing can encourage man to adapt Finance towards our goals, without allowing that our goals be driven by Finance. This means that fixing Finance, as already written in the last part of chapter 2, could be an opportunity to address social priorities as the climate change, cure for cancer, poverty and pandemics.

To conclude this thesis, I report the following citations of Lo about this argument from his book: “Adaptive Markets: Financial Evolution at the speed of thought” [2017]. Maybe Lo vision is too optimistic, but this is the reason for which I really enjoy study AMH and write this thesis.

“Our human intelligence will harness our collective fear and greed to solve our global problems”.

“Finance doesn’t have to be a zero-sum game if we don’t let it. We can do well by doing good, and if we all work together, we can do it now.”

APPENDIX

A) Matlab code

A.1 Matlab code for the elaboration of the first basic model

I present the case for f=0.75, clearly, the code remains the same for other values of f.

```
%Binary model investments evolution
f=0.75
%Probability of capital invested in stocks portfolio
T=120
%number of periods, i have considered interval of 1 month,
n=zeros(T,1) %array for the amount of capital
%n(1)=input('Enter amount of capital invested');
%T=input('Enter no. of iterations');
n(1)= 1000
%Capital invested at time t=1
xs1=0.035
%xstock1 is the % of return gained for a stock's portfolio, when market is
up-trend (for r=1)
xf1=0.005
%xfixedasset2 is the % of return gained for a fixed asset's portfolio, when
%market is up-trend
xs2=-0.04
%xs2 is the % of return for a stock's portfolio when market is downtrend
xf2=0.004
%xf1 is the % of return for a fixed asset's portfolio wh en market is
%downtrend
p=0.62
%is the probability of a positive market's trend (clearly, 1-p means that
%market's trend is negative
r=zeros(T,1)
%array for market's trend
t=zeros(T,1)
%array for the % of capital invested in stocks
%    ct=1
for i=1:T
r(i) = binornd(1,p);
%the output is a binomial casual array r(i) that determines if market is
uptrend (r =1) or downtrend (r=0)
end

for k=1:T
if(n(k)>0)
   n(k) = round(n(k))
t(k) = binornd(n(k),f);
% t(k) is an array which represents the percentage of capital invested in
stock according to probability f
n(k+1)=t(k)*(1+xs1)*r(k) + (n(k)-t(k))*(1+xf1)*r(k) + t(k)*(1+xs2)*(1-
(r(k))) + (n(k)-t(k))*(1+xf2)*(1-(r(k)))
%n(k+1) is the outcome array which represents the capital invested after t
period, it is equal to the sum of the returns.
end
%xlswrite('binary',n)
end
% filename='binarymodeleasy.xlsx';
% AP={f,T,n(k+1),t(k),xs1,xs2,xf1,xf2};
% pos=strcat('AP',num2str(ct))
```

```
% xlswrite(filename,AP,1,pos)
% ct=ct+1
```

A.2 Matlab code for the elaboration of Kolmogorov-Smirnov test

Case f=1

```
filename='binarymodeleasy_grafici.xlsx'
fzero=xlsread(filename,5,'J1:J100') %read the file excel containing the
sample data,fzero is a matrix that contains data on the fifth sheet, coloumn
from J1 to J100,
hone = kstest(fzero) %hone display the return of Kolmogorov-Smirnov test
[f,x_values] = ecdf(fzero); %construction of the empirical cumulative
distribution function
%draw the graphs
F = plot(x_values,f);
set(F,'LineWidth',2);
hold on;
G = plot(x_values,normcdf(x_values,0,1),'r-');
set(G,'LineWidth',2);
legend([F G ],...
       'Empirical CDF for terminal Portfolio value samples ','Standard Normal
CDF',...
       'Location','SE');
   title ('Kolmogorov-Smirnov test for f=1')
```

A.3 Matlab Code for the boxplot graph

```
filename='adaptiverebalancenew_graphs.xlsx'
box=xlsread(filename,7,'A2:E101')
f=xlsread(filename,7,'G2:G6')
boxplot(box,f)
title('terminal portfolio value for f')
xlabel('f')
ylabel('n(k+1)')
```

A.4 Matlab code for the re-balance model:

```
%Binary model investments evolution
f=0.5
%Probability of capital invested in stocks portfolio
T=120
%number of periods, i have considered interval of 1 month,
n=zeros(T,1) %array for the amount of capital
cap=zeros(T,1) %arrray to be conditioned for the rebalancing
%n(1)=input('Enter amount of capital invested');
%T=input('Enter no. of iterations');
n(1)= 1000
cap(1)=n(1)
%Capital invested at time t=1
rk=0.15
% investor is risk adverse, he place a hypothetic stop loss equal to price
fall by 15%
xs1=0.03
%xstock1 is the % of return gained for a stock's portfolio, when market is
up-trend (for r=1)
xf1=0.005
```

```
%xfixedasset2 is the % of return gained for a fixed asset's portfolio, when
%market is up-trend
xs2=-0.04
%xs2 is the % of return for a stock's portfolio when market is downtrend
xf2=0.004
%xf1 is the % of return for a fixed asset's portfolio when market is
%downtrend
p=0.62
%is the probability of a positive market's trend (clearly, 1-p means that
%market's trend is negative
r=zeros(T,1)
%array for market's trend
t=zeros(T,1)
%array for the % of capital invested in stocks
%y=zeros(T,1)
  filename='adaptiverebalancenew.xlsx';
nreb=0;
for i=1:T
r(i) = binornd(1,p);
%the output is a binomial casual array r(i) that determines if market is
uptrend (r =1) or downtrend (r=0)
end
    %end
%     ct=99
%     delete(filename)
y=zeros(T,1)
for k=1:T
    check=false
if(n(k)>0)
   n(k) = round(n(k))
t(k) = binornd(n(k),f);
% t(k) is an array which represents the percentage of capital invested in
stock according to probability f
n(k+1)=t(k)*(1+xs1)*r(k) + (n(k)-t(k))*(1+xf1)*r(k) + t(k)*(1+xs2)*(1-
(r(k))) + (n(k)-t(k))*(1+xf2)*(1-(r(k)))
% cap(k+1)=n(k+1)
%n(k+1) is the outcome array which represents the capital invested after t
period, it is equal to the sum of the returns.
end
if (n(k+1)<cap(1)*(1-rk))
    check=true
    disp ('rebalance more fixed assets')
    disp(n(k+1))
    Z=['n°of iteration is',num2str(k)]
    f=f-rk
  disp(k)
  disp (f)
  cap(1)= n(k+1)
end
if (n(k+1)>cap(1)*(1+rk))
    check=true
    disp ('rebalance more stocks')
    disp(n(k+1))
    Z=['n°of iteration is',num2str(k)]
    f=f+rk
    if f>1
        f=1
    end
  disp(k)
  disp (f)
  cap(1)=n(k+1)
```

```
end
   if (check)
       nreb=nreb+1
   end
   y(k)=k
end
% y(k+1)=k+1
% P={n(1),n(k+1),t(k),(k+1),nreb,f,cap(1)};
% pos=strcat('P',num2str(ct));
% ct=ct+1
% xlswrite(filename,P,1,pos)
%  plot(y,n)
```

A.5 Matlab Code for multiperiod model

```
%Binary model investments evolution
f=1
%Probability of capital invested in stocks portfolio
T=120
%number of periods, i have considered interval of 1 month,
n=zeros(T,1) %array for the amount of capital
%n(1)=input('Enter amount of capital invested');
%T=input('Enter no. of iterations');
n(1)= 1000
%Capital invested at time t=1
period=zeros(T,1)
%disp (vettore)
t=zeros(T,1);
k=1;
trend=rand(T,1);
j=0;
l=0;
m=0;
d=0;
e=0;
% avk=zeros(T,1)
% abavk=zeros(T,1)
% boomk=zeros(T,1)
% unavk=zeros(T,1)
% reck=zeros(T,1)
%    ct=88
for i = 1:T
    period(i) =  trend(i);
    disp(period(i))

    if (period(i) < 0.7 && period(i)>0.3)
disp('average return')
xs1=0.01
xf2=0.003
j=j+1
av=j
disp(av)
    end
    if (period(i)>0.7 && period(i)<0.9)
        disp('above average')
        xs1=0.02
        xf2=0.004
        l=l+1
abav=l
```

```
disp(abav)
    end
    if(period(i)>0.9)
        disp('boom')
        xs1=0.04
        xf2=0.008
        m=m+1
boom=m
disp(boom)
    end
    if(period(i)<0.3 && period(i)>0.1)
        disp('under average')
        xs1=-0.02
        xf2=0.002
        d=d+1
unav=d
disp(unav)
    end
if(period(i)<0.1)
    disp('recession')
    xs1=-0.05
    xf2=0.001
    e=e+1
rec=e
disp(rec)
end
if(n(k)>0)
   n(k) = round(n(k))
t(k) = binornd(n(k),f);
disp (t(k))
% t(k) is an array which represents the percentage of capital invested in
stock according to probability f
n(k+1)=t(k)*(1+xs1) + (n(k)-t(k))*(1+xf2) %+ t(k)*(1+xs2)*(1-(r(k))) +
(n(k)-t(k))*(1+xf1)*(1-(r(k)))
%n(k+1) is the outcome array which represents the capital invested after t
%period, it is equal to the sum of the returns.
if(k<120)
k=k+1
end

end


end
filename='binarychoice.xlsx';
A={f,k,n(k+1),t(k),av,abav,boom,unav,rec};
pos=strcat('A',num2str(ct))
xlswrite(filename,A,1,pos)
ct=ct+1
% A={n(k),};
% xlswrite(filename,A,1,'F101')
```

A.6 Matlab code for the hedging derivates model

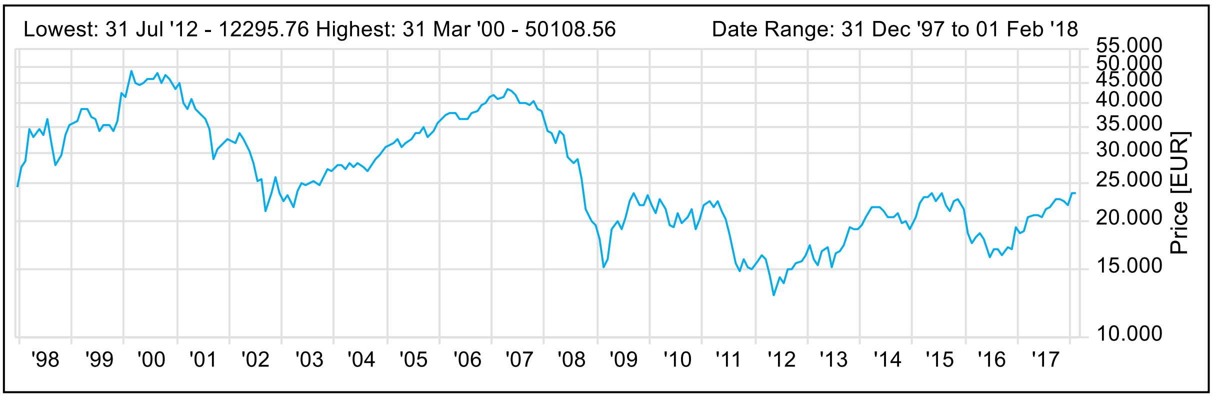
```
%investments evolution hedge derivates
pst=0.6
pfa=0.2
pder=0.2
f=pst+pfa+pder
%Probability of capital invested in stocks portfolio
T=36
%number of periods, i have considered interval of 1 month,
n=zeros(T,1)
st=zeros(T,1)
der=zeros(T,1)
fixass=zeros(T,1)%array for the amount of capital
%n(1)=input('Enter amount of capital invested');
%T=input('Enter no. of iterations');
n(1)= 1000
st(1)=n(1)*pst
der(1)=n(1)*pder
fixass(1)=n(1)*pfa
%Capital invested at time t=1
xs1=0.06 %xstock1 is the % of return gained for a stock's portfolio, when
market is up-trend (for r=1)
xf1=0.005%xfixedasset1 is the % of return gained for a fixed asset's
portfolio, when%market is up-trend
xd1=-0.04 %return for derivates when market is uptrend
%market downtrend
xs2=-0.07
%xs2 is the % of return for a stock's portfolio when market is downtrend
xf2=0.004
%xf1 is the % of return for a fixed asset's portfolio when market is
%downtrend
xd2=0.05
%return of derivates when markey is downtrend
p=0.62
%is the probability of a positive market's trend (clearly, 1-p means that
%market's trend is negative
r=zeros(T,1)
%array for market's trend
lev=pst/pder %leverage ratio
loan=(der(1)*lev)-der(1)
maintenance=zeros(T,1)
  ct=1
for i=1:T
r(i) = binornd(1,p);
%the output is a binomial casual array r(i) that determines if market is
uptrend (r =1) or downtrend (r=0)
end
    %end
for k=1:T
if(n(k)>0)
   n(k) = round(n(k))
   st(k+1)= (1+xs1)*r(k)*st(k)+ st(k)*(1+xs2)*(1-(r(k)))
   der(k+1)= (1+xd1)*r(k)*(der(k)+loan)+ ((der(k)+loan)*(1+xd2)*(1-(r(k))))
   der(k+1)=der(k+1)-loan
   fixass(k+1)= (1+xf1)*r(k)*fixass(k)+ fixass(k)*(1+xf2)*(1-(r(k)))
   if (der(k+1)/(der(k+1)+loan) < (0.25))
       disp 'margin call'
       st(k+1)=st(k+1)-(0.25*(der(k+1)+loan)-(der(k+1)))
       der(k+1)=der(k+1)+(0.25*(der(k+1)+loan)-(der(k+1)))
       maintenance(k)=1
```

```
    end
 n(k+1)=st(k+1)+der(k+1)+fixass(k+1)
% %pst*(1+xs1)*r(k)*n(k)  + pfa*(1+xf1)*r(k)*n(k)  +
pder*lev*(1+xd1)*r(k)*n(k)  + n(k)*pst*(1+xs2)*(1-(r(k))) +
n(k)*pfa*(1+xf2)*(1-(r(k)))+n(k)*pder*lev*(1+xd2)*(1-(r(k)))
% %n(k+1) is the outcome array which represents the capital invested after
t period, it is equal to the sum of the returns.
% n(k+1)=n(k+1)-((n(k)*pst)-(n(k)*pder))
% %margin call
end
%xlswrite('binary',n)
end
% filename='hedgederivates.xlsx';
%
A={f,T,n(k+1),der(k+1),st(k+1),fixass(k+1),pst,pfa,pder,maintenance(k),loan
};
% pos=strcat('A',num2str(ct))
% xlswrite(filename,A,5,pos)
% ct=ct+1
```

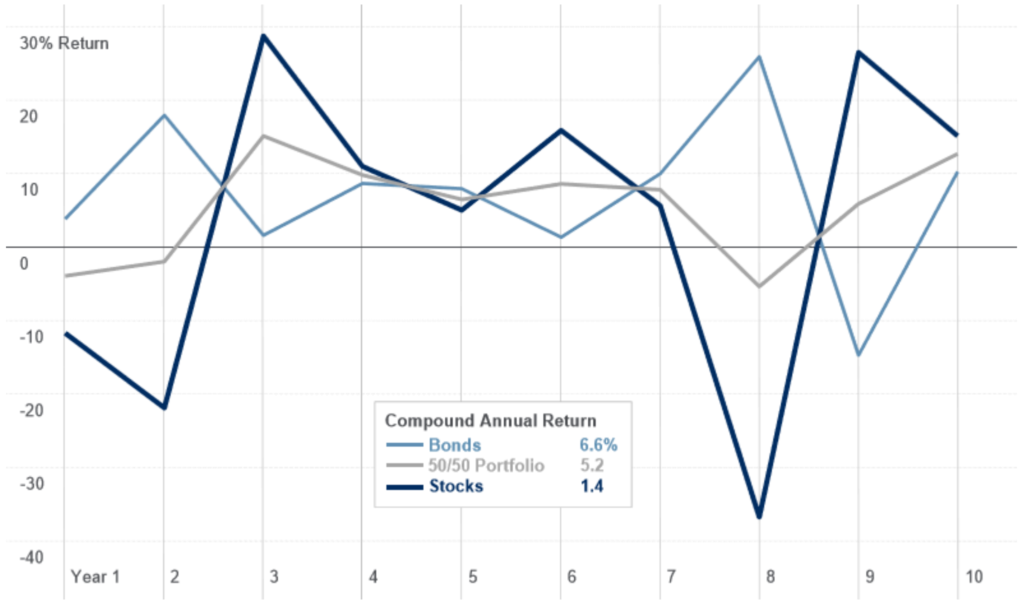
B) Tables and figures

This figures and tables were useful as source of information and data for the development of the evolutionary models in chapter 3. For reason of dimension I don't attach the historical data prices and returns for S&P500 and FTSE MIB

B.1 Historical data of stock prices trend and returns for FTSE MIB, for 1998 to 2018



B.2 Asset allocation for years 2001-2010 in U.S. market



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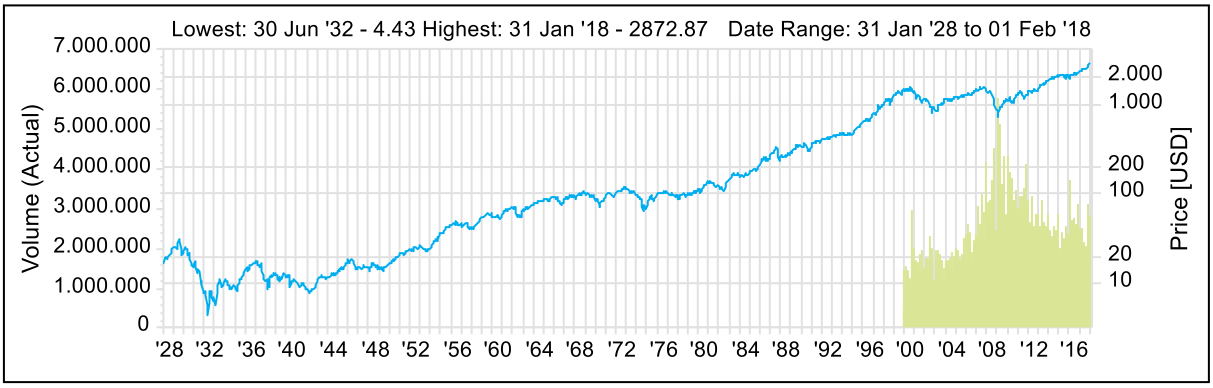
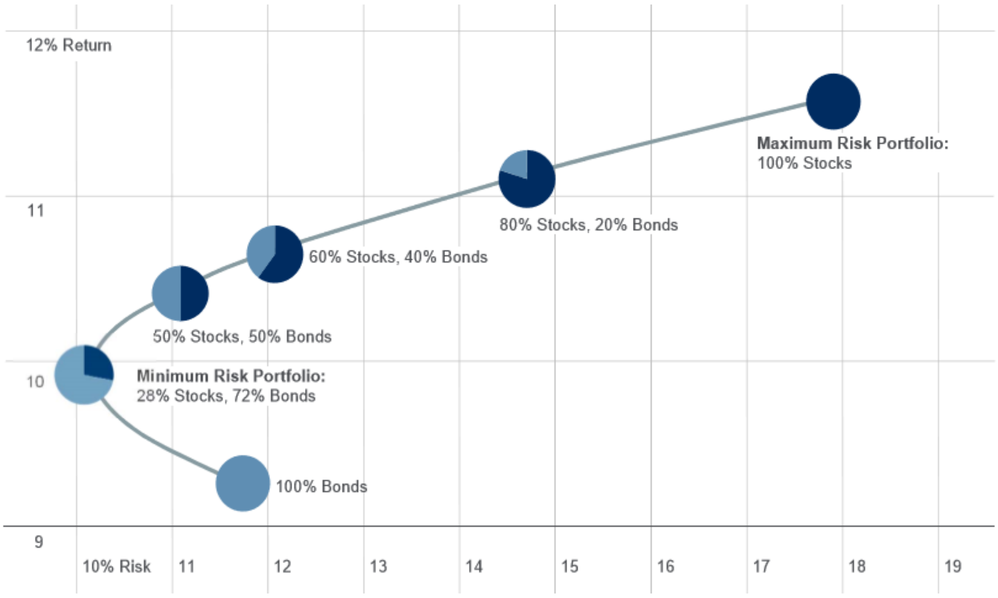
B.3 Historical data of stock prices and returns for S&P 500, for 1928 to 2018



B.4 Asset allocation: Risk Versus Return 1970-2010



Created by Raymond James using Ibbotson Presentation Materials © 2011 Morningstar

B.5 Table of the input (f, x_values,fzero) for Kolmogorov-Smirnov test

Where fzero is calculated thorugh the following expression:
(Terminal value of portfolio at T=120 – the average of the one-hundred terminal values for that simulation)/standard deviation of the-one hundred values =
(n(k+1)-average(n(k+1))/dev.std(n(k+1)

fzero	f	x values
-0,363975543365128	0	-1,57226923979818
-0,871215243609250	0,0100000000000000	-1,57226923979818
-0,221925314039121	0,0200000000000000	-1,45708817249384
-1,07249319567119	0,0300000000000000	-1,45376564170621
-0,0514008353544313	0,0400000000000000	-1,45360513297251
-0,977536228813258	0,0500000000000000	-1,24555371234836
1,50771680033043	0,0600000000000001	-1,23447860972294
2,46640336498396	0,0700000000000001	-1,16083720270060
-0,0483351185407290	0,0800000000000001	-1,15878269090922
-0,497984285132680	0,0900000000000001	-1,15775543501353
-0,0439050774905622	0,100000000000000	-1,07249319567119
0,517602625618105	0,110000000000000	-1,07167460112931
-0,976428718550716	0,120000000000000	-0,980040165059004
0,509850053780313	0,140000000000000	-0,977536228813258
0,313820737310423	0,160000000000000	-0,976428718550716
0,118898931103075	0,170000000000000	-0,975321208288174
1,24191433732041	0,180000000000000	-0,970891167238007
0,733567126813747	0,190000000000000	-0,969783656975465
-0,0494426288032706	0,200000000000000	-0,969767606102095
-0,208924106609283	0,210000000000000	-0,872322753871791
0,128866523465951	0,220000000000000	-0,871215243609250
1,49885671823010	0,230000000000000	-0,759180147485461

-0,754043868007006	0,240000000000000	-0,757125635694079
-0,872322753871791	0,250000000000000	-0,754043868007006
-1,45376564170621	0,260000000000000	-0,752711645517282
-0,757125635694079	0,270000000000000	-0,643100231272389
-1,16083720270060	0,290000000000000	-0,636423067950399
0,516671674962635	0,300000000000000	-0,633100537162773
0,979434405098015	0,310000000000000	-0,510584220728263
-0,976428718550716	0,320000000000000	-0,509059387758097
-0,636423067950399	0,330000000000000	-0,506844367233014
0,970574322997681	0,340000000000000	-0,502414326182847
1,50439426954280	0,350000000000000	-0,497984285132680
0,512562651379872	0,360000000000000	-0,372931930705683
-1,15878269090922	0,370000000000000	-0,371728115202920
0,976882316232158	0,390000000000000	-0,369513094677837
-0,510584220728263	0,400000000000000	-0,368822907122919
-0,0627327519537717	0,410000000000000	-0,363975543365128
-1,07167460112931	0,420000000000000	-0,362868033102586
-0,372931930705683	0,430000000000000	-0,361760522840044
0,729137085763579	0,440000000000000	-0,222214229759784
-1,57226923979818	0,450000000000000	-0,221925314039121
0,737997167863914	0,460000000000000	-0,213707266873594
0,972789343522765	0,470000000000000	-0,212680010977903
1,79765977688919	0,480000000000000	-0,212246637396908
-0,222214229759784	0,490000000000000	-0,208924106609283
0,126314434600094	0,500000000000000	-0,207543731499448
-1,24555371234836	0,510000000000001	-0,0627327519537717
0,307175675735173	0,520000000000001	-0,0571952006410633
-0,969783656975465	0,530000000000001	-0,0565371148328860
-1,45360513297251	0,540000000000001	-0,0527651595908959
-0,361760522840044	0,550000000000001	-0,0514008353544313
-0,369513094677837	0,560000000000001	-0,0494426288032706
-0,369513094677837	0,570000000000001	-0,0493463235630499
0,299423103897380	0,580000000000001	-0,0483351185407290
0,128866523465951	0,590000000000001	-0,0472918117716681
2,85444927958120	0,600000000000001	-0,0439050774905622
-0,969767606102095	0,610000000000001	0,114468890052908
2,47014321847921	0,620000000000001	0,117069131538876
-0,0472918117716681	0,630000000000000	0,118898931103075
0,303002448658929	0,640000000000000	0,126314434600094
2,46571317742904	0,660000000000000	0,126651502940867
-0,980040165059004	0,670000000000000	0,128368946391476
2,47568076979192	0,690000000000000	0,128866523465951
0,128368946391476	0,700000000000000	0,299423103897380
-1,15775543501353	0,710000000000000	0,303002448658929
-0,502414326182847	0,720000000000000	0,307175675735173
-0,0571952006410633	0,740000000000000	0,313820737310423
-0,0527651595908959	0,750000000000000	0,509850053780313
-0,636423067950399	0,760000000000000	0,510508139588490
-0,506844367233014	0,770000000000000	0,512562651379872
1,50550177980535	0,780000000000000	0,516671674962635
-0,509059387758097	0,790000000000000	0,517602625618105
-0,371728115202920	0,800000000000000	0,519817646143189
-0,0565371148328860	0,810000000000000	0,729137085763579
-0,643100231272389	0,820000000000000	0,733567126813747
-0,368822907122919	0,830000000000000	0,734449924849106
-0,212246637396908	0,840000000000000	0,737997167863914
0,313820737310423	0,850000000000000	0,970574322997681
-0,970891167238007	0,860000000000000	0,972789343522765
-1,45708817249384	0,870000000000000	0,976882316232158
-0,752711645517282	0,880000000000000	0,979434405098015
-0,759180147485461	0,890000000000000	1,24191433732041
0,126651502940867	0,900000000000000	1,49885671823010
3,25019961339612	0,910000000000000	1,50439426954280
0,510508139588490	0,920000000000000	1,50550177980535

-0,633100537162773	0,930000000000000	1,50771680033043
-0,212680010977903	0,940000000000000	1,79765977688919
-0,213707266873594	0,950000000000000	2,46571317742904
0,114468890052908	0,960000000000000	2,46640336498396
-1,23447860972294	0,970000000000000	2,47014321847921
-0,977536228813258	0,980000000000000	2,47568076979192
0,126651502940867	0,990000000000000	2,85444927958120
0,117069131538876	1	3,25019961339612

C) Outputs Data for the model simulations in Matlab

In this appendix I report the table containing the outcomes for the simulations of Matlab's models. For every model I report a table the data obtained for one-hundred runs of Matlab code

Basic model:

Outputs table for the one-hundred simulation of the basic model: I present the outputs for f= 0 and f=1. Table C1 and C2.

Adaptive Re-balance model:

Outputs table for the one-hundred simulation of the adaptive re-balance model: I present the outputs for f= 0 f=0,25 f=0,50 f=0,75 f=1. Tables C.6, C.7, C.8, C.9, C.10.

Multi-period model:

Outputs table for the one-hundred simulation of the adaptive re-balance model: I present the outputs for f= 0 f=0,25 f=0,50 f=0,75 f=1. Tables C.11, C.12, C.13, C.14, C.15.

Table C.1 basic model f=0

k	f	T	n(k+1)	t(k)	xs1	xs2	xf1	xf2		
1	0	120	1721,565	0	0,035	-0,04	0,005	0,004	average	
2	0	120	1727,595	0	0,035	-0,04	0,005	0,004		1734,12601
3	0	120	1759,755	0	0,035	-0,04	0,005	0,004	Variance	
4	0	120	1720,856	0	0,035	-0,04	0,005	0,004		103,0102911
5	0	120	1735,635	0	0,035	-0,04	0,005	0,004	dev standard	
6	0	120	1723,868	0	0,035	-0,04	0,005	0,004		10,14939856
7	0	120	1740,66	0	0,035	-0,04	0,005	0,004	MAX	
8	0	120	1738,65	0	0,035	-0,04	0,005	0,004		1759,755
9	0	120	1748,968	0	0,035	-0,04	0,005	0,004	MIN	
10	0	120	1733,625	0	0,035	-0,04	0,005	0,004		1707,804
11	0	120	1733,908	0	0,035	-0,04	0,005	0,004		
12	0	120	1735,635	0	0,035	-0,04	0,005	0,004		
13	0	120	1734,63	0	0,035	-0,04	0,005	0,004		
14	0	120	1726,88	0	0,035	-0,04	0,005	0,004		
15	0	120	1737,924	0	0,035	-0,04	0,005	0,004		
16	0	120	1720,856	0	0,035	-0,04	0,005	0,004		
17	0	120	1753,988	0	0,035	-0,04	0,005	0,004		
18	0	120	1742,67	0	0,035	-0,04	0,005	0,004		
19	0	120	1729,605	0	0,035	-0,04	0,005	0,004		
20	0	120	1734,63	0	0,035	-0,04	0,005	0,004		
21	0	120	1734,912	0	0,035	-0,04	0,005	0,004		
22	0	120	1722,57	0	0,035	-0,04	0,005	0,004		
23	0	120	1725,876	0	0,035	-0,04	0,005	0,004		
24	0	120	1707,804	0	0,035	-0,04	0,005	0,004		
25	0	120	1740,66	0	0,035	-0,04	0,005	0,004		
26	0	120	1740,936	0	0,035	-0,04	0,005	0,004		
27	0	120	1744,68	0	0,035	-0,04	0,005	0,004		
28	0	120	1732,904	0	0,035	-0,04	0,005	0,004		
29	0	120	1731,615	0	0,035	-0,04	0,005	0,004		
30	0	120	1726,88	0	0,035	-0,04	0,005	0,004		
31	0	120	1727,595	0	0,035	-0,04	0,005	0,004		
32	0	120	1730,61	0	0,035	-0,04	0,005	0,004		
33	0	120	1738,928	0	0,035	-0,04	0,005	0,004		
34	0	120	1739,655	0	0,035	-0,04	0,005	0,004		
35	0	120	1735,916	0	0,035	-0,04	0,005	0,004		
36	0	120	1733,625	0	0,035	-0,04	0,005	0,004		
37	0	120	1739,932	0	0,035	-0,04	0,005	0,004		
38	0	120	1752,72	0	0,035	-0,04	0,005	0,004		
39	0	120	1733,908	0	0,035	-0,04	0,005	0,004		
40	0	120	1745,685	0	0,035	-0,04	0,005	0,004		
41	0	120	1746,69	0	0,035	-0,04	0,005	0,004		
42	0	120	1743,675	0	0,035	-0,04	0,005	0,004		
43	0	120	1731,9	0	0,035	-0,04	0,005	0,004		
44	0	120	1742,67	0	0,035	-0,04	0,005	0,004		
45	0	120	1710,816	0	0,035	-0,04	0,005	0,004		
46	0	120	1728,6	0	0,035	-0,04	0,005	0,004		
47	0	120	1741,94	0	0,035	-0,04	0,005	0,004		
48	0	120	1748,7	0	0,035	-0,04	0,005	0,004		
49	0	120	1723,868	0	0,035	-0,04	0,005	0,004		
50	0	120	1749,705	0	0,035	-0,04	0,005	0,004		
51	0	120	1741,665	0	0,035	-0,04	0,005	0,004		
52	0	120	1725,585	0	0,035	-0,04	0,005	0,004		
53	0	120	1736,92	0	0,035	-0,04	0,005	0,004		
54	0	120	1723,868	0	0,035	-0,04	0,005	0,004		
55	0	120	1723,868	0	0,035	-0,04	0,005	0,004		
56	0	120	1735,635	0	0,035	-0,04	0,005	0,004		
57	0	120	1735,635	0	0,035	-0,04	0,005	0,004		
58	0	120	1745,685	0	0,035	-0,04	0,005	0,004		
59	0	120	1728,6	0	0,035	-0,04	0,005	0,004		
60	0	120	1736,64	0	0,035	-0,04	0,005	0,004		
61	0	120	1729,892	0	0,035	-0,04	0,005	0,004		
62	0	120	1713,828	0	0,035	-0,04	0,005	0,004		
63	0	120	1719,555	0	0,035	-0,04	0,005	0,004		
64	0	120	1736,64	0	0,035	-0,04	0,005	0,004		
65	0	120	1729,605	0	0,035	-0,04	0,005	0,004		
66	0	120	1728,888	0	0,035	-0,04	0,005	0,004		
67	0	120	1736,64	0	0,035	-0,04	0,005	0,004		
68	0	120	1748,7	0	0,035	-0,04	0,005	0,004		
69	0	120	1733,908	0	0,035	-0,04	0,005	0,004		
70	0	120	1744,952	0	0,035	-0,04	0,005	0,004		
71	0	120	1717,844	0	0,035	-0,04	0,005	0,004		
72	0	120	1746,96	0	0,035	-0,04	0,005	0,004		
73	0	120	1717,844	0	0,035	-0,04	0,005	0,004		
74	0	120	1725,585	0	0,035	-0,04	0,005	0,004		
75	0	120	1727,595	0	0,035	-0,04	0,005	0,004		
76	0	120	1725,876	0	0,035	-0,04	0,005	0,004		
77	0	120	1730,61	0	0,035	-0,04	0,005	0,004		
78	0	120	1712,52	0	0,035	-0,04	0,005	0,004		
79	0	120	1757,745	0	0,035	-0,04	0,005	0,004		
80	0	120	1744,952	0	0,035	-0,04	0,005	0,004		
81	0	120	1723,575	0	0,035	-0,04	0,005	0,004		
82	0	120	1737,645	0	0,035	-0,04	0,005	0,004		
83	0	120	1737,645	0	0,035	-0,04	0,005	0,004		
84	0	120	1719,555	0	0,035	-0,04	0,005	0,004		
85	0	120	1730,896	0	0,035	-0,04	0,005	0,004		
86	0	120	1735,635	0	0,035	-0,04	0,005	0,004		
87	0	120	1736,64	0	0,035	-0,04	0,005	0,004		
88	0	120	1737,924	0	0,035	-0,04	0,005	0,004		
89	0	120	1742,67	0	0,035	-0,04	0,005	0,004		
90	0	120	1728,6	0	0,035	-0,04	0,005	0,004		
91	0	120	1724,58	0	0,035	-0,04	0,005	0,004		
92	0	120	1733,908	0	0,035	-0,04	0,005	0,004		
93	0	120	1748,7	0	0,035	-0,04	0,005	0,004		
94	0	120	1732,62	0	0,035	-0,04	0,005	0,004		
95	0	120	1747,964	0	0,035	-0,04	0,005	0,004		
96	0	120	1744,68	0	0,035	-0,04	0,005	0,004		
97	0	120	1729,605	0	0,035	-0,04	0,005	0,004		
98	0	120	1732,62	0	0,035	-0,04	0,005	0,004		
99	0	120	1720,56	0	0,035	-0,04	0,005	0,004		
100	0	120	1731,9	0	0,035	-0,04	0,005	0,004		

Table C.2 basic model f=1

f	T	n(k+1)	t(k)	xs1	xs2	xf1	xf2		
1	120	1811,25	1750	0,035	-0,04	0,005	0,004	average	
1	120	1337,22	1292	0,035	-0,04	0,005	0,004		2151,39555
1	120	1944	2025	0,035	-0,04	0,005	0,004		
1	120	1149,12	1197	0,035	-0,04	0,005	0,004		
1	120	2103,36	2191	0,035	-0,04	0,005	0,004	dev std.	
1	120	1237,86	1196	0,035	-0,04	0,005	0,004		934,5285863
1	120	3560,4	3440	0,035	-0,04	0,005	0,004	MAX	
1	120	4456,32	4642	0,035	-0,04	0,005	0,004		5188,8
1	120	2106,225	2035	0,035	-0,04	0,005	0,004	MIN	
1	120	1686,015	1629	0,035	-0,04	0,005	0,004		682,065
1	120	2110,365	2039	0,035	-0,04	0,005	0,004		
1	120	2635,11	2546	0,035	-0,04	0,005	0,004	value less than 1000$	
1	120	1238,895	1197	0,035	-0,04	0,005	0,004		6
1	120	2627,865	2539	0,035	-0,04	0,005	0,004		
1	120	2444,67	2362	0,035	-0,04	0,005	0,004		
1	120	2262,51	2186	0,035	-0,04	0,005	0,004		
1	120	3312	3200	0,035	-0,04	0,005	0,004		
1	120	2836,935	2741	0,035	-0,04	0,005	0,004		
1	120	2105,19	2034	0,035	-0,04	0,005	0,004		
1	120	1956,15	1890	0,035	-0,04	0,005	0,004		
1	120	2271,825	2195	0,035	-0,04	0,005	0,004		
1	120	3552,12	3432	0,035	-0,04	0,005	0,004		
1	120	1446,72	1507	0,035	-0,04	0,005	0,004		
1	120	1336,185	1291	0,035	-0,04	0,005	0,004		
1	120	792,81	766	0,035	-0,04	0,005	0,004		
1	120	1443,84	1504	0,035	-0,04	0,005	0,004		
1	120	1066,56	1111	0,035	-0,04	0,005	0,004		
1	120	2634,24	2744	0,035	-0,04	0,005	0,004		
1	120	3066,705	2963	0,035	-0,04	0,005	0,004		
1	120	1238,895	1197	0,035	-0,04	0,005	0,004		
1	120	1556,64	1504	0,035	-0,04	0,005	0,004		
1	120	3058,425	2955	0,035	-0,04	0,005	0,004		
1	120	3557,295	3437	0,035	-0,04	0,005	0,004		
1	120	2630,4	2740	0,035	-0,04	0,005	0,004		
1	120	1068,48	1113	0,035	-0,04	0,005	0,004		
1	120	3064,32	3192	0,035	-0,04	0,005	0,004		
1	120	1674,24	1744	0,035	-0,04	0,005	0,004		
1	120	2092,77	2022	0,035	-0,04	0,005	0,004		
1	120	1149,885	1111	0,035	-0,04	0,005	0,004		
1	120	1802,88	1878	0,035	-0,04	0,005	0,004		
1	120	2832,795	2737	0,035	-0,04	0,005	0,004		
1	120	682,065	659	0,035	-0,04	0,005	0,004		
1	120	2841,075	2745	0,035	-0,04	0,005	0,004		
1	120	3060,495	2957	0,035	-0,04	0,005	0,004		
1	120	3831,36	3991	0,035	-0,04	0,005	0,004		
1	120	1943,73	1878	0,035	-0,04	0,005	0,004		
1	120	2269,44	2364	0,035	-0,04	0,005	0,004		
1	120	987,39	954	0,035	-0,04	0,005	0,004		
1	120	2438,46	2356	0,035	-0,04	0,005	0,004		
1	120	1245,105	1203	0,035	-0,04	0,005	0,004		
1	120	792,96	826	0,035	-0,04	0,005	0,004		
1	120	1813,32	1752	0,035	-0,04	0,005	0,004		
1	120	1806,075	1745	0,035	-0,04	0,005	0,004		
1	120	1806,075	1745	0,035	-0,04	0,005	0,004		
1	120	2431,215	2349	0,035	-0,04	0,005	0,004		
1	120	2271,825	2195	0,035	-0,04	0,005	0,004		
1	120	4818,96	4656	0,035	-0,04	0,005	0,004		
1	120	1245,12	1297	0,035	-0,04	0,005	0,004		
1	120	4459,815	4309	0,035	-0,04	0,005	0,004		
1	120	2107,2	2195	0,035	-0,04	0,005	0,004		
1	120	2434,56	2536	0,035	-0,04	0,005	0,004		
1	120	4455,675	4305	0,035	-0,04	0,005	0,004		
1	120	1235,52	1287	0,035	-0,04	0,005	0,004		
1	120	4464,99	4314	0,035	-0,04	0,005	0,004		
1	120	2271,36	2366	0,035	-0,04	0,005	0,004		
1	120	1069,44	1114	0,035	-0,04	0,005	0,004		
1	120	1681,875	1625	0,035	-0,04	0,005	0,004		
1	120	2097,945	2027	0,035	-0,04	0,005	0,004		
1	120	2102,085	2031	0,035	-0,04	0,005	0,004		
1	120	1556,64	1504	0,035	-0,04	0,005	0,004		
1	120	1677,735	1621	0,035	-0,04	0,005	0,004		
1	120	3558,33	3438	0,035	-0,04	0,005	0,004		
1	120	1675,665	1619	0,035	-0,04	0,005	0,004		
1	120	1804,005	1743	0,035	-0,04	0,005	0,004		
1	120	2098,56	2186	0,035	-0,04	0,005	0,004		
1	120	1550,4	1615	0,035	-0,04	0,005	0,004		
1	120	1806,72	1882	0,035	-0,04	0,005	0,004		
1	120	1953,045	1887	0,035	-0,04	0,005	0,004		
1	120	2444,67	2362	0,035	-0,04	0,005	0,004		
1	120	1244,07	1202	0,035	-0,04	0,005	0,004		
1	120	789,705	763	0,035	-0,04	0,005	0,004		
1	120	1447,965	1399	0,035	-0,04	0,005	0,004		
1	120	1441,92	1502	0,035	-0,04	0,005	0,004		
1	120	2269,755	2193	0,035	-0,04	0,005	0,004		
1	120	5188,8	5405	0,035	-0,04	0,005	0,004		
1	120	2628,48	2738	0,035	-0,04	0,005	0,004		
1	120	1559,745	1507	0,035	-0,04	0,005	0,004		
1	120	1952,64	2034	0,035	-0,04	0,005	0,004		
1	120	1951,68	2033	0,035	-0,04	0,005	0,004		
1	120	2258,37	2182	0,035	-0,04	0,005	0,004		
1	120	997,74	964	0,035	-0,04	0,005	0,004		
1	120	1237,86	1196	0,035	-0,04	0,005	0,004		
1	120	2269,755	2193	0,035	-0,04	0,005	0,004		
1	120	2260,8	2355	0,035	-0,04	0,005	0,004		
1	120	1812,285	1751	0,035	-0,04	0,005	0,004		
1	120	2105,28	2193	0,035	-0,04	0,005	0,004		
1	120	1957,44	2039	0,035	-0,04	0,005	0,004		
1	120	2637,18	2548	0,035	-0,04	0,005	0,004		
1	120	1239,93	1198	0,035	-0,04	0,005	0,004		
1	120	2837,76	2956	0,035	-0,04	0,005	0,004		

Tables: C.6 re-balance f=1

f	T	n(k+1)	t(k)	average	abav	boom	unav	rec
1	120	1620,78	1589	41	34	11	23	11
1	120	1528,13	1513	43	28	14	22	13
1	120	1034,28	1014	44	26	9	27	14
1	120	1436,16	1408	43	30	9	30	8
1	120	1104,46	1127	46	25	9	27	13
1	120	2022,72	2064	53	26	13	19	9
1	120	2089,69	2069	43	32	13	26	6
1	120	1426,9	1502	43	31	12	19	15
1	120	1128,17	1117	59	18	7	24	12
1	120	1865,47	1847	49	24	17	17	13
1	120	1012,96	974	41	21	16	23	19
1	120	1667,51	1651	51	21	14	25	9
1	120	1137,3	1115	50	23	11	19	17
1	120	1239,7	1265	46	16	16	31	11
1	120	1157,1	1218	49	24	10	22	15
1	120	1164,84	1142	45	25	11	25	14
1	120	1530,15	1515	41	23	19	23	14
1	120	1259,7	1235	57	26	5	19	13
1	120	1149,54	1127	40	29	9	31	11
1	120	1586,71	1571	45	23	16	25	11
1	120	1526,84	1558	49	24	10	31	6
1	120	1107,72	1086	47	26	9	23	15
1	120	1386,73	1373	60	18	10	20	12
1	120	1188,77	1177	51	21	10	26	12
1	120	1063,53	1053	54	22	7	23	14
1	120	1247,35	1235	58	16	9	28	9
1	120	735,98	751	45	19	10	27	19
1	120	1496,25	1575	49	26	13	17	15
1	120	1531,16	1516	48	34	8	16	14
1	120	1571,3	1654	45	22	16	27	10
1	120	1511,97	1497	44	22	18	22	14
1	120	1256,36	1282	40	30	11	26	13
1	120	1260,65	1327	48	21	14	22	15
1	120	1170,59	1159	51	25	7	25	12
1	120	921,12	912	47	22	7	32	12
1	120	1221,08	1246	48	20	13	26	13
1	120	1355,34	1383	54	19	11	26	10
1	120	1028,18	1018	46	22	11	26	15
1	120	1450,44	1422	43	26	14	25	12
1	120	1167,92	1123	42	17	18	29	14
1	120	871,63	863	48	17	12	25	18
1	120	2023,03	2003	62	28	6	17	7
1	120	1261,74	1237	47	21	12	30	10
1	120	1630,14	1614	44	26	17	18	15
1	120	1348,44	1322	39	31	14	19	17
1	120	1479,8	1510	50	24	10	28	8
1	120	1198,87	1187	45	25	11	26	13
1	120	1003,68	984	48	22	6	36	8
1	120	1385,1	1458	53	21	12	21	13
1	120	2084,3	2194	59	28	9	15	9
1	120	1451,37	1437	43	17	19	31	10
1	120	856,8	840	52	19	10	18	21
1	120	1208,97	1197	47	17	16	26	14
1	120	1867,49	1849	43	33	12	23	9
1	120	1379,04	1352	54	24	9	21	12
1	120	1742,25	1725	43	29	14	24	10
1	120	1763,46	1746	61	18	13	17	11
1	120	1267,3	1334	47	22	14	22	15
1	120	1496,25	1575	54	17	15	22	12
1	120	2329,68	2284	56	28	13	13	10
1	120	1314,01	1301	45	25	13	23	14
1	120	1204,42	1229	51	19	10	31	9
1	120	1666,5	1650	50	28	11	20	11
1	120	1108,98	1098	63	17	7	18	15
1	120	1130,5	1190	44	20	16	23	17
1	120	1212,2	1276	55	19	9	26	11
1	120	1530,15	1515	48	19	16	27	10
1	120	1036,45	1091	47	20	13	23	17
1	120	1324,96	1352	54	17	13	24	12
1	120	1253,58	1229	45	27	11	23	14
1	120	1495,81	1481	52	17	16	23	12
1	120	2229,5	2275	48	29	14	22	7
1	120	782,04	798	45	21	7	33	14
1	120	1871,53	1853	42	30	15	23	10
1	120	1149,54	1173	52	20	10	25	13
1	120	1014,9	995	51	26	6	21	16
1	120	1167,56	1156	41	28	9	32	10
1	120	1923,74	1963	52	27	12	20	9
1	120	1751,36	1684	48	30	14	12	16
1	120	1468,04	1498	48	22	16	19	15
1	120	1379,04	1352	48	26	9	28	9
1	120	1577,62	1562	49	22	14	25	10
1	120	1359,26	1387	52	22	11	23	12
1	120	2088,68	2068	48	23	16	28	5
1	120	2059,96	2102	46	31	14	19	10
1	120	1538,23	1523	52	22	12	24	10
1	120	1036,32	1016	44	23	12	25	16
1	120	1866,48	1848	48	21	18	23	10
1	120	2280,46	2327	54	20	20	14	12
1	120	1178,1	1155	48	26	10	20	16
1	120	1770,53	1753	47	25	15	23	10
1	120	789,88	806	44	18	8	39	11
1	120	1012,7	1066	44	29	7	25	15
1	120	773,66	766	50	23	5	24	18
1	120	835,27	827	40	22	10	34	14
1	120	1058,3	1114	47	24	8	29	12
1	120	1050,7	1106	51	20	10	24	15
1	120	1323,1	1310	45	26	12	24	13
1	120	1156,45	1145	44	27	10	25	14
1	120	1471,96	1502	52	21	12	25	10
			1386,63	48,27	23,56	11,82	23,95	12,4

C.7 re-balance f=0,75

f	T	n(k+1)	t(k)	Av	Abav	Boom	Unav	rec
0,75	120	1054,504	772	46	22	9	29	14
0,75	120	1522,496	1138	48	19	16	27	10
0,75	120	1631,422	1224	47	21	19	18	15
0,75	120	1193,764	879	47	23	12	21	17
0,75	120	975,923	717	44	29	3	31	13
0,75	120	1417,484	1038	44	27	13	22	14
0,75	120	1525,036	1127	57	19	11	24	9
0,75	120	1156,12	848	49	22	10	24	15
0,75	120	2104,258	1571	48	31	12	25	4
0,75	120	1711,356	1278	37	31	17	23	12
0,75	120	1186,67	877	50	20	10	28	12
0,75	120	1283,518	957	46	24	13	21	16
0,75	120	1690,534	1268	50	27	11	24	8
0,75	120	1300,128	938	43	19	18	24	16
0,75	120	750,136	563	49	20	7	19	25
0,75	120	1861,268	1390	50	26	13	24	7
0,75	120	1414,447	1034	58	13	14	24	11
0,75	120	1147,052	846	43	23	12	27	15
0,75	120	1282,04	937	46	23	12	26	13
0,75	120	1388,6	1067	56	15	14	22	13
0,75	120	1445,064	1086	53	22	12	20	13
0,75	120	1501,584	1176	42	38	8	18	14
0,75	120	1290,28	950	46	29	6	30	9
0,75	120	1488,032	1078	45	33	7	26	9
0,75	120	1382,316	1029	53	27	8	18	14
0,75	120	1180,652	877	43	33	5	26	13
0,75	120	1439,571	1041	52	26	10	18	14
0,75	120	1255,434	957	46	18	15	28	13
0,75	120	984,784	737	42	25	9	27	17
0,75	120	1625,47	1265	47	23	15	25	10
0,75	120	1248,021	901	49	20	13	23	15
0,75	120	1612,612	1204	48	29	13	15	15
0,75	120	1677,034	1308	57	16	17	17	13
0,75	120	1259,92	935	52	18	11	28	11
0,75	120	1125,952	845	46	21	12	25	16
0,75	120	1258,191	921	53	20	10	24	13
0,75	120	1193,719	881	44	22	13	26	15
0,75	120	1574,576	1210	42	26	15	27	10
0,75	120	1312,624	966	49	27	9	21	14
0,75	120	1301,502	947	52	22	9	26	11
0,75	120	1156,496	829	47	22	10	28	13
0,75	120	1324,72	969	49	24	9	28	10
0,75	120	1294,523	953	47	24	12	22	15
0,75	120	1075,556	770	40	24	13	25	18
0,75	120	1192,968	892	44	19	14	30	13
0,75	120	1241,035	906	41	25	14	24	16
0,75	120	1172,44	864	54	17	10	26	13
0,75	120	1794,664	1350	47	30	13	19	11
0,75	120	1231,264	901	45	21	12	31	11
0,75	120	1286,408	959	49	20	13	24	14
0,75	120	1380,536	1012	43	24	13	30	10
0,75	120	1282,333	931	39	29	12	26	14
0,75	120	1872,336	1395	54	24	12	24	6
0,75	120	1769,554	1327	51	19	18	21	11
0,75	120	1500,396	1093	45	25	12	30	8
0,75	120	1102,14	800	50	18	11	26	15
0,75	120	1401,422	1036	49	22	11	29	9
0,75	120	1754,292	1296	51	22	14	26	7
0,75	120	1987,049	1448	61	18	15	18	8
0,75	120	1780,958	1348	49	24	14	26	7
0,75	120	1214,22	944	42	22	15	25	16
0,75	120	1351,544	1009	52	24	9	23	12
0,75	120	2046,793	1529	46	20	22	23	9
0,75	120	1087,493	836	47	18	12	29	14
0,75	120	1531,768	1173	48	31	9	20	12
0,75	120	1460,236	1124	55	16	16	18	15
0,75	120	1632,946	1244	45	24	17	21	13
0,75	120	1011,476	781	42	25	9	28	16
0,75	120	1073,678	841	44	18	13	31	14
0,75	120	1763,726	1357	55	26	10	21	8
0,75	120	1337,568	975	51	17	16	20	16
0,75	120	1814,312	1323	52	20	17	21	10
0,75	120	1251,424	866	43	17	17	30	13
0,75	120	1183,598	868	49	22	7	34	8
0,75	120	1208,841	892	52	25	8	19	16
0,75	120	954,748	701	33	27	12	30	18
0,75	120	2127,744	1617	55	17	19	23	6
0,75	120	1083,136	821	49	18	10	30	13
0,75	120	1464,975	1088	40	28	13	29	10
0,75	120	1499,898	1143	52	23	10	27	8
0,75	120	1165,208	914	40	28	11	25	16
0,75	120	1788,457	1305	51	27	12	21	9
0,75	120	1207,152	900	44	27	8	30	11
0,75	120	1528,45	1166	50	20	14	27	9
0,75	120	1182,666	897	46	16	16	27	15
0,75	120	1564,752	1162	45	25	14	26	10
0,75	120	1472,024	1092	48	20	14	29	9
0,75	120	1477,972	1072	38	30	14	26	12
0,75	120	1169,535	865	49	21	11	24	15
0,75	120	1490,148	1102	56	22	9	24	9
0,75	120	957,128	710	42	21	11	29	17
0,75	120	1030,372	758	42	28	7	28	15
0,75	120	1509,394	1129	60	21	8	22	9
0,75	120	1470,288	1059	58	20	9	24	9
0,75	120	1773,328	1268	47	29	13	21	10
0,75	120	1207,078	925	46	25	9	28	12
0,75	120	1132,513	915	39	22	13	34	12
0,75	120	1491,86	1144	55	26	9	17	13
0,75	120	1787,236	1325	39	33	16	19	13
0,75	120	1171,516	869	50	24	6	30	10
			1035,71	47,71	23,23	12	24,77	12,29

Tables: C.8 re-balance f=0,5 C.9 re-balance f=0,25

0,5	120	1235,152	642	49	26	7	23	15			0,25	120	1415,922	385	49	22	11	25	13
0,5	120	1683,2	784	41	32	15	20	12			0,25	120	1372,808	335	47	28	6	29	10
0,5	120	1333,798	689	52	22	9	25	12			0,25	120	1665,112	423	50	26	16	17	11
0,5	120	1407,15	708	38	28	15	24	15			0,25	120	1522,45	415	47	19	18	23	13
0,5	120	1408,062	695	51	23	8	31	7			0,25	120	1370,358	336	51	27	4	29	9
0,5	120	1867,468	943	47	24	18	24	7			0,25	120	1537,25	380	46	28	12	24	10
0,5	120	1583,347	804	51	22	13	25	9			0,25	120	1561,175	359	51	28	10	22	9
0,5	120	1436,704	689	49	23	12	24	12			0,25	120	1486,153	388	48	29	11	17	15
0,5	120	1402,628	691	41	28	15	17	19			0,25	120	1678,316	416	45	23	20	21	11
0,5	120	1743,468	896	47	25	16	23	9			0,25	120	1450,684	336	52	21	11	25	11
0,5	120	1373,862	681	39	31	11	25	14			0,25	120	1720,304	430	50	25	16	23	6
0,5	120	1326,595	663	48	30	8	17	17			0,25	120	1566,432	378	49	29	10	25	7
0,5	120	1825,77	904	47	22	18	27	6			0,25	120	1461,432	352	50	27	8	26	9
0,5	120	1373,939	692	42	28	10	29	11			0,25	120	1265,012	312	53	21	5	27	14
0,5	120	1232,031	632	55	15	11	24	15			0,25	120	1586,558	403	59	18	13	23	7
0,5	120	1661,654	850	46	29	12	25	8			0,25	120	1561,273	373	52	20	16	20	12
0,5	120	1097,941	524	37	26	11	27	19			0,25	120	1369,623	362	44	27	11	21	17
0,5	120	1414,144	722	45	25	13	23	14			0,25	120	1594,505	392	49	27	12	25	7
0,5	120	1705,461	888	47	36	7	24	6			0,25	120	1463,954	369	53	23	11	19	14
0,5	120	1580,96	769	44	23	16	28	9			0,25	120	1421,758	368	51	19	12	26	12
0,5	120	1623,733	842	49	28	12	20	11			0,25	120	1320,456	326	52	18	9	29	12
0,5	120	1244,52	636	47	22	11	24	16			0,25	120	1548,395	396	53	17	15	27	8
0,5	120	1372,032	663	46	28	9	25	12			0,25	120	1561,427	395	42	27	16	22	13
0,5	120	1443,531	747	55	26	7	21	11			0,25	120	1516,276	353	47	28	10	27	8
0,5	120	1264,92	620	46	24	11	23	16			0,25	120	1671,642	396	52	25	14	22	7
0,5	120	1148,68	598	47	20	7	36	10			0,25	120	1478,915	357	55	28	6	21	10
0,5	120	1116,308	556	32	24	16	27	21			0,25	120	1409,4	363	34	32	13	25	16
0,5	120	1252,156	638	40	19	16	30	15			0,25	120	1565,341	381	41	35	11	23	10
0,5	120	2244,481	1113	40	30	21	26	3			0,25	120	1516,904	370	47	23	14	26	10
0,5	120	1156,196	539	42	27	9	24	18			0,25	120	1627,751	413	48	34	9	22	7
0,5	120	1445,742	740	53	18	13	25	11			0,25	120	1417,416	362	50	21	11	26	12
0,5	120	1579,244	750	45	22	17	25	11			0,25	120	1345,939	322	42	27	10	25	16
0,5	120	1545,684	786	44	26	14	25	11			0,25	120	1503,015	361	42	30	12	24	12
0,5	120	1307,391	642	50	28	4	29	9			0,25	120	1473,944	381	60	22	7	22	9
0,5	120	1486,032	732	56	21	11	20	12			0,25	120	1535,188	372	45	22	15	30	8
0,5	120	1432,134	695	49	25	10	25	11			0,25	120	1365,212	305	49	19	11	29	12
0,5	120	1620,409	797	50	28	13	15	14			0,25	120	1391,962	356	54	16	13	22	15
0,5	120	1487,754	760	36	28	17	25	14			0,25	120	1371,996	347	42	19	16	26	17
0,5	120	1126,624	575	44	20	10	32	14			0,25	120	1431,424	359	47	27	10	22	14
0,5	120	1514,138	769	44	20	18	26	12			0,25	120	1487,582	382	46	28	12	20	14
0,5	120	1478,064	764	58	18	11	22	11			0,25	120	1623,669	403	53	26	13	18	10
0,5	120	1458,275	704	51	27	10	18	14			0,25	120	1549,307	383	48	27	12	24	9
0,5	120	1258,939	598	60	16	8	22	14			0,25	120	1545,88	359	53	20	15	20	12
0,5	120	1457,265	703	43	29	12	23	13			0,25	120	1696,42	418	40	30	17	26	7
0,5	120	1228,444	638	46	17	13	31	13			0,25	120	1437,708	345	50	23	10	27	10
0,5	120	1240,377	645	38	28	11	28	15			0,25	120	1557,424	380	45	24	15	27	9
0,5	120	1444,345	720	53	22	10	25	10			0,25	120	1368,655	367	48	21	10	30	11
0,5	120	1355,364	688	42	27	13	22	16			0,25	120	1421,625	340	44	23	14	24	15
0,5	120	1531,872	758	54	25	9	23	9			0,25	120	1382,473	335	45	28	6	34	7
0,5	120	1207,012	573	47	23	10	23	17			0,25	120	1731,4	458	43	26	20	23	8
0,5	120	1475,452	722	52	22	11	25	10			0,25	120	1414,084	342	56	19	11	19	15
0,5	120	1392,007	694	52	19	13	22	14			0,25	120	1345,383	338	45	19	13	28	15
0,5	120	1459,264	702	43	26	15	21	15			0,25	120	1479,188	396	39	26	15	28	12
0,5	120	1206,853	608	47	26	7	25	15			0,25	120	1320,104	343	41	25	9	33	12
0,5	120	1338,687	671	40	28	12	25	15			0,25	120	1250,947	316	50	20	6	30	14
0,5	120	1685,028	858	58	19	14	19	10			0,25	120	1877,904	441	48	29	21	11	11
0,5	120	1497,076	760	48	22	15	22	13			0,25	120	1615,648	404	47	30	13	21	9
0,5	120	1837,549	987	51	19	19	24	7			0,25	120	1516,224	388	46	28	11	26	9
0,5	120	1053,83	527	49	13	11	31	16			0,25	120	1560,112	410	44	34	11	19	12
0,5	120	1621,332	830	46	28	13	23	10			0,25	120	1260,816	293	41	21	11	30	17
0,5	120	1304,8	644	47	20	13	26	14			0,25	120	1454,948	386	44	24	13	28	11
0,5	120	1238,386	596	44	21	13	26	16			0,25	120	1477,004	384	53	21	12	22	12
0,5	120	1294,308	637	39	19	17	31	14			0,25	120	1503,19	386	50	27	10	22	11
0,5	120	1696,917	837	64	20	9	21	6			0,25	120	1492,764	365	46	32	8	24	10
0,5	120	1322,864	673	38	31	10	28	13			0,25	120	1671,248	409	55	26	12	21	6
0,5	120	1432,59	741	46	30	9	23	12			0,25	120	1338,528	326	45	24	9	29	13
0,5	120	1660,85	841	51	30	11	17	11			0,25	120	1586,586	407	56	28	11	12	13
0,5	120	1710,11	859	58	24	10	21	7			0,25	120	1345,826	358	48	25	8	24	15
0,5	120	1581,288	815	49	27	12	21	11			0,25	120	1410,02	355	54	20	11	20	15
0,5	120	1504,44	390	48	21	15	24	12			0,25	120	1596,504	391	53	22	14	22	9
0,5	120	1988,956	1004	58	28	12	15	7			0,25	120	1414,737	359	50	23	9	29	9
0,5	120	1158,129	609	46	19	10	32	13			0,25	120	1407,772	367	56	16	11	25	12
0,5	120	1629,856	785	47	32	11	18	12			0,25	120	1379,266	341	53	20	9	26	12
0,5	120	1203,404	603	41	19	13	34	13			0,25	120	1262,022	322	54	21	5	24	16
0,5	120	1290,564	674	51	22	9	25	13			0,25	120	1595,204	367	43	27	16	23	11
0,5	120	1403,828	705	39	28	14	24	15			0,25	120	1429,698	347	51	25	10	20	14
0,5	120	1033,744	521	47	23	5	29	16			0,25	120	1588,788	435	48	24	14	27	7
0,5	120	1929,936	933	49	28	14	25	4			0,25	120	1476,56	382	50	23	13	21	13
0,5	120	1378,098	713	42	31	10	23	14			0,25	120	1452,534	381	40	25	14	30	11
0,5	120	1400,432	681	42	33	9	22	14			0,25	120	1399,89	360	48	27	8	25	12
0,5	120	1681,475	848	54	23	15	15	13			0,25	120	1511,214	386	49	26	12	21	12
0,5	120	1366,897	689	54	23	9	20	14			0,25	120	1433	358	42	27	13	23	15
0,5	120	1312,756	677	50	20	11	26	13			0,25	120	1522,388	394	45	26	13	25	11
0,5	120	1605,764	776	56	23	11	20	10			0,25	120	1235,594	313	49	18	6	35	12
0,5	120	1866,64	954	43	29	17	23	8			0,25	120	1612,643	404	55	23	12	24	6
0,5	120	1201,691	587	46	24	8	29	13			0,25	120	1432,604	370	51	25	9	23	12
0,5	120	1601,48	822	52	26	11	21	10			0,25	120	1510,112	382	44	21	16	29	10
0,5	120	1268,156	634	51	20	11	23	15			0,25	120	1303,948	321	45	14	14	32	15
0,5	120	1636,044	786	53	24	11	25	7			0,25	120	1297,085	316	52	17	9	28	14
0,5	120	927,116	479	43	23	5	30	19			0,25	120	1461,232	350	46	17	17	27	13
0,5	120	936,276	484	44	18	8	32	18			0,25	120	1517,197	381	55	21	12	22	10
0,5	120	1774,358	911	51	29	13	16	11			0,25	120	1470	373	49	21	15	19	16
0,5	120	1822,96	866	61	24	11	16	8			0,25	120	1564,527	408	53	21	14	22	10
0,5	120	1330,586	660	43	31	8	25	13			0,25	120	1578,328	379	43	32	14	17	14
0,5	120	1361,138	664	49	19	13	27	12			0,25	120	1585,868	411	47	36	8	20	9
0,5	120	1318,916	669	45	25	11	25	14			0,25	120	1184,5	300	47	17	7	33	16
0,5	120	1574,048	739	46	22	15	29	8			0,25	120	1405,406	337	49	19	12	29	11
0,5	120	1521,968	747	46	25	12	29	8			0,25	120	1448,804	354	45	24	13	26	12
0,5	120	1539,194	801	53	25	9	25	8			0,25	120	1332,172	341	50	19	10	27	14
0,5	120	1409,226	718	57	20	11	17	15			0,25	120	1365,457	340	51	22	9	24	14
			719,86	47,52	24,35	11,74	24,2	12,19						369,14	48,34	24,1	11,72	24,35	11,49

Tables: C.10 re-balance f=0

0	120	1453,902	0	46	26	10	22	16
0	120	1484,44	0	53	31	9	20	7
0	120	1417,239	0	52	21	6	28	13
0	120	1461,918	0	58	21	8	27	6
0	120	1479,744	0	41	26	13	25	15
0	120	1477,419	0	57	29	9	14	11
0	120	1457,359	0	53	30	7	20	10
0	120	1507,509	0	46	31	13	22	8
0	120	1495,986	0	49	19	16	24	12
0	120	1495,473	0	56	20	13	23	8
0	120	1435,434	0	42	25	9	30	14
0	120	1453,347	0	46	27	9	28	10
0	120	1532,104	0	54	23	17	19	7
0	120	1492,464	0	54	23	13	23	7
0	120	1501,998	0	51	27	12	25	5
0	120	1528,128	0	45	27	17	21	10
0	120	1409,408	0	45	22	8	24	21
0	120	1494,493	0	51	26	12	22	9
0	120	1485,443	0	47	26	14	16	17
0	120	1404,2	0	51	19	6	26	18
0	120	1468,392	0	48	31	9	19	13
0	120	1462,374	0	41	26	10	33	10
0	120	1489,455	0	55	22	13	17	13
0	120	1481,431	0	45	32	10	23	10
0	120	1491,978	0	40	26	15	24	15
0	120	1477,95	0	50	27	11	21	11
0	120	1444,884	0	49	21	10	26	14
0	120	1507,509	0	48	18	18	21	15
0	120	1499,485	0	53	23	14	18	12
0	120	1474,473	0	44	23	13	27	13
0	120	1489,974	0	49	26	12	20	13
0	120	1492,491	0	58	22	12	20	8
0	120	1473,407	0	43	28	11	27	11
0	120	1473,872	0	38	27	13	25	17
0	120	1474,41	0	57	24	10	21	8
0	120	1492,464	0	47	28	13	19	13
0	120	1504,944	0	45	19	18	24	14
0	120	1449,776	0	44	16	13	31	16
0	120	1516,536	0	51	20	17	22	10
0	120	1537,599	0	42	23	20	26	9
0	120	1491,461	0	52	23	13	22	10
0	120	1451,45	0	52	27	7	24	10
0	120	1503,497	0	49	22	15	23	11
0	120	1470,398	0	50	29	9	20	12
0	120	1495,473	0	50	30	12	16	12
0	120	1518,03	0	48	26	15	25	6
0	120	1565,683	0	45	24	22	20	9
0	120	1491,978	0	45	27	14	18	16
0	120	1515,024	0	49	25	17	18	11
0	120	1515,024	0	41	31	16	15	17
0	120	1474,944	0	42	26	12	27	13
0	120	1462,374	0	53	15	13	24	15
0	120	1493,467	0	51	22	14	23	10
0	120	1477,419	0	54	23	11	23	9
0	120	1464,463	0	47	24	11	23	15
0	120	1458,362	0	48	16	13	32	11
0	120	1483,437	0	52	19	13	28	8
0	120	1496,476	0	47	15	18	25	15
0	120	1497,968	0	45	22	15	27	11
0	120	1465,926	0	46	26	12	17	19
0	120	1503,497	0	44	22	16	28	10
0	120	1474,41	0	56	24	10	18	12
0	120	1445,323	0	49	31	6	22	12
0	120	1488,452	0	44	27	14	20	15
0	120	1464,836	0	51	25	9	27	8
0	120	1473,407	0	58	21	10	21	10
0	120	1448,332	0	54	22	8	25	11
0	120	1549,635	0	44	25	21	16	14
0	120	1456,908	0	51	22	10	25	12
0	120	1485,484	0	51	26	12	19	12
0	120	1428,852	0	51	23	6	27	13
0	120	1435,293	0	45	22	8	36	9
0	120	1553,647	0	47	23	20	22	8
0	120	1455,906	0	44	21	12	30	13
0	120	1488,932	0	45	25	14	24	12
0	120	1443,317	0	46	27	8	29	10
0	120	1460,368	0	49	20	11	28	12
0	120	1481,431	0	53	23	12	23	9
0	120	1480,956	0	42	29	11	31	7
0	120	1515,533	0	51	25	15	21	8
0	120	1417,416	0	38	24	8	35	15
0	120	1460,592	0	57	23	8	26	6
0	120	1440,876	0	51	23	8	28	10
0	120	1449,335	0	42	22	11	31	14
0	120	1530,578	0	43	26	19	17	15
0	120	1432,86	0	45	23	9	24	19
0	120	1528,088	0	43	34	15	15	13
0	120	1480,428	0	54	16	14	26	10
0	120	1472,471	0	54	24	10	20	12
0	120	1459,365	0	51	23	10	23	13
0	120	1460,82	0	47	27	10	22	14
0	120	1439,305	0	49	23	8	26	14
0	120	1498,972	0	50	27	12	24	7
0	120	1485,92	0	42	28	12	31	7
0	120	1462,828	0	49	24	11	22	14
0	120	1496,476	0	47	26	14	17	16
0	120	1450,338	0	53	20	10	25	12
0	120	1432,284	0	52	20	8	27	13
0	120	1513,527	0	45	17	19	28	11
0	120	1399,576	0	42	16	8	31	23
				48,39	24,02	12,12	23,63	11,84

C.11 Multiperiod model f=1

1000	1370,88	121	4	1	1311,19	0,000708	-0,19587	
1000	1319,43	121	4	1	1315,51	0,000692	-0,28893	
1000	1659,33	121	7	1	1534,7	0,000684	0,325818	
1000	1083,88	121	7	1	1083,88	0,000559	-0,71494	
1000	1197,26	121	3	0,85	1138,56	0,000634	-0,50988	
1000	1943,04	121	4	1	1867,39	0,000507	0,838936	
1000	943,94	121	6	0,7	906,552	0,000452	-0,96804	
1000	1121,67	121	5	1	1121,67	0,000585	-0,64659	
1000	1174,775	121	3	0,85	1145,28	0,00062	-0,55055	
1000	913,2	121	4	0,85	978,24	0,000427	-1,02363	
1000	1506,89	121	7	1	1524,4	0,000721	0,050115	
1000	1866,088	121	7	0,85	2076,48	0,000565	0,69976	
1000	1080,47	121	5	1	1076,495	0,000556	-0,72111	
1000	963,065	121	4	0,85	938,88	0,000467	-0,93345	
1000	844,185	121	3	0,85	827,52	0,000373	-1,14845	
1000	1792,2	121	6	1	1718,04	0,000615	0,566126	
1000	1363,72	121	2	1	1363,72	0,000706	-0,20882	
1000	1461,57	121	5	1	1540,355	0,000721	-0,03185	
1000	1807,68	121	6	0,85	1807,68	0,000605	0,594124	
1000	2228,92	121	5	1	2164,03	0,000288	1,355978	
1000	1295,74	121	5	1	1153,6	0,000683	-0,33177	
1000	2061,895	121	9	1	2061,895	0,000414	1,053897	
1000	1121,67	121	7	1	1038,24	0,000585	-0,64659	
1000	1807,96	121	6	0,85	1776,96	0,000605	0,59463	
1000	1020,8	121	3	0,85	1153,92	0,000512	-0,82903	
1000	1968,92	121	6	0,85	1790,4	0,000487	0,885742	
1000	1309,44	121	6	1	1313,25	0,000688	-0,30699	
1000	1081,5	121	7	1	1018,76	0,000557	-0,71925	
1000	1821,884	121	6	0,85	1867,2	0,000595	0,619813	
1000	2386,56	121	7	1	2189,78	0,000188	1,641086	
1000	866,325	121	5	0,85	801,6	0,00039	-1,10841	
1000	2187,332	121	9	0,7	2100,644	0,000318	1,280762	
1000	938,145	121	3	0,85	823,175	0,000447	-0,97852	
1000	1796	121	10	0,7	1690,04	0,000612	0,572999	
1000	870,815	121	3	0,85	802,575	0,000394	-1,10029	
1000	1077,38	121	5	1	1066,9	0,000554	-0,7267	
1000	2755,2	121	7	1	2980,82	5,03E-05	2,307808	
1000	1159,78	121	3	1	1120,64	0,000611	-0,57767	
1000	856,192	121	4	0,85	948,48	0,000382	-1,12674	
1000	840,012	121	3	0,85	816,67	0,00037	-1,156	
1000	958,568	121	4	0,85	936,96	0,000463	-0,94158	
1000	956,16	121	2	0,85	956,16	0,000461	-0,94594	
1000	927,215	121	4	0,85	931,2	0,000438	-0,99829	
1000	2761,43	121	6	1	2453,46	4,9E-05	2,319076	
1000	1106,795	121	2	0,85	978,24	0,000575	-0,6735	
1000	1567,66	121	5	1	1510,81	0,000712	0,160023	
1000	899,13	121	4	0,85	941,76	0,000416	-1,04908	
1000	1410,07	121	6	1	1350,33	0,000716	-0,12499	
1000	3397,97	121	8	1	3442,26	1,75E-06	3,470322	
1000	1029,12	121	3	1	1114,46	0,000518	-0,81398	
1000	2043,53	121	11	0,85	2071,68	0,000429	1,020682	
1000	1813,44	121	7	1	2120,77	0,000601	0,604541	
1000	2061,03	121	7	1	2016,74	0,000415	1,052332	
1000	1143,3	121	5	1	1110,335	0,0006	-0,60747	
1000	2887,09	121	9	1	2802,63	2,82E-05	2,546344	
1000	778,124	121	3	0,85	799,68	0,000323	-1,26793	
1000	1499,455	121	4	0,85	1348,8	0,000721	0,036668	
1000	1554,75	121	7	0,85	1539,84	0,000715	0,136674	
1000	2079,57	121	5	1	2165,06	0,0004	1,085864	
1000	1393,296	121	7	0,85	1519,68	0,000713	-0,15533	
1000	1078,41	121	5	1	1082,85	0,000555	-0,72483	
1000	956,87	121	4	1	951,335	0,000462	-0,94465	
1000	1432,93	121	6	0,85	1314,24	0,000719	-0,08365	
1000	1433,76	121	7	1	1468,78	0,000719	-0,08215	
1000	1467,75	121	7	1	1484,23	0,000721	-0,02067	
1000	1507,92	121	5	1	1525,43	0,000721	0,051978	
1000	3145,92	121	9	1	2924,17	7,67E-06	3,014464	
1000	1778,81	121	6	1	1752,03	0,000623	0,541909	
1000	1360,125	121	4	0,85	1324,8	0,000705	-0,21532	
1000	1360,612	121	6	0,85	1338,24	0,000705	-0,21444	
1000	1226,22	121	5	0,7	1128,46	0,00065	-0,4575	
1000	1277,2	121	4	1	1343,12	0,000675	-0,3653	
1000	1933,31	121	4	1	1822,07	0,000515	0,821338	
1000	1245,27	121	4	1	1301,265	0,00066	-0,42305	
1000	1040,19	121	4	0,85	927,36	0,000526	-0,79396	
1000	1812,48	121	8	1	1642,85	0,000602	0,602805	
1000	1065,02	121	2	1	982,06	0,000545	-0,74905	
1000	1648	121	6	1	1742,76	0,000689	0,305326	
1000	1685,08	121	5	1	1559,42	0,000673	0,372389	
1000	2395,78	121	5	1	2146,52	0,000183	1,657761	
1000	1056	121	3	1	1135,205	0,000538	-0,76536	
1000	1614,01	121	7	1	1506,89	0,0007	0,243852	
1000	2718,17	121	9	1	2718,17	5,86E-05	2,240836	
1000	790,215	121	5	0,7	786,28	0,000332	-1,24606	
1000	1251,45	121	5	1	1096,95	0,000663	-0,41187	
1000	1461,77	121	7	0,85	1479,36	0,000721	-0,03149	
1000	1106,24	121	4	0,85	981,12	0,000575	-0,6745	
1000	1271,02	121	1	1	1163,9	0,000672	-0,37648	
1000	1454,4	121	7	1	1444,46	0,000721	-0,04482	
1000	1512,845	121	9	0,85	1497,6	0,00072	0,060885	
1000	1630,49	121	7	1	1583,11	0,000695	0,273658	
1000	1939,49	121	4	1	1867,39	0,00051	0,832515	
1000	1624,31	121	7	1	1546,03	0,000697	0,262481	
1000	2228,92	121	5	1	2220,68	0,000288	1,355978	
1000	884,77	121	4	1	943,12	0,000405	-1,07505	
1000	1567,68	121	3	1	1568,69	0,000712	0,16006	
1000	767,696	121	5	0,85	792,4	0,000315	-1,28679	
1000	1368,87	121	4	1	1340,03	0,000707	-0,19951	
1000	906,34	121	6	0,85	933,665	0,000422	-1,03604	
1000	778,292	121	3	0,85	818,125	0,000323	-1,26763	

C.12 Multiperiod model f=0,75

1000	1294,71	1257	121	4	1	1309,22	0,000733	-0,50959
1000	1626,24	1694	121	5	1	1568,69	0,00082	0,183837
1000	2765,76	2881	121	9	1	2881,265	3,09E-05	2,567245
1000	834,492	521	121	1	0,6	843,708	0,000282	-1,47217
1000	928,996	655	121	4	0,7	940,856	0,00037	-1,27451
1000	1080,69	897	121	3	0,85	1132,8	0,000528	-0,95723
1000	2254,48	1900	121	10	0,85	2289,6	0,000272	1,497858
1000	2013,65	1955	121	7	1	2114,59	0,000509	0,99414
1000	1461,57	1419	121	7	1	1458,205	0,000824	-0,16058
1000	1921,92	2002	121	4	1	1824,13	0,000605	0,802279
1000	2208	2300	121	5	1	2122,83	0,000313	1,40064
1000	1300,8	1355	121	4	1	1320,285	0,000738	-0,49685
1000	2181,54	2118	121	5	1	2201,11	0,000338	1,345297
1000	1158,196	1082	121	3	0,9	1146,525	0,000608	-0,79512
1000	1292,972	1145	121	4	0,85	1303,68	0,000731	-0,51322
1000	1091,452	956	121	3	0,85	1128	0,000539	-0,93472
1000	919,944	701	121	2	0,75	963,1	0,000361	-1,29344
1000	1784,64	1859	121	4	1	1859,15	0,000731	0,515145
1000	1901,38	1846	121	6	1	1787,05	0,000625	0,759317
1000	1588,8	1655	121	5	1	1527,49	0,00083	0,105528
1000	1456,42	1414	121	4	1	1322,685	0,000822	-0,17136
1000	1132,18	826	121	2	0,75	992,415	0,000582	-0,84953
1000	1583,04	1649	121	7	1	1460,54	0,000831	0,093481
1000	2077,51	2017	121	7	1	2077,51	0,000442	1,127709
1000	3054,98	2966	121	7	1	2904,6	5,45E-06	3,172175
1000	1413,12	1472	121	5	1	1511,01	0,000806	-0,26192
1000	1521,31	1477	121	5	1	1521,31	0,000834	-0,03563
1000	815,288	638	121	3	0,6	815,288	0,000266	-1,51234
1000	2242,31	2177	121	9	1	2022,92	0,000282	1,472403
1000	1575,448	1388	121	5	0,85	1558,08	0,000832	0,077601
1000	1404,845	1160	121	8	0,85	1269,12	0,000803	-0,27923
1000	1252,652	1103	121	6	0,85	1282,56	0,000698	-0,59756
1000	1216,04	1085	121	5	0,9	1102,41	0,000665	-0,67413
1000	2285,76	2381	121	6	1	2565,73	0,000246	1,563282
1000	1671,69	1623	121	7	1	1519,805	0,000803	0,2789
1000	1889,28	1968	121	4	1	1858,12	0,000637	0,734009
1000	1700,53	1651	121	6	1	1787,86	0,000788	0,339222
1000	1077,5	890	121	5	0,85	1096,32	0,000524	-0,9639
1000	1340,03	1301	121	2	1	1347,985	0,000766	-0,4148
1000	1145,25	987	121	1	0,9	1155,265	0,000595	-0,8222
1000	1960,32	2042	121	7	1	2077,51	0,000565	0,882596
1000	1087,92	945	121	5	0,85	1125,12	0,000535	-0,94211
1000	1803,61	1513	121	5	0,85	1583,04	0,000715	0,554823
1000	1114,06	967	121	5	0,85	1084,8	0,000563	-0,88743
1000	1047,832	989	121	5	0,9	1087,185	0,000493	-1,02595
1000	1409,04	1368	121	4	1	1294,5	0,000804	-0,27046
1000	1081,94	944	121	5	0,85	1112,64	0,000529	-0,95461
1000	1036,915	754	121	2	0,75	979,452	0,000481	-1,04879
1000	1213,21	1012	121	5	0,85	1108,8	0,000662	-0,68005
1000	2099,52	2187	121	7	1	2155,475	0,000419	1,173745
1000	1040,372	976	121	7	0,9	1060,305	0,000485	-1,04156
1000	1913,74	1858	121	4	1	1798,38	0,000613	0,785169
1000	1059,455	731	121	5	0,7	1105,244	0,000505	-1,00164
1000	1703,62	1654	121	3	1	1601,65	0,000786	0,345685
1000	1781,9	1730	121	3	1	1561,48	0,000733	0,509414
1000	1179,244	1060	121	6	0,85	1272	0,000629	-0,75109
1000	1870,035	1557	121	12	0,85	1654,715	0,000656	0,693756
1000	1161,736	1070	121	1	0,9	1173,985	0,000612	-0,78771
1000	2447,28	2376	121	8	1	2447,28	0,000137	1,901116
1000	1698,47	1649	121	5	1	1566,63	0,000789	0,334913
1000	1126,275	831	121	3	0,9	1126,275	0,000576	-0,86188
1000	986,348	926	121	5	0,9	1118,89	0,000428	-1,15455
1000	2139,31	2077	121	5	1	2135,19	0,000379	1,256969
1000	1037,115	762	121	4	0,75	947,164	0,000482	-1,04837
1000	2516,29	2443	121	6	1	2509,08	0,000103	2,045457
1000	2282,88	2378	121	7	1	2076,48	0,000248	1,557259
1000	1451,27	1409	121	5	1	1565,6	0,000821	-0,18213
1000	2145,49	2083	121	4	1	1878,72	0,000373	1,269895
1000	1658,3	1610	121	4	1	1796,32	0,000809	0,250894
1000	1355,644	1181	121	4	0,85	1330,56	0,000776	-0,38214
1000	825,372	637	121	3	0,6	825,372	0,000274	-1,49125
1000	866,255	641	121	2	0,75	959,468	0,000311	-1,40574
1000	1502,26	1340	121	7	0,85	1463,04	0,000832	-0,07548
1000	937,224	719	121	4	0,75	920,416	0,000379	-1,2573
1000	929,84	864	121	4	0,75	929,84	0,000371	-1,27275
1000	1219,976	1047	121	3	0,85	1140,48	0,000668	-0,6659
1000	1547,06	1502	121	2	1	1358,61	0,000834	0,018225
1000	1636,67	1589	121	5	1	1578,04	0,000817	0,205653
1000	1535,91	1257	121	5	0,85	1496,64	0,000834	-0,0051
1000	2466,85	2395	121	8	1	2469,94	0,000127	1,942048
1000	1216,296	1085	121	4	0,85	1336,32	0,000665	-0,6736
1000	1920,96	2001	121	9	1	1937,09	0,000606	0,800271
1000	1929,6	2010	121	7	1	2065,79	0,000597	0,818342
1000	1195,944	1114	121	3	0,9	1143,16	0,000646	-0,71617
1000	1350,33	1311	121	4	1	1310,98	0,000772	-0,39325
1000	1128,232	1010	121	3	0,85	1126,08	0,000578	-0,85779
1000	2145,49	2083	121	5	1	2148,58	0,000373	1,269895
1000	1625,34	1578	121	3	1	1569,72	0,000821	0,181955
1000	1329,6	1385	121	6	1	1296,885	0,000759	-0,43661
1000	1260,765	1065	121	3	0,85	1160,64	0,000705	-0,58059
1000	1129,64	845	121	0	0,75	1000	0,000579	-0,85485
1000	1513,875	1260	121	7	0,85	1482,075	0,000833	-0,05118
1000	1156,26	945	121	3	0,85	1120,32	0,000606	-0,79917
1000	1364,775	1125	121	4	0,85	1335,36	0,000781	-0,36304
1000	1149,395	992	121	3	0,9	1119,97	0,000599	-0,81353
1000	1870,48	1816	121	4	1	1829,28	0,000656	0,694687
1000	1416	1475	121	5	1	1612,98	0,000808	-0,2559
1000	2216,56	2152	121	5	1	2136,22	0,000305	1,418544
1000	2128,32	2217	121	7	1	2151,67	0,00039	1,233983
1000	1444,8	1505	121	5	1	1533,67	0,000819	-0,19566

C.13 Multiperiod model f=0.5

1000	2195,96	2132	121	5	1	2132,1	0,000279	1,601693	
1000	1018,74	669	121	3	0,65	1141,352	0,000393	-1,37191	
1000	1597,428	1573	121	3	0,95	1571,365	0,001004	0,089828	
1000	2001,29	1943	121	4	1	1798,365	0,000544	1,109965	
1000	1210,78	955	121	2	0,8	1333,38	0,00068	-0,88683	
1000	1842,24	1919	121	6	1	1785,655	0,000784	0,708212	
1000	1513,205	1394	121	3	0,95	1564,785	0,001	-0,12292	
1000	1230,705	747	121	3	0,65	1119,448	0,00071	-0,8365	
1000	1398,925	1084	121	4	0,8	1284,936	0,000926	-0,41158	
1000	1272,792	1056	121	4	0,8	1314,16	0,000772	-0,73019	
1000	1146,604	775	121	3	0,65	1146,276	0,000581	-1,04893	
1000	1218,768	1006	121	2	0,8	1344,91	0,000692	-0,86665	
1000	1326,744	1313	121	5	0,95	1482,33	0,000845	-0,59391	
1000	1336,62	1044	121	2	0,8	1346,115	0,000857	-0,56896	
1000	2507,02	2434	121	6	1	2507,02	5,83E-05	2,387416	
1000	1543,805	1211	121	2	0,8	1354,155	0,001007	-0,04562	
1000	1565,484	1546	121	3	0,95	1573,325	0,001008	0,009139	
1000	1270,12	796	121	3	0,65	1122,628	0,000768	-0,73694	
1000	1333,13	1025	121	2	0,8	1360,535	0,000853	-0,57778	
1000	1420,78	1161	121	2	0,8	1351,565	0,000946	-0,35638	
1000	1523,02	1264	121	7	0,85	1492,925	0,001003	-0,09812	
1000	1615,025	1487	121	3	0,95	1578,625	0,000999	0,134277	
1000	1402,98	1383	121	3	0,95	1550,54	0,00093	-0,40134	
1000	1336,895	1055	121	4	0,8	1284,328	0,000857	-0,56827	
1000	1626,405	1500	121	3	0,95	1581,29	0,000994	0,163023	
1000	1376,312	1122	121	2	0,8	1346,895	0,000903	-0,4687	
1000	1447,71	1146	121	2	0,8	1343,355	0,000967	-0,28835	
1000	1412,79	1116	121	4	0,8	1338	0,000939	-0,37656	
1000	1826,63	1670	121	6	1	1826,63	0,000806	0,668782	
1000	1083,425	524	121	2	0,5	963,85	0,000485	-1,20852	
1000	3229,05	3135	121	7	1	3014,81	1,42E-07	4,211231	
1000	1201,224	994	121	4	0,8	1320,165	0,000665	-0,91097	
1000	1532,12	1427	121	3	0,95	1563,75	0,001005	-0,07514	
1000	1786,56	1861	121	4	1	1780,905	0,000858	0,567567	
1000	1405,896	1408	121	3	0,95	1551,62	0,000932	-0,39397	
1000	1289,715	816	121	1	0,65	1158,09	0,000796	-0,68744	
1000	2721,26	2642	121	6	1	2547,19	1,38E-05	2,928577	
1000	1061,29	523	121	0	0,5	1000	0,000453	-1,26443	
1000	1151,31	747	121	1	0,65	1155,605	0,000589	-1,03705	
1000	1415,305	1297	121	3	0,95	1567,7	0,000941	-0,37021	
1000	1375,71	1080	121	2	0,8	1341,77	0,000902	-0,47022	
1000	1118,808	745	121	1	0,65	1151,08	0,000539	-1,11914	
1000	1960,09	1903	121	4	1	1815,4	0,000608	1,005896	
1000	1121,68	771	121	1	0,65	1161,005	0,000543	-1,11189	
1000	1275,356	838	121	1	0,65	1163,015	0,000776	-0,72371	
1000	1288,585	1012	121	4	0,8	1350,26	0,000794	-0,6903	
1000	1221,888	821	121	1	0,65	1166,505	0,000697	-0,85877	
1000	1461,005	1115	121	4	0,8	1322,145	0,000976	-0,25477	
1000	1908,59	1853	121	4	1	1798,77	0,000687	0,875809	
1000	1328,912	1104	121	2	0,8	1358,55	0,000848	-0,58843	
1000	2369,28	2468	121	6	1	2467,88	0,000126	2,039491	
1000	1318,66	1009	121	4	0,8	1305,008	0,000834	-0,61433	
1000	1443,415	1135	121	2	0,8	1372,72	0,000964	-0,2992	
1000	1845,12	1922	121	4	1	1852,88	0,00078	0,715487	
1000	1598,47	1468	121	3	0,95	1558,025	0,001003	0,09246	
1000	1272,33	1005	121	4	0,8	1313,205	0,000771	-0,73135	
1000	1959,06	1902	121	6	1	1741,49	0,000609	1,003294	
1000	1769,595	1479	121	6	1	1769,595	0,000878	0,524714	
1000	1551,31	1270	121	5	0,85	1469,76	0,001007	-0,02666	
1000	1603,4	1597	121	3	0,95	1593,8	0,001002	0,104913	
1000	1193,168	812	121	3	0,65	1135,804	0,000653	-0,93131	
1000	1196,005	766	121	3	0,65	1108,545	0,000657	-0,92415	
1000	1500,745	1177	121	2	0,8	1349,23	0,000996	-0,15439	
1000	1224,432	786	121	1	0,65	1157,115	0,000701	-0,85234	
1000	1908,48	1988	121	4	1	1870,465	0,000687	0,875531	
1000	1579,2	1461	121	5	0,95	1464,995	0,001007	0,043785	
1000	1432,085	1124	121	4	0,8	1327,008	0,000955	-0,32782	
1000	1616,672	1592	121	3	0,95	1573,93	0,000998	0,138437	
1000	1307,3	1037	121	4	0,8	1302,4	0,00082	-0,64302	
1000	1522,475	1403	121	3	0,95	1575,71	0,001003	-0,0995	
1000	1798,715	1478	121	6	0,85	1779,84	0,000843	0,59827	
1000	2144,46	2082	121	5	1	2147,55	0,000341	1,471606	
1000	1286,496	1064	121	4	0,8	1345,79	0,000791	-0,69557	
1000	1339,41	1035	121	2	0,8	1335,715	0,000861	-0,56191	
1000	2180,51	2117	121	5	1	2180,51	0,000297	1,562667	
1000	1287,225	837	121	1	0,65	1153,57	0,000792	-0,69373	
1000	1734,52	1684	121	4	1	1794,245	0,000916	0,436116	
1000	1736,58	1686	121	4	1	1857,505	0,000914	0,44132	
1000	1500,72	1176	121	2	0,8	1335,865	0,000996	-0,15445	
1000	1246,45	814	121	1	0,65	1151,18	0,000734	-0,79673	
1000	1859,52	1937	121	4	1	1868,28	0,00076	0,75186	
1000	1207,664	802	121	3	0,65	1141	0,000675	-0,8947	
1000	1169,425	748	121	3	0,65	1147,99	0,000617	-0,99129	
1000	2132,1	2070	121	4	1	1878,63	0,000357	1,440385	
1000	1792,2	1740	121	4	1	1812,16	0,000851	0,581813	
1000	2391,66	2322	121	5	1	2155,79	0,000112	2,096022	
1000	1221,824	1005	121	2	0,8	1353,35	0,000697	-0,85893	
1000	1620,15	1491	121	3	0,95	1596,46	0,000997	0,147223	
1000	1576,31	1466	121	3	0,95	1537,25	0,001007	0,036485	
1000	1672,32	1742	121	4	1	1837,68	0,000969	0,279002	
1000	1252,404	812	121	1	0,65	1161,08	0,000742	-0,78169	
1000	2108,528	1891	121	8	0,85	2438,4	0,000388	1,380843	
1000	1319,51	842	121	1	0,65	1160,425	0,000836	-0,61218	
1000	1572,115	1459	121	5	0,95	1531,595	0,001007	0,025888	
1000	2004,38	1946	121	4	1	1803,365	0,00054	1,11777	
1000	1776,75	1725	121	4	1	1817,08	0,00087	0,542787	
1000	1185,505	748	121	3	0,65	1117,872	0,000641	-0,95067	
1000	1436,484	1192	121	2	0,8	1342,45	0,000958	-0,31671	
1000	1344,64	1317	121	3	0,95	1566,015	0,000867	-0,5487	
1000	2620,14	2175	121	8	0,85	2313,6	2,83E-05	2,673152	

C.14 Multiperiod model f=0.25

1000	1614,015	1125	121	3	0,7	1555,785	0,001624	0,061781
1000	1938,46	1882	121	5	1	2065,465	0,000623	1,385252
1000	1299,668	719	121	2	0,55	1330,025	0,000773	-1,2205
1000	1735,535	1202	121	3	0,7	1558,45	0,001393	0,557483
1000	1591,116	1146	121	3	0,7	1566,46	0,001627	-0,03163
1000	1527,695	1049	121	5	0,7	1508,284	0,00156	-0,29033
1000	1393,692	978	121	3	0,7	1556,665	0,001146	-0,83696
1000	1354,495	553	121	2	0,55	1354,495	0,00099	-0,99685
1000	1441,075	760	121	2	0,55	1333,22	0,001323	-0,64367
1000	1502,48	1084	121	3	0,7	1557,945	0,001506	-0,39319
1000	1218,88	475	121	1	0,4	1153,33	0,00049	-1,55005
1000	1595,1	1092	121	3	0,7	1575,255	0,001627	-0,01538
1000	1683,91	1147	121	3	0,7	1567,895	0,001532	0,346895
1000	1059,252	410	121	1	0,4	1158	0,000144	-2,2012
1000	1725,004	1503	121	4	0,85	1784,325	0,001426	0,514525
1000	1673,485	1132	121	3	0,7	1538,355	0,001554	0,30437
1000	1402,91	761	121	2	0,55	1353,845	0,001182	-0,79936
1000	1622,81	1115	121	3	0,7	1545,76	0,00162	0,097657
1000	1302,995	503	121	3	0,4	1133,155	0,000786	-1,20693
1000	1721,315	1196	121	3	0,7	1561,965	0,001437	0,499477
1000	1375,048	991	121	3	0,7	1555,91	0,001073	-0,91301
1000	1868,05	1558	121	4	0,85	1797,74	0,000891	1,098036
1000	1530,435	837	121	2	0,55	1351,43	0,001565	-0,27916
1000	1357,31	746	121	2	0,55	1335,325	0,001001	-0,98537
1000	1435,676	1005	121	3	0,7	1569,4	0,001304	-0,6657
1000	1415,404	1055	121	3	0,7	1547,14	0,00123	-0,74839
1000	1265,62	717	121	2	0,55	1341,485	0,000646	-1,35939
1000	1878,72	1957	121	5	1	2140,58	0,000848	1,141561
1000	1824,74	1514	121	6	0,85	1731,84	0,001064	0,921367
1000	1639,688	1183	121	3	0,7	1543	0,001605	0,166506
1000	1872,275	1526	121	4	0,85	1794,875	0,000874	1,115271
1000	1725,59	1166	121	3	0,7	1551,49	0,001424	0,516916
1000	1726,645	1168	121	3	0,7	1540,01	0,001421	0,521219
1000	1396,455	744	121	2	0,55	1345,98	0,001157	-0,82569
1000	1603,19	1094	121	3	0,7	1568,245	0,001627	0,017623
1000	1553,895	1092	121	3	0,7	1550,335	0,0016	-0,18346
1000	1710,21	1395	121	4	0,85	1846,075	0,001468	0,454178
1000	1711,615	1210	121	3	0,7	1549,88	0,001464	0,459909
1000	1675,912	1204	121	3	0,7	1538,525	0,001549	0,31427
1000	1771,912	1555	121	4	0,85	1785,43	0,001268	0,705872
1000	1505,97	783	121	2	0,55	1346,255	0,001515	-0,37895
1000	1912,336	1695	121	4	0,85	1808,09	0,000719	1,278687
1000	1506,108	1070	121	3	0,7	1564,905	0,001515	-0,37839
1000	1389,87	963	121	3	0,7	1560,435	0,001131	-0,85255
1000	1593,212	1395	121	4	0,85	1821,76	0,001627	-0,02308
1000	1612,61	1109	121	3	0,7	1549	0,001625	0,056049
1000	1735,176	1249	121	3	0,7	1557,37	0,001394	0,556019
1000	1289,796	738	121	4	0,55	1299,244	0,000735	-1,26077
1000	1237,32	690	121	2	0,55	1337,16	0,000548	-1,47483
1000	2542,04	2468	121	6	1	2415,35	9,94E-07	3,847366
1000	1774,968	1554	121	4	0,85	1812,36	0,001257	0,718338
1000	1561,535	1076	121	5	0,7	1542	0,001609	-0,1523
1000	1578,415	1108	121	3	0,7	1564,48	0,001622	-0,08344
1000	1314	501	121	1	0,4	1162,65	0,000828	-1,16204
1000	1904,83	1582	121	4	0,85	1807,815	0,000747	1,248069
1000	1750,235	1187	121	3	0,7	1543,095	0,001345	0,617447
1000	1149,288	463	121	3	0,4	1129,936	0,000303	-1,83393
1000	1582,315	1063	121	3	0,7	1547,62	0,001624	-0,06753
1000	2552,34	2478	121	6	1	2459,64	8,45E-07	3,889381
1000	1460,29	805	121	2	0,55	1330,905	0,001387	-0,56529
1000	1360,755	723	121	2	0,55	1342,965	0,001015	-0,97131
1000	1865,64	1542	121	4	0,85	1802,34	0,0009	1,088205
1000	1320,53	722	121	2	0,55	1326,91	0,000854	-1,1354
1000	1882,775	1544	121	4	0,85	1810,855	0,000832	1,158102
1000	1442,468	805	121	2	0,55	1329,2	0,001328	-0,63799
1000	1536,84	1053	121	3	0,7	1561,06	0,001576	-0,25303
1000	1591,248	1143	121	3	0,7	1561,115	0,001627	-0,03109
1000	1642,92	1178	121	3	0,7	1558,85	0,001601	0,179689
1000	1645,265	1370	121	4	0,85	1803,395	0,001598	0,189255
1000	1586,28	1101	121	3	0,7	1536,995	0,001625	-0,05136
1000	1482,304	858	121	2	0,55	1344,17	0,001453	-0,47549
1000	1611,155	1091	121	3	0,7	1554,53	0,001625	0,050114
1000	2074,42	2014	121	5	1	2121,01	0,000248	1,939858
1000	1786,175	1499	121	4	0,85	1795,555	0,001215	0,764053
1000	1501,828	1076	121	3	0,7	1537,595	0,001505	-0,39585
1000	1476,472	1059	121	3	0,7	1567,925	0,001437	-0,49928
1000	1374,395	947	121	3	0,7	1556,815	0,00107	-0,91567
1000	1715,06	1187	121	3	0,7	1559,325	0,001454	0,473962
1000	2032,73	1673	121	4	0,85	1839,295	0,00034	1,769796
1000	1421,855	755	121	4	0,55	1298,045	0,001254	-0,72208
1000	1448,78	827	121	2	0,55	1333,495	0,001349	-0,61224
1000	1318,67	728	121	2	0,55	1345,88	0,000847	-1,14299
1000	1539,364	1090	121	3	0,7	1563,725	0,00158	-0,24273
1000	2029,74	1674	121	4	0,85	1830	0,000347	1,7576
1000	1632,86	1115	121	3	0,7	1570,76	0,001612	0,138653
1000	1457,052	1044	121	3	0,7	1575,255	0,001377	-0,5785
1000	1441,225	766	121	2	0,55	1343,17	0,001323	-0,64306
1000	1425,195	768	121	4	0,55	1307,735	0,001266	-0,70845
1000	1833,228	1599	121	4	0,85	1778,92	0,00103	0,955991
1000	1434,788	797	121	2	0,55	1342,165	0,001301	-0,66932
1000	1283,984	505	121	1	0,4	1152,9	0,000713	-1,28448
1000	1823,364	1595	121	4	0,85	1810,355	0,00107	0,915754
1000	1638,715	1108	121	3	0,7	1573,82	0,001606	0,162536
1000	1826,9	1520	121	4	0,85	1793,47	0,001056	0,930178
1000	1643,51	1340	121	4	0,85	1785,105	0,001601	0,182096
1000	1648,01	1118	121	3	0,7	1554,455	0,001595	0,200452
1000	1386,355	742	121	2	0,55	1324,3	0,001118	-0,86689
1000	1722,256	1246	121	3	0,7	1556,535	0,001434	0,503316
1000	1678,105	1156	121	3	0,7	1536,695	0,001545	0,323216
1000	1628,815	1114	121	3	0,7	1568,77	0,001615	0,122153

C.15 Multiperiod model f=0

1000	1677,93	747	121	3	0,45	1542,495	0,002497	-0,05013
1000	1675,052	1041	121	4	0,6	1770,29	0,002494	-0,06816
1000	1576,96	688	121	3	0,45	1529,45	0,00198	-0,68285
1000	1634,015	719	121	3	0,45	1529,28	0,002371	-0,32532
1000	1779,064	1073	121	4	0,6	1786,235	0,002108	0,583622
1000	1451,14	439	121	2	0,3	1331,245	0,000847	-1,4713
1000	1815,27	1095	121	4	0,6	1779,275	0,0018	0,810505
1000	1703,48	764	121	3	0,45	1543,32	0,002485	0,109978
1000	1566,476	702	121	3	0,45	1540,66	0,001889	-0,74855
1000	1851,58	1060	121	4	0,6	1783,4	0,001459	1,03804
1000	1579,036	736	121	3	0,45	1544,78	0,001998	-0,66984
1000	1766,235	1023	121	4	0,6	1774,555	0,002203	0,50323
1000	1542,66	522	121	3	0,45	1542,66	0,001671	-0,89779
1000	1661,855	707	121	3	0,45	1535,585	0,002472	-0,15086
1000	1636,075	721	121	3	0,45	1532,315	0,002381	-0,31241
1000	1634,695	706	121	3	0,45	1554,145	0,002374	-0,32106
1000	1265,625	375	121	2	0,3	1335,545	7,79E-05	-2,63382
1000	1618,28	734	121	3	0,45	1537,315	0,002285	-0,42392
1000	1670,9	707	121	3	0,45	1538,42	0,002489	-0,09418
1000	1512,515	683	121	3	0,45	1531,16	0,001385	-1,08669
1000	1429,285	409	121	2	0,3	1337,705	0,000686	-1,60825
1000	1776,91	1048	121	4	0,6	1782,47	0,002125	0,570124
1000	1879,23	1161	121	4	0,6	1777,57	0,0012	1,211307
1000	1719,38	797	121	3	0,45	1553,82	0,002446	0,209615
1000	1797,335	1061	121	4	0,6	1765,79	0,001959	0,698116
1000	1641,88	712	121	3	0,45	1534,605	0,002407	-0,27603
1000	1900,39	1123	121	4	0,6	1800,26	0,001013	1,343906
1000	1814,295	1056	121	4	0,6	1778,575	0,001809	0,804395
1000	1648,172	762	121	3	0,45	1545,005	0,002431	-0,23661
1000	1834,69	1108	121	4	0,6	1773,345	0,001619	0,9322
1000	1470,535	451	121	2	0,3	1328,185	0,001005	-1,34976
1000	1538,672	695	121	3	0,45	1539,45	0,001633	-0,92278
1000	1581,475	748	121	3	0,45	1530,61	0,002018	-0,65456
1000	1678,88	785	121	3	0,45	1543,3	0,002498	-0,04418
1000	1438,984	405	121	2	0,3	1332,505	0,000755	-1,54747
1000	1534,8	690	121	3	0,45	1532,82	0,001597	-0,94705
1000	1632,405	735	121	3	0,45	1549,075	0,002363	-0,33541
1000	1710,552	759	121	3	0,45	1545,96	0,00247	0,154295
1000	1797,032	1121	121	4	0,6	1772,45	0,001962	0,696218
1000	1648,61	740	121	3	0,45	1545,205	0,002433	-0,23386
1000	1654,34	728	121	3	0,45	1544,125	0,002451	-0,19795
1000	2121,235	1555	121	5	0,75	2071,93	6,06E-05	2,72782
1000	1692,004	747	121	3	0,45	1536,11	0,002498	0,038065
1000	1532,01	699	121	3	0,45	1527,495	0,00157	-0,96453
1000	1608,665	710	121	3	0,45	1532,215	0,002223	-0,48418
1000	1855,42	1093	121	4	0,6	1799,605	0,001422	1,062103
1000	1827,135	1047	121	4	0,6	1768,575	0,00169	0,884857
1000	1512,215	671	121	3	0,45	1538,775	0,001382	-1,08857
1000	1717,05	744	121	3	0,45	1532,495	0,002453	0,195014
1000	1500,96	685	121	3	0,45	1545,685	0,001277	-1,1591
1000	2260,85	1994	121	6	0,9	2385,875	3,8E-06	3,602711
1000	1592,235	696	121	3	0,45	1539,4	0,002104	-0,58713
1000	1500,135	429	121	2	0,3	1334,085	0,001269	-1,16427
1000	1686,975	747	121	3	0,45	1528,9	0,0025	0,006551
1000	1498,925	662	121	3	0,45	1543,22	0,001258	-1,17186
1000	1748,63	761	121	3	0,45	1539,305	0,002314	0,392909
1000	1726,556	806	121	3	0,45	1548,75	0,00242	0,254583
1000	1968,555	1116	121	4	0,6	1763,48	0,000521	1,771058
1000	1673,16	717	121	3	0,45	1541,965	0,002492	-0,08002
1000	1756,745	764	121	3	0,45	1535,99	0,002266	0,443761
1000	1616,6	746	121	3	0,45	1534,075	0,002275	-0,43445
1000	1923,675	1170	121	4	0,6	1776,995	0,000824	1,48982
1000	1721,112	770	121	3	0,45	1531,91	0,00244	0,220469
1000	1871,245	1123	121	4	0,6	1783,955	0,001274	1,16127
1000	1508,772	690	121	3	0,45	1532,47	0,00135	-1,11015
1000	1910,688	1162	121	4	0,6	1782,675	0,000927	1,408438
1000	1976,952	1162	121	4	0,6	1786,715	0,000474	1,823678
1000	1820,12	1088	121	4	0,6	1788,725	0,001755	0,840897
1000	1777,848	1055	121	4	0,6	1775,865	0,002118	0,576002
1000	1524,704	693	121	3	0,45	1548,52	0,001501	-1,01031
1000	1505,385	438	121	2	0,3	1331,375	0,001318	-1,13137
1000	2031,368	1591	121	5	0,75	2054,645	0,00024	2,164673
1000	1658,704	728	121	3	0,45	1550,735	0,002464	-0,17061
1000	1640,668	750	121	3	0,45	1530,08	0,002401	-0,28363
1000	1857,93	1113	121	4	0,6	1766,49	0,001399	1,077832
1000	1567,365	666	121	3	0,45	1542,29	0,001897	-0,74298
1000	1756,19	782	121	3	0,45	1539,205	0,002269	0,440283
1000	1695,29	758	121	3	0,45	1534,425	0,002496	0,058656
1000	1544,572	675	121	3	0,45	1537,565	0,001689	-0,88581
1000	1524,052	685	121	3	0,45	1548,975	0,001494	-1,0144
1000	1850,544	1137	121	4	0,6	1788,045	0,001468	1,031548
1000	1725,46	1000	121	4	0,6	1770,635	0,002424	0,247715
1000	1592,435	704	121	3	0,45	1540,685	0,002106	-0,58588
1000	1795,875	1083	121	4	0,6	1779,855	0,001972	0,688967
1000	1492,22	635	121	3	0,45	1542,995	0,001197	-1,21387
1000	1663,735	742	121	3	0,45	1534,75	0,002476	-0,13908
1000	1825,16	1075	121	4	0,6	1792,645	0,001709	0,87248
1000	1732,02	780	121	3	0,45	1540,835	0,002398	0,288823
1000	1452,028	679	121	3	0,45	1548,395	0,000854	-1,46573
1000	1573,395	666	121	3	0,45	1544,93	0,00195	-0,70519
1000	1743,375	792	121	3	0,45	1543,475	0,002343	0,359979
1000	1741,385	1034	121	4	0,6	1778,025	0,002353	0,347508
1000	1691,42	764	121	3	0,45	1529,18	0,002498	0,034405
1000	1535,93	695	121	3	0,45	1534,955	0,001607	-0,93997
1000	1548,588	675	121	3	0,45	1541,165	0,001726	-0,86064
1000	1852,544	1160	121	4	0,6	1786,19	0,00145	1,044081
1000	1780,385	785	121	4	0,6	1780,385	0,002098	0,5919
1000	1477,604	440	121	2	0,3	1330,02	0,001066	-1,30546
1000	1863,71	1103	121	4	0,6	1790,66	0,001344	1,114052
1000	1695,756	753	121	3	0,45	1530,86	0,002495	0,061576

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